PHYSICAL REVIEW C VOLUME 24, NUMBER 2 AUGUST 1981

Separable potential model for the off-shell πN amplitude

Charles Coronis and Rubin H. Landau
Department of Physics, Oregon State University, Corvallis, Oregon 97331
(Received 19 June 1980)

A simple model is used to reproduce modern πN phase shifts in the S, P, D, and F waves for $T_\pi < 2$ GeV (except P_{11}). In these channels a rank one, two-channel separable potential model is used and the inverse scattering problem is solved directly. In the P_{11} channel the phase shifts are fit with a separable t matrix which contains a pole at the nucleon mass and an explicit sign change in the phase shift. This model is intended for use in theories of the low and medium energy pion-nucleus interaction where it supersedes the obsolete LT and LMM potentials. The explicit pole in the P_{11} channel should facilitate the inclusion of pion annihilation into the many body problem.

[NUCLEAR REACTIONS $\pi^{\pm}N$; 0–2 GeV phase shifts reproduced; S, P, D, F waves; separable potential with inverse scattering problem; nucleon pole and sign change in P_{11}.]

I. INTRODUCTION

Most theoretical descriptions of the pion-nucleus interaction are based on some form of multiple scattering theory. This theory in turn originates from the assumption that the pion-nucleon (πN) interaction can be described by a potential. Consequently it should not be surprising that a great deal of effort[1-10] has been expended trying to find potential-like models of the pion-nucleon interaction. In this way—at the least—it is possible to obtain consistency in the two- and many-body problems and to include unitarity[11] into the theory.

Although these two-body, potential descriptions are not as fundamental as field theories, they have proved to be quite capable of reproducing elastic data over a wide range of energies and are equivalent to the interaction produced by heavy meson exchange.[10] While this does not imply any particular uniqueness to the solution found, it does increase our confidence that it is a useful tool in generating an off-energy shell πN t matrix with analytic behavior approximating that of a field theory—except possibly in the P wave. To construct a P wave interaction consistent with field theory and the many body problem including pion annihilation, it is necessary to have a t matrix which includes a subthreshold pole at the nucleon mass.[7,12,13] Yet the existence of this pole permits the repeated emission and annihilation of a pion, which in turn produces a strongly energy-dependent t matrix, which in turn is not similar to that produced by a separable potential model.[10]

In practice, when these separable πN potential models are used in an optical potential to generate elastic scattering from many nucleon systems (the LPT potential[14]) there is a comfortingly low sensitivity to the details of the two-body potential.[15] In part, this insensitivity of pion-nucleus studies to the πN potentials arises from the relatively long ranges of these potentials ($R \simeq 0.7$ fm) producing rapid cutoffs in momentum space. In contrast, the popular $\vec{r}$-space pion-nucleus potentials (Kisslinger, local Laplacian) are derived from zero-range hypotheses for the πN interaction and do produce results significantly different from each other and from the LPT potential.

As important as the range of the πN potential may be in determining the properties of the lowest order pion-nucleus interaction, it now appears to be crucial in determining the importance of higher order processes. For example, the effectiveness of correlations within the nucleus in reducing multiple scattering (ξ)—and thus the importance of the Ericson-Ericson Lorentz-Lorenz effect—is greatly reduced ($\xi \simeq 0.2$) by a finite range πN force ($R \simeq 0.6$ fm).[16]

The question of the range of the πN interaction, and the related analytic structure of $t^{\pi N}$, has been a difficult one to answer with confidence since field theory and potential theory appear to give quite different results (even the different field theoretic models appear to disagree with each other at times). Specifically, the field theoretic fits to the πN interaction have found[10,11,17,18] that the elementary vertex (cutoff) function or "form factor" $v(p)$ corresponds to a much shorter range, $R \simeq 0.2$ fm, than found for the separable potential ($R \simeq 0.7$ fm). Ernst and Johnson[17] and Miller[18] explain this difference by noting that the field theory amplitudes contain the combination $v(p)^2/\omega_\pi(p)^2$. The $\omega_\pi(p)^2 (=m_\pi{}^2 + p^2)$ thus acts as an extra cutoff and permits the vertex function $v(p)$ to have a longer tail in $\vec{p}$ space (shorter range in $\vec{r}$ space) than the potential function. A related explanation for the different sizes of the potential theory and field theory form factors is given by Myhrer and Thomas.[19] They argue that the *po-*

tential function itself should only be associated with the left hand (crossing) cut of a Chew-Low type t matrix, and that the Fredholm determinant calculated from the potential should be associated with the right hand (elastic) cut. The appropriate *parts* of the t matrices produced by both descriptions will then be consistent with each other.

Since there exists the above connection between separable potential models and field theory, an examination of recent developments in the latter may be illuminating for the former. In particular, the original Chew-Low model field theory generated the πN P_{33} resonance only for a static nucleon and then, by using an arbitrary cutoff function. Johnson and Ernst *et al.*,[10,17,20] in an extensive number of calculations, have indicated the extensions needed to permit this model to reproduce actual scattering data. In particular, it has been possible to apply the inverse problem with inelastic channel coupling to determine the cutoff function.[20]

In the most recent work of Ernst and Johnson,[10] a nonstatic model is used which (in all channels except P wave) is equivalent to a separable potential model including coupling to the inelastic channels. The amplitudes in the P wave channels are similar to those of Chew-Low theory (they all have a nucleon pole term) with a special, rank two N/D model used for the P_{11} channel. Very good fits to data are obtained. The P wave part of that model is very similar to the work of Liu and Shakin[8] who also include a clever (but rather complicated) separation of background and pole terms. Although none of these models attempt to account for the crossing cut explicitly, they—as do the potential models—implicitly approximate the left hand structure by fitting the theory to data.

Although most investigators have considered the Chew-Low picture as the microscopic essence of the P wave interaction, some very recent work by Wei and Banerjee,[21] calls into question the validity of the simple picture of the interaction as being generated by the iterated annihilation and emmission of pions. In particular, the inclusion of nucleon recoil terms caused sufficient reduction of rescattering to remove the P_{33} resonance. Only after including higher order graphs—of the types used by Cammarata and Banerjee[22] for their theory of the S wave—is the P_{33} resonance restored. Although not directly applicable to the potential-based models, these results may permit an improved and possibly more correct fit to the P waves within a field theory model, or possibly a different phenomenological fit.

In our paper, a simple separable potential model of the πN interaction is presented. It is an extension of the work done earlier by Landau and Tabakin (LT) (Ref. 2) and Londergan, McVoy, and Moniz (LMM) (Ref. 3) on the inverse scattering problem.[23] In particular, we use the LMM rank one, two channel, separable potential model for all S, P, D, and F channels except the P_{11}. In the P_{11} channel we explicitly include the subthreshold nucleon pole and a low energy sign change in the phase shift. In contradistinction to the LT and LMM potentials which are not valid at lower energies (they used the same obsolete low energy phases), the present potentials use modern fits to the low energy[24] and medium energy[25] phases. Since it has been shown a number of times that low energy pion-nucleus scattering is very sensitive to the input πN phases,[26,27] this is a substantial and needed update. As indicated in Sec. IV, we are able to fit the data over a relatively large range of energies ($0 \lesssim T_\pi \lesssim 2$ GeV).

Other similar works on the πN interaction have been published since the introduction of the LT and LMM potentials. In particular, Schwarz, Zingl, and Mathelitsch[5] have fit the S and P waves up to 250 MeV with completely analytic parametrizations within the framework of the Blankenbecler-Sugar formalism. We differ by our direct "inversion" of the phase shifts by fitting the data up to ~2 GeV and by using a Lippmann-Schwinger equation with relativistic kinematics.

The basic physical similarity of all these models is their separability and finite range for the πN interactions; their difference is in the degree of analyticity, the degree of off-shell unitarity, and the assumed dynamics for the P waves. In all cases it is beneficial to have several models (all of which fit πN data) available when performing a pion-nucleus calculation. For example, we are comparing several of them in our present calculation of pion-nucleus scattering lengths.[28] Unfortunately, the quark bag models—while very encouraging[29]—are not yet at the stage where they can produce the long range πN interaction or be incorporated into a many body theory. We hope enough progress will occur in that area to make the present works obsolete in the near future.

II. THE INVERSE SCATTERING PROBLEM

For completeness and clarity, in this section we outline the solution of the inverse scattering problem for the single channel and multichannel models. Details of the solution and original references can be found in Refs. 30, 2, and 3, with extensive discussion of the applicability and existence of solutions given in Refs. 11 and 23.

A. Single channel model

We wish to solve the Lippmann-Schwinger equation with relativistic kinematics,

$$T(\vec{k}',\vec{k};E^+) = V(\vec{k}',\vec{k}) + \int \frac{d^3p\, V(\vec{k}',\vec{p})T(\vec{p},\vec{k};E)}{E - E(p) + i\epsilon}, \tag{1}$$

where

$$E(p) = E_\pi(p) + E_N(p) = (m_\pi^2 + p^2)^{1/2} + (m_N^2 + p^2)^{1/2} \tag{2}$$

is the two-body energy in the πN center of mass. If T and V are decomposed into spin and isospin states labeled α, Eq. (1) reduces to the one dimensional equation

$$T_\alpha(k',k;E) = V_\alpha(k',k) + \frac{2}{\pi}\int_0^\infty \frac{dp\, p^2 V_\alpha(k',p)T_\alpha(p,k;E)}{E - E(p) + i\epsilon}. \tag{3}$$

If the potential $V_\alpha(k',k)$ is separable,

$$V_\alpha(k',k) = \sigma_\alpha g_\alpha(k')g_\alpha(k) \tag{4}$$

($\sigma_\alpha = \pm 1$ for repulsion/attraction), the off-energy-shell t matrix can be determined algebraically,

$$T_\alpha(k',k;E^+) = \sigma_\alpha g_\alpha(k')g_\alpha(k) \Big/ \left[1 - \frac{2\sigma_\alpha}{\pi}\int_0^\infty \frac{dp\, p^2 g_\alpha^2(p)}{E^+ - E(p)}\right]. \tag{5}$$

Given the on-shell t matrix expressed in terms of the *complex* phase shift $\gamma_\alpha(k)$,

$$T_\alpha[k,k,E(k)] = -\exp[i\gamma_\alpha(k)]\sin\gamma_\alpha(k)/2\mu(k)k, \tag{6a}$$

$$\mu(k) = E_\pi(k)E_N(k)/[E_\pi(k) + E_N(k)], \tag{6b}$$

and the phase shifts for all energies, it is possible to exactly invert Eqs. (5) and (6) and solve for the "potential" or form factor,[2,30]

$$g_\alpha^2(k) = -\sigma_\alpha \exp[-\Delta_\alpha(k)]\sin\gamma_\alpha(k)/2\mu(k)k, \tag{7a}$$

$$\Delta_\alpha(k) = \frac{1}{\pi}\,\mathrm{P}\int_{m_\pi+m_N}^\infty \frac{d\omega\,\gamma_\alpha[k(\omega)]}{\omega - E(k)}. \tag{7b}$$

The theoretical difficulty with this procedure, which led LMM to generalize it, is that because $\gamma_\alpha(k)$ is complex for energies above pion production threshold, Δ_α in Eq. (7b) is always complex, and hence so is $g_\alpha(p)$. In addition to this small—yet still unphysical—imaginary part below any inelastic thresholds, the strong *energy* dependence in the complex phase shift $\gamma_\alpha(\omega)$, which occurs when a new channel opens, gets inverted into a rapid *momentum* dependence of the potential via Eq. (7). Yet since these new channels contribute to unitarity, their effect should appear in the energy variable of the t matrix and not the momentum variable. Physically, the rapid momentum dependence (peaks and valleys) so introduced may transform into an unphysically long-ranged πN potential in coordinate space.

B. Two channel model

If a formal second channel is introduced into the scattering problem, LMM (Ref. 3) showed that the strong energy dependence of the phase shift arising from the opening of new channels can be described by a strong *energy* dependence of the potential strength, $\sigma_\alpha \to \sigma_\alpha \xi_\alpha(E)$. This procedure should leave the potential in the elastic channel quite smooth (and therefore of shorter range in r space). However, since these elastic potentials are now pure real, it is not possible to describe the imaginary part of the phase shifts with this one function. The energy dependent coupling constant $\xi_\alpha(E)$ must be simultaneously given.

The effective, energy-dependent one channel potential is thus

$$V_\alpha(k',k;E) = \sigma_\alpha \xi_\alpha(E) v_\alpha(k')v_\alpha(k), \tag{8}$$

which yields the algebraic solution

$$T_\alpha(k',k;E) = v_\alpha(k)v_\alpha(k')/D_\alpha(E), \tag{9a}$$

$$D_\alpha(E) = 1/\sigma_\alpha\xi_\alpha(k) - \frac{2}{\pi}\int_0^\infty \frac{dp\, p^2 v_\alpha^2(p)}{E^+ - E(p)}. \tag{9b}$$

One now defines a new "elastic" or pseudophase $\hat{\delta}_\alpha(p)$ by

$$f_\alpha(p) = [\eta_\alpha(p)e^{2i\delta_\alpha(p)} - 1]/2ip \equiv \left[\frac{1}{\xi_\alpha(p)}\right](e^{2i\hat{\delta}_\alpha(p)} - 1) \equiv \hat{f}_\alpha(p)/\xi_\alpha(p) \equiv \hat{\eta}_\alpha(p)\hat{f}_\alpha(p), \tag{10}$$

or more explicitly,

$$\tan\hat{\delta}_\alpha = \tan\delta_\alpha\left(1 + \frac{1-\eta_\alpha}{2\eta_\alpha \sin^2\delta_\alpha}\right), \tag{11}$$

$$\xi_\alpha(E) = \frac{2[1 - \eta_\alpha(E)\cos 2\delta_\alpha(E)]}{1 + \eta_\alpha^2 - 2\eta_\alpha\cos 2\delta_\alpha}. \tag{12}$$

Note that below the first inelastic threshold, $\eta_\alpha = 1$, $\hat{\delta}_\alpha = \delta_\alpha$, $\xi_\alpha(E) = 1$, and $\hat{f}_\alpha = f_\alpha$. By writing a dispersion relation for $1/\xi_\alpha(E)$, LMM show that it is possible to apply the inverse scattering procedure, Eq. (7), to the pseudoscattering amplitude $\hat{f}_\alpha$ and obtain the pseudopotential $g_\alpha(p)$ and the "actual" potential $v_\alpha(p)$:

$$g_\alpha{}^2(k) = -\sigma_\alpha \exp[-\hat{\Delta}_\alpha(k)]\sin\hat{\delta}_\alpha(k)/2\mu(k)k\,, \qquad (13)$$

$$v_\alpha(k) = g_\alpha(k)/[\xi_\alpha(k)]^{1/2}\,, \qquad (14)$$

where the pure real, "pseudo" phase shifts $\hat{\delta}_\alpha$ are now used to calculate pure real $\hat{\Delta}_\alpha(k)$ and consequently pure real $g_\alpha(k)$.

The on-shell $\hat{t}$ matrix is now related to the pseudophases $\hat{\delta}$ by the usual relation

$$\hat{T}_\alpha(k,k;E) = \frac{g_\alpha{}^2(k)}{1/\sigma_\alpha - \frac{2}{\pi}\int_0^\infty \frac{dp\,p^2 g_\alpha{}^2(p)}{E^+ - E(p)}} = \frac{-1}{2k\mu(k)}\exp[i\hat{\delta}_\alpha(k)]\sin\hat{\delta}_\alpha(k)\,. \qquad (15)$$

The full off-shell t matrix then requires a tabulation of both of these $g_\alpha(k)$'s, Eq. (13), [or $v_\alpha(k)$] *and* the $\xi_\alpha(E)$, Eq. (12),

$$T_\alpha(k',k;E) = v_\alpha(k')v_\alpha(k)/\hat{D}_\alpha(E)\,, \qquad (16a)$$

$$\hat{D}_\alpha(E) = 1/\sigma_\alpha - \frac{2}{\pi}\int_0^\infty \frac{dp\,p^2 g_\alpha{}^2(p)}{E^+ - E(p)}\,. \qquad (16b)$$

It is important to observe in Eqs. (16) that the "actual" potential $v_\alpha(k)$ appears in the numerator, but the pseudopotential $g_\alpha(k)$ appears in the denominator. [The two are simply related by Eq. (14).] Note that *above* the elastic threshold, Eqs. (16) relate the off-to-on shell to matrices by

$$T_\alpha(k',k;E) = T_\alpha[k_0,k_0;E(k_0)]v_\alpha(k')v_\alpha(k)/v_\alpha{}^2(k_0) = \left[\frac{\xi_\alpha(k_0)^2}{\xi_\alpha(k')\xi_\alpha(k)}\right]^{1/2} T_\alpha[k_0,k_0;E(k_0)]g_\alpha(k')g_\alpha(k)/g_\alpha{}^2(k_0)\,, \qquad (17)$$

whereas below threshold the Fredholm determinant $\hat{D}_\alpha(E)$, Eq. (16b), must be explicitly calculated.

III. THE P_{11} CHANNEL

As indicated in Sec. I, a simple potential model is no longer useful for the P_{11} channel since we want to have an explicit pole in $t^{\pi N}$ at the nucleon mass to account for πN annihilation and emission, and to incorporate π annihilation into a many body theory. We do this by assuming Eq. (16a) is still valid, but that the Fredholm determinant $\hat{D}_\alpha$ has a zero at $E = m_N$.[5] This is then equivalent to the use of an energy-dependent coupling constant $\sigma_\alpha \to \xi_\alpha(E)\sigma_\alpha$. In addition, we explicitly account for the sign change (zero) in the P_{11} phase shift at $E_0 = 1210.7$ MeV by requiring $\hat{D}_\alpha$ to have a pole at E_0. If these two requirements

$$\hat{D}(m_N) = 0\,, \quad \hat{D}(E_0) = \infty\,, \qquad (18)$$

are then placed on the Fredholm determinant

$$\hat{D}_\alpha(E) = \frac{1}{\sigma_\alpha \xi_\alpha(E)} - \frac{2}{\pi}\int_0^\infty \frac{dp\,p^2 g_\alpha{}^2(p)}{E^+ - E(p)}\,, \qquad (19)$$

we can solve for $\xi_\alpha(E)$ and rewrite $\hat{D}(E)$,

$$\hat{D}_\alpha(E) = \frac{1}{\sigma_\alpha}\frac{E - m_N}{E_0 - E} - \frac{E_0 - m_N}{E_0 - E}\frac{2}{\pi}\int_0^\infty \frac{dp\,p^2 g_\alpha{}^2(p)}{E(p) - m_N} - \frac{2}{\pi}\int_0^\infty \frac{dp\,p^2 g_\alpha{}^2(p)}{E^+ - E(p)}\,. \qquad (20)$$

To remain consistent with the work of the previous sections, we then fit the on-shell pseudo t matrix,

$$\hat{T}_\alpha[p,p;E(p)] = g_\alpha{}^2(p)/\hat{D}_\alpha[E(p)] = -\frac{1}{2\mu(k)k}e^{i\hat{\delta}_\alpha(p)}\sin\hat{\delta}_\alpha(p)\,, \qquad (21)$$

to the pseudophase shifts $\hat{\delta}_\alpha(p)$ for the P_{11} channel. Although it is possible to express the t matrix as an inverse problem, we reduced the complications by assuming an analytic form for the pseudopotential,

$$g_\alpha(p) = \frac{\alpha_1 p}{(p^2 + \beta_1{}^2)^2} + \frac{\alpha_2 p^3}{(p^2 + \beta_2{}^2)^3}\,, \qquad (22)$$

and then adjusting the four parameters, α_1, α_2, β_1, and β_2 to obtain a best fit to the phase shifts.

IV. CALCULATIONAL DETAILS

The phase shifts used in the preceding calculations were from three different sources. At low energies ($p \lesssim 250$ MeV/c) the phases were from the Salomon analytic fit[24]; at intermediate energies (250 MeV/$c < p <$ 1000 MeV/c) the Almehed-Lovelace tabulation[25] was used; above 1000 MeV/c the phase shifts were determined by partial-wave analyzing a Regge-pole fit to very high energy πN scattering.[2] The Regge phases $[\eta^R(p), \delta^R(p)]$ for the S_{31}, P_{31}, and P_{13} channels were then slightly adjusted to fit smoothly onto the tabulated phase shifts at $p = 1$ GeV/c.

Since a potential model is only consistent with phase shifts which vanish at infinite energy, the very highest ($p > K = 2$ GeV/c) Regge phases were damped according to the prescription

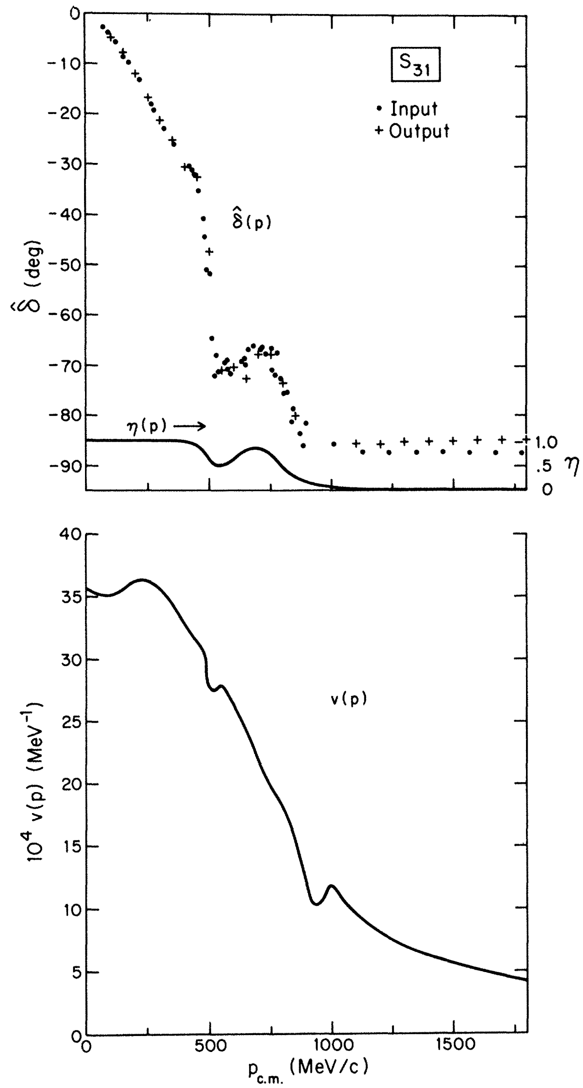

FIG. 1. Upper graph: The πN absorption parameter η and the pseudoscattering phase shift $\hat{\delta}$, Eq. (11), in the S_{31} channel versus pion center-of-mass momentum. The dots are the input values used to calculate the separable potentials $v(p)$; the crosses are the prediction of the model. For $0 \leq p_{\text{c.m.}} \leq 250$ MeV/c the Salomon phases (Ref. 24) are used; for $250 < p_{\text{c.m.}} < 1000$ MeV/c the Almhed-Lovelace phases (Ref. 25) are used; and for $p_{\text{c.m.}} > 1000$ MeV/c phase derived from Regge amplitudes are used (see Sec. V). Lower graph: The actual separable potential $v(p)$ calculated from the above phase shifts by first solving for $g(p)$ via direct inversion, Eqs. (13) and (7), and then determining $v(p)$ by dividing out the energy-dependent coupling constant Eq. (14).

$$\delta(p) = \delta^R(\infty) + \frac{[\delta^R(p) - \delta^R(\infty)]\Lambda}{[\Lambda + (p-K)^{1/2}]}, \tag{23}$$

$$\eta(p) = 1 + \frac{[\eta^R(p) - 1]\Lambda^2}{\Lambda^2 + (p-K)^2}, \tag{24}$$

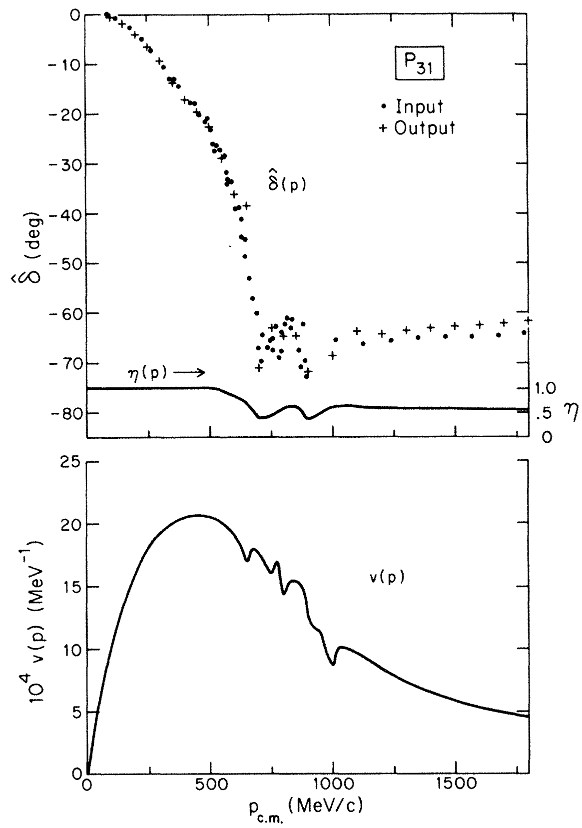

FIG. 2. The same as Fig. 1, except for the P_{31} channel.

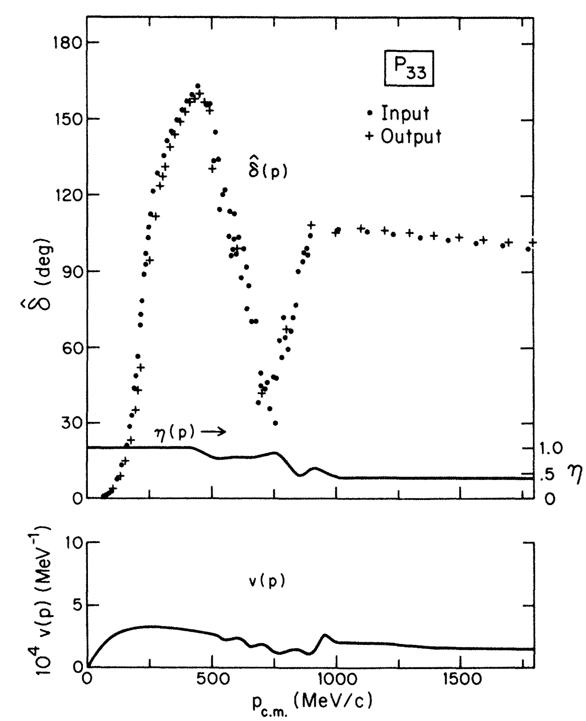

FIG. 3. The same as Fig. 1, except for the P_{33} channel.

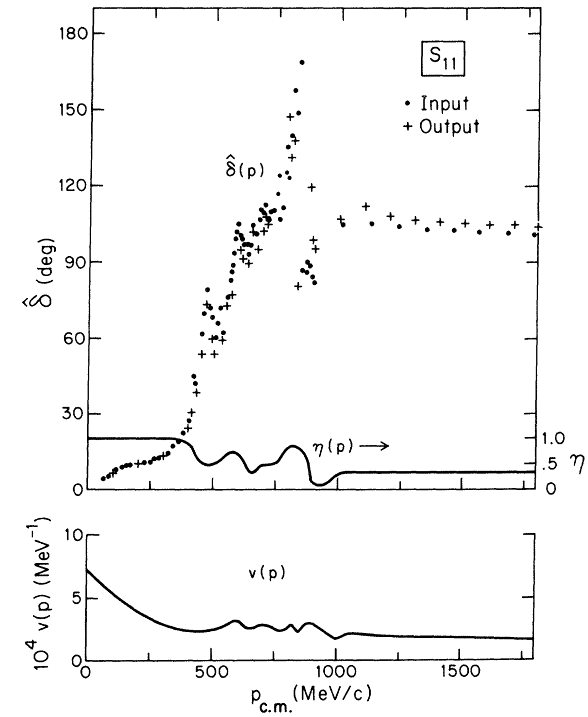

FIG. 4. The same as Fig. 1, except for the S_{11} channel.

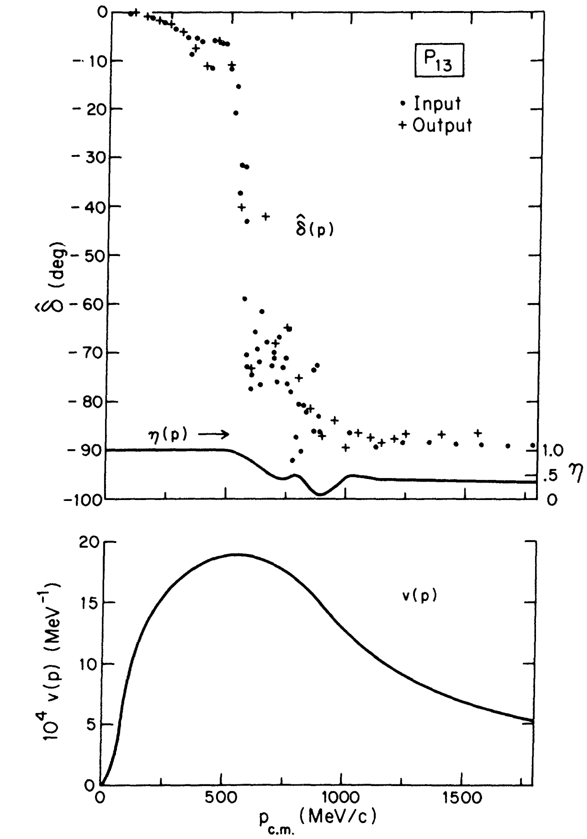

FIG. 5. The same as Fig. 1, except for the P_{13} channel. Some of the statistical noise in the potential function has been reduced to show more clearly the structure of the potential function.

where $\Lambda = 1$ GeV/c (except for P_{33}, where 0.5 GeV/c was used). The above cutoff, and even the use of Regge phases, may appear somewhat arbitrary. It is important to remember, however, that the main effect on the off-shell t matrix will be for $T_\pi \simeq 2$ GeV. Since we are interested in applying this model at much, much lower energies ($T_\pi \lesssim 300$ MeV), there is little practical significance to these cutoffs. Although the assumptions made for the high energy phase shifts will affect the magnitude of the low momentum potentials,[1,31] this is essentially just an additive change in the value of $\Delta_\alpha(k)$ in Eq. (17), and therefore only renormalizes the magnitude of all the $g_\alpha(k)$'s.[1] Since the off-shell behavior involves just ratios, Eq. (17), the renormalization simply cancels.

V. RESULTS

The actual potentials $v_\alpha(p)$ calculated by Eqs. (13) and (14) are displayed in the lower part of Figs. 1–7. The upper parts of these figures display the input pseudophase shifts $\hat{\delta}_\alpha(p)$ and absorption parameters $\eta_\alpha(p)$, along with the output phases obtained by reinversion (see Sec. V for more details about the phase shifts used).

Although in general the potentials calculated with the multichannel method used here are smoother than those obtained with the single channel model of LT,[2] they are not as smooth as the potentials displayed by LMM.[3] In many cases a fine structure is introduced into the $v(p)$'s when the smooth $g(p)$'s obtained by inversion are converted to $v(p)$ [Eq. (14)] since $\xi_\alpha(p)$ contains the rapid variation associated with the opening of new channels. We cannot explain why these structures were not obtained previously. Specifically, we have traced the structures in the P_{31}, P_{33}, S_{11}, and F_{35} channels as being related to those in $\xi_\alpha(p)$.

Since the inversion procedure produces potentials directly from phase shifts, we have checked the potentials by "reinverting" them to calculate the phase shifts via Eq. (15). The results of this reinversion are displayed in the upper parts of Figs. 1–7. It can be seen that the data points calculated in this way (the crosses) correspond rather closely to the input phase shifts, especially at the lower energies. This is rather remarkable considering the noise present in the input.

A potential $v(p)$ and the pseudopotential $g(p)$ for

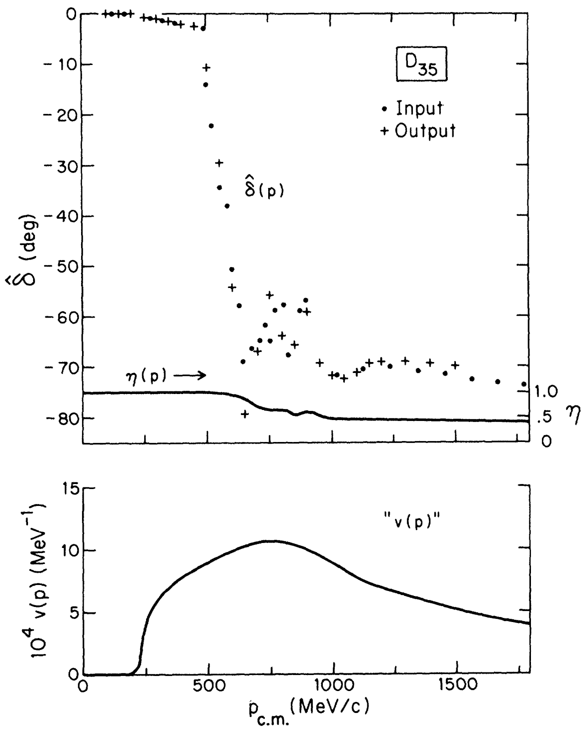

FIG. 6. The same as Fig. 5, except for the D_{35} channel.

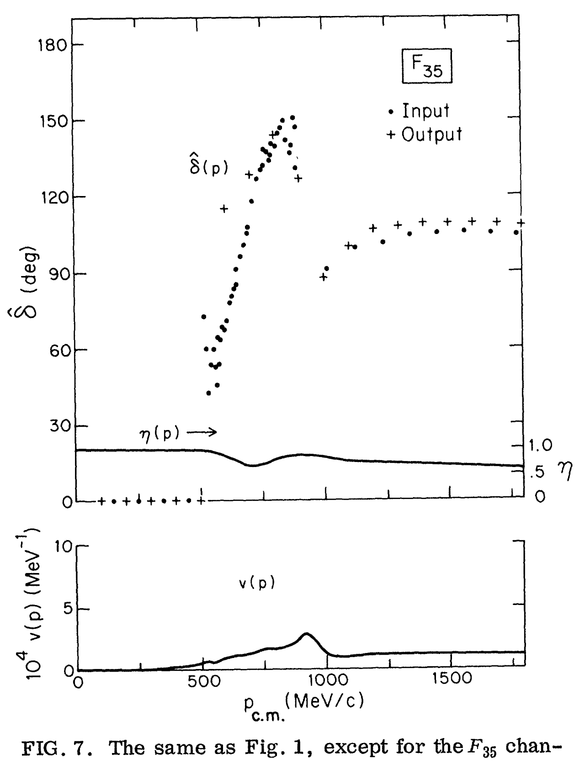

FIG. 7. The same as Fig. 1, except for the F_{35} channel.

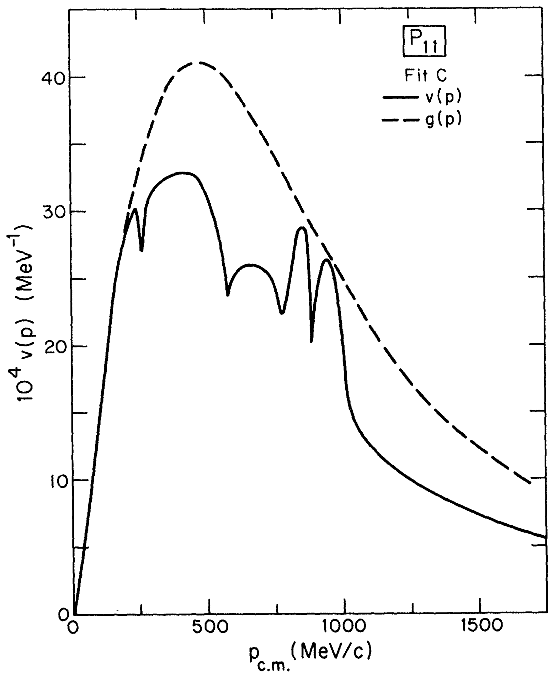

FIG. 8. A separable potential function $v(p)$ and the pseudopotential $g(p)$ for the P_{11} channel obtained by fitting the pseudophase shifts with a t matrix containing a pole at m_N and a sign change.

the P_{11} channel are shown in Fig. 8. As occurred for the inversion problem, the structures in $v(p)$ at $p=250$ and 800 MeV/c are not present in $g(p)$, but rather are introduced by $\xi_\alpha(p)$ via Eq. (14).

The quality of the fit to the pseudophase $\hat{\delta}_\alpha(p)$ in this P_{11} channel is shown in Fig. 9. The fit

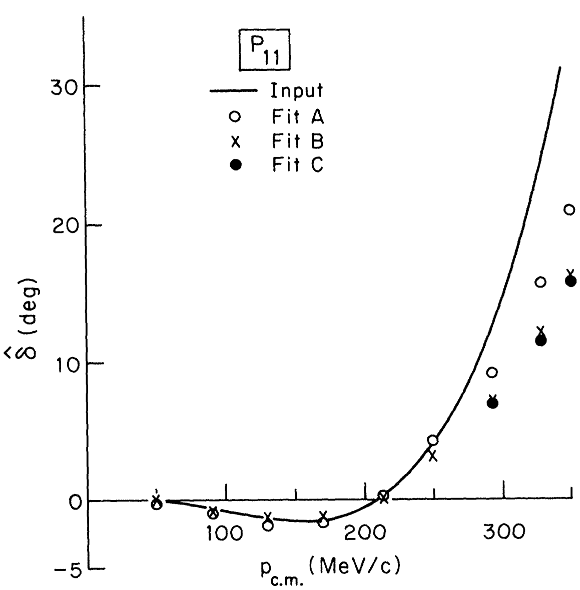

FIG. 9. The pseudophase shifts in the P_{11} channel and the predictions of the model. The different parameters for the three fits are given in Table I.

TABLE I. Parameters of the P_{11} potential.

Fit	α_1 ($\times 10^6$ MeV^2)	β_1 (MeV)	α_2 (MeV^4)	β_2 (MeV)
A	7.01	875.8	-3.341×10^8	1240
B	7.01	947.0	-3.341×10^8	1240
C	7.01	821.1	-1.370	782.2

is excellent for $0<p<250$ MeV including both the sign change and the subthreshold nucleon pole. However, our simple form for the t matrix, and in particular for its denominator, Eqs. (20)–(22), is not capable of producing a good fit at both lower and higher energies. In particular, our searching has found a number of equally "good fits," with the ones which fit well at low (high) energy not fitting well at high (low) energy. The fits A, B, and C in Table I and Fig. 8 are examples. Although the parameters in fits A and B are very similar, B is better at low energy; in contrast, the parameters of fits B and C appear quite different, yet produce very similar fits.

VI. CONCLUSIONS

We have presented a separable potential model which reproduces the most up-to-date πN phase shifts in all the S, P, D, and F waves for $T_\pi \lesssim 2$ GeV (~0.2 GeV for P_{11}). Its main application will be in low and medium energy studies of the pion-nucleus problem within a multiple scattering formalism. In this way the two- and many-body problem can be treated with the same dynamical equation. The pure real potentials account for the coupling to inelastic channels by use of an energy-dependent coupling constant. In the P_{11} channel, the t matrix explicitly contains the nucleon pole, arising from π annihilation and creation, and a low energy sign change. This facilitates the inclusion of pion annihilation into the many body problem.

Since we directly invert the experimental phase shifts, at very high momentum our potentials contain a fine structure which is related to the opening of inelastic channels. This is true even within the multichannel formulation. At low and medium energies the potentials are smooth and we are currently using them and other models in a study of pion-nucleus scattering lengths.[28] The change in pion-nucleus scattering due to the very different off-shell behavior of the πN P_{11} amplitude is an open and interesting question.

This research was supported in part by the National Science Foundation Grant No. PHY79-090994, the Oregon State University Computer Center, and the Research Corporation.

[1]M. G. Piepho and G. E. Walker, Phys. Rev. C 9, 1352 (1974).
[2]R. Landau and F. Tabakin, Phys. Rev. D 5, 2746 (1972).
[3]J. T. Londergan, K. W. McVoy, and E. J. Moniz, Ann. Phys. (N.Y.) 86, 147 (1974).
[4]W. R. Gibbs, B. F. Gibson, A. T. Hess, G. J. Stephenson, Jr., and W. B. Kaufman, Phys. Rev. C 13, 2433 (1976).
[5]K. Schwarz, H. F. K. Zingl, and L. Mathelitsch, Phys. Lett. 83B, 297 (1979).
[6]P. Desgrolard and T. F. Hammann, Phys. Rev. C 6, 482 (1972).
[7]A. W. Thomas and I. R. Afnan, Phys. Lett. 45B, 437 (1973); I. R. Afnan and A. W. Thomas, Phys. Rev. C 10, 109 (1974); A. W. Thomas, Nucl. Phys. A258, 417 (1976).
[8]L. C. Liu and C. M. Shakin, Phys. Rev. C 18, 604 (1978).
[9]L. L. Foldy and J. A. Lock, Phys. Rev. D 17, 3065 (1978).
[10]D. J. Ernst and M. B. Johnson, Phys. Rev. C 22, 651 (1980).
[11]A. W. Thomas and R. H. Landau, Phys. Rep. 58, 121 (1980) and work cited in that review.
[12]D. S. Koltun and T. Mizutani, Ann. Phys. (N.Y.) 109, 1 (1978); J. L. Ballot and F. Becker, Phys. Rev. 164, 285 (1967); V. B. Mandelzweig, H. Garcilazo and J. M. Eisenberg, Nucl. Phys. A256, 163 (1976); A. S. Rinat, E. Hammel, and A. W. Thomas, Phys. Lett. 80B, 166 (1979); A. S. Rinat, Nucl. Phys. A287, 399 (1977); K. L. Kowalski, E. R. Siciliano, and R. M. Thaler, Phys. Rev. C 19, 1813 (1979).
[13]I. R. Afnan and B. Blankleider, Phys. Rev. C 22, 1638 (1980).
[14]R. H. Landau, S. C. Phatak, and F. Tabakin, Ann. Phys. (N.Y.) 78, 299 (1973).
[15]R. H. Landau, and F. Tabakin, Nucl. Phys. A23, 445 (1974).
[16]G. Fäldt, Nucl. Phys. A206, 176 (1970); J. Eisenberg, J. Hüfner, and E. J. Moniz, Phys. Lett. 47B, 381 (1973); F. Iachello and A. Lande, *ibid.* 50B, 313 (1974).
[17]D. J. Ernst and M. B. Johnson, Phys. Rev. C 17, 247 (1978).
[18]G. A. Miller, Phys. Rev. C 18, 579 (1978); 18, 915 (1978); 14, 2230 (1976).

[19] F. Myhrer and A. W. Thomas, Nucl. Phys. A326, 497 (1979).
[20] C. B. Dover, D. J. Ernst, R. A. Friedenberg, and R. M. Thaler, Phys. Rev. Lett. 33, 728 (1974).
[21] N.-C. Wei and M. K. Banerjee, Phys. Rev. C 22, 2052 (1980); 22, 2061 (1980).
[22] M. K. Banerjee and J. B. Cammarata, Phys. Rev. D 16, 1334 (1977); 17, 1125 (1978).
[23] A further discussion and generalization of the assumptions underlying the application of the inverse scattering problem techniques in a multichannel setting can be found in the work of D. J. Ernst, J. T. Londergan, E. J. Moniz, and R. M. Thaler, Phys. Rev. C 10, 1708 (1974).
[24] G. Rowe, M. Salomon, and R. H. Landau, Phys. Rev. C 18, 584 (1978).
[25] S. Almehed and C. Lovelace, Nucl. Phys. B40, 157 (1972).
[26] M. McMillan and R. H. Landau, TRIUMF Report No. TRI-74-1 (1974).
[27] R. H. Landau and A. W. Thomas, Phys. Lett. 41B, 361 (1976); Nucl. Phys. A302, 461 (1978).
[28] K.-B. Yoo, A. W. Thomas, and R. H. Landau (unpublished).
[29] S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Lett. 91B, 192 (1980); and Phys. Rev. D (to be published).
[30] F. Tabakin, Phys. Rev. 177, 1443 (1969).
[31] R. A. Friedenberg and D. Weiss, Phys. Rev. C 14, 204 (1976); D. J. Ernst and G. Miller, *ibid.* 12, 1462 (1975).