Lepton violating double β decay in modern gauge theories

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The neutrinoless lepton violating double β decay is investigated in the context of modern gauge theories, whereby it is mediated by a Majorana neutrino. Transition operators appropriate for calculations of the relevant nuclear matrix elements are constructed. In addition, some of the approximations of the pregauge theories of double β decay are investigated. Explicit shell model calculations are performed in the case of the $A = 48$ system.

[RADIOACTIVITY Double β decay. Gauge theories. Majorana neutrinos. Lepton nonconservation. Shell model calculations.]

I. INTRODUCTION

All physical processes observed thus far seem to be consistent with the conservation of the lepton quantum number $\mathfrak{L}$. The question arises as to whether such conservation laws follow exact symmetries of nature or whether they merely hold at the present level of experimental accuracy. Most of the currently fashionable gauge theories,[1] including the standard one of Weinberg-Salam,[2] can accommodate or can easily be extended to accommodate lepton nonconservation. They differ, of course, in the level at which such nonconservation shows up, or in the mechanism by which it is produced. Lepton conservation is also not expected to hold in grand unification theories.

Many lepton violating processes are possible.[3] The oldest among them is the neutrinoless double β decay[4,5]

$$A(Z)\to A(Z+2)+e^-+e^- , \quad (1)$$

which is expected to take place simultaneously with the lepton-conserving double β-decay process

$$A(Z)\to A(Z+2)+e^-+e^-+\tilde{\nu}_e+\tilde{\nu}_e . \quad (2)$$

Clearly, for the above extremely slow process to have a chance to be observed, the nuclear system must be judiciously chosen so that the usual β-decay process

$$A(Z)\to A(Z+1)+e^-+\tilde{\nu}_e \quad (3)$$

is completely suppressed due to angular momentum selection rules or energy conservation. Many such systems actually exist.[5,6] The experiments which have hitherto been done are of two classes: firstly, those involving measurements of relative abundances of the $A(Z+2)$ nucleus in geologic ores,[7,8] and secondly, experiments performed in the laboratory[9-12] which attempt to measure the energy E_1+E_2 of the two electrons and plot the number of simultaneously observed electrons against E_1+E_2. A peak at the available energy Δ will signal the observation of process (1). On the other hand, a broad distribution around $\Delta/2$ will signal process (2). In practice one must overcome tremendous background problems since both processes are extremely slow. Also if process (1) is much slower than process (2) it may be difficult to identify the former in the background of the latter. In spite of this, experiments have now reached the point where such measurements are feasible and in fact new results have been recently reported.[12]

The theoretical analysis of the neutrinoless double β decay has in the past been made more or less phenomenologically in the spirit of the pregauge weak interaction theory.[4,13,14] In this treatment process (1) was assumed to take place via a small mixture of right-handed currents in the usual left-handed Lagrangian. It was viewed as a second order process mediated by a massless Majorana neutrino. The net result was that lepton nonconservation could be described in terms of a single parameter η. Furthermore, some approximations were made regarding the nuclear transition operator which had the net result that processes (1) and (2) could be described in terms of a single nuclear matrix element. By exploiting these results Pontecorvo[15] suggested that the parameter η could be extracted from a total lifetime measurement, as obtained, e.g., in geologic ore measurements, in a way completely independent of nuclear physics (see Sec. V for details).

We feel, however, that the above analysis must be reviewed for two reasons. First the above mentioned assumptions were never really tested explicitly. As a consequence the extraction of the parameter η from the experimental data may be dependent on the nuclear models. Second we feel it is imperative to investigate double β decay in the spirit of modern gauge theories since these theories had such a tremendous impact on the

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physics of the last decade.[1,2,16]

In the present paper we will address ourselves to the questions raised above. We hope that our work will be of help to the analysis of the important double β-decay experiments currently under way. It is also going to be useful in the analysis of another lepton nonconserving process, namely,[16,17]

$$\mu^- + A(Z) \to A(Z-2) + e^+ . \qquad (4)$$

At the end we will apply our general results by performing numerical shell model calculations in the case of the $^{48}Ca \to {}^{48}Ti$ transition, which is the simplest to investigate from a theoretical point of view.

II. DESCRIPTION OF THE GAUGE MODEL

The gauge theory model we are going to use has already been discussed elsewhere.[16,17] We will briefly mention its basic features here in order to establish the notation and to make the paper as self-sufficient as possible.

In order to accommodate reaction (1) the standard Weinberg-Salam model[2] must be extended to accommodate one more lepton pair in a left-handed doublet of weak isospin, i.e., the left-handed leptons of the theory are

$$\begin{pmatrix} \nu_1 \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_2 \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_3 \\ \tau^- \end{pmatrix}_L \begin{pmatrix} \nu_4 \\ N^- \end{pmatrix}_L , \qquad (5)$$

where[18] $L = \frac{1}{2}(1-\gamma_5)$ and ν_i, $i=1,2,3,4$, are the eigenstates of weak interactions which are linear combinations of the well known neutrinos ν_e, ν_μ, ν_τ and the new Majorana[19] neutrino N_0 with mass M_σ. Process (1) takes place via Cabbibo-type mixing between N_0 and ν_e and it cannot take place if $M_\sigma = 0$ (there is no mixing). For our purposes it is adequate to write

$$(\nu_1)_L \approx (\nu_e)_L + \beta(N_0)_L \quad |\beta| \ll 1 . \qquad (6)$$

Since the mixing mechanism is not currently adequately understood the mixing angle cannot be calculated from the fundamental parameters of the theory and it will thus be treated as a free parameter. [The rate for process (1) will turn out to be proportional to β^4]. Due to the Majorana nature of N_0 there can be a mass term in the Lagrangian of the form

$$M_\sigma \bar{N}_0 N_0 \qquad (7)$$

which will violate the lepton number by two units. Thus the interaction Lagrangian takes the form

$$\mathcal{L} = f\beta \bar{e}_L \gamma^\lambda N_L^0 W_\lambda + M_\sigma \bar{N}_0 N_0 + hc , \qquad (8)$$

where f is the usual coupling constant associated with the SU(2) of Weinberg-Salam given[2] by

$$\frac{f^2}{M_\omega{}^2} = 4\sqrt{2}\, G_F \qquad (9)$$

and W_λ is the field associated with the charged W boson with mass $M_W \approx 80$ GeV. G_F is the usual Fermi coupling constant. In this treatment right handed leptons do not contribute to the charge changing process (1) since they belong to weak isosinglets. Hence the two unknown parameters of the theory are the quantities β and M_σ.

With the Lagrangian density (8) one can immediately write down Feynman diagrams in perturbation theory. This can be done either at the quark or the nucleon level. Process (1) must involve two such particles since it is forbidden to occur on a single particle due to charge conservation. One possibility is that only nucleons are involved, as indicated in Fig. 1. Other possibilities[17] may occur such as the double β decay of a neutron into a virtual Δ^{++} which subsequently gets converted into a proton via charge exchange or the double charge exchange of a π^- in flight between the two nucleons, i.e.,

$$\begin{aligned} &n \to \Delta^{++} + e^- + e^- , \\ &\pi^- \to \pi^+ + e^- + e^- . \end{aligned} \qquad (10)$$

Reactions (10) have, however, been found to contribute to the total (μ^-, e^+) process significantly less[17] than the mode involving only nucleons (Fig. 1). We expect their contribution to be even smaller in our process (1), which involves specific initial and final nuclear states, and we will neglect them completely in the present work.

III. EFFECTIVE OPERATORS IN THE GAUGE MODEL

With the Feynman diagram of Fig. 1 the invariant transition amplitude can immediately be written as follows:

$$\begin{aligned} \mathfrak{M}_{12} = \frac{f^4}{16}\beta^2 \int d^4x \int d^4y \int \frac{d^4k}{(2\pi)^4} &\frac{F[(p_1+k)^2]}{(p_1+k)^2 - M_w{}^2} \frac{F[(p_2-k)]^2}{(p_2-k)^2 - M_w{}^2} \frac{1}{(k^2 - M_\sigma{}^2)^2} \\ &\times \phi_1^* \phi_2^* \bar{u}(p_2)\gamma^\mu (1-\gamma_5) e^{-ik(x-y)} (\not{k} + M_\sigma) M_\sigma (1-\gamma_5)(\not{k} + M_\sigma)\gamma^\nu (1-\gamma_5) u^c(p_1) \\ &\times \sum_n e^{-i(E_i - E_n)y_0} e^{-i(E_n - E_f)x_0} \langle f | J_\mu(\vec{x}) | n \rangle \langle n | J_\nu(\vec{y}) | i \rangle , \end{aligned} \qquad (11)$$

where $F[(p_1+k)^2]$, $F[(p_2-k)^2]$ are the relevant nucleon form factors, $|i\rangle$, $|n\rangle$, and $|f\rangle$ represent the initial, intermediate, and final nuclear states with charges Z, $Z+1$, and $Z+2$ and energies E_i, E_n, and E_f, respectively. p_1 and p_2 are the outgoing electron momenta and $u^c(p_1)$ is the charge conjugate state associated with e_1. ϕ_1^* and ϕ_2^* are the electron wave functions which in general are distorted in the presence of the nuclear field. To a good approximation they can be written as

$$\phi^* = \phi^*(z,\epsilon,p_e\cdot\vec{x}) \approx [F(z,\epsilon)]^{1/2} e^{ip_e x}, \tag{12a}$$

where $F(z,\epsilon)$ is the Fermi function[4] given by

$$F(z,\epsilon) \approx \frac{2\pi\alpha z}{1-e^{-2\pi\alpha z}} \frac{\epsilon+1}{[\epsilon(\epsilon+2)]^{1/2}}. \tag{12b}$$

ϵ is the electron kinetic energy in units of m_ec^2. $J_\mu(\vec{x})$ and $J_\nu(\vec{x})$ are the usual hadronic currents appearing in ordinary β decay and they will be defined explicitly later. Equation (11) can be simplified as follows

$$\mathfrak{M}_{12} = \frac{f^4}{4}(\beta M_\sigma)^2[F(z+1,\epsilon_1)F(z+2,\epsilon_2)]^{1/2}$$
$$\times \int d^4x \int d^4y \int \frac{d^4k}{(2\pi)^4} \frac{F[(p_1+k)^2]}{(p_1+k)^2-M_\omega^2} \frac{F[(p_2-k)^2]}{(p_2-k)^2-M_\omega^2} \frac{1}{(k^2-M_\sigma^2)^2} \bar{u}(p_2)\gamma_\mu \not{k} \gamma_\nu (1-\gamma_5) u^c(p_1)$$
$$\times e^{-ik(x-y)} e^{i(p_1x+p_2y)} \sum_n e^{-i(E_i-E_n)y_0} e^{-i(E_n-E_f)x_0} \langle f|J_\mu(x)|n\rangle\langle n|J_\nu(y)|i\rangle. \tag{13}$$

The nucleon form factor $F(q^2)$ can be approximated by the dipole shape

$$F(q^2) = \frac{1}{(1-q^2/M_A{}^2)^2}. \tag{14}$$

The quantity M_A is determined by a fit to the experimental data and it is found to be[20]

$$M_A = 0.84 \text{ GeV}/c^2.$$

In order to proceed further it is necessary to make some further approximations. These are:

(i) The sum over the intermediate hadronic states is performed by invoking the closure approximation,[4,16] i.e., we write

$$\sum_n e^{-i(E_i-E_n)y_0} e^{-i(E_n-E_F)x_0} \langle f|J_\mu(\vec{x})|n\rangle\langle n|J_\nu(y)|i\rangle$$
$$\approx e^{-i(E_i-\langle E_n\rangle)y_0} e^{-i(\langle E_n\rangle - E_f)x_0} \times\langle f|J_\mu(\vec{x})J_\nu(\vec{y})|i\rangle \tag{15}$$

with $\langle E_n\rangle$ being some appropriate average energy of the intermediate nuclear states. This approximation has been adopted in the earlier work on double β decay and was found to be good. In addition, the obtained results[4] were found to be approximately independent of $\langle E_n\rangle$. We will see later that our results will also be independent of $\langle E_n\rangle$ as long as M_σ is much larger than $\langle E_n\rangle$.

(ii) Drop all the external momenta in the propagators and the form factors. This is an excellent approximation for the W propagators, since the external momenta are much smaller than $M_\omega c \approx 80$ GeV/c^2. It is also expected to be reasonably good for the nucleon form factors where $p_e \lesssim 10$ MeV/c and $M_A \sim 1$ GeV/c^2. This approximation tremendously simplifies the calculation.

With the above approximations the integration over x_0, y_0, and k can be performed using standard contour integral techniques.[16,17] We find it useful to distinguish two cases:

(i) $M_A = \infty$, i.e., the momentum dependence of the nucleon form factors is neglected. Then the amplitude takes the form

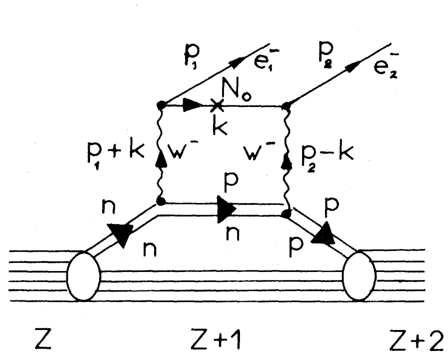

FIG. 1. Schematic representation of the leading diagram at the loop level associated with neutrinoless double β decay. In the hadronic sector two fermions (nucleons or quarks) must participate. $|i\rangle$, $|n\rangle$, and $|f\rangle$ stand for the initial, intermediate, and final nuclear systems. Z indicates the charge of the nucleus.

$$\mathfrak{M}_{12}=\frac{f^4}{16}(\beta M_\sigma)^2F(z+1,\epsilon_1)F(z+2,\epsilon_2)\int d^3\vec{x}\int d^3\vec{y}\,e^{-i(\vec{p}_1\cdot\vec{x}+\vec{p}_2\cdot\vec{y})}[\tilde{F}_t(r)\bar{u}(p_2)\gamma^\mu\gamma^0\gamma^\nu(1-\gamma_5)u^c(p_1)\langle f|J_\lambda(\vec{x})J_\nu(\vec{y})|i\rangle$$
$$+\tilde{F}_s(r)\bar{u}(p_2)\gamma^\mu\vec{\gamma}\cdot\hat{r}\gamma^\nu(1-\gamma_5)u^c(p_1)\langle f|J_\lambda(\vec{x})J_\nu(\vec{y})|i\rangle] \quad (16)$$

with

$$\vec{r}=\vec{x}-\vec{y}\,,$$

$$\tilde{F}_t(r)=\frac{1}{(M_w^2-M_\sigma^2)^2}\left\{\frac{\Delta}{M_\sigma}\left[1+\frac{4M_\sigma^2}{(M_\sigma^2-M_\omega^2)M_\sigma r}\right]e^{-M_\sigma r}+M_\omega\leftrightarrow M_\sigma\right\}, \quad (17a)$$

$$\tilde{F}_s(r)=\frac{1}{(M_\omega^2-M_\sigma^2)^2}\left\{\left[1+\frac{4(1-M_\sigma r)}{(M_\sigma^2-M_\omega^2)r^2}\right]e^{-M_\sigma r}+M_\omega\leftrightarrow M_\sigma\right\}, \quad (17b)$$

$$\Delta=E_i-\langle E_n\rangle-E_1\,.$$

Since, as it has already been mentioned, $\Delta \ll M_\sigma$, the term containing $\tilde{F}_t(r)$ is negligible in front of that containing $\tilde{F}_s(r)$. Thus the time component was neglected. The following limiting cases are of interest:

$$\tilde{F}_t(r)\approx\frac{1}{M^4}\frac{\Delta}{m}e^{-mr}\,,\quad \tilde{F}_s(r)=\frac{1}{M^4}e^{-mr}\,,\quad m\ll M \quad (17c)$$

$$\left.\begin{aligned}\tilde{F}_t(r)&\approx\frac{1}{M^4}\frac{\Delta}{m}\frac{1}{24}[(mr)^2+3mr+3]e^{-mr}\\ \tilde{F}_s(r)&\approx\frac{1}{M^4}\frac{1}{24}(1+mr)e^{-mr}\end{aligned}\right\},\quad m\approx M \quad (17d)$$

where $m=\min(M_\sigma,M_w)$ and $M=\max(M_\sigma,M_w)$.

(ii) $M_A \ll M_w$, i.e., use the momentum dependent nucleon form factor of Eq. (14). Again the amplitude takes the form of Eq. (16) except that in this case we have

$$\tilde{F}_t(r)=\frac{1}{M_w^4}\frac{\Delta}{M_\sigma}\frac{1}{\left[1-\left(\frac{M_\sigma}{M_A}\right)^2\right]^2}\times\left\{\frac{8}{\left[1-\left(\frac{M_\sigma}{M_A}\right)^2\right]^3}\frac{e^{-\chi_A}-e^{-\chi_\sigma}}{\chi_A}+\frac{3Y_0(\chi_A)+Y_0(\chi_\sigma)}{[1-(M_\sigma/M_A)^2]^2}+\frac{1}{2}\frac{\chi_A^2Y_1(\chi_A)}{1-(M_\sigma/M_A)^2}+\frac{1}{24}\chi_A^4Y_2(\chi_A)\right\} \quad (18)$$

$$\tilde{F}_s(r)=\frac{1}{M_w^4}\frac{1}{[1-(M_\sigma/M_A)^2]^2}\times\left\{\frac{8}{[1-(M_\sigma/M_A)^2]^3}\frac{(\chi_A+1)e^{-\chi_A}-(\chi_\sigma+1)e^{-\chi_\sigma}}{\chi_A^2}+\frac{3e^{-\chi_A}+e^{-\chi_\sigma}}{[1-(M_\sigma/M_A)^2]^2}+\frac{1}{2}\frac{\chi_A^2Y_0(\chi_A)}{1-(M_\sigma/M_A)^2}+\frac{\chi_A^4}{24}Y_1(\chi_A)\right\}, \quad (19)$$

where

$$\chi_A=M_Ar\,,\quad \chi_\sigma=M_\sigma r\,, \quad (20a)$$

$$Y_0(x)=e^{-x}/x\,,$$
$$Y_1(x)=\left(\frac{1}{x^2}+\frac{1}{x}\right)Y_0(x)\,, \quad (20b)$$
$$Y_2(x)=\left(\frac{3}{x^4}+\frac{3}{x^3}+\frac{1}{x^2}\right)Y_0(x)\,.$$

Again we observe that $F_t(r)$ is negligible in front of $F_s(r)$ provided that $\Delta \ll M_A$. In the above expression the hadronic current is normalized as follows:

$$J_\mu(x)=\tfrac{1}{2}\psi_p(\vec{x})\gamma_\mu(f_V-f_A\gamma_5)\psi_n(x)\,,\quad f_A/f_V=1\cdot 24\,. \quad (21)$$

The hadronic current density inside the nucleus takes the form

$$J_\mu(\vec{x})=\tfrac{1}{2}\sum_i\mathcal{L}_\mu(i)\delta(\vec{x}-\vec{r}_i)\,, \quad (22)$$

where $\vec{r}_i$ is the coordinate of the ith nucleon and $\mathcal{L}_\mu(i)$ will be given below [see Eq. (28)].

Since the wave function describing the two electrons must be antisymmetric with respect to the interchange of the labels 1 and 2, the physical amplitude[4] is

$$\mathfrak{M}=(1-P_{12})\mathfrak{M}_{12}\,, \quad (23)$$

where P_{12} is the permutation operator. Noting that, in the approximation of Eq. (12b), the product of the two Fermi functions is symmetric under the action of P_{12} we obtain

$$\mathfrak{M}=\beta^2G\,\delta(E_i-E_f-E_1-E_2)\times[F(z+1,\epsilon_1)F(z+2,\epsilon_2)]^{1/2}\langle f|\Omega|i\rangle \quad (24)$$

with

$$G=\tfrac{1}{2}(G_Fm_p)^2\,,\quad \Omega=(1-P_{12})\Omega_{12}\,, \quad (25)$$

$$\Omega_{12}=\sum e^{-i(\vec{p}_1\cdot\vec{r}_i+\vec{p}_2\cdot\vec{r}_j)}\left(\frac{M_\sigma}{m_p}\right)^2F_s(r)L_\mu(i)L_\nu(i)\times\bar{u}(p_2)\gamma^\mu\vec{\gamma}\cdot\hat{r}\gamma^\nu(1-\gamma_5)u^c(p_1)\,, \quad (26)$$

where

$$F_s(r) = M_\omega{}^4 \tilde{F}_s(r) \quad \text{(dimensionless)}. \tag{27}$$

For nonrelativistic nucleons the weak hadronic current takes the form

$$L_\mu(i) = \begin{cases} f_V \tau_-(i) & \mu = 0 , \\ -f_A \tau_-(i)\vec{\sigma}_i & \mu \neq 0 . \end{cases} \tag{28}$$

Thus after some algebra involving γ matrices we find

$$\Omega_{12} = l_0 \Omega_{12}^0 + \vec{l} \cdot \vec{\Omega}_{12} \tag{29}$$

with $l_\mu = \bar{u}(p_2)\gamma_\mu(1-\gamma_5)u^c(p_1)$,

$$\Omega_{12}^0 = \sum_{i \neq j} e^{-i(\vec{p}_1 \cdot \vec{r}_1 + \vec{p}_2 \cdot \vec{r}_2)} F_s(r) \tau_-(i) \tau_-(j) \omega_{ij}^0 , \tag{30}$$

$$\vec{\Omega}_{12} = \sum_{i \neq j} e^{-i(\vec{p}_1 \cdot \vec{r}_i + \vec{p}_2 \cdot \vec{r}_j)} F_s(r) \tau_-(i) \tau_-(j) \vec{\omega}_{ij} , \tag{31}$$

$$\omega_{12}^0 = f_V f_A (\vec{\sigma}_i + \vec{\sigma}_j) \cdot \frac{\hat{r}}{i} - f_A f_v (\vec{\sigma}_i \times \vec{\sigma}_j) \cdot \hat{r} , \tag{32}$$

$$\vec{\omega}_{12} = (f_V{}^2 - f_A{}^2 \vec{\sigma}_i \cdot \vec{\sigma}_j) \frac{\hat{r}}{i} + f_A{}^2 \left(\frac{\vec{\sigma}_i \cdot \hat{r}}{i} \vec{\sigma}_j + \vec{\sigma}_i \frac{\vec{\sigma}_j \cdot \hat{r}}{i} \right) - f_v f_A (\vec{\sigma}_i - \vec{\sigma}_j) \times \hat{r} . \tag{33}$$

The above operators are fairly complicated. They are quite a bit simplified if one considers $0^+ \to 0^+$ transitions, which is the case of actual experimental interest, and then one retains the leading nonvanishing terms[4] (linear in the electron momenta). One finds

$$\Omega = -\frac{f_A}{9} \frac{m_e}{m_p{}^2 R_0} \left\{ \Omega_B \left[\bar{u}(p_2) \frac{\vec{\gamma} \cdot \vec{p}}{m_e} (1-\gamma_5) u^c(p_1) + u(p_1) \frac{\vec{\gamma} \cdot \vec{p}}{m_e} (1-\gamma_5) u^c(p_2) \right] \times \Omega_A \left[\bar{u}(p_2) \frac{\gamma \cdot \vec{q}}{m_e} (1-\gamma_5) u^c(p_1) - u(p_1) \frac{\gamma \cdot \vec{q}}{m_e} (1-\gamma_5) u^c(p_2) \right] \right\} , \tag{34}$$

where

$$\vec{p} = \tfrac{1}{2}(\vec{p}_1 - \vec{p}_2) , \quad \vec{q} = (\vec{p}_1 + \vec{p}_2) ,$$

R_0 is the nuclear radius, and

$$\Omega_A = 3 \left(\frac{M_\sigma c^2 R_0}{\hbar c} \right)^2 (f_V / f_A)^2 \sum_{i \neq j} F_s(r) \frac{R}{R_0} \tau_-(i) \tau_-(j) (\vec{\sigma}_i - \vec{\sigma}_j)(i\hat{r} \times \hat{R}) , \tag{35}$$

$$\Omega_B = \left(\frac{M_\sigma c^2 R_0}{\hbar c} \right)^2 \sum_{i \neq j} F_s(r) \frac{r}{R_0} \tau_-(i) \tau_-(j) O(\vec{\sigma}_i, \vec{\sigma}_j, \hat{r}) \tag{36}$$

with

$$O(\vec{\sigma}_i, \vec{\sigma}_j, \hat{r}) = 3 \frac{f_V{}^2}{f_A{}^2} - \vec{\sigma}_i \vec{\sigma}_j + \frac{2\sqrt{2 \cdot 3}}{\sqrt{5}} \sqrt{4\pi}\, Y^2(\hat{r}) \cdot (\vec{\sigma}_i \otimes \vec{\sigma}_j)^2 \tag{37}$$

$$\vec{r} = \vec{r}_i - \vec{r}_j , \quad \vec{R} = \tfrac{1}{2}(\vec{r}_i + \vec{r}_j) .$$

The above operators Ω_A and Ω_B were normalized in such a way that comparison with the earlier work[4] can become easier.

After averaging over all lepton spins and integrating over all electron directions we obtain

$$\langle \mathfrak{M}^2 \rangle = \Lambda \beta^4 \frac{(\epsilon_1 + 1)(\epsilon_2 + 1)}{[\epsilon_1(\epsilon_1 + 2)\epsilon_2(\epsilon_2 + 2)]^{1/2}} \{ g_A(\epsilon_1, \epsilon_2) |\langle f | \Omega_A | i \rangle|^2 + g_B(\epsilon_1, \epsilon_2) |\langle f | \Omega_B | i \rangle|^2 \} , \tag{38}$$

where

$$g_A(\epsilon_1, \epsilon_2) = \tfrac{2}{3} \{ 6[\epsilon_1(\epsilon_1 + 2) + \epsilon_2(\epsilon_2 + 2)][(\epsilon_1 + 1)(\epsilon_2 + 1) - 1] - \epsilon_1 \epsilon_2 (\epsilon_1 + \epsilon_2 + 4)^2 + 6\epsilon_1 \epsilon_2 (\epsilon_1 + 2)(\epsilon_2 + 2) - 3(\epsilon_1 + 2)(\epsilon_2 + 2)(\epsilon_1{}^2 + \epsilon_2{}^2) \} , \tag{39}$$

$$g_B(\epsilon_1, \epsilon_2) = \tfrac{1}{6} \{ 6[\epsilon_1(\epsilon_1 + 2) + \epsilon_2(\epsilon_2 + 2)][(\epsilon_1 + 1)(\epsilon_2 + 1) + 1] + \epsilon_1 \epsilon_2 (\epsilon_1 - \epsilon_2)^2 + 3(\epsilon_1 + 2)(\epsilon_2 + 2)(\epsilon_1{}^2 + \epsilon_2{}^2) - 18\epsilon_1 \epsilon_2 (\epsilon_1 + 2)(\epsilon_2 + 2) \} , \tag{40}$$

with ϵ_1 and ϵ_2 are the electron kinetic energies (in units of $m_e c^2$) and

$$\Lambda = G^2 \frac{m_e{}^2 f_A{}^2}{(9 m_p{}^2 R_0)^2} \chi(Z+1) \chi(Z+2) , \tag{41}$$

$$\chi(Z) = \frac{2\pi\alpha Z}{1 - e^{-2\pi\alpha Z}} , \quad \alpha = \text{fine structure constant} .$$

All nuclear physics is contained in the matrix elements of the operators Ω_A and Ω_B, which will be referred to as nuclear matrix elements.

IV. EFFECTIVE OPERATORS IN THE EARLIER TREATMENT

In the treatment of the double β decay which prevailed in the pregauge era, process (1) has been described by a phenomenological Lagrangian which contained both left-handed and right-handed currents.[4,13] In present day notation this can be written as

$$\mathcal{L} = \frac{2f}{(1+\eta^2)^{1/2}}(\bar{e}_L\gamma^\lambda\nu_L + \eta\bar{e}_R\gamma^\lambda\nu_R)W_\lambda\ , \tag{42}$$

where η is a dimensionless quantity which provides a measure of the right handed currents present in the Lagrangian. A Feynman diagram analogous to that of Fig. 1 can be written (without the mass insertion term). In this case the intermediate neutrino propagator is that of a massless particle. Thus the analog of Eq. (11) now becomes

$$\mathfrak{M}_{12} = \frac{f^4}{4}\eta\int d^4x\int d^4y\int\frac{d^4k}{(2\pi)^4}\,\frac{F[(p_1+k)^2]}{(p_1+k)^2-M_W^2}\,\frac{F[(p_2+k)^2]}{(p_2-k)^2-M_W^2}\,\frac{k_\alpha}{k^2}\,\phi_1^*\phi_2\bar{u}(p_2)\gamma^\mu\gamma^\alpha\gamma^\nu u^c(p_1)$$
$$\times e^{-ik(x-y)}\sum_n e^{-i(E_i-E_n)y_0}e^{-i(E_n-E_f)x_0}\langle f|J_\mu(\vec{x})|n\rangle\langle n|J_\nu(\vec{y})|i\rangle\ . \tag{43}$$

The fact that there is no mass in the intermediate neutrino propagator has two important consequences:

(i) The results do not sensitively depend on the nucleon form factor $F(q^2)$.

(ii) The effective transition operator is of very long range.

Proceeding as in the previous section we find

$$\langle\mathfrak{M}^2\rangle = \Lambda\eta^2\frac{(\epsilon_1+1)(\epsilon_2+2)}{[\epsilon_1(\epsilon_1+2)\epsilon_2(\epsilon_2+2)]^{1/2}}g_c(\epsilon_1,\epsilon_2)|\langle f|\Omega_c|i\rangle|^2\ , \tag{44}$$

where

$$\Omega_c = \sum_{i\neq j}\frac{R_0}{r}\tau_-(i)\tau_-(j)O(\vec{\sigma}_i,\vec{\sigma}_j,\hat{r}) \tag{45}$$

with O given by Eq. (37) and $g_c(\epsilon_1,\epsilon_2)$ given by

$$g_c(\epsilon_1,\epsilon_2) = [\epsilon_1(\epsilon_1+2)+\epsilon_2(\epsilon_2+2)][(\epsilon_1+1)(\epsilon_2+1)+1] - \epsilon_1\epsilon_2(\epsilon_1+2)(\epsilon_2+2)\ . \tag{46}$$

Now we note the following:

(i) The operator Ω_c contains a tensor component which was omitted in the earlier work[4] in which the nuclear matrix element was evaluated using the operator

$$\Omega_D = \sum_{i\neq j}\frac{R_0}{r}\tau_-(i)\tau_-(j)\left(\frac{-f_V^2}{f_A^2}+\vec{\sigma}_i\cdot\vec{\sigma}_j\right). \tag{47}$$

(ii) The radial dependence of the operator Ω_c is the same with that of Primakoff and Rosen[4] even though they used a nonrelativistic formulation. In the earlier calculations the Primakoff-Rosen approximation was employed,[4] i.e.,

$$\Omega_D \approx \sum_{i\neq j}\frac{R_0}{\langle r\rangle}\tau_-(i)\tau_-(j)\left(\frac{-f_V^2}{f_A^2}+\vec{\sigma}_i\cdot\vec{\sigma}_j\right)$$
$$\approx \sum_{i\neq j}\tau_-(i)\tau_-(j)\left(\frac{-f_V^2}{f_A^2}+\vec{\sigma}_i\cdot\vec{\sigma}_j\right).$$

For double β-decay transitions of experimental interest, the isospin of the final state is different from that of the initial state. Then

$$\Omega_D \approx \vec{Y}\cdot\vec{Y}$$

with (48)

$$\vec{Y} = \sum_i\tau_-(i)\vec{\sigma}_i\quad\text{(Gamow-Teller operator)}\ .$$

(iii) In this treatment there is no analog of the operator Ω_A which is separately antisymmetric in the spin and space indices.

V. TRANSITION RATES

The transition rate for the no-neutrino double β decay [Eq. (1)] is easily obtained once the basic amplitude is known. One finds

$$dW(i\to fe^-e^-) = \rho_B\frac{(2\pi)^4}{V^3}\frac{m_1}{E_1}\frac{m_2}{E_2}\frac{m_f}{E_f}\times|\mathfrak{M}|^2\delta(p_i-p_f-p_1-p_2)\ , \tag{49}$$

where ρ_B is the density of final states

$$\rho_B = \frac{Vd^3p_f}{(2\pi)^3}\frac{Vd^3p_1}{(2\pi)^3}\frac{Vd^3p_2}{(2\pi)^3}\ . \tag{50}$$

After averaging over the final lepton spin and integrating over all possible lepton directions one finds

$$dW(i\to fe^-e^-) = \frac{4}{(2\pi)^3}\delta(E_i-E_f-E_1-E_2)\times p_1^2dp_1p_2^2dp_2\langle\mathfrak{M}^2\rangle\ . \tag{51}$$

The total transition probability can be obtained by integrating over the electron energy spectrum. Then from Eqs. (44) and (51) we find

$$W(i\to fe^-e^-) = g_2(\epsilon_0,Z,A)\eta^2|\langle f|\Omega_c|i\rangle|^2 \tag{52}$$

with

$$g_2(\epsilon_0, Z, A) = KA^{-2/3}\chi(Z+1)\chi(Z+2)f_c(\epsilon_0) \quad (53a)$$

and

$$K = c\frac{m_e c^2}{\hbar c}\frac{(G_F M_p^2)^4}{(2\pi)^3}\left(\frac{m_e}{m_p}\right)^6\left(\frac{\hbar c}{m_p c^2 r_0}\right)^2\frac{f_A^4}{81}$$
$$= 4.0\times 10^{-22}\ \mathrm{sec}^{-1} = 1.3\times 10^{-16} y^{-1}, \quad (53b)$$

$$R_0 = r_0 A^{1/3},\quad r_0 = 1.1\ \mathrm{fm},$$

$$f_c(\epsilon_0) = \int_0^{\epsilon_0}(\epsilon+1)(\epsilon_0-\epsilon+1)g_c(\epsilon,\epsilon_0-\epsilon)d\epsilon, \quad (54)$$

while Eq. (39) (gauge model) yields

$$W(i\to fe^-e^-) = g_2(\epsilon_0,Z,A)\beta^4\{I_A(\epsilon_0)|\langle f\Omega_A|i\rangle|^2 + I_B(\epsilon_0)|\langle f\Omega_B|i\rangle|^2\} \quad (55)$$

with

$$I_A(\epsilon_0) = \frac{f_A(\epsilon_0)}{f_c(\epsilon_0)},\quad I_B(\epsilon_0) = \frac{f_B(\epsilon_0)}{f_c(\epsilon_0)}, \quad (56)$$

$$f_A(\epsilon_0) = \int_0^{\epsilon_0}(\epsilon+1)(\epsilon_0-\epsilon+1)g_A(\epsilon,\epsilon_0-\epsilon)d\epsilon, \quad (57)$$

$$f_B(\epsilon_0) = \int_0^{\epsilon_0}(\epsilon+1)(\epsilon_0-\epsilon+1)g_B(\epsilon,\epsilon_0-\epsilon)d\epsilon, \quad (58)$$

where ϵ_0 is the available energy in unites of $m_e c^2$. The functions $f_A(\epsilon_0)$, $f_B(\epsilon_0)$, $f_C(\epsilon_0)$ are seventh degree polynomials given by

$$f_A(\epsilon_0) = \frac{2}{5\cdot 9}\epsilon_0^3\{\epsilon_0^4 + 11\epsilon_0^3 + 61\epsilon_0^2 + 120\epsilon_0 + 75\}, \quad (59)$$

$$f_B(\epsilon_0) = \frac{1}{2\cdot 7\cdot 9}\epsilon_0^2\{\epsilon_0^5 + 15\epsilon_0^4 + 133\epsilon_0^3 + 504\epsilon_0^2 + 840\epsilon_0 + 504\}, \quad (60)$$

$$f_C(\epsilon_0) = \frac{1}{2\cdot 3\cdot 5\cdot 7}\epsilon_0^2\{\epsilon_0^5 + 14\epsilon_0^4 + 154\epsilon_0^3 + 630\epsilon_0^2 + 1260\epsilon_0 + 840\}. \quad (61)$$

The lepton conserving two-neutrino double β decay can also be treated as a second order process involving the usual weak interaction Hamiltonian.[4,13] In the allowed approximation the transition rate for $0^+\to 0^+$ transitions has previously been written[13] as

$$W(i\to fe^-e^-\tilde{\nu}_e\tilde{\nu}_e) = g_4(\epsilon_0,Z,A,\langle E\rangle)|(ME)_4|^2, \quad (62)$$

where

$$(ME)_4 = \sum_n \frac{\langle E\rangle - E_i + \Delta/2}{E_n - E_i + \Delta/2}\times\left\{\frac{f_V^2}{f_A^2}\langle f|T_-|n0^+\rangle\langle n0^+|T_-|i\rangle - \tfrac{1}{3}\langle f||\vec{\mathrm{Y}}||n1^+\rangle\langle n1^+||\vec{\mathrm{Y}}||i\rangle\right\}, \quad (63)$$

where Δ is the available energy, $\langle E\rangle$ is the average energy of $Z+1$ nuclear system, and n labels the possible contributing intermediate nuclear states. We note that in this case the transition rate depends on the energy and structure of the intermediate states. In the cases of experimental interest $T_i \neq T_f$ and the first term (vector part) in the curly bracket makes no contribution. We note further that, because of the energy denominators, closure approximation cannot be invoked in this case.

Once the nuclear matrix elements have been evaluated one can compute the lifetime for process (2) quite easily. If the experiment cannot discriminate between the no-neutrino and the two-neutrino modes, the total lifetime against both decay modes can be written as

$$\frac{1}{T_{1/2}} = \frac{1}{(T_{1/2})_{0\nu}} + \frac{1}{(T_{1/2})_{2\nu}}, \quad (64)$$

where $(T_{1/2})_{0\nu}$ and $(T_{1/2})_{2\nu}$ are the no-neutrino and two-neutrino half-lives given by

$$\frac{1}{(T_{1/2})_{2\nu}} = \frac{g_4(\epsilon_0,Z,A,\langle E\rangle)}{\ln 2}|(ME)_4|^4 \quad (65)$$

and

$$\frac{1}{(T_{1/2})_{0\nu}} = \frac{g_2(\epsilon_0,Z,A)}{\ln 2}\begin{cases}\eta^2|\langle f|\Omega_c|i\rangle|^2 & (66)\\ \beta^4[I_A(\epsilon_0)|\langle f|\Omega_A|i\rangle|^2 + I_B(\epsilon_0)|\langle f|\Omega_B|i\rangle|^2], & (67)\end{cases}$$

where the upper case refers to the old treatment and the lower case refers to the present gauge model.

VI. COMPARISON OF VARIOUS NUCLEAR MATRIX ELEMENTS

As it has already been mentioned earlier the relationship between $|\langle f|\Omega_c|i\rangle|^2$ and $|(ME)_4|^2$ played an essential role in the extraction of the parameter η from total lifetime measurements. Let us for the moment assume that $|\langle f|\Omega_c|i\rangle|^2 = |(ME)_4|^2$. Then Eq. (64) yields

$$\frac{T_{1/2}(128)}{T_{1/2}(130)} = \frac{\eta^2 g_2(130) + g_4(130)}{\eta^2 g_2(128) + g_4(128)}\frac{|ME(130)|^2}{|ME(128)|^2}, \quad (68)$$

where the labels 130 and 128 refer to the corresponding quantities associated with the two similar double β-decay transitions $^{130}Te\to{}^{130}Xe$ and $^{128}Te\to{}^{128}Xe$. Pontecorvo[15] exploited the above equation by noting that for these presumably similar systems the nuclear matrix elements must be the same. This last assumption was later found to be correct by explicit shell model calculations.[13] Thus one obtains

$$\frac{T_{1/2}(128)}{T_{1/2}(130)}=\frac{\eta^2 g_2(130)+g_4(130)}{\eta^2 g_2(128)+g_4(128)}. \tag{69}$$

Equation (69), except for the dependence of g_4 on the average excitation energy, allows the determination of η^2 in a way which is independent of the nuclear structure. This way, from total lifetimes of geologic ore type experiments[7,8] and the kinematical functions g_2 and g_4, the extracted value of the parameter n was found[13] to be

$$\eta \approx 0.5\times 10^{-4}. \tag{70}$$

We must stress once more that the assumption which leads to Eq. (70) was the equality

$$|\langle f|\Omega_c|i\rangle|^2=|(ME)_4|^2. \tag{71}$$

For this equation to hold the following conditions are required:

(i) The Primakoff-Rosen approximation must hold, i.e.,

$$\Omega_c \approx \vec{Y}\cdot\vec{Y}. \tag{72}$$

(ii) One must be able to involve closure in the evaluation of $(ME)_4$. Thus neglecting the energy denominators in Eq. (63) and setting $E_n=\langle E\rangle$ one finds that

$$(ME)_4 \approx -\tfrac{1}{3}\sum_n \langle f||\vec{Y}||n1^+\rangle\langle n1^+||\vec{Y}||i\rangle \tag{73}$$

or

$$(mE)_4 \cong \langle f|\vec{Y}\cdot\vec{Y}|i\rangle. \tag{74}$$

Both assumptions must be checked by explicit nuclear structure calculations. Results of such calculations for $^{48}Ca \to {}^{48}Ti$ will be presented later. We note that the parameter β^2 can be similarly extracted from an equation analogous to Eq. (69). However because of the short range nature of the operators Ω_A and Ω_B, it is almost impossible to have

$$I_A|\langle f|\Omega_A|i\rangle|^2+I_B|\langle f|\Omega_B|i\rangle|^2=|(ME)_4|^2. \tag{75}$$

Thus if the present gauge model adequately describes reality there is no hope of extracting the parameter β^2 from the data this way. Thus it ap pears necessary to investigate the matrix elements $|\langle f|\Omega_A|i\rangle|^2$ and $|\langle f|\Omega_B|i\rangle|^2$. For this we have to use realistic shell model wave functions and take into account short range correlations as well. Thus the two-nucleon density necessary for the evaluation of the nuclear matrix elements will be written as follows:

$$\rho(\vec{r}_1,\vec{r}_2)=\rho_0(\vec{r}_1,\vec{r}_2)[1-G(|\vec{r}_1-\vec{r}_2|)], \tag{76}$$

where $\rho_0(\vec{r}_1,\vec{r}_2)$ is the usual shell model two body density and $G(r)$ the two nucleon short range correlation function. The function $G(r)$, of course, is not known very well, but many forms of it are available. For our purpose we have chosen to work with the simple correlation function[21]

$$G(r)=\begin{cases}1 & r>r_c\\ 0 & r<r_c\end{cases}, \tag{77}$$

i.e., we have no correlation beyond a certain distance r_c and complete correlation for shorter distances (r_c is of the order of the hard core radius, i.e., $r_c\approx 0.4$ fm). We will investigate the dependence of our results on the parameter r_c.

Before we conclude this section we mention that the parameter β^2 will, among other things, depend on M_σ. The nuclear matrix elements themselves are expected to be very sensitive functions of M_σ and r_c. Furthermore, the relation between η and β^2 is going to be a complicated function of the gross nuclear parameters A, R_0, and Z, as we will see in the next section.

VII. A RELATION BETWEEN η AND β^2

As has already been mentioned, the analysis of the experimental data in the past has been made in terms of the parameter η, while in the model discussed in this work the relevant parameter is β^2. It is also quite likely that both descriptions are applicable. Even though the physical origins of η and β^2 are very different it may be useful for the analysis of experimental data to extract a relationship between them, in particular one which depends on the gross properties of the nucleus, e.g., R_0, A, r_c, etc.

For such a relation, following Halprin *et al.*[21] we write the two-body density $\rho_0(\vec{r}_1,\vec{r}_2)$ as follows

$$\rho_0(\vec{r}_1,\vec{r}_2)=\rho(\vec{r}_1)\rho(\vec{r}_2)$$

$$\rho(\vec{r})=\begin{cases}\text{constant}\neq 0, & r<R_0\\ 0, & r>R_0.\end{cases} \tag{78}$$

Since the spin-isospin structure of the operators Ω_B and Ω_c is similar, neglecting for the moment the contribution of the operator Ω_A, we obtain

$$\sqrt{I_B}\beta^2\left(\frac{M_\sigma c^2 R_0}{\hbar c}\right)^2\left\langle F_s(r)\frac{r}{R_0}\right\rangle=\eta\left\langle\frac{R_0}{r}\right\rangle \tag{79}$$

with the above two nucleon densities [Eqs. (77) and (78)] we get

$$\left(\frac{\eta}{\beta^2}\right)_B=\tfrac{1}{2}\sqrt{I_B}\left(\frac{M_\sigma c^2}{\hbar c R_0}\right)^2\int_{r_c}^{2R_0}F_s(r)r^3dr. \tag{80}$$

Finally using the functions $F_s(r)$ given by Eqs. (17a)–(17d) we obtain

$$\left(\frac{\eta}{\beta^2}\right)_B=\frac{\sqrt{I_B}}{A^{2/3}}\phi_B(M_\sigma,r_c), \tag{81}$$

where

$$\phi_B(M_\sigma r_c)=\frac{1}{2}\left(\frac{\hbar c}{M_\sigma c^2 r_0}\right)^2\chi_B\left(\lambda,\frac{M_A c^2 r_c}{\hbar c},\frac{M_\sigma c^2 r_c}{\hbar c}\right), \quad (82a)$$

$$\chi_B(\lambda,\xi,\zeta)=\frac{8\Lambda_1(\xi)-\lambda^2\Lambda_1(\zeta)}{\lambda^2\left(1-\frac{1}{\lambda^2}\right)^5}+\frac{3\Lambda_2(\xi)+\lambda^4\Lambda_2(\zeta)}{\lambda^4\left(1-\frac{1}{\lambda^2}\right)^4}$$

$$+\frac{1}{2}\frac{\Lambda_3(\xi)}{\left(1-\frac{1}{\lambda^2}\right)^3\lambda^4}+\frac{1}{24}\frac{\Lambda_4(\xi)}{\lambda^4\left(1-\frac{1}{\lambda^2}\right)^2}, \quad (82b)$$

and

$$\Lambda_1(\xi)=(\xi^2+3\xi+3)e^{-\xi},$$
$$\Lambda_2(\xi)=(\xi^3+3\xi^2+6\xi+6)e^{-\xi},$$
$$\Lambda_3(\xi)=(\xi^4+4\xi^3+12\xi^2+24\xi+24)e^{-\xi}, \quad (82c)$$
$$\Lambda_4(\xi)=(\xi^5+6\xi^4+24\xi^3+72\xi^2+144\xi+144)e^{-\xi}.$$

Without detailed information about the two-body spin density it is not possible to give an accurate estimate of the ratio η/β^2 in the case when the operator Ω_A dominates the transition. Just to give an idea of the order of magnitude of the ratio η/β^2 expected in this case we will assume that on the average the spin dependence of the matrix elements of the operators Ω_A and Ω_C is similar. Owing to its complicated tensorial character the matrix element of Ω_A is expected to be suppressed for $0^+\to 0^+$ transitions. But the matrix element of Ω_C for the ground state transition is also suppressed[13] due to its spin character. Thus the above assumption may not be completely unrealistic. With this assumption, proceeding exactly as before, we get

$$\left(\frac{\eta}{\beta^2}\right)_A=\frac{3}{16}\left(\frac{M_\sigma c^2}{\hbar c}\right)^2\frac{\sqrt{I_A}}{R_0}\int_{r_c}^{2R_0}F_s(r)r^2dr \quad (83)$$

or

$$\left(\frac{\eta}{\beta^2}\right)_A=\frac{\sqrt{I_A}}{A^{1/3}}\phi_A(M_\sigma,r_c). \quad (84)$$

From this equation we see that in this case the ratio η/β^2 is proportional to $A^{-1/3}$ instead of the $A^{-2/3}$ found in the previous case. It thus appears that the A term becomes more important as the mass number increases. The quantity $\Phi_A(m_\sigma,r_c)$ is given as follows

$$\phi_A(M_\sigma,r_c)=\frac{3}{4}\left(\frac{\hbar c}{M_\sigma c^2 r_c}\right)\chi_B\left(\lambda,\frac{M_A c^2 r_c}{\hbar c},\frac{M_\sigma c^2 r_c}{\hbar c}\right), \quad (85a)$$

where χ_B is of the same form as Eq. (83b) the only modification being that the functions Λ_i are replaced by the functions K_i defined as follows:

$$K_1(\xi)=(\xi+2)e^{-\xi},\quad K_2(\xi)=(\xi^2+2\xi+2)e^{-\xi}, \quad (85b)$$
$$K_3(\xi)=(\xi^3+3\xi^2+6\xi+6)e^{-\xi},$$
$$K_4(\xi)=(\xi^4+5\xi^3+15\xi^2+30\xi+30)e^{-\xi}.$$

We note that if $r_c\neq 0$ the functions Φ_A and Φ_B contain an exponential factor. They are thus dominated by the smallest of M_A, M_w, and M_σ (the expressions for $M_A=\infty$ are similar).

Considering both the A and the B terms we obtain

$$\frac{\eta}{\beta^2}=\left\{\left[\frac{\sqrt{I_A}}{A^{1/3}}\phi_A(M_r,r_c)\right]^2+\left[\frac{\sqrt{I_B}}{A^{2/3}}\phi_B(M_\sigma,r_c)\right]^2\right\}^{1/2}. \quad (86)$$

In the special case of the $A=130$ we find

$$\epsilon_0=5.0,\quad I_A=2.1,\quad I_B=1.3.$$

Thus for the most reasonable choice of the correlation parameter, i.e., $r_c=0.4$ fm, and for $M_A=\infty$ we obtain

$$(\eta/\beta^2)_A=0.20,\ 0.97\times10^{-1},\ 0.74\times10^{-1},\ 0.13\times10^{-1},\ 0.60\times10^{-4},\ 0.50\times10^{-8},\ 0.00,\ 0.00,$$
$$(\eta/\beta^2)_B=0.29\times10^{-1},\ 0.89\times10^{-2},\ 0.64\times10^{-2},\ 0.8\times10^{-3},\ 0.31\times10^{-5},\ 0.24\times10^{-10},\ 0.00,\ 0.00,$$
$$(\eta/\beta^2)=0.22,\ 0.12,\ 0.10,\ 0.37\times10^{-1},\ 0.27\times10^{-2},\ 0.30\times10^{-1},\ 0.00,\ 0.00,$$

for $M_\sigma=0.5$, 0.85, 1.0, 2.0, 5.0, 10, 80, 100, GeV/c^2, respectively. On the other hand, if we adopt the more reasonable value of $M_A=0.85$ GeV/c^2 the corresponding quantities become

$$(\eta/\beta^2)_A=0.14,\ 0.78\times10^{-1},\ 0.18,\ 0.15\times10^{-1},\ 0.22\times10^{-2},\ 0.55\times10^{-3},\ 0.90\times10^{-5},\ 0.55\times10^{-5}$$
$$(\eta/\beta^2)_B=0.34\times10^{-1},\ 0.42\times10^{-2},\ 0.80\times10^{-2},\ 0.21\times10^{-2},\ 0.34\times10^{-3},\ 0.85\times10^{-4},\ 0.13\times10^{-5},\ 0.85\times10^{-7} \quad (87)$$
$$(\eta/\beta^2)=0.15,\ 0.79\times10^{-1},\ 0.18,\ 0.15\times10^{-1},\ 0.22\times10^{-2},\ 0.56\times10^{-3},\ 0.91\times10^{-5},\ 0.56\times10^{-5}.$$

From the above results we see that the double β decay cannot be described in terms of a single "universal" value of η, i.e., 0.5×10^{-4} as it is commonly accepted, because for high values of M_σ it will lead to unacceptable values of β^2 (greater than unity). Adopting, however, the value of $\eta=0.5\times10^{-4}$ discussed earlier we get

$$\beta^2 = 3.3\times10^{-4} \text{ for } M_A = 0.85 \text{ GeV}/c^2 \text{ and } M_\sigma = 0.5 \text{ GeV}/c^2,$$
$$\beta^2 = 6.3\times10^{-4} \text{ for } M_A = 0.85 \text{ GeV}/c^2 \text{ and } M_\sigma = 1.0 \text{ GeV}/c^2, \quad (88)$$
$$\beta^2 = 2.2\times10^{-4} \text{ for } M_A = \infty \text{ and } M_\sigma = 0.5 \text{ GeV}/c^2,$$
$$\beta^2 = 4.2\times10^{-4} \text{ for } M_A = \infty \text{ and } M_\sigma = 1.0 \text{ GeV}/c^2.$$

The above extracted values are an order of magnitude smaller than those previously[16] obtained. If, however, one retains only the space-symmetric term, i.e., one uses the value $(\eta/\beta^2)_B$ given above, one finds for β^2 the values 1.4×10^{-3}, 1.2×10^{-2}, 1.7×10^{-3}, 5.6×10^{-3}, which are in essential agreement with those of the earlier work.

VIII. MICROSCOPIC CALCULATIONS IN THE CASE OF ^{48}Ca

The ideas and the formalism developed in the previous sections are going to be applied in the case of $^{48}Ca \rightarrow {}^{48}Ti$ double β decay. This system has previously been studied both experimentally[9] and theoretically.[13] From a theoretical point of view this is the simplest nucleus which can undergo double β decay. It is believed that ^{48}Ca can fairly well be described as a closed $1f_{7/2}$ shell.

The main feature of the previous theoretical investigations has been the fact that the nuclear matrix element appears anomalously small compared to that of other systems.[13] This perhaps has discouraged experimentalists from further pursuing its study. We shall see later that this retardation is less severe if the tensor component of the transition operator is retained. In addition, this relative retardation may not exist if process (1) is mediated by a massive Majorana neutrino, as in our model, since the radial function $F_s(r)$ becomes complicated.

In the present work we will use the simple shell model space of the earlier work[13] but we will adopt a slightly different two-body interaction, namely the bare G-matrix Kuo-Lee-Brown matrix elements.[22] Thus within the $if_{7/2}$ shell we obtain the following wave functions:

$$\begin{aligned}
{}^{48}\text{Ca(g.s.)} &= |1f_{7/2}{}^8(n)\rangle,\\
{}^{48}\text{Ti(g.s.)} &= 0.8551|1\rangle - 0.5233|2\rangle + 0.1094|3\rangle - 0.0009|4\rangle,\\
{}^{48}\text{Ti}(0_2^+) &= 0.4036|1\rangle + 0.5000|2\rangle - 0.7404|3\rangle + 0.1936|4\rangle,\\
{}^{48}\text{Ti}(0_3^+) &= 0.2926|1\rangle + 0.5447|2\rangle + 0.3441|3\rangle - 0.7059|4\rangle,\\
{}^{48}\text{Ti}(0_4^+) &= 0.1890|1\rangle + 0.4226|2\rangle + 0.5669|3\rangle + 0.6814|4\rangle,
\end{aligned} \quad (89)$$

where

$$|i\rangle = |f_{7/2}{}^6(n)J_i\,; f_{7/2}{}^2(p)J_i\,, 0^+\rangle,$$
$$J_i = 0, 2, 4, 6 \text{ for } i = 1, 2, 3, 4.$$

We note that, within the above model, the ^{48}Ca(g.s.) and ^{48}Ti(0_4) states are independent of the two-nucleon effective interaction used. The last state is characterized by isospin quantum numbers $T=4$ and $T_z=2$ and is the double analog of the ^{48}Ca(g.s.). We also mention that the double β-decay transitions to the excited states of ^{48}Ti are not energetically allowed. The pertinent nuclear matrix elements to those states have been calculated, however, to clarify some of the theoretical considerations which follow. Nuclear matrix elements of the operators Ω_C and Ω_D for various situations of physical interest are presented in Table I. We should mention that in all cases presented

TABLE I. The nuclear matrix elements $|\langle f|\Omega_C|i\rangle|^2$ and $|\langle f|\Omega_D|i\rangle|^2$ for $A=48$. For comparison purposes the nuclear matrix elements for transitions to excited states of ^{48}Ti are presented even though transitions to such states are forbidden energetically.

	With radial dependence				Rosen-Primakoff approximation			
	$r_c=0.0$ fm		$r_c=0.4$ fm		$r_c=0.0$ fm		$r_c=0.4$ fm	
	Ω_C	Ω_D	Ω_C	Ω_D	Ω_C	Ω_D	Ω_C	Ω_D
g.s.	14.8	1.63	0.205	0.184	7.47	5.51×10^{-3}	7.47	7.43×10^{-3} [a]
0_2^+	11.2	4.28	2.14	2.30	3.72	0.471	3.72	1.320
0_3^+	51.8	44.7	34.6	34.8	45.5	41.5	45.5	37.6
0_4^+	645	441	403	393	764	453	764	452
g.s. %	2.0	0.33	0.047	0.043	0.91	0.0011	0.91	0.0011

[a] This matrix element has changed sign relative to that with $r_c=0.0$ fm.

in Table I the ground state matrix elements are retarded and they exhaust a small portion of the sum rule. The retardation is greatest when the tensor component of Ω_C is neglected.

From Table I one can immediately draw the following conclusions:

(i) The tensor component of the transition operator dominates in the case of the physically interesting transitions to the ground state. This is due to the fact that there are strong cancellations involving the matrix element of the rest of the operator. This seems to be related to the basic properties of the nucleon-nucleon interaction and is expected to be true for all nuclear systems. Thus only a small portion of the sum rule is exhausted in such transitions (see last row of Table I). This retardation may not be as large in other systems as in ^{48}Ca. Even in the case of ^{48}Ca, however, it may be less dramatic if more configurations are included. The tensor contribution is, as expected, negligible in the case of the strong transitions, i.e., to the 0_3^+ (spin-isospin mode) and 0_4^+ (isospin mode).

(ii) The Primakoff-Rosen approximation is not very accurate (especially if the tensor component is neglected) for precisely the same reasons as above.

(iii) The results seem to be sensitive to the short range correlations.

At first glance these may appear to be curious results, but they are to be understood as being caused by the retardation mentioned above. Effects which are otherwise small become dominant in the case of weak transitions.

The above mentioned trends persist even if the radial part of the operator becomes more complicated as in our gauge model. For lack of space only results for ground state transition are presented in Tables II–IV. We present our results as function of M_σ and r_c for two choices of the harmonic oscillator parameter, i.e., $\hbar\omega = 7$ and 11 MeV. We note that even for a relatively light nucleus like ^{48}Ca the A term is more important.

In all cases the nuclear matrix element is a decreasing function of M_σ even if we start with values of M_σ as low as 0.25 GeV/c^2. As expected the dependence on M_σ is much more dramatic when the nucleon form factors are neglected, i.e., $M_A = \infty$. This clearly happens, since for large M_σ the transition operator is characterized by very short range $\lambda_\sigma = \hbar c / M_\sigma c^2 \ll 1$ fm. It thus appears that the double β decay will be too slow to be observed for $M_\sigma \geq 2$ GeV/c^2.

The inclusion of nucleon form factors somewhat cures this pathology since for $M_\sigma \geq 1$ GeV/c^2 the range of the transition operator is set by the nucleon size, i.e., $\lambda_A = \hbar c / M_A c^2 \approx 0.2$ fm. The appearance in this case of short range correlations is not surprising since such effects were found even in the case of operator Ω_c. For $M_\sigma \gg M_A$ the rate roughly falls as $1/M_\sigma^4$. Even for M_σ as high as 10 GeV/c^2 the nuclear matrix elements do not become much smaller than previously expected in the framework of the Primakoff-Rosen approximation. However, even for $M_A = 0.85$ GeV/c^2 the no-neutrino double β decay will become undetectable if the Majorana mass is much larger than $M_w = 80$ GeV/c^2 as in grand unification schemes.[23] In ordinary gauge theories, though, there is no reason for M_σ to be greater than M_w.

Before proceeding further with the extraction of the parameters β^2 and η we state the important conclusion of this work. The no-neutrino and the two-neutrino nuclear matrix elements are not equal. They are not even simply related. Thus a relationship of the kind of Eq. (69) does not hold.

TABLE II. The nuclear matrix elements $|ME|_A{}^2 = \langle f | \Omega_A | i \rangle^2$ and $|ME|_B{}^2 = \langle f | \Omega_B | i \rangle^2$ for the $A=48$ system presented as a function of M_σ and r_c. A dipole nucleon form factor with $M_A = 0.85$ GeV/c^2 was employed. $\hbar\omega = 7$ MeV was used for the Harmonic oscillator parameter.

	M_σ (GeV/c^2) \ r_c (fm)	0.000	0.125	0.250	0.375	0.500	0.625
$\|ME\|_B{}^2$	0.25	0.270×10^{-1}	0.256×10^{-1}	0.180×10^{-1}	0.676×10^{-2}	0.261×10^{-3}	0.698×10^{-3}
	0.5	0.104×10^{-2}	0.128×10^{-2}	0.297×10^{-2}	0.297×10^{-2}	0.724×10^{-1}	0.546×10^{-2}
	1.0	0.680×10^{-3}	0.727×10^{-3}	0.133×10^{-2}	0.215×10^{-2}	0.237×10^{-2}	0.106×10^{-2}
	2.0	0.689×10^{-4}	0.806×10^{-3}	0.136×10^{-3}	0.190×10^{-3}	0.168×10^{-3}	0.633×10^{-4}
	5.0	0.202×10^{-5}	0.239×10^{-5}	0.400×10^{-5}	0.518×10^{-5}	0.422×10^{-5}	0.163×10^{-5}
	10	0.128×10^{-6}	0.153×10^{-6}	0.255×10^{-6}	0.327×10^{-6}	0.262×10^{-6}	0.820×10^{-7}
$\|ME\|_A{}^2$	0.25	1.79	1.79	1.30	0.635	0.128	0.290×10^{-2}
	0.5	0.167×10^{-1}	0.233×10^{-1}	0.778×10^{-1}	0.238	0.446	0.537
	1.0	0.244×10^{-1}	0.286×10^{-1}	0.518×10^{-1}	0.886×10^{-1}	0.102	0.770×10^{-1}
	2.0	0.271×10^{-2}	0.321×10^{-2}	0.565×10^{-2}	0.823×10^{-2}	0.758×10^{-2}	0.455×10^{-2}
	5.0	0.814×10^{-4}	0.972×10^{-4}	0.229×10^{-3}	0.229×10^{-3}	0.193×10^{-3}	0.107×10^{-3}
	10	0.519×10^{-5}	0.625×10^{5}	0.108×10^{-4}	0.144×10^{-4}	0.119×10^{-4}	0.657×10^{-5}

TABLE III. The same quantities as in Table II except that the nucleon form factor is neglected here ($M_A=\infty$). The dash indicates quantities smaller than 10^{-40}.

	M_σ (GeV/c^2) \ r_c (fm)	0.000	0.125	0.250	0.375	0.500	0.625
$\|ME\|_B^2$	0.5	0.311	0.307	0.271	0.199	0.119	0.350×10^{-1}
	1.0	0.383×10^{-1}	0.311×10^{-1}	0.194×10^{-1}	0.649×10^{-1}	0.148×10^{-2}	0.217×10^{-3}
	2.0	0.293×10^{-2}	0.159×10^{-2}	0.164×10^{-3}	0.633×10^{-5}	0.139×10^{-6}	0.413×10^{-8}
	5.0	0.793×10^{-4}	0.155×10^{-5}	0.122×10^{-9}	0.304×10^{-19}	0.376×10^{-19}	0.203×10^{-23}
	10	0.500×10^{-5}	0.115×10^{-11}	0.370×10^{-31}	0.310×10^{-31}	—	—
	80	0.122×10^{-9}	—	—	—	—	—
$\|ME\|_A^2$	0.5	13.9	13.7	12.2	8.94	5.39	2.70
	1.0	1.83	1.67	0.945	0.322	0.756×10^{-1}	0.134×10^{-1}
	2.0	0.143	0.788×10^{-1}	0.831×10^{-2}	0.333×10^{-3}	0.770×10^{-5}	0.124×10^{-6}
	5.0	0.392×10^{-2}	0.577×10^{-4}	0.628×10^{-8}	0.162×10^{-12}	0.213×10^{-18}	0.185×10^{-22}
	10	0.248×10^{-3}	0.432×10^{-9}	0.189×10^{-18}	0.163×10^{-28}	—	—
	80	0.674×10^{-8}	—	—	—	—	—

The parameters β^2 and η can perhaps be determined from an understanding of the Higgs mechanism or a phenomenology of the gauge models but they cannot be extracted from the double β-decay data in a way which is independent of the nuclear structure.

From the numerical results presented in Tables I–IV and Eqs. (65) and (66) one can compute the no-neutrino half-lives in terms of β^2 or η for ^{48}Ca and compare them with the experimental ones. In the context of the old theory using[11] $\epsilon_0=8.4$ we get

$$\eta^2(T_{1/2})_{0\nu}=2.2\times10^{10},\ 2.0\times10^{11},\ 1.6\times10^{12},\ 1.8\times10^{12},\ 4.4\times10^{10},\ \text{and}\ 6.0\times10^{13}. \quad (90)$$

for the various nuclear matrix elements in the order appearing in the first row of Table I.

In the framework of the present gauge model for the most realistic choice of the parameters, i.e., $\hbar\omega=11$ MeV, $M_A=0.85$ GeV/c^2, and $r_c=0.4$ fm, we get

$$\beta^4(T_{1/2})_{0\nu}=1.3\times10^{10},\ 9.4\times10^{11},\ 4.1\times10^{11},\ 3.9\times10^{12},\ 1.1\times10^{14},\ 1.8\times10^{15}\ \text{yr} \quad (91)$$

for $M_\sigma=0.25$, 0.5, 1.0, 2.0, 5.0, 10 GeV/c^2, respectively. For illustration purposes we also present results for $r_c=0.0$ fm:

$$\beta^4(T_{1/2})_{0\nu}=5.6\times10^{9},\ 2.5\times10^{11},\ 7.4\times10^{12},\ 2.2\times10^{13},\ 6.0\times10^{14},\ 9.2\times10^{14}\ \text{yr}. \quad (92)$$

For the two-neutrino double β decay we obtain[13]

$$(T_{1/2})_{2\nu}=\frac{f_4}{|(ME)_4|^2},\quad f_4=8.0\times10^{18}\ \text{yr}. \quad (93)$$

In the simple shell model description we obtain

TABLE IV. The same data as in Table II except that now the harmonic oscillator parameter is chosen to be $\hbar\omega=11$ MeV.

	M_σ \ r_c	0.000	0.125	0.250	0.375	0.500	0.625
$\|ME\|_B^2$	0.25	0.181	0.174	0.131	0.633×10^{-1}	0.144×10^{-1}	0.446×10^{-2}
	0.5	0.240×10^{-2}	0.169×10^{-2}	0.685×10^{-2}	0.402×10^{-2}	0.126×10^{-1}	0.265×10^{-2}
	1.0	0.420×10^{-3}	0.609×10^{-3}	0.186×10^{-3}	0.401×10^{-3}	0.446×10^{-2}	0.121×10^{-2}
	2.0	0.874×10^{-4}	0.118×10^{-3}	0.275×10^{-3}	0.434×10^{-3}	0.359×10^{-3}	0.802×10^{-4}
	5.0	0.300×10^{-5}	0.405×10^{-5}	0.885×10^{-5}	0.124×10^{-4}	0.927×10^{-4}	0.234×10^{-5}
	10	0.195×10^{-6}	0.264×10^{-6}	0.571×10^{-6}	0.786×10^{-6}	0.578×10^{-6}	0.935×10^{-6}
$\|ME\|_A^2$	0.25	10.7	10.3	8.17	4.64	1.70	0.317
	0.5	0.243	0.196	0.310×10^{-1}	0.625×10^{-1}	0.361	0.518
	1.0	0.802×10^{-2}	0.135×10^{-1}	0.579×10^{-1}	0.147	0.181	0.124
	2.0	0.266×10^{-2}	0.376×10^{-2}	0.102×10^{-1}	0.156×10^{-1}	0.156×10^{-1}	0.820×10^{-2}
	5.0	0.986×10^{-4}	0.138×10^{-3}	0.339×10^{-3}	0.518×10^{-3}	0.415×10^{-3}	0.197×10^{-3}
	10	0.648×10^{-5}	0.909×10^{-5}	0.220×10^{-4}	0.329×10^{-4}	0.259×10^{-4}	0.122×10^{-4}

$$|(ME)_4|^2 = |\langle f|\Omega_0|i\rangle|^2 = \begin{cases} 5.5\times10^{-3} & r_c = 0.0 \text{ fm}, \\ 7.4\times10^{-3} & r_c = 0.4 \text{ fm}. \end{cases} \tag{94}$$

Hence

$$(T_{1/2})_{2\nu} = \begin{cases} 1.4\times10^{21} \text{ yr} & r_c = 0.0 \text{ fm}, \\ 1.1\times10^{21} \text{ yr} & r_c = 0.4 \text{ fm}. \end{cases} \tag{95}$$

For such long lifetimes the experimental search for double β decay in the laboratory becomes difficult. Thus even the most refined recent experiments[9] could only set lower limits on $T_{1/2}$, i.e.,

$$\{(T_{1/2})_{0\nu}\}_{\text{exp}} \geq 2\times10^{21} \text{ yr}, \quad \{(T_{1/2})_{2\nu}\}_{\text{exp}} \geq 3.6\times10^{19} \text{ yr} \tag{96}$$

Thus we see that the predicted lifetime for two neutrino double β decay is 1000 longer than the present experimental lower limit.

Combining Eqs. (90) or (91) and (96) we obtain

$$\eta \text{ or } \beta^2 = \left[\frac{(T_{1/2})_{0\nu}}{\{(T_{1/2})_{0\nu}\}_{\text{exp}}}\right]^{1/2} \leq \left[\frac{(T_{1/2})_{0\nu}}{2\times10^{21}}\right]^{1/2} \tag{97}$$

thus we have

$\eta \leq 3.3\times10^{-6}$, 1.1×10^{-5}, 1.0×10^{-4}, 5.5×10^{-4}, 6.8×10^{-4}

[in the same order as in Eq. (90)]

or

$\beta^2 \leq 2.5\times10^{-6}$, 2.2×10^{-5}, 1.4×10^{-5}, 4.1×10^{-5}, 2.2×10^{-4}, 9.5×10^{-4}

[in the order of Eq. (91)].

From the above results we see that the most realistic value of η, associated with the calculation that does not ignore the presence of the tensor force, is quite a bit smaller than the previously adopted value of 0.5×10^{-4}. The extracted values of β^2 are also quite a bit smaller than those obtained in Sec. VII. Obviously some of the approximations made in Sec. VII are unrealistic. We note, however, that the lower experimental limit on $(T_{1/2})_{0\nu}$ is consistent with values of $\beta^2 \approx 10^{-3}$ but with Majorana mass $m_\sigma \approx 10$ GeV/c^2, i.e., quite a bit heavier than that assumed in the earlier work. Needless to say, however, that on the basis of present experimental evidence the values of the parameters η and β^2 quoted above are merely limits.

IX. CONCLUSIONS

In the present paper we investigated double β decay in the context of modern gauge theories which can accommodate right handed Majorana neutrinos. The formalism has been applied in the case of the ^{48}Ca $\to$ ^{48}Ti double β decay. We find that the transition rates depend crucially on the mass M_σ of the Majorana neutrino which was treated as a free parameter. From the lower experimental limit of the neutrinoless double β-decay lifetime, upper limits for the lepton violating parameter β were obtained, i.e., $\beta^2 \leq 2.5\times10^{-6}$, 2.2×10^{-5}, 1.4×10^{-5}, 4.1×10^{-5}, 2.3×10^{-4}, and 9.5×10^{-4} for $M_\sigma = 0.25$, 0.5, 1.0, 2.0, 5.0, and 10 GeV/c^2. For $m_\sigma \cong M_\omega = 80$ GeV/c^2 we find $\beta^2 \leq 0.06$. These values are somewhat smaller than those obtained in the semiphenomenological global treatment of Sec. VII as well as in the earlier work.[16] It does not seem likely that one will be able to extract β^2 from the double β-decay data in a way which is independent of the nuclear physics.

We also studied the various approximations adopted in the earlier theoretical investigations of double β decay. We find that two of these approximations are not accurate, i.e.,

(i) The radial part of the transition operator cannot be treated in an average fashion (Primakoff-Rosen approximation).

(ii) The tensor contribution to the transition is very important and it should not be neglected. In the ^{48}Ca $\to$ ^{48}Ti transition investigated in the present work it is the most dominant.

As a consequence of (i) and (ii) above, if the dominant mode of the neutrinoless double β decay indeed proceeds via a mixing of left-handed and right-handed currents, the nuclear matrix elements are predicted to be quite a bit larger than previously expected. Thus the transition rates are expected to be larger provided that the value of η remains the same, i.e., $\sim 0.5\times10^{-4}$. It has, however, recently been estimated by Mohapatra and Senjanović[25] that the value of η is much smaller then previously believed, i.e., $\eta \lesssim 10^{-7}$.

It is the role of the experiments to determine η once the nuclear matrix elements are known. If the value of η turns out to be this small the Majorana neutrino mechanism discussed in this work may be the most dominant. The transition rates are in this case predicted to be high enough to be detectable only if $M_\sigma < M_w{}^2 = 80$ GeV/c^2.

Double β-decay experiments are very important in settling very fundamental questions of physics. Lepton nonconservation is linked to very fashionable current ideas of grand unification schemes which respect neither lepton nor baryon number. Observation of double β decay will also rule out all theories[24] that predict superheavy Majorana neutrinos, e.g., 10^4 GeV. Taking the pessimistic point of view, i.e., if such neutrinos are indeed so heavy, the neutrinoless double β-decay experiments may be doomed to failure, unless such a process proceeds via a yet unknown mechanism.

In this respect the recent experiments of Moe and Lowenthal[13] in the case of ^{82}Se → ^{82}Kr are extremely interesting even if the lifetime they measure is attributed to the two-neutrino mode. Hopefully exciting no-neutrino measurements will come soon. In the meantime realistic shell model calculations for the admittedly complicated (theoretically) nuclear systems of experimental interest are very much desired. They will aid in the analysis of the data and facilitate the extraction of the lepton violating parameters from them.

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