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# Coupling of collective quadrupole and monopole pairing vibrations in the Ge nuclei

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The coupling of the solutions to the random phase approximation pairing vibration and the collective quadrupole oscillation, obtained from a boson expansion approach, is used to describe simultaneously many properties of the even-even isotopes of Ge. Low lying energies, $E0$ and $E2$ transitions, two nucleon transfer amplitudes, and quadrupole moments are calculated and fairly good agreement is obtained compared to experiment. In particular the behavior of the $0_2^+$ state energy as a function of mass is accurately described including the prediction of its being the lowest energy excited state of $^{72}$Ge. The shape of the nucleus is concluded to be in a transitional region between prolate and oblate.

[NUCLEAR STRUCTURE $^{68-76}$Ge boson expansion and RPA calculations. Quadrupole moments, $B(E2)$; $(p,t)$ and $(t,p)$ spectroscopic amplitudes and $E0$ transitions.]

## I. INTRODUCTION

The Ge nuclei ($Z=32$, $N=36-44$) are extremely interesting from both experimental and theoretical viewpoints. Because of the spin-orbit interaction the $g_{9/2}$ neutron orbit is pushed down in the shell model, such that the ${p_{1/2}}^2$ and ${g_{9/2}}^2$ configurations compete at the $N=40$ shell closure. This results in turn in a competition between the (rather ubiquitous) quadrupole-type collective motion with other modes of excitation, in particular the pairing vibrational mode. The most conspicuous consequence of this competition is the strong dependence on $A$ (mass number) of the energy of the first excited $0^+$ ($0_2^+$) state. $^{72}$Ge is one of very few even-even nuclei, which have the $0_2^+$ state as the first excited state.

Recently[1-4] we have successfully applied a boson expansion procedure[5] to microscopically describe the collective quadrupole behavior for even-even nuclei between closed shells and with mass ranging from $A=100$ to 200. As a first attempt at extending our formalism to describe more complicated shell model configurations, we have chosen the RPA (random phase approximation) formalism for the pairing vibration problem and intend to describe the Ge region using a coupled representation of both the quadrupole and pairing vibrations.

Previous theoretical descriptions of the Ge region utilized the Hartree-Fock-Bogoliubov (HFB) method,[6] coupling of vibrator to deformed rotor,[7] the dynamic deformation theory of Kumar,[8] shell model calculations of varying complexity,[9] the generator coordinate method,[10] coupling of the pairing vibration, and the $0^+$ two phonon state.[11] This last approach is the most similar in spirit to our own as will be made clear in the next section. As emphasized by Vergnes,[12] who reviews all these methods, the various methods described above meet with varying successes but more importantly vary in their description of the $0_2^+$ state in $^{72}$Ge, some describing it mainly as a proton excitation while others as a neutron excitation. In our approach we will find that the collective quadrupole behavior of this state is dominantly proton in character (for $^{72}$Ge) and this is mixed almost equally with the neutron pairing vibration.

## II. FORMULATION

The fermion Hamiltonian consisting of single particle, monopole, and quadrupole pairing, and quadrupole particle hole terms is given by[1-4]

$$H=H_{\mathrm{sp}}+H_{0\text{-pair}}+H_{2\text{-pair}}+H_{2\text{-ph}}. \tag{1}$$

This is written in terms of quasiparticles by the Bogoliubov transformation. We consider $H'$,

$$H'=H_{\mathrm{sp}}+H_{0\text{-pair}}, \tag{2}$$

and look for solutions to the equation of motion[13-16]

$$[H',\Gamma_n^\dagger]=\omega_n\Gamma_n^\dagger. \tag{3}$$

Here

$$\Gamma_n^\dagger=\sum_j (a_j^{(n)}\Gamma_j^\dagger+b_j^{(n)}\Gamma_j)$$

with

$$\Gamma_j^\dagger=\frac{\sqrt{2}}{\hat{j}}\sum_{m>0}\alpha_{jm}^\dagger\alpha_{j\tilde{m}}^\dagger. \tag{4}$$

Defining further

 

$$C_j^\dagger = \frac{1}{\hat{j}} \sum_m \alpha_{jm}^\dagger \alpha_{jm}\,, \quad (5)$$

we have that

$$[\Gamma_j, \Gamma_{j'}^\dagger] = \delta_{jj'}(1 - 2C_j^\dagger/\hat{j})\,, \quad (6)$$

$$[C_j^\dagger, \Gamma_{j'}^\dagger] = 2\Gamma_j^\dagger \delta_{jj'}/\hat{j}\,. \quad (7)$$

In these equations, $\alpha_{jm}^\dagger$ creates a quasiparticle in the orbit $j$ with $j_z = m$, while $\hat{j} = (2j+1)^{1/2}$. If we restrict our attention to just the lowest excitation, the approximation $[\Gamma_n, \Gamma_{n'}^\dagger] = \delta_{nn'}$, i.e., the RPA, is justified. The non-Hermitian matrix to be solved is

$$E_{\text{RPA}}^{(n)} \begin{pmatrix} a_j^{(n)} \\ b_j^{(n)} \end{pmatrix} = \begin{pmatrix} A_{jk} & B_{jk} \\ -B_{jk} & -A_{jk} \end{pmatrix} \begin{pmatrix} a_j^{(n)} \\ b_j^{(n)} \end{pmatrix}, \quad (8)$$

$$A_{jk} = (2E_j - 2G_0 u_j^2 v_j^2)\delta_{jk} - \frac{G_0}{2}\hat{j}\hat{k}(u_j^2 v_k^2 + u_k^2 v_j^2)\,, \quad (9a)$$

$$B_{jk} = 2G_0 u_j^2 v_j^2 \delta_{jk} - \frac{G_0}{2}\hat{j}\hat{k}(u_j^2 v_k^2 + v_j^2 u_k^2)\,, \quad (9b)$$

and $E_j$ is the quasiparticle energy, $u_j$ and $v_j$ are the BCS amplitudes, and $G_0$ is the strength of the monopole interaction.

The lowest energy solution of this method and their characteristics have been discussed previously.[13,14] We will denote the wave function as $\Gamma_{\text{RPA}}^\dagger|0\rangle$ with energy $E_{\text{RPA}}$, and will drop the superscripts on the $a_j$ and $b_j$ amplitudes.

As regards the remainder of the Hamiltonian, we replace the quadrupole fermion operators by expansions in terms of quadrupole bosons. The solutions to the boson expanded collective Hamiltonian are linear combinations of the basis wave functions $|N v \gamma I\rangle$ with energies $E_{\text{coll}}$. The $N$ is the number of quadrupole bosons (which physically correspond to a coherent-linear superposition of quadrupole particle-hole excitations), $v$ is the boson seniority, $\gamma$ is an additional quantum number, and $I$ is the angular momentum.[17] Because of the weak collectivity in this region we have restricted $N$ to be less than 11 phonons in order to avoid mixing in spurious states.[5]

In our approximate method of solution of the fermion Hamiltonian, we truncate the Hamiltonian to the part which depends only on the quadrupole modes and obtain the lowest energy eigenfunctions using the boson expansion technique (BET).[1-4] The RPA monopole solutions are found in the proton and neutron spaces separately. The proton solution lies at an energy of approximately 2.7 MeV for all isotopes in this study. It couples very weakly to the other modes and henceforth when we speak of the RPA solution we will always be referring to the neutron solution. Since we are including the collective monopole excitation we must retain that part of (1) which couples the monopole and quadrupole degrees of freedom. The coupling Hamiltonian ($H_{\text{coup}}$), the collective quadrupole boson, and the monopole pairing parts of the Hamiltonian are then diagonalized, in the space of states $\Gamma_{\text{RPA}}^\dagger|0\rangle$ and $|0_{\text{coll}}^{+(i)}\rangle$, where $|0_{\text{coll}}^{+(i)}\rangle = \sum_{Nv\gamma} Z_{Nv\gamma}^{(i)}|Nv\gamma, I=0\rangle$ and $Z_{Nv\gamma}^{(i)}$ are the coefficients of the quadrupole collective phonon solutions. We have assumed $[\Gamma_j, (\alpha^\dagger\alpha^\dagger)_{2\mu}] = 0$ and hence that the ground state of the RPA solution is the same as the ground state of the quadrupole boson calculation. This approximation which simplifies the calculations enormously is at the root of the few difficulties which remain in our description of the Ge nuclei. We shall come back to this later.

There are three sources for coupling between the two modes. The first (and the most important for the strong coupling seen in $^{72}$Ge) is from the $Q$-$Q$ force (keeping only terms quadratic in $d^\dagger$)

$$\begin{aligned} H_{\text{coup}}^{Q\text{-}Q} = -\sqrt{2}\chi_2 \sum_{j_1 \le j_2} q_0 r_{j_1 j_2} \psi_{j_1 j_2} D_{j_1 j_2}^{-1} \{ & (\Gamma_{\text{RPA}}^\dagger + \text{H.c.})(d^\dagger d)_0 z^2 (a_{12} - b_{12}) + [\Gamma_{\text{RPA}}^\dagger (dd)_0 + \text{H.c.}] \\ & \times [(a_{12} - b_{12})\psi\phi + a_{12}\phi^2 - b_{12}\psi^2] + [\Gamma_{\text{RPA}}^\dagger (d^\dagger d^\dagger)_0 + \text{H.c.}] \\ & \times [(a_{12} - b_{12})\psi\phi + a_{12}\psi^2 - b_{12}\phi^2] \}\,, \end{aligned} \quad (10)$$

where the notation $q_0$, $r_{j_1 j_2}$, $D_{j_1 j_2}$, and $\psi_{j_1 j_2}$ (the Tamm-Dancoff amplitudes) were defined in Ref. 1, $\chi_2$ is as usual[1-4] the $Q$-$Q$ strength found from fitting the first $2^+$ energy in the boson expansion calculations, $d^\dagger$ (in Ref. 1 it was denoted $\alpha^\dagger$) is the *correlated* quadrupole boson operator and $z$, $\psi$, and $\phi$ are constants calculated as in Ref. 1 (they have roughly the values $z \approx 1.2$, $\psi \approx 1.02$, $\phi \approx 0.2$) and,

$$a_{12} \equiv a_{j_1}/\hat{j}_1 + a_{j_2}/\hat{j}_2 \quad b_{12} \equiv b_{j_1}/\hat{j}_1 + b_{j_2}/\hat{j}_2\,. \quad (11)$$

The coupling from the monopole pairing term is

$$H_{\text{coup}}^{0\text{-pair}} = -\frac{G_0}{\sqrt{2}} \sum_{jj'j''} \hat{j} D_{j'j''}^{\ 2} \psi_{j'j''}^{\ 2} u_{j'} v_{j'} (u_j^2 - v_j^2)(a_j - b_j)(\Gamma_{\text{RPA}}^\dagger + \text{H.c.})\,[\psi^2 + \phi^2) d^\dagger d + \psi\phi(d^\dagger d^\dagger + \text{H.c.})] \quad (12)$$

This term originated from the $H_{\text{res}}$ part[13] of $H_{0\text{-pair}}$ which is dropped[13,14] in the RPA procedure. The coupling of the quadrupole pairing is fairly weak and is given by

$$H^{2\text{-pair}}_{\text{coup}} = -2\sqrt{2}G_2(\Gamma^\dagger_{\text{RPA}} + \text{H.c.})(d^\dagger d)_0[(Q^{uu}R_{21} + Q^{vv}R_{12})(\psi^2 + \phi^2) - (Q^{uu}R_{12} + Q^{vv}R_{21})(2\psi\phi)]$$
$$- 2G_2(\Gamma_{\text{RPA}}(d^\dagger d^\dagger)_0 + \text{H.c.})[\psi\phi(Q^{uu}R_{21} + Q^{vv}R_{12}) + \psi^2(Q^{uu}R_{a_{12}} - Q^{vv}R_{a_{21}}) - \phi^2(Q^{uu}R_{b_{12}} - Q^{vv}R_{b_{21}})]$$
$$- 2G_2(\Gamma^\dagger_{\text{RPA}}(d^\dagger d^\dagger)_0 + \text{H.c.})[\psi\phi(Q^{uu}R_{21} + Q^{vv}R_{12}) + \phi^2(Q^{uu}R_{a_{12}} - Q^{vv}R_{a_{21}}) - \psi^2(Q^{uu}R_{b_{12}} - Q^{vv}R_{b_{21}})], \quad (13)$$

where

$$R^{uv}_{a_1} = \sum_{j_1 \le j_2} \psi_{j_1 j_2} D_{j_1 j_2} Q_{j_1 j_2} u_{j_1} v_{j_2} a_{j_1}/\hat{j}_1,$$
$$Q_{j_1 j_2} = \langle j_1 \| r^2 Y_2 \| j_2 \rangle / \sqrt{5}, \quad Q^{uu} = \sum_{j_1 \le j_2} \psi_{j_1 j_2} D_{j_1 j_2} Q_{j_1 j_2} u_{j_1} u_{j_2}, \quad (14)$$
$$R_{a_{12}} = R^{uv}_{a_1} + R^{vu}_{a_2}, \quad R_{12} = R_{b_{12}} - R_{a_{12}}, \quad R_{21} = R_{a_{21}} - R_{b_{21}},$$

and similarly for $R_{b_{12}}$, $R_{a_{21}}$, $R^{vu}_{a_1}$, $R^{uv}_{b_1}$, etc.

The single particle basis used is given in Table I and is fixed for all the Ge isotopes. We take $G_0^{(n)} = 0.252$ MeV and $G_0^{(p)} = 0.34$ MeV. In the process of solving for the collective quadrupole solutions using BET, the $Q$-$Q$ particle-hole and $Q$-$Q$ pairing strengths are chosen so as to reproduce the first $2^+$ energy. We write[1-4] $\chi_2 = f_2 \chi_2^{sc}$ and $G_2 = g_2 \chi_2^{sc}$, where $\chi_2^{sc} = 240A^{-5/3}$ MeV. The values of $f_2$ and $g_2$ which were required were $f_2 = 0.57, 0.64, 0.95, 0.80, 0.84$ and $g_2 = 0.53, 0.62, 0.72, 0.67, 0.67$ for $^{68\text{-}76}$Ge, respectively.

The $B(E2)$ operator, for transitions between collective quadrupole states, was given previously.[1] For transitions from an RPA $0^+$ state to a collective state, we have (only the terms which have the largest matrix elements are shown)

$$Q_2 = -\sqrt{2} \sum_{j_1 \le j_2} r_{j_1 j_2} \psi_{j_1 j_2} D_{j_1 j_2}^{-1} [(\psi a_{12} - \phi b_{12})(\Gamma^\dagger_{\text{RPA}} d + \text{H.c.}) + (\phi a_{12} - \psi b_{12})(d^\dagger \Gamma^\dagger_{\text{RPA}} + \text{H.c.})]. \quad (15)$$

We utilize the concept of effective charge in the same way as previously.[1-4] The values used here are $(1.0, 1.1, 1.1, 1.2, 1.1)e$ for $^{64\text{-}76}$Ge, respectively.

For two particles coupled to angular momentum zero we have the spectroscopic operator (dropping higher order terms)

$$\frac{1}{\sqrt{2}}[a_j^\dagger a_j^\dagger]_0 = (u_j^2 a_j + v_j^2 b_j)\Gamma^\dagger_{\text{RPA}} - (u_j^2 b_j + v_j^2 a_j)\Gamma_{\text{RPA}} - \frac{\hat{j}}{\sqrt{2}} u_j v_j$$
$$+ \sqrt{2} u_j v_j \hat{j} \sum_{j'} \frac{D_{jj'}{}^2 \psi_{jj'}{}^2}{2j+1} [(\psi^2 + \phi^2)(d^\dagger d)_0 + \psi\phi(d^\dagger d^\dagger + dd) + 5\phi^2]. \quad (16)$$

For two particles coupled to angular momentum two we have

$$D_{j_1 j_2}{}^{-1}[a^\dagger_{j_1} a^\dagger_{j_2}]_2 = d^\dagger[v_{12}h_1 - u_{12}h_2] + d[v_{12}h_2 - u_{12}h_1] - 5P_{12}[(\psi^2 + \phi^2)[d^\dagger \tilde{d}]_2 + \psi\phi([d^\dagger d^\dagger]_2 + \text{H.c.})], \quad (17)$$

where

$$P_{12} = (u_{j_1} v_{j_2} + u_{j_2} v_{j_1}) \sum_j \psi_{j_1 j} \psi_{j j_2} W(j_2 j_1 22; 2j) \times D_{j_1 j} D_{j_2 j} D_{j_1 j_2}{}^{-1},$$
$$v_{12} = v_{j_1} v_{j_2} \psi_{j_1 j_2},$$
$$u_{12} = u_{j_1} u_{j_2} \psi_{j_1 j_2},$$
$$h_1 = \phi + s(F_a(2\phi^3 + 5\psi^2\phi) + F_b(\psi^2\phi + 6\phi^3) - 6\phi F_a),$$
$$h_2 = \psi + 7s(F_a + F_b)\psi\phi^2.$$

Here,[5] $s = -\frac{1}{4}$, while $F_a$, $F_b$, and $F_c$ are the products of geometrical factors and the Tamm-Dancoff amplitudes ($\psi_{j_1 j_2}$). These operator representations have been used to calculate the two neutron transfer spectroscopic amplitudes, to be used in calculating form factors. Finally the operator for $E0$ transitions between an RPA state and the ground state is given in lowest order by

$$\hat{Q}_0 = \sum_j u_j v_j \hat{j} (N_j + \tfrac{3}{2})\sqrt{2}(b_j - a_j)(\Gamma^\dagger_{\text{RPA}} + \text{H.c.}), \quad (18)$$

where $N_j$ is the principal oscillator quantum number for orbit $j$. The operator for transitions between collective states was given in Ref. 2.

## III. COMPARISON WITH EXPERIMENT

In Table II, we summarize our predicted energies for $^{68\text{-}76}$Ge. We find that most of the positive parity states with energies below 3 MeV are predicted fairly well, with the main discrepancy being that the predicted energy of the first $3^+$ state for all the isotopes and the $2_2^+$ state for

TABLE I. Valence shells used in the calculation and relative single particle energies in units of $41/A^{1/3}$ MeV

| Proton | | Neutron | |
|---|---|---|---|
| $f_{7/2}$ | −0.66 | $f_{7/2}$ | −0.63 |
| $f_{5/2}$ | −0.218 | $p_{3/2}$ | −0.213 |
| $p_{3/2}$ | −0.175 | $f_{5/2}$ | −0.111 |
| $p_{1/2}$ | 0.0 | $p_{1/2}$ | 0.0 |
| $g_{9/2}$ | 0.218 | $g_{9/2}$ | 0.183 |
| | | $d_{5/2}$ | 0.64 |

$^{74-76}$Ge are not predicted as low as experiment. The excellent predictions for the $0_2^+$ state are evident with the only drawback being the slightly too low energy for $^{72}$Ge. This is seen more clearly in Fig. 1 where the pairing vibration (RPA), boson expansion predictions (BET), and $2\Delta_N$ are plotted as a function of $N$. By comparing the RPA and BET-RPA predicted energies, we note that the coupling is fairly strong for $^{72}$Ge, whereas it is much weaker for the other isotopes. The mass trends of a few low lying states are shown with the corresponding experimental trends in Fig. 2. The agreement for the $0_3^+$ states is another indication that the coupling of the two modes is valid.

Regarding the coupling terms one can see immediately from Eq. (12), (recalling the smallness of $\phi$ and the two phonon nature of the BET $0_2^+$ state) that $H^{\rm coup}_{0\text{-pair}}$ is not very effective in coupling the two modes. $H^{2\text{-pair}}_{\rm coup}$ reduces somewhat the coupling but is not very important compared to $H^{Q\text{-}Q}_{\rm coup}$. The strong coupling for $^{72}$Ge as opposed to the other isotopes, originates from $H^{Q\text{-}Q}_{\rm coup}$. In that nucleus, the neutron Fermi surface lies between the $p_{1/2}$ and $g_{9/2}$ orbits. The Tamm-Dancoff amplitudes, $\psi_{1/2,3/2}$, $\psi_{1/2,5/2}$, and $\psi_{9/2,9/2}$ are all significant in $^{72}$Ge. In $^{70}$Ge, the $\psi_{9/2,9/2}$ amplitude is a factor of 4 smaller than in $^{72}$Ge, whereas for $^{74}$Ge the $\psi_{1/2,3/2}$ and $\psi_{1/2,5/2}$ are

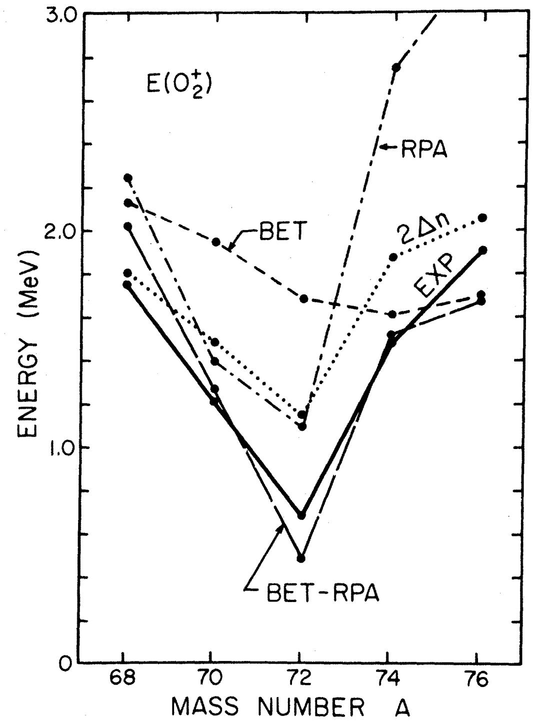


FIG. 1. Energies of the BET first excited $0^+$ state (BET), the RPA neutron pairing vibration $0^+$ state (RPA), twice the neutron gap ($2\Delta_N$), the coupled first excited $0^+$ state (BET-RPA), and experiment for $^{68-76}$Ge.

reduced. The coefficient $r_{j_1j_2}$ in (12) goes as $u_{j_1}u_{j_2}-v_{j_1}v_{j_2}$ and is significant for all three of the above components in $^{72}$Ge. Because of the lower (higher) Fermi surface in $^{70}$Ge ($^{74}$Ge) the $r_{1/2,3/2}$ ($r_{9/2,9/2}$) factor is less. Thus the position of the Fermi surface between the $p_{1/2}$ and $g_{9/2}$ orbits for $^{72}$Ge allows $\psi_{j_1j_2}r_{j_1j_2}a_{12}$ to add coherently for several neutron orbits as well as contributing to the strong pairing vibration which is spread among the $(f_{5/2})_0^{\,2}$, $(p_{1/2})_0^{\,2}$, and $(g_{9/2})_0^{\,2}$ configurations.

TABLE II. Comparison of theoretical (BET coupled to RPA) and experimental Refs. 18–24 energies for $^{68-76}$Ge in units of keV.

| | $^{68}$Ge | | $^{70}$Ge | | $^{72}$Ge | | $^{74}$Ge | | $^{76}$Ge | |
|---|---|---|---|---|---|---|---|---|---|---|
| $I$ | Th | Exp | Th | Exp | Th | Exp | Th | Exp | Th | Exp |
| $0_2$ | 2022 | 1753 | 1271 | 1212 | 473 | 690 | 1510 | 1485 | 1665 | 1912 |
| $0_3$ | 2341 | 2617 | 2052 | 2311 | 2191 | 2029 | 2440 | 2228 | 2932 | 2908 |
| $0_4$ | 3128 | | 2830 | 2891 | 2419 | 2756 | 2831 | 2755 | 3521 | |
| $2_1$ | 928 | 1017 | 905 | 1037 | 809 | 833 | 615 | 598 | 543 | 563 |
| $2_2$ | 1973 | 1779 | 1854 | 1708 | 1604 | 1467 | 1459 | 1203 | 1628 | 1107 |
| $2_3$ | 3310 | 2942 | 2957 | 2160 | 2655 | 2406 | 2499 | 2198 | 2538 | 2506 |
| $3_1$ | 3157 | 2429 | 2909 | 2452 | 2572 | 2062 | 2417 | 1700 | 2629 | 1540 |
| $4_1$ | 2090 | 2269 | 2023 | 2155 | 1851 | 1725 | 1411 | 1464 | 1277 | 1409 |
| $4_2$ | 3262 | 2834 | 3034 | 2806 | 2717 | 2466 | 2423 | 2165 | 2593 | 2739 |
| $5_1$ | 4611 | | 4210 | | 3842 | | 3548 | | 3834 | |
| $6_1$ | 3483 | | 3344 | 3298 | 3123 | | 2375 | | 2193 | |

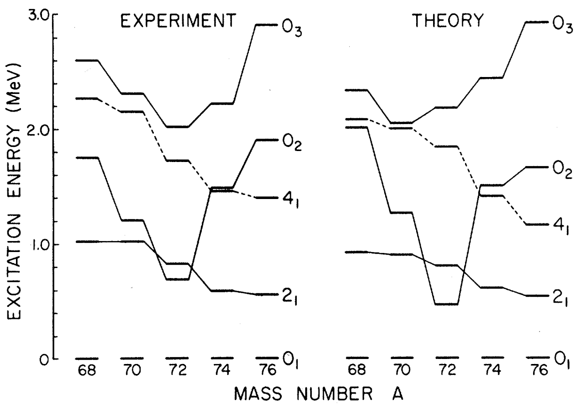


FIG. 2. Comparison of theoretical and experimental energy trends for the $2_1^+$, $0_2^+$, $4_1^+$, and $0_3^+$ states.

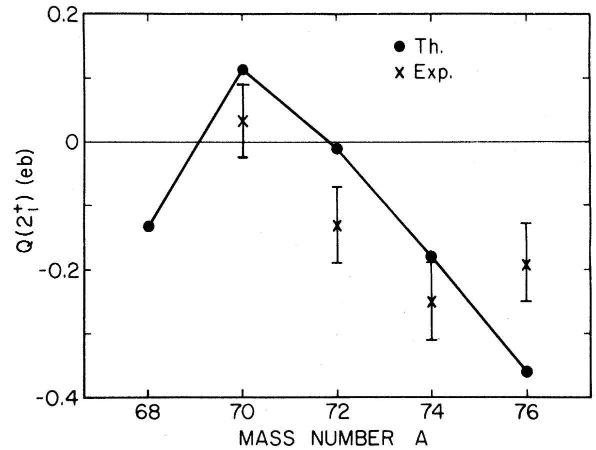


FIG. 3. Comparison of the theoretical and experimental $Q(2_1^+)$. For the latter, those of Ref. 29, assuming $P_3>0$ throughout, are plotted.

In Table III, we present calculated $B(E2)$'s, and compare them with experiment, although available data are yet very much limited. It is seen that the $B(E2;4_1\rightarrow 2_1)$ in $^{70}$Ge, which reveals in it a slightly reduced two-phonon nature, is well accounted for by the calculation. In both $^{70}$Ge and $^{72}$Ge, the smallness of the crossover $B(E2;2_2\rightarrow 0_1)$ is well explained, but the theory predicts a similarly small $B(E2)$ in $^{74}$Ge, whereas the experimental $B(E2)$ is somewhat larger. As for the $B(E2;2_2\rightarrow 2_1)$ in $^{72}$Ge, the discrepancy seen in Table III might only be apparent, because the experimental $B(E2)$ given there may include a sizable mixture of $B(M1)$.

The transitions in which the largest discrepancy is encountered between theory and experiment are those originating from the $0_2$ state. In both $^{70}$Ge and $^{72}$Ge, the experimental $B(E2;0_2\rightarrow 2_1)$ reveals in it a strong collectivity and, although the theory predicts fairly large (but still too small by a factor of 4) $B(E2)$ for this transition in $^{72}$Ge, the corresponding value in $^{70}$Ge is far too small. As for the $B(E2;0_2\rightarrow 2_2)$, the theory predicts in $^{72}$Ge a sufficiently small value, but fails to predict the large experimental value seen in $^{70}$Ge. Overall, the theory is doing rather poorly for $^{70}$Ge, although the extremely peculiar behavior of the $0_2$ state should be kept in mind. Compared with this, the situation in $^{72}$Ge is much better, although there still remains large room for improvement.

Table III also includes the static quadrupole moments $Q(2_1)$ of the $2_1$ states. They are also presented in Fig. 3 and are compared with recent data obtained by the Montreal group.[29] The data for $^{70-76}$Ge shows that a transition from oblate to prolate shapes takes place within these isotopes,[29] and the theory reproduces this very nicely. It is interesting to note further that the theory predicts a prolate nature for $^{68}$Ge, although no data is available; $^{68}$Ge is unstable. It appears

TABLE III. Same as Table II except for $B(E2;I_i\rightarrow I_f)$ in units of $e^2\,\mathrm{fm}^2$ and $Q_{2_1^+}$ in units of $e$b. Experiment is taken from Refs. 18, 21, and 25–31. The + or − superscript on experimental (theoretical) $Q_{2_1^+}$ refers to assumed (calculated) $P_3>0$ or $P_3<0$, respectively.

| | | $^{68}$Ge | | $^{70}$Ge | | $^{72}$Ge | | $^{74}$Ge | | $^{76}$Ge | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I_i$ | $I_f$ | Th | $E$ | Th | $E$ | Th | $E$ | Th | $E$ | Th | $E$ |
| $2_1$ | $0_1$ | 2.57 | 2.45(82) | 3.61 | 3.58(6) | 4.40 | 4.16(6) | 5.94 | 6.10(6) | 6.00 | 5.56(6) |
| $4_1$ | $2_1$ | 3.84 | 1.80(49) | 5.23 | 5.5 | 6.36 | 6.40(67) | 8.97 | 6.67(64) | 8.91 | 7.3(1.3) |
| $6_1$ | $4_1$ | 4.30 | 1.97(49) | 5.67 | | 6.87 | | 10.47 | | 10.11 | |
| $2_2$ | $2_1$ | 3.76 | | 5.09 | 5.0(1.9) | 6.75 | 11.4(1.3) | 7.98 | 10(2) | 5.20 | 7.4(9) |
| $2_2$ | $0_1$ | 0.033 | | 0.022 | 0.044 | 0.032 | 0.035(9) | 0.030 | 0.13(2) | 0.16 | 0.17(3) |
| $3_1$ | $2_2$ | 3.19 | | 4.11 | | 5.21 | | 7.21 | | 5.86 | |
| $0_2$ | $2_1$ | 2.03 | | 0.26 | 6.0(1.5) | 3.00 | 13.0(2.5) | 3.94 | <4.0 | 2.67 | <1.7 |
| $0_2$ | $2_2$ | 1.09 | | 0.22 | 3.08 | $10^{-5}$ | 0.05(2) | 1.10 | | 4.23 | |
| $4_2$ | $4_1$ | 2.12 | | 2.73 | | 3.61 | | 4.51 | | 3.12 | |
| $Q_{2_1^+}$ | | $-0.11^+$ | | $0.115^-$ | $0.03(6)^+$ | $-0.002^-$ | $-0.13(6)^+$ | $-0.18^+$ | $-0.25(6)^+$ | $-0.34^+$ | $-0.19(6)^+$ |
| | | | | | $0.09(6)^-$ | | $-0.05(6)^-$ | | $-0.05(6)^-$ | | $-0.03(6)^-$ |

TABLE IV. Same as Table III except for relative $B(E2)$'s for $^{72}$Ge. Experiment is taken from Ref. 25.

| $I_i \rightarrow I_f$ | | Th | Exp |
|---|---|---|---|
| $3_1$ | $2_2$ | 100 | 100 |
| | $4_1$ | 44.5 | 35 |
| | $2_1$ | 0.69 | 0.74 |
| $2_3$ | $2_2$ | 100 | 100 |
| | $0_2$ | 152 | 17 |
| | $2_1$ | 0.4 | 14 |
| | $0_1$ | 8.3 | 0.2 |
| $4_2$ | $4_1$ | 100 | 100 |
| | $2_2$ | 94 | 46 |
| | $2_1$ | 0.72 | 0.16 |

that the 38 neutrons in $^{70}$Ge are close enough to the $N=40$ shell, exerting a strong tendency for an oblate shape. However, this tendency is weaker in $^{68}$Ge, having 36 neutrons, and cannot overcome the prolate favoring tendency of protons. For $^{74}$Ge and $^{76}$Ge, the $g_{9/2}$ neutrons enhance the prolate tendency. We note that our calculated sign of the interference term[29] $P_3(=M_{0_1 2_1}M_{0_1 2_2}M_{2_1 2_2}$, where $M_{0_1 2_1}$ is the reduced matrix element between the ground $0^+$ state and the first $2^+$ state) is (+, −, −, +, +) for $^{68-76}$Ge, respectively. In Table III we see that these choices for experiment agree better with our results than if one assumes $P_3>0$ for all the nuclei as was done in Fig. 3.

In Table IV, we compare the predicted branching ratios with experiment for $^{72}$Ge, and it is seen that the agreement is good, particularly for the $3_1$ and $4_2$ states, indicating that they retain collectivity to a large extent, in spite of their rather high energies which might make them feel the effects of noncollective states nearby.

We have also calculated the $\rho(E0; 0_2 \rightarrow 0_1)$. This transition is unusually strong in this mass region, but we have successfully reproduced it. The theoretical (experimental) $\rho$ values[32] are 0.04 (0.085) (Ref. 33) and 0.18 (0.10) (Ref. 34) for $^{70}$Ge and $^{72}$Ge, respectively.

In the course of the above calculations of the energies and the electromagnetic properties, spectroscopic amplitudes for two-nucleon transfer reactions were also calculated by using the method of Bayman and Kallio.[35] These amplitudes were then used to obtain the form factors to be used for zero-range distorted wave Born approximation (DWBA) calculations, and the ratio of the square of these form factors at the nuclear surface were compared with the ratios of experimental cross sections.

It is seen in Fig. 4, that our prediction of the $0_1$ cross section is very good, reproducing nicely the dips seen for the $^{74}$Ge$(p,t)^{72}$Ge and

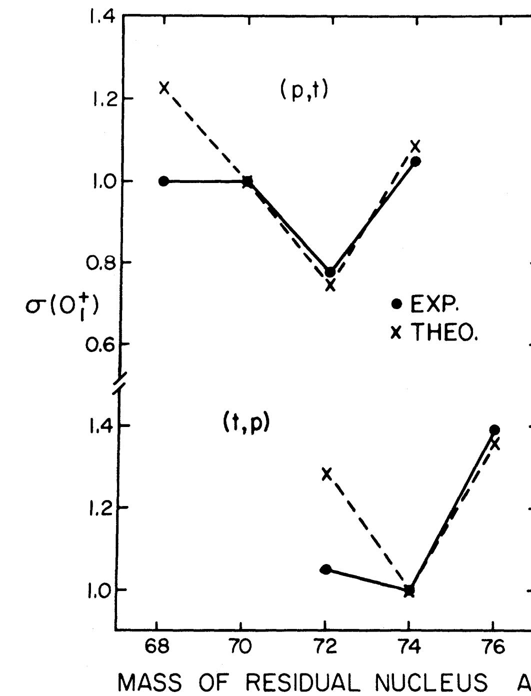


FIG. 4. Comparison of the theoretical and experimental (Refs. 36–40) $(p,t)$ and $(t,p)$ ground to ground cross sections normalized to the $^{72}$Ge$(p,t)^{70}$Ge and $^{72}$Ge$(t,p)^{74}$Ge reactions.

$^{72}$Ge$(t,p)^{74}$Ge reactions. The dips certainly reflect the fact that the low-lying $0_2$ state in $^{72}$Ge has had mixed into it a significant fraction of the pairing correlation, which is normally concentrated in the $0_1$ state.

We then expect that $\sigma(0_2)/\sigma(0_1)$ will peak when $^{72}$Ge appears as the residual nucleus, both for $(p,t)$ and $(t,p)$ reactions. As is seen in Fig. 5, this expectation is fulfilled theoretically for the $(p,t)$ reaction; indeed the predicted $A$ dependence of the above ratio agrees very nicely with that of experiment. For the $(t,p)$ reaction, however, the above expectation is not realized experimentally, and thus the theoretical prediction is in somewhat poor agreement with the data.

The information about the ratio $\sigma(2_n)/\sigma(2_1)$ is given in Fig. 6, for $n=2$ and 3. For $n=3$ theory reproduces rather nicely the experimental $A$ dependence, for both $(p,t)$ and $(t,p)$ reactions, although the predicted values are too small by a factor of several. The $n=2$ results for the $(t,p)$ reaction are in nearly perfect agreement with experiment. However, the agreement is rather poor in the $(p,t)$ reaction.

## IV. DISCUSSION OF RESULTS

In Table V we give a summary of the structure of our $0_2^+$ state. We see that the RPA neutron

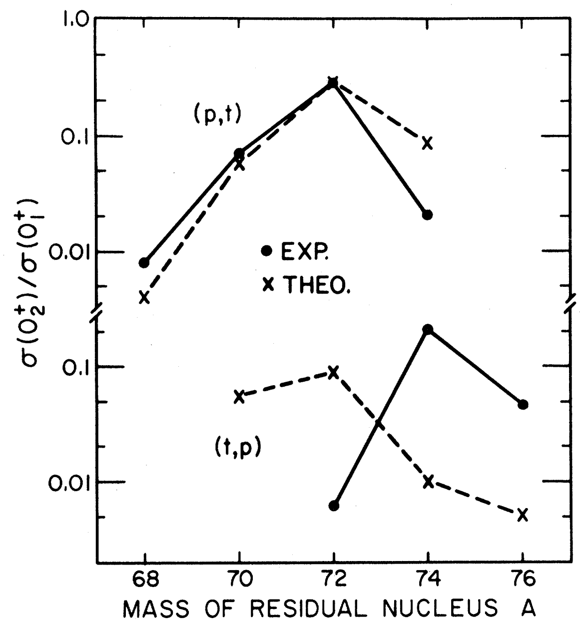


FIG. 5. Comparison of the theoretical and experimental (Refs. 36–40) $(p,t)$ and $(t,p)$ ground to $0_2^+$ state cross sections relative to the ground to ground cross section.

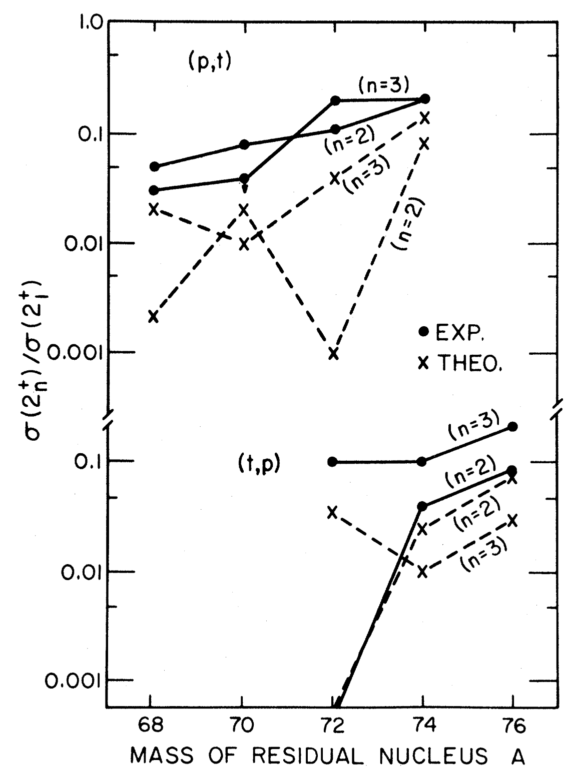


FIG. 6. Comparison of the theoretical and experimental (Refs. 36–40) $(p,t)$ and $(t,p)$ ground to $2_n^+$ ($n=2$ or 3 cross sections relative to the ground to $2_1^+$ cross section.

TABLE V. The first two rows give the % composition of the first $0^+$ excited state from the BET and the neutron RPA excited state, in the coupled first excited $0^+$ state ($0_2^+$). The last two rows give the strengths of the neutron and proton contributions to the collective quadrupole oscillation.

| | 68 | 70 | 72 | 74 | 76 |
|---|---|---|---|---|---|
| $0_{2\ \mathrm{BET}}^+$ | 69 | 14 | 31 | 92 | 99 |
| $0_{\mathrm{RPA}}^+$ | 30 | 85 | 67 | 7 | 0 |
| $q_{0_n}$ | 1.7 | 1.37 | 0.50 | 1.3 | 1.56 |
| $q_{0_p}$ | 1.1 | 1.33 | 2.08 | 1.6 | 1.44 |

pair state is a significant component except for $^{74,76}$Ge. Our collective $0_2^+$ BET state is roughly 80% of two phonon nature (only 65% for $^{76}$Ge because of the increasing quadrupole deformation). The quantity $q_0(=q_{0_n}+q_{0_p})$ is a measure of the quadrupole collectivity strength for the nucleus. We find it is relatively constant and almost split equally between proton and neutron components, except for $^{72}$Ge. Concentrating on $0_2^+$ for $^{72}$Ge (since it is the most interesting) we find that it is a mixture of mainly a pair of proton dominated quadrupole collective excitations coupled to zero and the neutron pairing vibration.

The coupling which produced this mixing came mainly from the $Q$-$Q$ particle-hole force which is five times bigger for $^{72}$Ge than for either $^{70}$Ge or $^{74}$Ge. The coupling from the other interactions helped produce some cancellations but the final results for $^{68-76}$Ge would not have changed significantly had they been ignored. We note that we used the same $\chi_2$ and $G_2$ strengths in the coupling Hamiltonian as were fixed from the BET calculation. Had we decreased $\chi_2$ in $^{72}$Ge and increased it for $^{70,74}$Ge in the coupling Hamiltonian by roughly 10–20%, thereby changing the mixing, we could have fit the $0_2^+$ and $0_3^+$ energies extremely accurately and at the same time improved the $B(E2;0_2^+\to 2_1^+)$ for $^{70}$Ge (cross section ratios would have been changed negligibly).

An important part of the analysis was the neutron $p_{1/2}$-$g_{9/2}$ energy separation (roughly 1.8 MeV). If this energy separation is different, the major predicted characteristics of the $0_2^+$ state can still be reproduced by choosing a new $G_0^{(n)}$. However, whether or not $^{70}$Ge has a strongly collective neutron pairing vibration when the same $G_0^{(n)}$ strength is used as for $^{72}$Ge requires that this gap not become too small. A further point is that the results were insensitive to reasonable variations of the proton single particle energy spectrum as well as the other neutron single particle energies. It is important to note (regarding Fig. 1) that, though one could reduce $E_{\mathrm{RPA}}$ in $^{72}$Ge to the experimental value by reducing $G_0^{(n)}$ (i.e., making $\Delta \sim 0.3$ MeV) one would find that the cross sections to this state would be equal to the cross sections to the ground state. Thus we know that the pure RPA state is not the solution to the $0_2^+$ state in $^{72}$Ge, especially when we know that the coupling with the collective quadrupole branch is fairly large for $^{72}$Ge.

Finally the result that the coupling is very weak for cases other than where a pairing vibrational state is present justifies the explicit neglect of both the coupling and the excited $0^+$ pair states in our previous calculations[1-4] where no pairing vibrational features were believed to be very important at low energies.

The present method has relied on the coupling of the pairing and quadrupole vibrational modes to successfully describe many features of the Ge region. As pointed out above, we in effect ignored the nonzero difference in the commutator of the RPA and quadrupole collective excitations. The best way to improve the calculations especially $B(E2)$'s is to calculate the entire problem in a boson expansion framework and thereby avoid the RPA. The commutation relations between the two modes can then be more accurately preserved. Encouraged by the success found here using the RPA monopole mode coupled to the boson expanded quadrupole mode we are planning more accurate treatments.

The calculations of Kumar[8] were perhaps the most successful of those reviewed by Vergnes.[12] Better agreement with experiment was obtained for some of the $B(E2)$'s for which we encountered difficulty. It appears, however, that the $0_2^+$ state of Kumar is basically a $\beta$-vibrational state. It is possible this leads to some difficulties in fitting some of the two-nucleon transfer reactions. Another point is that the predicted $Q(2_1^+)$ is positive for $A = 70-74$ (with an approximate value of 0.2 $e$b) which is in disagreement with a recent experiment.[29]

Phenomenological calculations of Gneuss and Greiner[41] using a quadrupole phonon collective model correctly predicted the $0^+$ energy behavior for $^{70,72}$Ge. Their $0^+$ state is basically a $\beta$-vibrational state and thus there might be trouble reconciling this with the two neutron transfer data. It is of interest to note that the Hamiltonian used in this analysis has the same general form as the pure quadrupole Hamiltonian which we derive from the microscopic fermion Hamiltonian (1) through the BET. We have found, however, that if we start with (1) and assume that the single particle levels *closest* to the Fermi surface are those in Table I, then even if unreasonable choices of single particle energy positions are made, the boson Hamiltonian which is derived through BET does not lead to the prediction of a low lying $0^+$ state. Previously,[1-4] we had found that our microscopically derived boson Hamiltonian produced results which were qualitatively as good, in reproducing experiment, as those obtained through a Gneuss-Greiner calculation.[42] Of course for some fine details the phenomenological calculation is much better. $^{72}$Ge is the first case where the BET derived quadrupole boson Hamiltonian cannot reproduce a significant low energy feature of experiment which the phenomenological quadrupole boson Hamiltonian can. This indicates that it is possible to reproduce some of the effects of modes other than the quadrupole by renormalization of the collective quadrupole boson coefficients. This renormalization is introduced in the process of finding the collective quadrupole boson symmetry which best fits the data. In a calculation which begins with a fermion Hamiltonian, the only recourse is to extend the calculations to include more configurations. As far as our BET procedure is concerned, the present work is our first attempt to do so.

de Lima *et al.*[31] compared the results of an interacting boson approximation (IBA) calculation to the spectra of $^{68}$Ge. The parameters were determined through a least squares search and excellent agreement with experiment was obtained. It would be of interest to see this procedure extended to $^{70-76}$Ge.

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