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Adiabatic description of dissipative processes in heavy-ion reactions and fission. I. Microscopic theory: Statistics of matrix elements

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The statistical properties of matrix elements which couple the collective (or shape) degrees of nuclear motion with the intrinsic degrees of freedom, are evaluated in the adiabatic representation. The results will be used in a future paper for the calculation of transport coefficients which describe the irreversible (dissipative) transfer of energy from the collective degrees of freedom to the intrinsic ones. As input we use a random-matrix model for the residual interaction, and salient properties of single-particle levels in nonspherical potentials.

[NUCLEAR REACTIONS Random-matrix model applied to heavy-ion reaction and fission.]

I. INTRODUCTION

Dissipative processes play an important role in heavy-ion induced nuclear reactions and in induced nuclear fission. During the final stage of a deeply inelastic heavy-ion reaction (i.e., before the nuclear complex breaks up), and during the fission process, shape deformations of large amplitudes occur. The characteristic times associated with these processes are of the order of 10^{-21} sec or larger, and thus about one or two orders of magnitude bigger than the characteristic times of single-particle motion in the nucleus. This suggests that a theory of these processes might advantageously use a decomposition of the Hamiltonian into a "collective" part (describing the dynamics of the shape degrees of freedom) and an "intrinsic" part (describing the remaining degrees of freedom which we loosely associate with the single-particle degrees of freedom, cf. below). The intrinsic motion should be well approximated by an adiabatic treatment in which the intrinsic degrees of freedom adjust instantaneously to the nuclear shape. If one assumes, moreover, that the intrinsic motion loses memory very quickly and therefore acts, qualitatively speaking, as a heat bath, one is in a position to derive transport equations for the collective degrees of freedom. Various transport equations have been derived along these lines.[1-3]

It is the purpose of the present paper to provide the microscopic input needed for the evaluation of the transport coefficients in an adiabatic basis (the actual derivation and evaluation of these coefficients is the topic of a future paper). The procedure we use is patterned after the work of Barrett *et al.*[4]: The intrinsic motion is modeled as single-particle motion with residual interactions, the latter being represented by a random-matrix model. Because of the adiabatic basis chosen, the details of the derivations are quite different from those of Ref. 4. In Ref. 4, a sudden basis was used, and the coupling between collective and intrinsic degrees of freedom was given by the matrix elements of the potential containing both kinds of variables. In the present paper, the coupling between collective and intrinsic degrees of freedom is caused by the collective velocity, and the coupling matrix elements are those of the kinetic-energy operator.

The aim of this paper is the deduction of the statistical properties of the coupling matrix elements; more precisely, of their joint probability density. Once this is known, one can use the method of Ref. 5 to derive the transport equation and explicit expressions for the transport coefficients. In doing so, we shall determine the limits of usefulness of the adiabatic representation, and the limit of validity of linear response theory.

We do not pay any attention in this paper to the problem of how to determine all the relevant collective variables, and how to set up the collective part of the Hamiltonian. The first problem has had no definitive solution up to now. For the second, well-known methods are at hand, once the choice of a collective variable has been made; the Strutinsky averaging procedure[6] yields the potential energy, and generalized cranking procedures,[7] the inertia parameters. In our numerical examples and estimates, we use a constrained single-particle motion to simulate the adiabaticity condition. The constraint consists in keeping the shape of the single-particle potential fixed. By taking the motion of the A nucleons as being other-

wise independent, we obviously do not fully take into account the existence of collective variables, and we encounter the usual overcounting problem. We do not believe, however, that this has any noticeable effect on the problem of interest in the present paper, namely, the statistical properties of the coupling matrix elements.

In Sec. II we establish the notation and introduce the adiabatic basis in the framewrok of both a classical and a quantum description of the collective motion. In Sec. III we derive the statistical distribution of the coupling matrix elements, and we estimate the relevant parameters which characterize this distribution. Section IV contains a brief summary.

II. THE ADIABATIC BASIS

A. Collective variables

This paper does not aim at a broad discussion of how to define collective variables, a problem on which there exists a vast body of literature (see Ref. 7 and the papers cited therein). We simply wish to describe one approach which is particularly convenient for our purpose, without relating it to other approaches.

Let $\vec{x}_i$, $i=1,\ldots,A$ be the operators of the Cartesian coordinates of the A nucleons. A collective variable Q (in our case, a shape deformation degree of freedom) is a function of the $\vec{x}_i$. In order to be able to separate the Hamiltonian into intrinsic and collective parts in a manner consistent with the exclusion principle, we must require Q to be a *symmetric* function of the $\vec{x}_i$. Trivial examples are the c.m. coordinate, the operator for quadrupole deformation, etc. For simplicity we consider a *single* collective variable in the sequel; the generalization to several variables is straightforward. Aside from the collective variable and the three degrees of freedom of the c.m. motion (which we disregard in the sequel), we introduce further independent variables η_j, $j=1,\ldots,3A-4$ which we do not specify in detail, save for condition (2.3) below.

For the Hamiltonian we write

$$H=-\frac{\hbar^2}{2m}\sum_{i=1}^{A}(\vec{\nabla}_i)^2+\sum_{i<j}V(y_i,y_j), \tag{2.1}$$

where the y_i stand for the space, spin, and isospin variables of each nucleon (in the sequel the latter pair of variables is suppressed). Given Q and the η_j, we can write the potential part of H as a function of these variables, $V(Q,\eta_1,\ldots,\eta_{3A-4})$. For the kinetic energy, we find

$$-\frac{2m}{\hbar^2}(T-T_{\text{c.m.}})=\sum_{i=1}^{A}\sum_{\alpha=1}^{3}\frac{\partial}{\partial x_{i\alpha}}\left(\frac{\partial Q}{\partial x_{i\alpha}}\frac{\partial}{\partial Q}+\sum_{j=1}^{3A-4}\frac{\partial\eta_j}{\partial x_{i\alpha}}\frac{\partial}{\partial\eta_j}\right)$$

$$=\sum_{i=1}^{A}\sum_{\alpha=1}^{3}\left[\frac{\partial^2 Q}{\partial x_{i\alpha}^{\,2}}\frac{\partial}{\partial Q}+\left(\frac{\partial Q}{\partial x_{i\alpha}}\right)^2\frac{\partial^2}{\partial Q^2}+\sum_{j=1}^{3A-4}\frac{\partial\eta_j^{\,2}}{\partial x_{i\alpha}^{\,2}}\frac{\partial}{\partial\eta_j}+\sum_{j,k=1}^{3A-4}\frac{\partial\eta_j}{\partial x_{i\alpha}}\frac{\partial\eta_k}{\partial x_{i\alpha}}\frac{\partial^2}{\partial\eta_j\partial\eta_k}\right.$$

$$\left.+2\sum_{j=1}^{3A-4}\frac{\partial Q}{\partial x_{i\alpha}}\frac{\partial\eta_j}{\partial x_{i\alpha}}\frac{\partial^2}{\partial Q\partial\eta_j}\right]. \tag{2.2}$$

This expression simplifies if we use the orthogonality condition[2]

$$\sum_{i=1}^{A}\sum_{\alpha=1}^{3}\frac{\partial Q}{\partial x_{i\alpha}}\frac{\partial\eta_j}{\partial x_{i\alpha}}=0,\quad \text{all } j=1,\ldots,3A-4. \tag{2.3}$$

Geometrically this condition means that the hypersurfaces $Q=$ constant and the hypersurface $\eta_j=$ constant, $j=1,\ldots,3A-4$, are orthogonal. With this condition, the kinetic energy becomes a sum of two terms,

$$T-T_{\text{c.m.}}=T_{\text{coll}}+T_{\text{intrinsic}}\ , \tag{2.4}$$

where

$$T_{\text{coll}}=-t_1(Q,\eta_1,\ldots,\eta_{3A-4})\frac{\partial}{\partial Q}-t_2(Q,\eta_1,\ldots,\eta_{3A-4})\frac{\partial^2}{\partial Q^2}\ , \tag{2.5}$$

and

$$t_1=\frac{\hbar^2}{2m}\sum_{i=1}^{A}\sum_{\alpha=1}^{3}\frac{\partial^2 Q}{\partial x_{i\alpha}^{\,2}}\ \text{and}\ t_2=\frac{\hbar^2}{2m}\sum_{i=1}^{A}\sum_{\alpha=1}^{3}\left(\frac{\partial Q}{\partial x_{i\alpha}}\right)^2. \tag{2.6}$$

The intrinsic part of the Hamiltonian (which depends upon Q only parametrically) is given by

$$H_{\text{intrinsic}}=T_{\text{intrinsic}}+V(Q,\eta_1,\ldots,\eta_{3A-4}). \tag{2.7}$$

B. Adiabatic states

They are defined as antisymmetric eigenstates of $H_{\text{intrinsic}}$ for a *fixed* value of the collective variable Q,

$$H_{\text{intrinsic}}\,\psi_\mu(Q;\eta_1,\ldots,\eta_{3A-4})=E_\mu(Q)\psi_\mu(Q;\eta_1,\ldots,\eta_{3A-4}), \tag{2.8}$$

and obey the orthonormality conditions

$$\langle\psi_\nu(Q)|\psi_\mu(Q)\rangle = \delta_{\nu\mu}. \tag{2.9}$$

The bracket symbolizes integration over $\eta_1, \ldots, \eta_{3A-4}$ and *not* over Q. The full wave function Ψ, an eigenfunction of H, can be written as

$$\Psi = \sum_\mu a_\mu(Q)\psi_\mu(Q;\eta_1,\ldots,\eta_{3A-4}). \tag{2.10}$$

The expansion coefficients obey the equation

$$[E - E_\mu(Q)]a_\mu(Q) = \sum_\nu M_{\mu\nu}(Q)a_\nu(Q), \tag{2.11}$$

where we have introduced the operators

$$M_{\mu\nu}(Q) = \langle\psi_\mu|T_{\text{coll}}|\psi_\nu\rangle. \tag{2.12}$$

Deviation from adiabaticity, i.e., changes in the coefficients $a_\mu(Q)$, are caused by these operators $M_{\mu\nu}(Q)$. Explicitly, they are found by inserting Eq. (2.5) into Eq. (2.12),

$$-M_{\mu\nu}(Q) = \langle\psi_\mu|t_1|\psi_\nu\rangle\frac{\partial}{\partial Q} + \langle\psi_\mu|t_2|\psi_\nu\rangle\frac{\partial^2}{\partial Q^2} + \langle\psi_\mu|t_1|\partial_Q\psi_\nu\rangle + 2\langle\psi_\mu|t_2|\partial_Q\psi_\nu\rangle\frac{\partial}{\partial Q} + \langle\psi_\mu|t_2|\partial_Q^2\psi_\nu\rangle. \tag{2.13}$$

If Q denotes a multipole operator of low multipolarity, the dependence of Q on the $x_{i\alpha}$ is very smooth, and so is, therefore, the dependence of t_1 and t_2 on the η_j, $j = 1, \ldots, 3A-4$. We therefore expect the main contribution to the nondiagonal parts ($\eta \neq \nu$) of $M_{\mu\nu}$ to arise from the terms involving $\partial_Q\Psi_\nu$ and $\partial_Q{}^2\Psi_\nu$.

C. Classical approximation for Q

Collective motion in nuclei is often treated classically. One reason is that the inertial parameters are expected to be large. The transition from the Hamiltonian $H = T_{\text{coll}} + H_{\text{intrinsic}}$ to a new Hamiltonian H^{cl} depending on the classical variable Q and its conjugate momentum P can be made in the usual way, transition from the Schrödinger to the Heisenberg picture, and replacement of the commutator brackets by Poisson brackets. The quantities η_j, $j = 1, \ldots, 3A-4$ remain quantum-mechanical operators. The Hamiltonian H^{cl} can again be used to construct an adiabatic approximation. This Hamiltonian consists of two pieces, the intrinsic Hamiltonian $H_{\text{intrinsic}}$ defined in Eq. (2.7) which depends parametrically upon $Q(t)$, and the classical collective kinetic energy $T^{\text{cl}}_{\text{coll}}(Q, P)$ which depends parametrically upon η_j, $j = 1, \ldots, 3A-4$. The adiabatic wave functions Ψ_μ and eigenvalues E_μ are again defined by Eq. (2.8) and obey Eq. (2.9). It is consistent with the remarks made at the end of Sec. II B to assume that the inertia parameter for the collective kinetic energy depends smoothly on the variables η_j, $j = 1, \ldots, 3A-4$, so that the off-diagonal contributions ($\mu \neq \nu$) can be neglected, and the diagonal contributions ($\mu = \nu$) can be assumed to be independent of μ for the range of μ values that are of physical interest. Deviations from adiabaticity are then caused only by the time dependence of Q. Expanding the wave function $\Psi(Q, t, \eta_1, \ldots, \eta_{3A-4})$ into the adiabatic wave functions Ψ_μ, we find for the expansion coefficient a^{cl}_μ the well-known equations

$$i\hbar\dot{a}^{\text{cl}}_\mu = E_\mu a^{\text{cl}}_\mu - i\hbar\sum_\nu \dot{Q}\langle\psi_\mu|\partial_Q|\psi_\nu\rangle a^{\text{cl}}_\nu. \tag{2.14}$$

We see that the terms causing deviations from adiabaticity are the nondiagonal matrix elements of ∂_Q. This is the same situation as in Eq. (2.13) where, under the assumption of smoothness of the inertia parameters t_α, $\alpha = 1, 2$, and with $\langle\Psi_\mu|t_\alpha|\Psi_\nu\rangle \cong \delta_{\mu\nu}\langle\Psi_\mu|t_\alpha|\Psi_\mu\rangle \cong \delta_{\mu\nu}\langle\Psi_\nu|t_\alpha|\Psi_\nu\rangle$, we also find that the principal contributions to deviation from adiabaticity arise from the nondiagonal matrix elements of ∂_Q and $\partial_Q{}^2$.

It is therefore our aim in the sequel to study the statistical properties of the matrix elements $\langle\Psi_\mu|\partial_Q|\Psi_\nu\rangle$ and $\langle\Psi_\mu|\partial_Q{}^2|\Psi_\nu\rangle$.

III. STATISTICS OF THE MATRIX ELEMENTS

In the spirit of statistical nuclear spectroscopy, we consider the adiabatic states Ψ_μ and the eigenvalues E_μ as stochastic quantities. This is permissible if (i) the excitation energies E_μ are sufficiently large (typically $E_\mu > 1$ MeV for heavy nuclei), and (ii) all collective features of the nuclear system are described by Q. If condition (ii) is not met, further collective variables must be introduced.

In order to work out the statistical properties of the adiabatic wave functions Ψ_μ and thereby those of the matrix elements $\langle\Psi_\mu|\partial_Q|\Psi_\nu\rangle$ and $\langle\Psi_\mu|\partial_Q{}^2|\Psi_\nu\rangle$, we introduce a model. We replace $H_{\text{intrinsic}}$ of Eq. (2.7) by a single-particle Hamiltonian with residual interactions, the latter being given by a random-matrix model.

Because of the introduction of the collective variable(s) Q, not all the single-particle degrees of freedom are really independent. We disregard this problem and take the value of Q only as a constraint on the *form* of the single-particle potential. Our results on the statistical properties of the functions Ψ_μ do not depend on the number of particles present, and on the number of degrees of freedom involved. Therefore, we do not believe that a more complete inclusion of the constraints imposed by Q would alter our results in a qualitative fashion. A similar comment applies

to our neglect of self-consistency in the definition of the single-particle field. The reader will observe that the main ingredient in our derivation is the characteristic dependence of the single-particle energies on the value of Q. The characteristic features which we use are the same for a self-consistent and a phenomenological single-particle potential.

A. Statistical properties of the wave functions Ψ_μ

For Q fixed, let $\tilde{\varphi}_\alpha(Q,\vec{x})$ be a complete set of single-particle wave functions (i.e., solutions of the Schrödinger equation with a deformed potential), and $\tilde{\epsilon}_\alpha(Q)$ the associated eigenvalues. As we change Q, we follow these wave functions *diabatically:* At each Landau-Zener crossing of eigenvalues $\tilde{\epsilon}_\alpha$ and $\tilde{\epsilon}_\beta$, we stay on the branch which gives maximum overlap with the original eigenfunction. This procedure introduces a slight discontinuity of $\tilde{\varphi}_\alpha$ and $\tilde{\epsilon}_\alpha$ at the Landau-Zener crossing which we remove by a suitable averaging procedure which involves a small interval ΔQ around each Landau-Zener crossing. We denote the averaged functions and eigenvalues by φ_α and ϵ_α, respectively. They are approximate eigenfunctions and eigenvalues of the original single-particle Hamiltonian $H_{\text{s.p.}}(Q)$; we disregard the difference between $H_{\text{s.p.}}(Q)$ and $\sum_\alpha \epsilon_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$.

A complete set of Slater determinants $\Phi_m(Q;\vec{x}_1,\ldots,\vec{x}_A)$ is constructed from the functions $\varphi_\alpha(Q,\vec{x})$, with eigenvalues $E_m(Q)$ given as an appropriate sum over the $\epsilon_\alpha(Q)$. The functions $\Psi_\mu(Q)$ are obtained by diagonalizing the full $H_{\text{intrinsic}}$ (including the residual interaction) in the space of functions Φ_m. Therefore, the Ψ_μ have the form

$$\psi_\mu = \sum_m A_\mu^m(Q)\,\Phi_m(Q;\vec{x}_1,\ldots,\vec{x}_A). \tag{3.1}$$

While the Φ_m do not have stochastic properties, the Ψ_μ do, since they are obtained by diagonalizing a Hamiltonian given by a random-matrix model. The stochastic features are contained in the expansion coefficients $A_\mu^m(Q)$. The stochastic properties of the Ψ_μ determine the statistical distribution of the matrix elements $\langle\Psi_\mu|\partial_Q|\Psi_\nu\rangle$ and $\langle\Psi_\mu|\partial_Q{}^2|\Psi_\nu\rangle$. We shall show below that the matrix elements have a Gaussian distribution characterized by its first and second moments. These moments can be evaluated once the distribution of the A_μ^m is known.

Usual arguments used in random-matrix theory[8] show[4] that the $A_\mu^m(Q)$ have a Gaussian distribution with zero mean value, and with a second moment given by

$$\overline{A_\mu^m(Q)A_{\mu'}^{*m'}(Q')} = \delta_{mm'}\delta_{\mu\mu'}C_\mu^m(Q,Q'), \tag{3.2}$$

while $\overline{A_\mu^m(Q)A_\mu^m(Q')} = 0$ due to the randomness of phases occurring in the unitary matrix A_μ^m. The Kronecker symbols in Eq. (3.2) arise from the same source.

Postponing a discussion of the correlation functions C_μ^m to Sec. III C, we notice that the Gaussian distribution and the form (3.2) alone suffice to deduce that the mean value of the nondiagonal matrix elements $\langle\Psi_\mu|\partial_Q|\Psi_\nu\rangle$ and $\langle\Psi_\mu|\partial_Q{}^2|\Psi_\nu\rangle$ $(\mu\neq\nu)$ is zero, and that due to the law of large numbers, these matrix elements have a Gaussian distribution. It remains to determine the second moments. These are expected to contain a correlation length in Q. This correlation length is determined either by the length σ_0 over which the basis states Φ_m change, or by the correlation length σ_1 contained in $C_\mu^m(Q,Q')$, whichever is smaller. In the next three subsections, we estimate σ_0 and σ_1 and show that $\sigma_1 \ll \sigma_0$. This implies that the correlation length of the second moment of the matrix elements is given by σ_1.

B. Estimate of the length σ_0

Since each Φ_m is a Slater determinant, its Q dependence is given by that of the constituent single-particle wave functions $\varphi_\alpha(Q,\vec{x})$. We have evaluated the change with Q of the latter as follows. We have used a Nilsson scheme[9] (which we take to be representative of any deformed single-particle potential), in which we have varied the quadrupole deformation Q. (Q corresponds to the unitless deformation parameter δ as defined by Nilsson.[9]) Generally, we can write

$$\varphi_{\alpha'}(Q',\vec{x}) = \sum_\alpha \langle\varphi_\alpha(Q,\vec{x})|\varphi_{\alpha'}(Q',\vec{x})\rangle\varphi_\alpha(Q,\vec{x}); \tag{3.3}$$

we can therefore confine the discussion to the Q-Q' dependence of the overlap matrix elements $\langle\alpha,Q\,|\,\alpha',Q'\rangle = \langle\varphi_{\alpha'}(Q',\vec{x})\rangle$. In Fig. 1 we show, for a number of Nilsson orbitals as indicated in the figure, the change of the diagonal terms $\langle\alpha,Q|\alpha,Q'\rangle$ with increasing distance between Q and Q'. The average of these can be fitted with a Gaussian with a variance $\sigma_0 = 0.225$. The Gaussian approximates the mean value of $\langle\alpha,Q|\alpha,Q'\rangle$; this approximation is meaningful only if the variance of the variation of the matrix elements is sufficiently small. We have calculated this number; for $\Delta Q < 0.4$ we found it to be about 15%. The behavior of the totality of the nondiagonal matrix elements is in each case given by orthonormality; some cases calculated explicitly are displayed in Fig. 2 (and Table I).

We take a ratio of two of the major to the minor axes of the spheroid as a measure of the maximum physically possible value of Q, denoted by

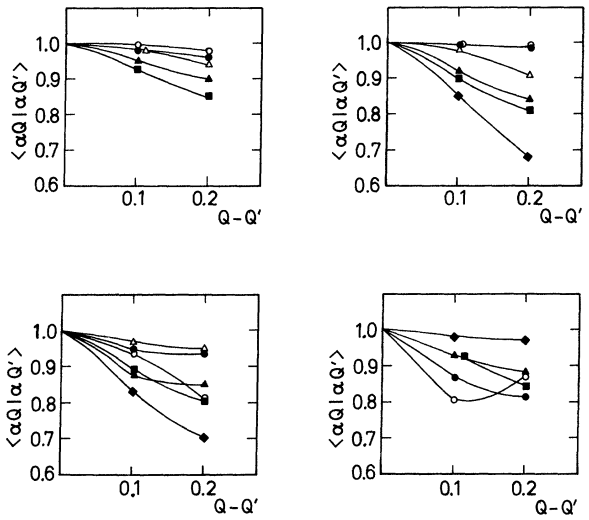

FIG. 1. Dependence of the diagonal elements $\langle\alpha Q|\alpha Q'\rangle$ on Q-Q'. The orbitals $|\alpha Q\rangle$ are specified in terms of the Nilsson (Ref. 9) quantum numbers N and Ω and by their energy E for a deformation $\eta=2$ (in the units given in Ref. 9). A change of Q by 0.1 corresponds to a change of η by 2.0. Upper right: $N=5$, $\Omega=\frac{3}{2}$; $\bullet E=0.918$, $\circ E=1.865$, $\triangle E=-9.169$, $\blacktriangle E=-10.230$, $\blacksquare E=-20.692$. Upper left: $N=5$, $\Omega=\frac{1}{2}$; $\bullet E=3.162$, $\circ E=2.149$, $\triangle E=-4.868$, $\blacktriangle E=-10.324$, $\blacksquare E=-11.269$, $\blacklozenge E=-21.152$. Lower right: $N=6$, $\Omega=\frac{3}{2}$; $\bullet E=3.270$, $\circ E=-1.003$, $\triangle E=-6.198$, $\blacktriangle E=-14.358$, $\blacksquare E=-15.386$, $\blacklozenge E=-27.524$. Lower left: $N=6$, $\Omega=\frac{5}{2}$; $\bullet E=-26.794$, $\circ E=-3.487$, $\blacktriangle E=-13.048$, $\blacksquare E=-14.039$, $\blacklozenge E=-1.132$.

Q_{max}. The variable $q=Q/Q_{max}$ thus typically ranges in the interval $|+\frac{1}{2},+1|$. Expressed in these units, the variance σ_0 of the Gaussian introduced in the previous paragraph has the value $\sigma_0=0.33$. Therefore, typical lengths over which the quantities $\langle\alpha Q|\alpha' Q'\rangle$ change are 0.33 or so in these units.

This number can be used to estimate the variation with Q of the matrix elements of the residual

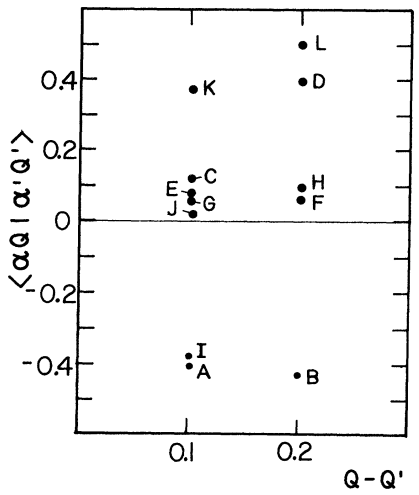

FIG. 2. Dependence of the nondiagonal elements $\langle\alpha Q|\alpha' Q'\rangle$ on Q-Q'. We use the same notation as in Fig. 1. The labels A – L denote the overlaps between the pairs of levels listed in Table I.

interaction. We take a random two-body interaction with antisymmetrized two-body matrix elements written as $\langle\alpha\beta|V|\gamma\delta\rangle$. For Q fixed, these are supposed[10] to have a Gaussian distribution with zero mean value and a second moment of the form

$$\overline{\langle\alpha\beta|V|\gamma\delta\rangle\langle\alpha'\beta'|V|\gamma'\delta'\rangle} = (\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{\gamma\gamma'}\delta_{\delta\delta'}+\cdots)\overline{|\langle\alpha\beta|V|\gamma\delta\rangle|^2}. \quad (3.4)$$

Here, the dots indicate terms obtained by the exchange of indices in an obvious fashion.

The correlation function of the two matrix elements taken at different deformations Q and Q' can be evaluated with the help of the transformation matrix $\langle\alpha Q|\alpha' Q'\rangle$. We find, using relation (3.4),

TABLE I. Tabulation of the pairs of levels, here distinguished by the indices 1 and 2, the overlaps of which are denoted by A–L on Fig. 2.

	A	B	C	D	E	F	G	H	I	J	K	L
E_1	−11.269	−11.269	−11.269	−11.269	−4.868	−4.868	0.918	0.918	−15.386	−6.868	−10.324	−10.324
η_1	2	2	2	2	2	2	2	2	2	2	2	2
N_1	5	5	5	5	5	5	5	5	6	6	5	5
Ω_1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
E_2	−10.324	−16.571	−24.834	−29.436	−13.537	−16.571	−1.323	−0.438	−16.881	−16.881	−15.732	−20.742
η_2	4	6	4	6	4	6	4	6	4	4	4	6
N_2	5	5	5	5	5	5	5	5	6	6	5	5
Ω_2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$

$$\overline{\langle\alpha\beta|V|\gamma\delta\rangle_Q\langle\alpha'\beta'|V|\gamma'\delta'\rangle_{Q'}}$$
$$=\{\langle\alpha'Q'|\alpha Q\rangle\langle\beta'Q'|\beta Q\rangle\langle\gamma Q|\gamma'Q'\rangle\langle\delta Q|\delta'Q'\rangle+\cdots\}$$
$$\times\overline{|\langle\alpha\beta|V|\gamma\delta\rangle|^2}\,. \tag{3.5}$$

The dots have the same origin as in Eq. (3.4). We now use the fact that for $\alpha\neq\alpha'$, the elements of the transformation matrix $\langle\alpha,Q|\alpha',Q'\rangle$ are small and have either sign, while for $\alpha=\alpha'$, they fall off with the Gaussian introduced above. Accordingly, we find that, on the average,

$$\overline{\langle\alpha\beta|V|\gamma\delta\rangle_Q\langle\alpha'\beta'|V|\gamma'\delta'\rangle_{Q'}}$$
$$\cong(\delta_{\alpha\alpha},\delta_{\beta\beta},\delta_{\gamma\gamma},\delta_{\delta\delta},+\cdots)\,\overline{|\langle\alpha\beta|V|\gamma\delta\rangle|^2}$$
$$\times\exp[-2(Q-Q')^2/\sigma_0^2]\,. \tag{3.6}$$

C. Estimate of the correlation length σ_1

Having estimated the typical length over which the Φ_m and the matrix elements $\langle\alpha\beta|V|\gamma\delta\rangle_Q$ change, we now return to the correlation function $C_\mu^m(Q,Q')$. It is defined as the mean value of the product of two random functions, and we therefore expect it to have the form

$$C_\mu^m(Q,Q')=\overline{|A_\mu^m(Q)|^2}\exp[-(Q-Q')^2/(2\sigma_1^2)]\,. \tag{3.7}$$

It is our aim to estimate the correlation length σ_1. We are going to show that $\sigma_1\ll\sigma_0$, where σ_0 was introduced in the previous subsection as the typical length over which the function Φ_m and the matrix elements $\langle\alpha\beta|V|\gamma\delta\rangle$ change. We therefore proceed by assuming that the Φ_m and the matrix elements $\langle\alpha\beta|V|\delta\gamma\rangle$ are independent of Q over an interval of length σ_1, and estimate σ_1 under this assumption. If the resulting value indeed fulfills $\sigma_1\ll\sigma_0$, our proof is complete.

The coefficients $A_\mu^m(Q)$ are the elements of the unitary matrix which diagonalizes the Hermitian Hamiltonian matrix

$$H_{mn}=E_m(Q)\delta_{mn}+\langle m|V|n\rangle\,. \tag{3.8}$$

The matrix elements $\langle m|V|n\rangle$ of the residual interaction with respect to the functions Φ_m can be trivially expressed in terms of the $\langle\alpha\beta|V|\gamma\delta\rangle$ introduced earlier.

Let $A_\mu^m(Q_0)$ be given at some point $Q=Q_0$. The average distribution of these quantities describes the spreading of a state Ψ_μ over the states Φ_m or, conversely, that of a state Φ_m over the states Ψ_μ. We therefore write

$$\overline{|A_\mu^m(Q_0)|^2}\cong\frac{\Gamma^\downarrow d/(2\pi)}{[E_m(Q_0)-E_\mu(Q_0)]^2+\frac{1}{4}(\Gamma^\downarrow)^2}\,. \tag{3.9}$$

This form fulfills the orthogonality conditions

$$\sum_m|A_\mu^m(Q_0)|^2=1=\sum_\mu|A_\mu^m(Q_0)|^2\,. \tag{3.10}$$

The spreading width is $\Gamma^\downarrow$, and d is the mean level spacing, taken to be the same for the states Φ_m and Ψ_μ. In order to estimate σ_1, we write H_{mn} in the form $H_{mn}=H_{mn}(Q_0)+\Delta H_{mn}$ and estimate the change of $A_\mu^m(Q)$ with ΔH_{mn}. Using the assumed independence of $\langle m|V|n\rangle$ of Q, and writing $E_m(Q)=E_m(Q_0)+f(Q)+F_m(Q)$, where $f(Q)$ is a suitably defined average value of $E_m(Q)-E_m(Q_0)$ (averaged over many levels in an interval of width $\Gamma^\downarrow$), we transcribe H_{mn} by multiplication with $A_\mu^m(Q_0)$ and $A_\nu^{*n}(Q_0)$ into the form

$$H_{\mu\nu}=[E_\mu(Q_0)+f(Q)]\delta_{\mu\nu}+\sum_m A_\mu^m(Q_0)F_m(Q)A_\nu^{*m}(Q_0)\,. \tag{3.11}$$

The last term on the right-hand side (rhs) is the cause of the change of $A_\mu^m(Q)$ with Q. We have extracted from $F_m(Q)$ the average value $f(Q)$: A uniform change of all $E_m(Q)$ with Q leads to a diagonal contribution and does *not* change $A_\mu^m(Q)$.

The eigenfunctions Ψ_μ [and the coefficients $A_\mu^m(Q)$] will change qualitatively over a distance ΔQ over which the perturbation series with respect to the last term on the rhs of Eq. (3.11) diverges. This series has the form

$$\psi_\mu(Q)=\psi_\mu(Q_0)+\sum_{\nu\neq\mu}\{[E_\nu(Q_0)-E_\mu(Q_0)]^{-1}\Big[\sum_m A_\nu^{*m}(Q_0)A_\mu^m(Q_0)F_m(Q)\Big]\psi_\nu(Q_0)\}+\cdots\,. \tag{3.12}$$

Convergence terminates, or the wave function changes qualitatively, when the sum of the squares of the 1st-order expansion coefficients is about 1,

$$1\cong\sum_{\nu\neq\mu}[E_\nu(Q_0)-E_\mu(Q_0)]^2\overline{\Big|\Big(\sum_m A_\nu^{*m}(Q_0)A_\mu^m(Q_0)F_m(Q)\Big)\Big|^2}\,. \tag{3.13}$$

We evaluate the average using Eqs. (3.2) and (3.9) and find, after a straightforward calculation,

$$\langle F_m^{\,2}(Q)\rangle\cong\Gamma^\downarrow d\,, \tag{3.14}$$

where the brackets denote an average over an interval of width $\Gamma^\downarrow$.

Equation (3.14) is the central result of this paper: The correlation length σ_1 is given by that distance ΔQ, over which the spread in eigenvalues E_m becomes comparable with the product

$\Gamma^{\downarrow} d$. This result is physically plausible; recalling that $\Gamma^{\downarrow} = 2\pi V^2/d$ (where V^2 is the mean square matrix element of the residual interaction), we see that it can be read as a comparison between the strength of the residual interaction, and the reordering of the eigenvalues E_m caused by the change of Q.

D. Numerical estimate

To estimate σ_1 numerically, we use $d = 30$ eV (a conservative value even at neutron threshold) and $\Gamma^{\downarrow} = 10$ MeV (which we also believe to be a conservative estimate up to excitation energies of several 10 MeV; we recall that giant resonances have spreading widths of several MeV). Then, σ_1 is given by that value of ΔQ for which the variance of the energies E_m has roughly the value 20 keV. Our remaining task consists in relating the variance of the E_m to that of the single-particle energies $\epsilon_\alpha(Q)$ and in evaluating the latter in terms of a Nilsson model.

It is preferable to use the excitation energies $E_m - A\epsilon_F$ (where ϵ_F denotes the Fermi energy) instead of the energies E_m themselves; a significant part of the mean value $f(Q)$ is thereby taken into account. For the variance of $E_m - A\epsilon_F$ we find

$$\sigma[E_m(Q) - A\epsilon_F(Q) - E_m(Q_0) + A\epsilon_F(Q_0)] = \sqrt{2N}\ \sigma[\epsilon_\alpha(Q) - \epsilon_\alpha(Q_0)], \tag{3.15}$$

where $\sigma(\epsilon_\alpha)$ is the variance of the s.p. energies ϵ_α, and N the number of particle-hole pairs excited on the mean at a given excitation energy. Equation (3.15) is intuitively reasonable on statistical grounds; it can be verified by using combinatorial arguments which we do not reproduce here. Taking for N a value between 2 and 20 (typical for excitation energies up to several 10 MeV in heavy nuclei), we find

$$\delta[\epsilon_\alpha(Q) - \epsilon_\alpha(Q_0)] \approx \text{several keV} \tag{3.16}$$

as the condition which determines σ_1. According to the statistical model, N is proportional to the root of the excitation energy E^* and σ_1 thus proportional to $(E^*)^{1/4}$. Hence, σ_1 does not depend sensitively upon E^*.

To estimate $\sigma[\epsilon_\alpha(Q) - \epsilon_\alpha(Q_0)]$, we take a simple model with N single-particle energies, their slopes versus Q being distributed equally between $-k$ and $+k$, where k is the maximum slope versus a change of Q. This yields in a straightforward fashion at $|Q - Q_0| = \sigma_1$ for $\sigma[\epsilon_\alpha(Q) - \epsilon_\alpha(Q_0)]$ the value $(1/\sqrt{3})\sigma_1 k$. Hence, we find

$$\delta_1 \approx \text{several keV}/k. \tag{3.17}$$

The constant k can be determined from numerical evaluations of s.p. energies in deformed potentials. We have used Ref. 11 for this purpose, especially Fig. 5. In the units given above ($q = Q/|Q_{\max}|$), we find typically $k \cong 1$ MeV$/0.06$, which yields for σ_1 the approximate value

$$\sigma_1 \approx 10^{-3}. \tag{3.18}$$

Comparing this with $\sigma_0 \cong 0.2$ we see that $\sigma_1 \ll \sigma_0$ so that the assumptions introduced above are indeed met. Using Fig. 12 of Ref. 11 for the neck parameter, or Fig. 13 for the asymmetry parameter, we find similar values.

E. Statistical properties of the matrix elements

For simplicity, we consider the nondiagonal elements $\langle \Psi_\mu | \partial_Q | \Psi_\nu \rangle$, $\mu \neq \nu$, which appear in the semiclassical approximation of Sec. IIC. (The arguments apply similarly to the matrix elements $\langle \Psi_\mu | \partial_Q{}^2 | \Psi_\nu \rangle$ used in the full quantum context.) It was shown in Sec. IIA that because of the properties of the expansion coefficients A_μ^m and the law of large numbers, these matrix elements have a Gaussian distribution with zero mean value.

To calculate the second moment, we use Eqs. (3.2) and (3.7) and the inequality $\sigma_0 \gg \sigma_1$, so that we execute the differentuation ∂_Q always with respect to $A_\mu^m(Q)$, not $\phi_m(Q)$. This yields in a straightforward fashion (recall that we put $\mu \neq \nu$ and $\rho \neq \tau$)

$$\begin{aligned} &- \overline{\langle \psi_\mu | \partial_Q | \psi_\nu \rangle_Q \langle \psi_\rho | \partial_Q | \psi_\tau \rangle_Q}, \\ &\cong \delta_{\mu\tau}\delta_{\nu\rho} \frac{(Q - Q')^2}{\sigma_1{}^4} \exp[-(Q - Q')^2/\sigma_1{}^2] \\ &\quad \times \sum_m \overline{|A_\mu^m(Q)|^2 |A_\nu^m(Q)|^2}. \end{aligned} \tag{3.19}$$

Equation (3.19) yields the value zero at $Q = Q'$. This is a consequence of the Gaussian correlation between the $A_m^\mu(Q)$; the Gaussian has a vanishing derivative at $Q = Q'$. The other, neglected terms arising from $\partial_Q \phi_m$ do not have this feature. The last term on the rhs of Eq. (3.19) can be evaluated using Eq. (3.9), and the factor $(Q - Q')^2$ can be approximated by $\sigma_1{}^2$. This yields finally

$$\begin{aligned} &- \overline{\langle \psi_\mu | \partial_Q | \psi_\nu \rangle_Q \langle \psi_\rho | \partial_Q | \psi_\tau \rangle_Q}, \\ &\cong + \delta_{\mu\tau}\delta_{\nu\rho} \exp[-(Q - Q')^2/\sigma_1{}^2] \\ &\quad \times \frac{1}{\sigma_1{}^2} \frac{\Gamma^{\downarrow} d/\pi}{(E_\mu - E_\nu)^2 + (\Gamma^{\downarrow})^2}. \end{aligned} \tag{3.20}$$

This shows that the correlation length in Q is given by $\sqrt{2}\,\sigma_1$, and the energy range of the possible deviation from adiabaticity by the spreading width $\Gamma^{\downarrow}$. Larger jumps in $|E_\mu - E_\nu|$ than given by $\Gamma^{\downarrow}$

are suppressed because when expanded into the shell-model states ϕ_m, the functions ψ_μ and ψ_ν do not have any significant overlap. This is completely analogous to the results of Ref. 4.

The occurrence of the factor $(Q-Q')^2$ in Eq. (3.19) is caused by our assuming a Gaussian form for the correlation function C_μ^m. However, C_μ^m could also be an exponential. Moreover, the behavior of C_μ^m over very short distances $(Q-Q')$ is irrelevant; what matters is some *integral* of C_μ^m over the relevant values of $(Q-Q')$, as will be shown in a future paper. This is why we replace $(Q-Q')^2$ by σ_1^2.

For $\mu=\tau$, $\nu=\rho$, the rhs of Eq. (3.20) is positive. This is as it should be since orthogonality implies $\langle\psi_\mu|\partial_Q|\psi_\nu\rangle^* = -\langle\psi_\nu|\partial_Q|\psi_\mu\rangle$, showing that the left-hand side (lhs) of Eq. (3.20) is an absolute square.

The Lorentzian factor $(\Gamma' d/\pi)|(E_\mu - E_\nu)^2 + \Gamma'^2|^{-1}$ on the rhs of Eq. (3.20) originates from the spreading of the states ψ_μ and is normalized to 1 if we sum over E_μ, keeping d constant. In later applications, the level spacing d changes significantly over distances $\Gamma' \approx 5-10$ MeV. In such a situation, the use of a Lorentzian is not useful, since the exponential rise of the level density with excitation energy causes sums over intermediate states μ to diverge, if the Lorentzian is the sole cutoff factor in the summation. For this reason, we proceed as in Ref. 4 and replace the Lorentzian by a Gaussian, $\exp[-(E_\mu - E_\nu)^2/(2\Gamma')^2]$. To preserve normalization, the Gaussian must be multiplied by $(\sqrt{2\pi}\,\Gamma')^{-1} \times [d(E_\mu)d(E_\nu)]^{1/2} \exp[-\beta^2(\Gamma')^2/8]$, where $\beta = (kT)^{-1}$ and the nuclear temperature T is that appropriate at the excitation energy $E_\mu = E_\nu$. The factor $[d(E_\mu)d(E_\nu)]^{1/2}$ is a (symmetrized) version of the factor d appearing in Eq. (3.20), and the exponential arises if one replaces in the summation over μ the ratio $[d(E_\nu)/d(E_\mu)]^{1/2}$ by $\exp[(\beta/2)(E_\nu - E_\mu)]$. In summary, we replace Eq. (3.20) by the formula

$$\overline{-\langle\psi_\mu|\partial_Q|\psi_\nu\rangle_Q\langle\psi_\rho|\partial_Q|\psi_\tau\rangle_{Q'}} \cong \delta_{\mu\tau}\delta_{\nu\rho}\frac{1}{\sigma_1^2}\exp[-(Q-Q')^2/\sigma_1^2]\frac{[d(E_\mu)d(E_\nu)]^{1/2}}{\sqrt{2\pi}\,\Gamma'}\exp[-\beta^2\Gamma'^2/8]\exp[-(E_\mu - E_\nu)^2/(2\Gamma')^2]. \tag{3.21}$$

IV. SUMMARY

We have evaluated the statistical distribution of the matrix elements $\langle\psi_\mu|\partial_Q|\psi_\nu\rangle_Q$, where the ψ_ν are the eigenfunctions of the *full* intrinsic Hamiltonian including the residual interaction and subject to the constraint that one (or several) collective variable(s) Q exist. We have shown that, using a random-matrix model for the residual interaction, these matrix elements have, for $\mu \neq \nu$, a Guassian distribution with zero mean value and a second moment given by Eq. (3.21). The Kronecker symbols in this equation are a direct consequence of the random nature of the residual interaction and of the associated random signs of the wave functions ψ_μ. The occurrence of the Gaussian is due to the spreading of the functions ψ_μ over the shell-model wave functions ϕ_m; large values of $|E_\mu - E_\nu|$ are suppressed because the associated functions ψ_μ, ψ_ν have orthogonal constituents. We estimate $\Gamma' \lesssim 10$ MeV.

The Gaussian with correlation length $(\sqrt{2})^{-1}\sigma_1$ arises from the correlation function of the expansion coefficients $A_\mu^m(Q)$ of the ψ_μ in terms of the ϕ_m. Among the possible factors which lead to a finite correlation length—the change of the s.p. wave functions and of the s.p. energies with Q—we have identified the change of the s.p. energies as the most important factor. A redistribution of the s.p. energies over an energy interval roughly given by the root mean square matrix element of the residual interaction destroys the correlation. Because of the strong dependence of (diabatic) s.p. energies upon the shape of the s.p. potential, this happens over very short intervals $|Q-Q'|$. When expressed in units of the dimensionless quantity $q = Q/|Q_{max}|$, σ_1 takes the very small value of $\approx 10^{-3}$. We note that this small value of σ_1 pertains independent of whether the $E_\mu(Q)$ as functions of Q do or do not show Landau-Zener behavior.

[1] H. Hofmann and P. J. Siemens, Nucl. Phys. A257, 165 (1976); Nucl. Phys. A275, 464 (1977).
[2] D. Agassi, Nucl. Phys. A330, 125 (1979).
[3] S. Ayik, GSI report (unpublished).
[4] B. Barrett, S. Shlomo, and H. A. Weidenmüller, Phys. Rev. C 17, 544 (1978); S. Shlomo, B. Barrett, and H. A. Weidenmüller, *ibid.* 20, 20 (1979).
[5] D. Agassi, C. M. Ko, and H. A. Weidenmüller, Ann. Phys. (N.Y.) 107, 140 (1977).
[6] V. Strutinsky, Nucl. Phys. A95, 420 (1967); A112, 1 (1968).
[7] B. Buck, L. C. Biedenharn, and R. Y. Cusson, Nucl. Phys. A317, 205 (1979).
[8] C. E. Porter, *Statistical Theories of Spectra: Fluctuations* (Academic, New York, 1965).
[9] S. G. Nilsson, K. Dan. Viedensk. Selsk. Mat. Fys. Medd. 29, No. 16 (1955).
[10] K. K. Mon and J. B. French, Ann. Phys. (N.Y.) 95, 90 (1975).
[11] H. C. Pauli, Phys. Rep. C7, 35 (1973).