# Determination of the $D_2$ parameter for (d,t) reactions

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Measurements of the tensor analyzing powers have been obtained for  $(\vec{d},t)$  reactions on <sup>91</sup>Zr, <sup>118</sup>Sn, <sup>119</sup>Sn, and <sup>208</sup>Pb for deuteron energies both above and below the Coulomb barrier. The measurements are sensitive to the presence of D-state components in the triton wave function and allow the determination of a parameter  $D_2$ . This parameter is a measure of the importance of triton wave function components in which one neutron moves with orbital angular momentum L = 2 relative to the deuteron center of mass. Values of  $D_2$  are extracted from the tensor analyzing power measurements by making use of distorted-wave Born approximation calculations. Analysis of the sub-Coulomb measurements leads to  $D_2 = -0.279 \pm 0.012$  fm<sup>2</sup>, which is somewhat larger in magnitude than recent theoretical predictions.

NUCLEAR REACTIONS <sup>91</sup>Zr(d,t),  $E_d = 6.0$ , 7.5 MeV, <sup>118</sup>Sn(d,t),  $E_d = 12.0$ MeV, <sup>119</sup>Sn(d,t),  $E_d = 6.0$ , 7.5, 9.0 MeV, <sup>208</sup>Pb(d,t),  $E_d = 10.0$ , 12.3 MeV; measured polarization parameters  $T_{20}(\theta)$ ,  $T_{21}(\theta)$ ,  $T_{22}(\theta)$ ; deduced  $D_2$ . Enriched targets, DWBA analysis.

#### I. INTRODUCTION

It has long been recognized that the threenucleon problem plays a central role in nuclear physics. Although the study of any light nucleus is interesting in its own right, the three nucleon system is of special importance, because in this case, it is possible to carry out quantum mechanical calculations which are essentially exact. This in turn allows one to investigate the question of whether nucleon-nucleon interactions determined from two-body experiments are able to explain the properties of complex nuclei. Unfortunately, experiments have not provided us with a great deal of useful information about the properties of the <sup>3</sup>H and <sup>3</sup>He wave functions. For the most part, comparisons between three-nucleon boundstate calculations and experiment are limited to the binding energy and the charge form factor.

It has been demonstrated<sup>1</sup> that one can obtain experimental information about the *D*-state components of the triton wave function by measuring the tensor analyzing powers for  $(\vec{a}, t)$  reactions. In particular, the measurements allow one to determine the value of a single parameter,  $D_2$ . In general, (d, t) reactions are sensitive to those components of the triton wave function which look like a neutron coupled to a deuteron, and the parameter  $D_2$  is a measure of the importance of the component in which the neutron moves with orbital angular momentum L = 2 relative to the deuteron center of mass.

Measurements of the tensor analyzing powers have previously been reported for a number of  $(\tilde{d}, t)$  reactions.<sup>2-5</sup> The values of  $D_2$  extracted from these data range from -0.24 to -0.30 fm<sup>2</sup>. The empirical  $D_2$  results agree quite well with the theoretical value,  $-0.24 \text{ fm}^2$ , which has been obtained<sup>6</sup> from a Faddeev calculation of the triton wave function.

In this paper we present the results of a series of  $(\tilde{d}, t)$  experiments. Measurements of the three tensor analyzing powers  $(T_{20}, T_{21}, \text{ and } T_{22})$  have been obtained for  ${}^{91}\text{Zr}(\tilde{d}, t){}^{90}\text{Zr}$  at 6.0 and 7.5 MeV, for  ${}^{118}\text{Sn}(\tilde{d}, t){}^{117}\text{Sn}$  at 12.0 MeV, for  ${}^{119}\text{Sn}(\tilde{d}, t){}^{118}\text{Sn}$ at 6.0, 7.5, and 9.0 MeV, and for  ${}^{208}\text{Pb}(\tilde{d}, t){}^{207}\text{Pb}$ at 10.0 and 12.3 MeV. The measurements will be analyzed to obtain new empirical  $D_2$  values.

In Sec. II we present some background information, including a rigorous definition of  $D_2$  and a discussion of the advantages which result from the use of sub-Coulomb energies. The experimental details are given in Sec. III, and the analysis of the measurements is presented in Sec. IV. In Sec. V the results are discussed and a comparison between the experimental and theoretical  $D_2$  values is presented. Some of the measurements described in this paper have previously been reported elsewhere.<sup>1,3</sup>

### **II. BACKGROUND**

It is well known that (d, t) reactions on mediumand heavy-weight nuclei can usually be understood in terms of the standard distorted-wave Born approximation (DWBA), in which one assumes that the reaction occurs through a simple one-step process. In this approximation the deuteron is treated as an inert particle which, as it passes the target nucleus, picks up a neutron to form the trition. It is easily seen that two distinct angular momentum coupling schemes are allowed in the n+d-t process. The first possibility is an S state in which the n-d relative orbital angular momentum L is zero and the total spin, defined as the vector sum of the neutron and deuteron spins, is  $\frac{1}{2}$ . Second, the neutron and deuteron may combine in a D-state configuration in which the orbital angular momentum, L = 2, and the total spin, which is  $\frac{3}{2}$  in this case, couple to give a  $j^{*} = \frac{1}{2}^{*}$  final state.

The relative importance of L = 0 and L = 2 transfers in a (d, t) reaction will, of course, depend on the extent to which the corresponding configurations are present in the triton wave function. Since the triton consists primarily of three nucleons in a relative S state, the L = 0 configuration will be dominant; however, reactions with L = 2 also occur, since the triton wave function contains Dstate components which have the appropriate configuration. It can be shown<sup>1</sup> that the tensor analyzing powers for a (d, t) reaction are sensitive to the presence of L = 2 contributions, and thus measurements of these quantities can provide information about the D-state components of the triton wave function.

To extract quantitative information about the triton from measured tensor analyzing powers, one makes use of the DWBA. In addition we employ the local-energy approximation (LEA) as introduced by Johnson and Santos.<sup>7</sup> Within the context of DWBA and the LEA, the magnitude of the *D*-state effect depends on the value of a single parameter,  $D_2$ . This parameter depends on the nature of the deuteron and triton internal wave functions,  $\phi_d$  and  $\phi_t$ , respectively. To define  $D_2$  we begin by projecting out the neutron-deuteron part of the triton wave function:

$$G(\mathbf{\tilde{r}}) = \langle \chi_n^{\sigma_n} \phi_d^{\sigma_d}(\mathbf{\tilde{\rho}}) | \phi_t^{\sigma_t}(\mathbf{\tilde{r}}, \mathbf{\tilde{\rho}}) \rangle , \qquad (1)$$

where  $\chi_n^{o_n}$  is a spin wave function for the neutron,  $\vec{\mathbf{r}}$  is the *n*-*d* separation, and  $\vec{\rho}$  is the internal coordinate of the deuteron. The quantity  $G(\vec{\mathbf{r}})$  can be thought of as a wave function which specifies the *n*-*d* relative motion in the triton. From angular momentum and parity considerations, one knows that  $G(\vec{\mathbf{r}})$  can be written in the form

$$G(\vec{\mathbf{r}}) = \sum_{L=0,2} \sum_{\Lambda,\sigma} \langle L\Lambda, s\sigma | \frac{1}{2}\sigma_t \rangle \langle 1\sigma_d, \frac{1}{2}\sigma_n | \frac{1}{2}\sigma_t \rangle \\ \times u_L(r) Y_L^{\Lambda}(\hat{r}) , \qquad (2)$$

where  $s = \frac{1}{2}$  for L = 0 and  $s = \frac{3}{2}$  for L = 2. Here  $u_0$ and  $u_2$  are radial wave functions for the S- and D-state parts of G. The parameter  $D_2$  is defined in terms of the relative strengths of  $u_2$  and  $u_0$  in the zero-momentum limit. More specifically,  $D_2$  is given in terms of  $u_2$  and  $u_0$  by

$$D_{2} = \frac{1}{15} \int_{0}^{\infty} u_{2}(r) r^{4} dr / \int_{0}^{\infty} u_{0}(r) r^{2} dr .$$
 (3)

Further details are given in Ref. 8.

Since DWBA is used to determine  $D_2$  from the measurements, the reliability of this approximate reaction theory is an important issue. In general, DWBA calculations reproduce the main qualitative features of cross section and analyzing power measurements, but are not accurate in a quantitative sense. Thus, it seems unlikely that tensor analyzing power measurements for arbitrarily chosen (d, t) transitions would lead to an accurate determination of  $D_2$ . However, it is known<sup>9</sup> that the reliability of DWBA calculations improves significantly for (d, t) reactions carried out below the Coulomb barrier. The calculations are expected to be particularly accurate for transitions which have Q values close to zero. Under these conditions the (d, t) reactions occur well outside the nuclear surface. This results in part from the use of sub-Coulomb energies, which prevents the deuteron and triton from penetrating into the nuclear interior. The importance of the  $Q \simeq 0$ condition is that it leads to a situation in which the elastic scattering wave functions are well matched in the region of the turning point of the classical Coulomb trajectories. This increases the likelihood that the reaction will take place outside the nucleus.

It is easy to see that the accuracy of DWBA calculations should be greatly improved under these conditions.<sup>9</sup> First, the wave functions which enter the calculations are known accurately in the region outside the nucleus. The deuteron and triton optical model wave functions are essentially Coulomb wave functions while the bound neutron wave function is (except for an overall normalization factor) just the appropriate Hankel function. A second reason for expecting the calculations to be more reliable for sub-Coulomb reactions is that the DWBA theory itself is intrinsically more accurate under these conditions. In particular, for energies below the Coulomb barrier, reaction cross sections are small compared to the cross section for elastic scattering and consequently the fundamental DWBA assumption of weak coupling between channels is well satisfied. In addition, the DWBA approximation in which one discards various nuclear potential terms<sup>10</sup> is also reasonable for sub-Coulomb reactions since these terms contribute only in the nuclear interior. Another advantage of using sub-Coulomb energies is that the reactions are insensitive to nuclear spin-dependent forces. Consequently, the observed tensor analyzing powers result almost entirely from D-state effects.

The method of using sub-Coulomb tensor analyzing power measurements to study projectile wave functions has previously been employed for the (d, p) reaction.<sup>11</sup> In this case the tensor ana-

lyzing powers result from the *D*-state component of the deuteron wave function. The value of  $D_2$ for (d, p) reactions is fairly well known from other work  $(D_2$  depends on the long-range parts of the wave function and most realistic nucleon-nucleon potentials predict nearly the same  $D_2$  value), and the result derived from the tensor analyzing power measurements<sup>11</sup> is in good agreement with the expected value. Based on this experience we believe that accurate determinations of  $D_2$  are also possible for (d, t) reactions.

## **III. EXPERIMENTAL DETAILS**

The tensor analyzing power measurements were carried out using the deuteron beam from the University of Wisconsin Lamb-shift polarized ion source.<sup>12</sup> After being accelerated and momentum analyzed by a 90° bending magnet, the incident beam was focused through 1 mm wide by 2 mm high rectangular slits located about 15 cm upstream of the target. An automatic feedback system was used to keep the beam centered on the slits. The targets were enriched, self-supporting foils with thicknesses of from 1 to 4 mg/cm<sup>2</sup>.

Reaction products were detected by an array of four  $\Delta E$ -E counter telescopes located to one side of the beam. An on-line particle identification computer program was used to distinguish the various reaction products. For the experiments at 6.0 and 7.5 MeV the thickness of the  $\Delta E$ detectors was approximately 60  $\mu$ m, while at the higher energies detectors of approximately 100  $\mu$ m were used. Figure 1 shows a typical particle identification spectrum for 6 MeV deuterons on <sup>119</sup>Sn. This spectrum gives the number of counts as a function of a parameter calculated from the energies deposited in the  $\Delta E$  and E detectors.<sup>13</sup>

 $H^{19}Sn = E_d = 6.0 \text{ MeV } \Theta_{1ab} = 132^{\circ}$ 

FIG. 1. A typical particle identification spectrum for 6 MeV deuterons on  $^{119}$ Sn.

In spite of the fact that the ratio of deuterons to tritons is large, the triton peak is well separated.

A typical triton energy spectrum for the reaction  $^{118}Sn(d, t)$  at 12.0 MeV is shown in Fig. 2. Note that the various triton peaks are well resolved and virtually free of background. For  $^{91}Zr$ ,  $^{119}Sn$ , and  $^{208}Pb$  the peaks of interest were easily resolved, since in all cases the states are separated by at least 300 keV.

Beam integration was accomplished by observing deuteron elastic scattering with a pair of monitor detectors located symmetrically to the left and right of the beam at an angle of  $13.1^{\circ}$ . To a good approximation, the count rate in the monitor detectors is independent of the beam polarization, since the elastic scattering analyzing powers are essentially zero at this angle.

The procedure used to determine the tensor analyzing powers is similar to that described by Rohrig and Haeberli.<sup>14</sup> The method involves obtaining relative measurements of the polarizedbeam cross section for a variety of polarization states of the incident beam. The beam polarization was monitored continuously during the experiments by a polarimeter located downstream of the target. For the measurements at 12.0 and 12.3 MeV the polarimeter described in Ref. 15 was used. For the experiments at lower energies, which were performed somewhat later, we used a more accurate polarimeter.<sup>16</sup>

The measured tensor analyzing powers will be presented in a number of subsequent figures. In all cases, the error bars shown in the figures include contributions from counting statistics, from uncertainties in background subtraction and from the statistical uncertainty in the determination of the beam polarization. In addition to the displayed errors, the measurements are subject to an overall normalization uncertainty which is estimated to be 10% for the measure-



FIG. 2. A typical triton energy spectrum for the reaction  $^{118}Sn(d,t)$  at 12 MeV. The peaks are identified by the excitation energy of the residual nucleus.

ments at 12.0 and 12.3 MeV and 2% for the measurements at lower energies.<sup>16</sup> The analyzing powers presented here are defined according to the Madison convention.<sup>17</sup>

### **IV. RESULTS**

### A. Sub-Coulomb measurements

As discussed in Sec. II, it is expected that the most reliable determinations of  $D_2$  are obtained from measurements for sub-Coulomb transitions which have Q values close to zero. Four of the transitions studied in the present experiment satisfy these conditions. The Q value and the angular momentum transfer for each of these transitions are given in Table I.

The measured differential cross sections for the sub-Coulomb transitions are given in Fig. 3. Note that in all cases the cross sections show the smooth angular dependence and backward peaking which are characteristic of sub-Coulomb reactions. The curves in Fig. 3 are DWBA calculations of the cross sections which have been normalized to the measurements. The DWBA calculations do not reproduce the measured cross sections as well as one might have expected; the predicted cross sections tend to be too small in magnitude at back angles. This is particularly true for the  ${}^{91}$ Zr(d, t) transition. Further calculations have shown that the shape of the predicted cross sections is sensitive to the strength and range of the absorptive terms in the deuteron and triton optical model potentials.

Since the analysis of the tensor analyzing power measurements relies heavily on the accuracy of the DWBA calculations, an effort was made to find the best possible optical model potentials. A number of DWBA calculations were carried out for each transition using different combinations of deuteron<sup>18-21</sup> and triton<sup>22-24</sup> optical model potentials. Overall, the best fit to the cross section data for the sub-Coulomb transitions was obtained by using the deuteron potentials of Ref. 18 and the triton potentials of Ref. 22. The cal-



FIG. 3. Cross section measurements for sub-Coulomb (d,t) reactions on  $^{91}$ Zr,  $^{119}$ Sn, and  $^{208}$ Pb. The residual nucleus and the excitation energy are indicated for each angular distribution. The curves are DWBA calculations which have been normalized to obtain the best overall fit to the measurements.

culations shown in Fig. 3 correspond to this choice of potentials.

The measured tensor analyzing powers for the sub-Coulomb transitions are shown in Fig. 4. The curves in the figure show the DWBA results obtained with the deuteron and triton optical model potentials from Refs. 18 and 22. The calculations shown include the effects of L = 2 transfers; if the D-state effects are not included, the resulting tensor analyzing powers for these transitions are typically three orders of magnitude smaller than the measurements. For calculations which include D-state effects, the predicted tensor analyzing powers are, to a good approximation, directly proportional to the value of  $D_2$  used in the calculation. For each of the four sub-Coulomb transitions, the value of  $D_2$  has been adjusted to obtain the best overall fit to the tensor analyzing power data, and the curves shown in Fig. 4 correspond to these best-fit values. The  $D_2$  value for each transition and the corresponding chi squared per degree of freedom  $(\chi^2/N)$  for the fit to the tensor analyzing power data are given in Table I. Note

 $E_d$  $E_{x}$ Q  $D_2$ j"  $(fm^2)$ (MeV)  $\chi^2/N$ Target (MeV) (MeV)  $\frac{5}{2}^{+}$  $^{91}$ Zr 6.0 0.0 -0.94 $-0.260 \pm 0.011$ 1.20 <sup>119</sup>Sn  $\frac{1}{2}$ 6.0 -0.22 $-0.302 \pm 0.015$ 0.0 1.16 <sup>208</sup>Pb 10.0 -1.12 $-0.278 \pm 0.014$ 1.11 0.0 <sup>208</sup>Pb  $\frac{3}{2}$ 10.0 0.90 -2.02 $-0.278 \pm 0.013$ 0.62

TABLE I. Properties of the sub-Coulomb (d,t) transitions studied in this experiment.



FIG. 4. Measurements of the tensor analyzing powers for sub-Coulomb  $(\vec{d}, t)$  reactions on <sup>91</sup>Zr, <sup>119</sup>Sn, and <sup>208</sup>Pb. The curves are DWBA calculations corresponding to the  $D_2$  values listed in Table I.

that in all four cases the  $\chi^2/N$  value is close to 1.0, indicating that the DWBA calculations reproduce the measurements as accurately as can be expected. The uncertainties in  $D_2$  quoted in Table I reflect only the statistical errors in the analyzing power measurements. Sources of systematic error in the determination of  $D_2$  will be considered below. It is encouraging to note that the results obtained from the four transitions are in reasonably good agreement.

Additional DWBA calculations have been carried out for the purpose of determining the extent to which the  $D_2$  values are sensitive to the choice of optical model parameters. It is found that the calculated tensor analyzing powers are less sensitive than the differential cross sections to changes in the potentials. The results of these calculations are summarized in Table II, which lists the  $D_2$  values obtained for six different combinations of deuteron and triton optical model potentials. The last line in Table II gives the standard deviation among the six calculations for each transition. The quality of the fit to the tensor analyzing power data was essentially identical for all of the optical model potentials considered.

Using the  $D_2$  values obtained with the optical model potentials of Refs. 18 and 22 (see Table I), we find that the average value of  $D_2$  is

$$D_2 = -0.279 \pm 0.012 \text{ fm}^2. \tag{4}$$

In obtaining this average, the  $D_2$  value for each transition has been given an equal weight. The  $D_2$  value given in Eq. (4) differs slightly from the result (-0.275 fm<sup>2</sup>) quoted in Ref. 3. The reason for the discrepancy is that, in Ref. 3, we used the optical model potentials of Refs. 19 and 24.

The quoted uncertainty in Eq. (4) includes contributions from several sources. Systematic errors in the measurement of the beam polarization lead to an uncertainty of approximately 2% (see Ref. 16) in the overall normalization of the measured tensor analyzing powers. The corresponding uncertainty in  $D_2$  is  $\pm 0.0056$  fm<sup>2</sup>. The choice of optical model potentials also affects the value of  $D_2$ . From Table II we see that, for a given transition, changing potentials produces variations of typically  $\pm 0.003$  fm<sup>2</sup> in  $D_2$ . Somewhat arbitrarily, the uncertainty associated with the choice of optical model potentials is assigned a value of  $\pm 0.006 \text{ fm}^2$ , which is twice the typical standard deviation. The statistical errors in the tensor analyzing power measurements also make a significant contribution to the uncertainty in  $D_2$ . The statistical errors in the  $D_2$  values for the four sub-Coulomb transitions range from  $\pm 0.010$  to  $\pm 0.015$  fm<sup>2</sup> (see Table I). These errors are somewhat smaller than the standard deviation among the four  $D_2$  values, 0.017 fm<sup>2</sup>. This suggests that the  $D_2$  determinations may be subject to systematic errors (of unknown origin) in addition to the statistical errors. To allow for this possibility, instead of using the statistical errors listed in Table I, we assign an uncertainty of  $\pm 0.017$  to each of the four  $D_2$  values. The corresponding contribution to the uncertainty in the average  $D_2$  value is  $\pm 0.0085$  fm<sup>2</sup>. The error quoted in Eq. (4) is obtained by adding this uncertainty in quadrature with the contributions arising from the beam polarization measurements and the optical model potentials.

## B. Measurements at higher energies

As the deuteron energy is raised above the Coulomb barrier the characteristics of the (d, t)reactions change. In particular, the shape of the differential cross section changes rapidly with

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Deuteron potential	Triton potential	$^{91}{ m Zr}(\frac{5}{2}^{+})$	$^{119}$ Sn $(\frac{1}{2}^{+})$	$^{208}$ Pb $(\frac{1}{2}$ )	$^{208}$ Pb $(\frac{3}{2})$	
Ref. 18	Ref. 22	-0.260	-0.302	-0.278	-0.278	
Ref. 19	Ref. 22	-0.258	-0.302	-0.277	-0.278	
Ref. 20	Ref. 22	-0.260	-0.304	-0.283	-0.281	
Ref. 18	Ref. 23	-0.262	-0.301	-0.287	-0.282	
Ref. 18	Ref. 24	-0.259	-0.305	-0.279	-0.279	
Ref. 21	Ref. 24	-0.253	-0.296	-0.276	-0.275	

0.003

0.003

TABLE II. Values of  $D_2$  obtained with various deuteron and triton optical model potentials. The  $D_2$  values are in fm<sup>2</sup>.

increasing energy. This is illustrated in Fig. 5 which shows the cross section measurements for  $^{119}$ Sn(d, t) at  $E_d$  = 6.0, 7.5, and 9.0 MeV. At 6 MeV the cross section is small and backward peaked. However, at 7.5 MeV the peak cross section occurs at approximately  $90^{\circ}$ , while at 9 MeV the peak has moved to forward angles. This forward peaking is characteristic of reactions which take place at the nuclear surface, and thus we expect that the reactions at the higher energies will show a greater sensitivity to the projectile-nucleus interactions. We expect that, as a result, the DWBA calculations will be less reliable than for the sub-Coulomb energies.

Standard deviation:

Tensor analyzing power measurements for the (d, t) reactions on <sup>91</sup>Zr at 7.5 MeV, <sup>118</sup>Sn at 12.0 MeV,  $^{119}$ Sn at 7.5 and 9.0 MeV, and  $^{208}$ Pb at 12.3 MeV are shown in Figs. 6 and 7. The curves in the figures are DWBA calculations which were obtained with the optical model potentials of Refs. 18 and 22. For the solid curves, D-state effects were included with  $D_2$  set equal to  $-0.279 \text{ fm}^2$ , the value obtained from the analysis of the sub-Coulomb data. Calculations in which the D-state



FIG. 5. Cross section measurements for  $^{119}Sn(d,t)$  at three energies. The curves are guides to the eye.

effects were neglected are shown for the  $^{118}$ Sn(d, t)transitions (dashed curves in Fig. 7). For all of the other transitions, the calculations which include only the S state lead to tensor analyzing powers which are too small to be seen in the figures, typically two orders of magnitude smaller than the measurements.

0.002

0.004

The *D*-state calculations do a reasonably good job of reproducing the tensor analyzing power measurements; however, the calculations are definitely inferior to those obtained for the sub-Coulomb reactions. For several of the transitions shown in Figs. 6 and 7 it is clear that one could improve the fit by using either a larger or a smaller  $D_2$  value. For each transition we have determined the value of  $D_2$  which would have provided the best fit to the data. The resulting  $D_2$  values, along with the best fit  $\chi^2/N$  value for each transition, are given in Table III. The uncertainties quoted in Table III reflect only the statistical errors in the measured tensor analyzing powers. It should be noted that for many of the transitions the best fit  $D_2$  value is inconsistent with the value derived from the sub-Coulomb data. In addition we see that the  $\chi^2/N$  values are uniformly larger than 1. These results would appear to indicate that for energies above the Coulomb barrier, DWBA calculations are no longer sufficiently accurate to allow a reliable determination of  $D_2$ .

#### V. DISCUSSION AND CONCLUSIONS

Measurements of the tensor analyzing powers have been presented for (d, t) reactions both above the below the Coulomb barrier. By making use of DWBA calculations we have extracted experimental values of  $D_2$  from the measurements. The parameter  $D_2$  is defined by Eq. (3) and contains information about *D*-state components of the triton wave function. We have argued that the DWBA theory should be most reliable for the sub-Coulomb



FIG. 6. Measurements of the tensor analyzing powers for  $(\tilde{d}, t)$  reactions on <sup>91</sup>Zr, <sup>119</sup>Sn, and <sup>208</sup>Pb for energies above the Coulomb barrier. The curves are DWBA calculations corresponding to  $D_2 = -0.279$  fm<sup>2</sup>.



FIG. 7. Measurements of the tensor analyzing powers for  ${}^{118}\text{Sn}(\overline{d},t){}^{119}\text{Sn}$ . The solid curves are DWBA calculations corresponding to  $D_2 = -0.279 \text{ fm}^2$ , while the dashed curves are calculations which include only the S state.

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Target	$E_d$ (MeV)	$E_x$ (MeV)	jπ	D <sub>2</sub> (fm <sup>2</sup> )	$\chi^2/N$
<sup>91</sup> Zr	7.5	0.0	5 +	$-0.331 \pm 0.009$	1.55
<sup>118</sup> Sn	12.0	0.0	$\frac{\frac{1}{1}}{\frac{1}{2}}$ +	$-0.278 \pm 0.010$	1.15
<sup>118</sup> Sn	12.0	0.16	$\frac{3}{2}^{+}$	$-0.270 \pm 0.025$	1.54
<sup>118</sup> Sn	12.0	1.02	$\frac{5}{2}^{+}$	$-0.228\pm0.006$	2.06
<sup>118</sup> Sn	12.0	1.18	$\frac{5}{2}$ +	$-0.238 \pm 0.009$	1.60
<sup>119</sup> Sn	7.5	0.0	$\frac{1}{2}^{+}$	$-0.360 \pm 0.013$	1.13
<sup>119</sup> Sn	9.0	0.0	$\frac{1}{2}^{+}$	$-0.396 \pm 0.009$	1.73
$^{208}$ Pb	12.3	0.0	$\frac{1}{2}$	$-0.254\pm0.013$	1.72
$^{208}\mathrm{Pb}$	12.3	0.57	$\frac{5}{2}$	$-0.328\pm0.020$	1.98
<sup>208</sup> Pb	12.3	0.90	$\frac{3}{2}$ -	$-0.233 \pm 0.006$	1.29

TABLE III. Best-fit  $D_2$  values for the (d,t) measurements obtained at energies above the Coulomb barrier.

transitions, and that consequently, the derived  $D_2$  values are most accurate in these cases. Our experimental results support this argument. For the sub-Coulomb reactions, the DWBA fits to the data are of good quality, and the  $D_2$  values obtained from different transitions (see Table I) are in reasonably good agreement. On the other hand, the  $D_2$  values for the measurements at higher energies (see Table III) are clearly not consistent to within the statistical errors.

The primary result of the present work is the empirical  $D_2$  value given in Eq. (4). This value is the average of the results obtained for the four sub-Coulomb transitions.

The quoted uncertainty for the final  $D_2$  value includes estimates of the systematic errors arising from the overall normalization uncertainty in the tensor analyzing power measurements and from the uncertainties in the optical model potentials. However, several possible sources of error have not been included. First, tensor terms in the deuteron optical model potential have been neglected in the DWBA calculations. The presence of tensor potentials has been indicated in a number of recent elastic scattering experiments (see, for example, Ref. 25). Although the effect of these potentials on the tensor analyzing powers are probably small for the sub-Coulomb transitions, it may not be completely negligible. Second, the DWBA calculations have been carried out in the local-energy approximation, in which the finite range of the n-d interaction is treated in an approximate way. Although the approximation is expected to be valid for reactions with low momentum transfer, exact DWBA calculations will be required to determine the importance of the

finite-range effects. Finally, systematic errors in the determination of  $D_2$  could result from the neglect of multistep processes in the DWBA calculation. For sub-Coulomb energies, conventional multistep processes (e.g., inelastic excitation of the target followed by neutron transfer) are probably unimportant since the coupling between various reaction channels is weak. Of somewhat greater concern is the effect of distortion (i.e., virtual excitation) of the deuteron and triton by the Coulomb field of the nucleus. In the DWBA calculations one assumes that at the point of transfer the internal wave functions of the deuteron and triton are just the free-projectile wave functions. Of course, distortion of the projectile wave functions will affect the determination of  $D_2$  only

functions will affect the determination of  $D_2$  only to the extent that the effects are spin dependent. Recently, Tostevin and Johnson<sup>26</sup> have calculated the effects of deuteron distortion for sub-Coulomb (d,p) reactions and found that the tensor analyzing powers change by only a few percent. Thus one might expect that these effects will be small for (d,t) reactions as well. As noted above, the consistency of our results

is quite poor for the reactions which have energies above the Coulomb barrier. To some extent, this is to be expected. However, it is disturbing that the extracted  $\boldsymbol{D}_{\rm 2}$  values change so rapidly with increasing energy. In particular, we note that for  ${}^{91}$ Zr and  ${}^{119}$ Sn, the derived  $D_2$  value increases in magnitude by 0.06 fm<sup>2</sup> or more as the energy is changed from 6.0 to 7.5 MeV. This behavior raises some uncertainty about the reliability of the 6 MeV  $D_2$  values. However, in spite of this problem, we believe (based primarily on the consistency of the sub-Coulomb results) that the 6 MeV values are correct. Of course, this conjecture can be tested by obtaining additional tensor analyzing power measurements at energies below those of the present experiment.

It is of interest to compare the empirical  $D_2$ value given in Eq. (4) with theory. Unfortunately, few useful calculations of  $D_2$  are available. Results have been obtained for a number of variational triton wave functions<sup>8,27</sup> but in all cases it is found that the variational wave functions do not behave properly in the asymptotic region. As a result, the calculated  $D_2$  values are not very meaningful. Kim and Muslim<sup>6</sup> have calculated the asymptotic normalization constants of the triton for a wave function obtained by solving the Faddeev equations. This calculation leads to a  $D_2$  value of  $-0.24 \text{ fm}^2$ . which agrees fairly well with the measured value. More recently, Ioannides et al.<sup>28</sup> have calculated the  $D_2$  values for the triton wave functions of Phillips and Roig,<sup>29</sup> and have obtained values between -0.20 and -0.22 fm<sup>2</sup>.

It is clear that additional theoretical work on various aspects of this subject would be valuable. In particular, it would be of interest to determine whether including finite-range effects, the effects of tensor forces, or the effects of projectile distortion in the DWBA calculations would lead to improved agreement between experiment and theory. Another important question which has not yet been answered is whether triton wave functions obtained from various nucleon-nucleon potentials lead to different  $D_2$  values. In addition, it would be interesting to determine whether  $D_2$  might be sensitive to the presence of three-body forces in the triton.

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