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$(\pi,\pi n)$ puzzle above the (3,3) resonance

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We calculate the corrections due to pion production, Fermi motion, and the Pauli principle on the ratio of $^{12}C(\pi^{\pm},\pi N)^{11}C$ total cross sections for particle-stable final states. These corrections are important above 400 MeV. When they are combined with those estimated earlier for final state nucleon charge exchange interactions, good agreement is obtained with the data, which cover the energy region from 40 to 550 MeV.

[NUCLEAR REACTIONS $^{12}C(\pi^{\pm},\pi N)^{11}C$, calculated ratio of knockout cross sections 40 — 1000 MeV, impulse approximation, nucleon final state charge exchange interactions, pion production corrections, Pauli principle and Fermi motion effects.]

INTRODUCTION

A few years ago,[1] we found that final state charge exchange interactions of the outgoing nucleons could explain some striking features of $(\pi,\pi n)$ cross sections in the (3,3) resonance region. Dropesky *et al.*[2] had studied the $^{12}C(\pi^{\pm},\pi N)^{11}C$ reaction at several energies using activation techniques, i.e., by observing the decays of the residual ^{11}C nuclei. Their results for the neutron knockout cross section ratio $R = \sigma^-/\sigma^+$ differed substantially from the ratio of the corresponding $\pi^{\pm}$-free neutron cross sections (impulse approximation). We found that this ratio had the characteristic energy variation predicted by the nucleon charge exchange mechanism. Since this charge exchange effect diminishes with increasing energy, and is very small above 400 MeV, we might expect the observed ratio to approach the impulse approximation value at higher energies. However, at 500 MeV these ratios disagree[3] in ^{11}C by about a factor of 2. We will show here that this disagreement can be significantly reduced by properly including the effects of pion production, Fermi motion, and the Pauli principle.

NUCLEON CHARGE EXCHANGE EFFECTS

We first briefly review the effects of final state nucleon charge exchange interactions[1,4] on the ratio R. When a pion knocks out a nucleon, the nucleon has a cross section for charge exchange on another nucleon which is quite large at low energies. In the (3,3) region, a π^+ has a big cross section for scattering by a proton. Since a proton can charge exchange and exit as a neutron, this tends to increase σ^+. A similar effect tends to decrease σ^-, so the ratio decreases markedly. For a $Z = N$ nucleus, if P is the probability of a nucleon charge exchanging before it leaves the nucleus, and $(1-P)$ is the probability that it emerges in its original charge state, then

$$R \equiv \frac{\sigma^-}{\sigma^+} = \frac{\sigma_{\pi^- n}(1-P) + \sigma_{\pi^- p\to\pi^- p}P}{\sigma_{\pi^+ n}(1-P) + \sigma_{\pi^+ p}P} \simeq \frac{9-8P}{3+6P}\,. \quad (1)$$

Here the approximate form includes only the (3,3) amplitude. Note when $P = 0$, the approximate ratio is 3, but if $P = 0.2$, it is only 1.76. This illustrates the importance of charge exchange effects.

To estimate P, we used[1] a model in which the pion and struck nucleon move straight ahead in the nuclear medium. P is determined from the nucleon mean free path[5] and its charge exchange cross section $\sigma_{N,\mathrm{ex}}$. The latter can be obtained in principle from the measured free np backscattering cross section at the average nucleon recoil energy. Since the Pauli principle restricts the final nucleon states, the cross section is reduced significantly from its free value by a factor which depends on the local nuclear density (or momentum distribution). In our simple model, it is difficult to estimate this reduction reliably, so we elected instead to take only the energy variation from the free *np* data. Thus we wrote

$$\sigma_{N,\mathrm{ex}} = \beta T_N^{-1.9} \qquad (2)$$

and chose the parameter β so that the measured value of the ratio R was obtained at one energy, 180 MeV. The value of β so obtained was consistent with crude estimates of the range of possible Pauli corrected values.

The model successfully predicted the measured values of R in the resonance region for several nuclei.[1,6] However, data are also available in the region from 300 to 550 MeV for one nucleus, ^{12}C. The predictions of this model are plotted in Fig. 1, along with the impulse approximation predictions. The data shown are the smoothed values recommended in Ref. 3. The πN cross sections required were calculated from the phase shifts of Rowe *et al.*[7] at c.m. momenta up to 250 MeV/*c* and from the *CERN* − *TH* phases[8] at higher energies. The parameter β was chosen so that $P = 0.22$ at 180 MeV. (In Ref. 1, we used $P = 0.24$ at this energy.) Note that the measured and theoretical values gradually diverge about 300 MeV, differing by a factor of 2 at the last data point, or at 550 MeV. Also, for low pion energies the present predictions agree slightly better with the data than in Ref. 1. This improvement is due in part to the use of better πN cross sections, but it is mostly attributable to refinements of the knockout data.

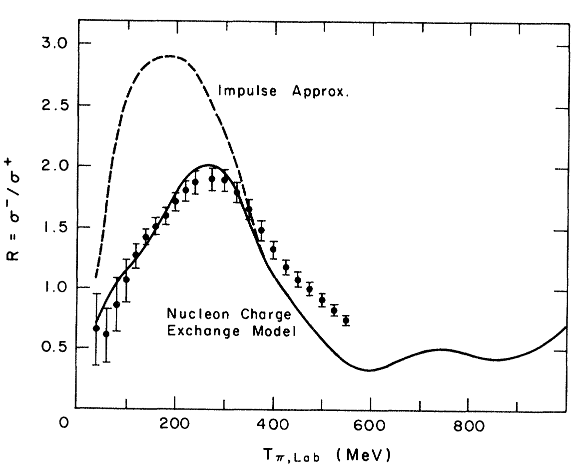

FIG. 1. Impulse approximation and nucleon charge exchange model predictions of the neutron knockout ratio R for ^{11}C. The two curves coincide to within 2% above 400 MeV. Data are taken from Ref. 3.

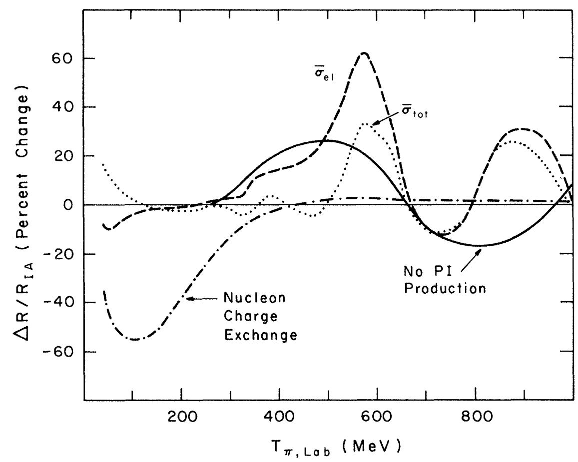

FIG. 2. Percentage changes due to various effects in the ratio R from the impulse approximation value based on free total pi-neutron cross sections.

The nucleon charge exchange mechanism has no significant effect at the higher energies. At 550 MeV P is merely 0.018; fast nucleons do not have much of a chance of charge exchange. As shown in Fig. 2, the effects of nucleon charge exchange, as calculated in this model, do not exceed 2% above 400 MeV. Furthermore, Nishi *et al.*[9] have shown that the effects of nucleon charge exchange are not large above 500 MeV, even if one allows for the formation of isospin $\frac{1}{2}$ isobars followed by subsequent interactions leading to slow nucleons. Clearly some other mechanism must play a role at these energies.

PION PRODUCTION EFFECTS

Suppose a pion with a kinetic energy of 500 MeV produces a second pion in a collision with a stationary nucleon. The two pions and the nucleon will have a total kinetic energy of 360 MeV. Since the lighter pions will share most of the kinetic energy, they will both typically have an energy close to the peak of the (3,3) resonance at 180 MeV, while the nucleon will have very little energy. Slow nucleons and resonance-energy pions both have very large cross sections for scattering on nucleons, so it is unlikely that all three particles will escape the nucleus without ejecting an additional nucleon. In that case the event will not be detected in radiochemistry experiments which see only nuclei with a single neutron removed.

The cross section for π production in π^+-n collisions rises rapidly above 400 MeV, while the production cross section in π^--n collisions remains small up to 600 MeV. (See Fig. 3.) As can be seen

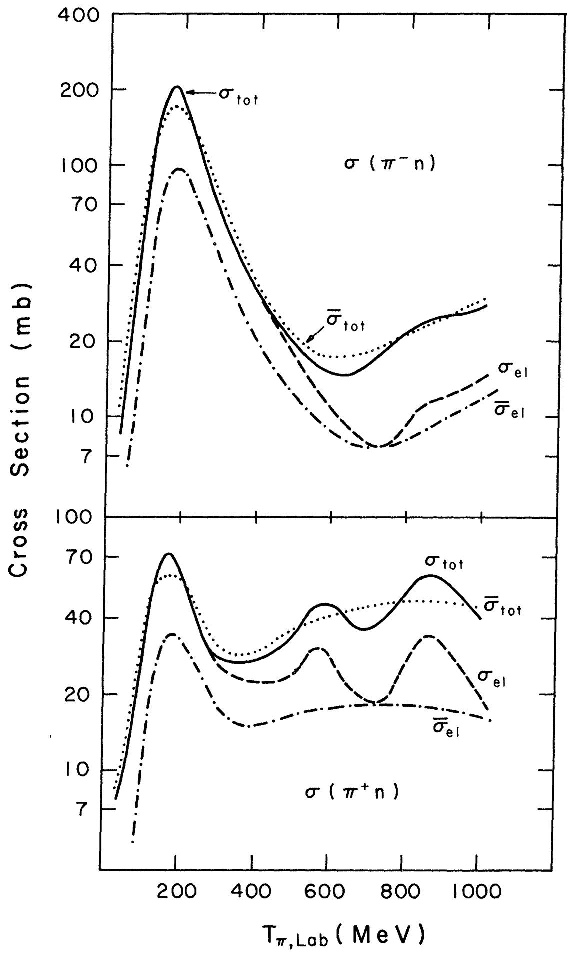

FIG. 3. Free $\pi^{\pm}$-n cross sections, with and without pion production and Fermi averaging.

in Fig. 2, if we include only the elastic and charge exchange $\pi^{\pm}$-n cross sections, the impulse approximation value of R is increased by up to 25% in the 400 to 600 MeV region. This change is in the right direction but does not by itself eliminate the discrepancy. Note that the effects of this exclusion are energy dependent, and actually go in the other direction, decreasing R above 700 MeV.

FERMI MOTION AND THE PAULI PRINCIPLE

The isospin $\frac{1}{2}$ resonances at 600 and 900 MeV in the πN system cause large and rapid variations in the π^{+}-n cross sections which are absent in π^{-}-n scattering (Fig. 3). Motion of a nucleon bound within a nucleus smears out the effective energy in a quasifree collision over a range of energy values which is much larger than the kinetic energy of the nucleons. For example, if the nucleon has a momentum of 250 MeV/c, and the incident pion momentum is 500 MeV/c, the net momentum of the two particles can be anywhere from 250 to 750 MeV/c, depending on the relative directions. Thus the marked structure seen in the π^{+}-n cross sections will largely be washed out for a bound neutron. This will be reflected in the energy variation of the ratio of the π^{-} and π^{+} cross sections.

A Fermi averaging procedure which has often been used[9–11] is to average the total cross sections for $\pi^{\pm}$. If the incident pion momentum is $\vec{p}$ and the struck nucleon momentum is $\vec{k}$, then one finds the c.m. energy $E_{\text{c.m.}}(\vec{p},\vec{k})$ for this pair of particles, and evaluates the total cross section at that energy. (This ignores all delicate questions about how to extrapolate the scattering amplitudes for bound nucleons.) Assuming a Fermi gas momentum distribution ρ, the Fermi averaged cross section is then

$$\bar{\sigma}_{\text{tot}} = \int \rho(\vec{k})\sigma[E_{\text{c.m.}}(\vec{p},\vec{k})]d^{3}k \quad . \tag{3}$$

Some authors[9,11] use a p-wave momentum distribution for ρ instead of a zero-temperature Fermi gas momentum distribution in Eq. (3).

This procedure is reasonable if we want to know the total cross section for removal of a nucleon. However, as we have seen, if we are to look only at events leaving a particle-stable nucleus, it is only the elastic and charge exchange cross sections which are relevant; pi production is to be excluded. The Pauli principle will also be important in determining the ratio R whenever the $\pi^{\pm}$ angular distributions are different, since it tends to suppress small angle scatterings. Thus we generalize Eq. (3) to read

$$\bar{\sigma}_{\text{el}} = \int \rho(\vec{k})\frac{d\sigma}{d\Omega_{\text{c.m.}}}(E_{\text{c.m.}},\theta_{\text{c.m.}})J\Theta d^{3}k \quad . \tag{4}$$

Here $d\sigma/d\Omega_{\text{c.m.}}$ is the sum of the elastic and charge-exchange differential cross sections. $\theta_{\text{c.m.}}$ is the scattering angle in the c.m. frame and J is the Jacobean for the c.m. to lab transformation. The theta function restricts the integral to nucleon final state momenta above the Fermi momentum and also requires an energy transfer to the nucleon greater than the neutron separation energy.

The effects of both averaging procedures on the cross sections are shown in Fig. 3. As expected, with either procedure much of the structure has disappeared in the π^{+}-n cross section. Also, inclusion of the Pauli principle reduces the cross sections significantly even at the highest energies shown. This occurs because the scattering is increasingly forward peaked as the energy increases so that the mean momentum transfer to the neutron tends to remain small.

Figure 2 shows the changes in the impulse approximation ratios resulting from the Fermi-averaging procedures. In the (3,3) resonance region neither procedure causes the ratio to change significantly. This is because the dominance of the (3,3) phase shift reduces the cross section ratio to a quotient of Clebsch-Gordan coefficients which is unaffected by the averaging. Below 100 MeV the changes resulting from the two procedures are of opposite sign. The total cross section averaging procedure effectively brings one closer to the resonance, where R is larger. In contrast, the differences in the $\pi^{\pm}$-n differential cross sections resulting from s- and p-wave interference imply different amounts of small-angle suppression. Above the (3,3) resonance, the Fermi-averaging effects are large at most energies. For example, at 550 MeV Fermi averaging the total cross sections increases R by 25%. The exclusion of pion production discussed above also increases R by 25% at this energy. Fermi averaging the differential cross sections increases R by 50%, suggesting that omitting pion production and Fermi averaging are additive corrections. However, inspection of the figure at 900 MeV shows that the situation is not so simple.

RESULTS AND DISCUSSION

The result of including all the corrections discussed in the knockout ratio R is shown in Fig. 4. The agreement with the data is improved significantly at the higher energies, although the fit is not perfect. Interestingly, at the lowest energies (around 50 MeV), the fit is also slightly improved by including the Fermi and Pauli corrections.

In summary, we have seen that the knockout ratio is modified by several factors. Since they have rather different energy variations, it is possible to see their effects more or less independently. The nucleon charge exchange effect is predominant up to about 300 MeV. The pion production becomes significant at about 400 MeV. Fermi motion and the Pauli principle are important near threshold and above 500 MeV. We must fit one constant in calculating the nucleon charge exchange corrections, but the other effects can be estimated without introducing additional free parameters. It should be very interesting to have data at higher energies and on other targets to more fully test these ideas.

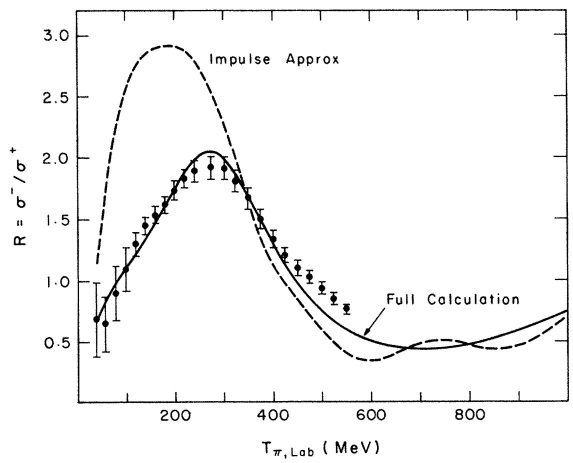

FIG. 4. Neutron knockout ratio in ^{12}C with the effects of nucleon charge exchange, pion production exclusion, Fermi averaging, and the Pauli principle all included. Data are from Ref. 3.

ACKNOWLEDGMENTS

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