Isospin mixing in the $^{2}H(\alpha,\alpha p)n$ reaction

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We calculate the $^{2}H(\alpha,\alpha)^{2}H^{*}$ isospin forbidden transition in a nucleon-alpha impulse approximation which includes higher order terms. The results agree qualitatively with the interpretations of experiments which favor a departure from isospin conservation of a few percent at $E_{\alpha} \simeq 24$ MeV and of the order of 30% at $E_{\alpha} \simeq 13$ MeV. The interplay of Coulomb and spin-orbit effects causing the transition to the singlet deuteron is elucidated and the energy and angular dependence of the d^{*} production is presented. A comparison is made with the intermediate coupling shell model of ^{6}Li and isospin mixing in the $2^{+}(4.31, T=0)$ and $2^{+}(5.36, T=1)$ states is calculated to be of the order of 0.5%. This suggests that the production of the d^{*} occurs through a direct reaction.

[NUCLEAR REACTIONS Deuteron breakup by alphas, theoretical prediction of singlet deuteron production in $^{2}H(\alpha,\alpha p)n$; isospin violation explained by differences in n alpha and p alpha on shell T matrices.]

I. INTRODUCTION

In experiments with the observation of the d^{*} a stated goal there exist a series of conflicting results[1-5] on the presence of isospin symmetry breaking in the $^{2}H(\alpha,\alpha p)n$ reaction. However, most of these experiments are not directly comparable because of differing energies and angles. There are other experiments[6,7] where the d^{*} seemingly appears but no claims as to the observation of its production are made because the analyses of the data are based on a model[8] that neglects transitions to the $S=0$, $T=1$ S state of the n-p pair. (In this regard, see Fig. 9 of Ref. 7). Koike[9] has obtained good predictions for the d-α breakup spectrum for $E_{\alpha} < 42$ MeV through solving the Faddeev equations using separable s- and p-wave N-α potentials and a separable s-wave n-p potential. Since the n-α and p-α potentials are the same in the multiple scattering terms no transitions to the d^{*} state are generated. However, to better fit that part of the three body spectrum dominated by sequential decay through the $^{5}Li_{g.s.}$, the final p-α t matrix in the terms with p-α final state interaction (FSI) is approximated by an on-shell t matrix including Coulomb effects.[10]

The formation of d^{*} in the breakup of the deuteron is interesting because it cannot be interpreted in terms of a pure Coulomb effect since the intrinsic total spin of the np pair must also be changed. In order for such a transition to occur an effective isovector spin-orbit potential with respect to the α particle must be present. From the spectroscopic point of view the existence of such a term can be related to the difference in energy splitting between the $P_{3/2}$ and $P_{1/2}$ levels in the mirror nuclei ^{5}He and ^{5}Li. While the splitting is of the order of 6 MeV the difference is about 0.5 MeV. Nevertheless, we show below that in a shell model picture this difference can only produce an admixture of 0.5% $T=0$ into the $2^{+}(5.36)$ state and a corresponding admixture of $T=1$ into the $2^{+}(4.31)$ state of ^{6}Li. The subsequent admixtures of $^{1}D_{2}$ and $^{3}D_{2}$ are insufficient to mediate the transition between the triplet and singlet np states.

While the angular distribution of emerging d^*'s may be strongly affected "on resonance" we stress that a high probability of their production exists away from identifiable resonances. We demonstrate this through a simple scattering model which is restricted to use of on-shell nucleon-alpha T matrices. Our calculations are to be understood as an approximation to a multiple scattering approach in which all effects due to virtual excitations of the alpha particle are neglected. Similar ideas are also applicable to ^{2}H$(\pi,\pi p)n$. Our procedure is equivalent to summing the diagrams shown in Fig. 1 in an approximate way. Essentially, the recoil of the intermediate alpha is neglected, the propagator between the two successive N-alpha scatterings is replaced by an energy delta function (thus, intermediate np interactions are ignored), and the final state interactions for $E_{np} = 0$ are approximated by Jost function enhancement factors. The production of the d^* is presumably due to the Coulomb caused difference in the nuclear amplitudes for $n-\alpha$ and p-α scattering superimposed on the P-wave spin-orbit splitting. This type of process was first studied by Noble[11] as a mechanism for isospin breaking in ^{12}C$(d,\alpha)^{10}$B$(0^+,T=1)$ through the production of d^* in intermediate states.

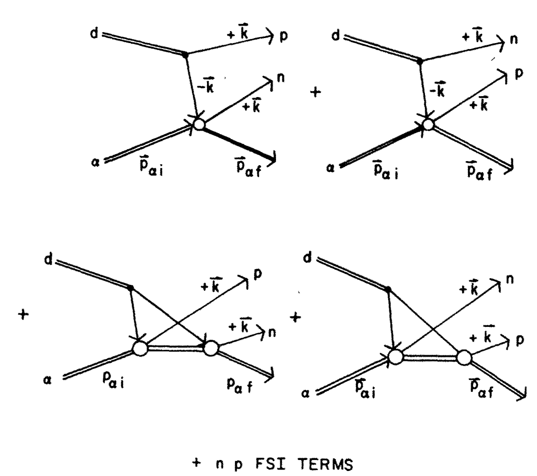

FIG. 1. Multiple scattering diagrams which are summed approximately in our model. The open circles represent T matrices for nucleon-alpha scattering, evaluated on-shell for the final nucleon-alpha relative energies. The final state correction is accomplished by multiplying the graphs shown by singlet and triplet enhancement factors.

Since the N-alpha T matrices are dominated by $\frac{3}{2}^-$ and $\frac{1}{2}^-$ resonances of ^{5}Li and ^{5}He our first order terms are a generalization of a process which Beam and Valkovic[12] suggested could allow the isospin breaking reaction ^{12}C$(d,d^*)^{12}$C to occur. The diagram they selected which contained a ^{13}N$^* + n$ intermediate state, was one of the triangle graphs evaluated by Aitchison and Kacser[13] in their study of final state interactions in three body final states. We have not used their formalism since there seems to be an ambiguity over the role of the N-alpha interaction.

Since Coulomb effects can now be incorporated in the Faddeev equations[14] the intricate interplay between the Coulomb potential and the nuclear spin-orbit potential necessary for d^* formation should soon be examined in a more fundamental way. Here we would like to exhibit the qualitative aspects such as the variation of order of magnitude estimates with respect to energy and angle.

II. SCATTERING MODEL

It is an emprical fact in ^{2}H$(\alpha,\alpha p)n$ that in the kinematic region of the three-body phase space where the proton can be considered a spectator particle there is a strong final state interaction of the neutron and α particle through the ground state of ^{5}He and the shape of the proton spectrum is reasonably well fit[4,6] by the simple impulse approximation for the triply differential cross section

$$\frac{d^3\sigma}{d\Omega_\alpha d\Omega_p dE_p} = \frac{2\pi}{v_0}\Psi_d{}^2(k)\left[\frac{d\sigma}{d\Omega}(\Omega_{n\alpha},E_{n\alpha})\right]\rho\,. \tag{1}$$

Here $d\sigma/d\Omega$ is the free $n-\alpha$ differential cross section, $\Psi_d{}^2(k)$ gives the probability for the proton to be moving with the observed final proton momentum within the deuteron, and ρ is the three-body phase space density. This formula cannot be used to describe the proton spectrum around $E_{np}=0$ because the neutron and proton momenta must appear on the same footing; neither the neutron nor the proton can be alone the spectator particle. Thus, to the first diagram in Fig. 1 where the proton is the spectator a second diagram must be added in which the neutron is the spectator and the proton undergoes a "specular" reflection such that its final momentum is exactly equal to the neutron's. This suggests the neutron and proton amplitudes must be added coheently, a procedure that follows from the impulse approximation in any case. In spite of the extended nature of the deuteron, the large P-wave amplitude for nucleon-alpha scattering implies that simultaneous n-α and p-α scattering cannot be ignored. This second order term contributes a large effective D-wave amplitude to d^* formation. Such an amplitude must be present because of the known even parity states of ^{6}Li relatively near to the $d+\alpha$ threshold. Such a term is represented by the third and fourth diagrams in Fig. 1.

Since isospin invariance is broken by the differing nucleon-alpha scattering amplitudes and not by the np interaction, a model which ignores this interaction except for initial state or final state rescattering should reproduce, in some average sense, the isospin breaking (d,d^*) reaction. We, therefore, neglect the np interaction in intermediate states. For two free nucleons incident onto a fixed scattering center the S matrix for the system factors. Thus, in a momentum basis (suppressing spins, for the moment)

$$\langle \vec{p}_{1f},\vec{p}_{2f}\,|S\,|\,\vec{p}_1,\vec{p}_2\rangle = \langle \vec{p}_{1f}\,|S\,|\,\vec{p}_1\rangle_1 \langle \vec{p}_{2f}\,|S\,|\,\vec{p}_2\rangle_2 \ , \quad (2)$$

where

$$\langle \vec{p}_f\,|S\,|\,\vec{p}\,\rangle = \delta(\vec{p}_f-\vec{p}\,) - 2\pi i\,\delta(E_f-E)\,t(\vec{p}_f\leftarrow\vec{p}\,) \ .$$

Here $t(\vec{p}_f\leftarrow\vec{p}_i)$ is the on the energy shell T matrix for nucleon scattering from the fixed cluster. For a moving scattering center Galilean invariance prescribes that all momenta undergo the transformation $\vec{p}\to\vec{p}-M\vec{V}$, where $\vec{V}$ is the velocity of the scattering center and M the nucleon mass. Restricting ourselves to S states the initial momentum space state of a deuteron at rest in the laboratory is

$$|\Psi_i\rangle = \int dp^3\psi_d(\vec{p}\,)\,|-\vec{p}\,\rangle_1\,|+\vec{p}\,\rangle_2 \ . \quad (3)$$

Since $\psi_d(\vec{p}\,)=\psi_d(-\vec{p}\,)$ the initial state is symmetric with respect to the exchange of the labels 1 and 2. Then the S matrix for the breakup of a deuteron by an incoming scattering center becomes

$$\begin{aligned}\langle \vec{p}_{1f},\vec{p}_{2f}\,|S\,|\,\psi_i\rangle &= \delta(\vec{p}_{1f}+\vec{p}_{2f})\psi_d(\vec{p}_{2f}) \\ &\quad - 2\pi i\{\,\delta[E(\vec{p}_{1f}-M\vec{V})-E(-\vec{p}_{2f}-M\vec{V})]\psi_d(\vec{p}_{2f})t_1(\vec{p}_{1f}-M\vec{V}\leftarrow\vec{p}_{2f}-M\vec{V}) \\ &\qquad + \delta[E(\vec{p}_{2f}-M\vec{V})-E(-\vec{p}_{1f}-M\vec{V})]\psi_d(-\vec{p}_{1f})t_2(\vec{p}_{2f}-M\vec{V}\leftarrow-\vec{p}_{1f}-M\vec{V}) \\ &\qquad - 2\pi i\int dp^3\psi_d(\vec{p}\,)\delta[E(\vec{p}_{1f}-M\vec{V})-E(-\vec{p}-M\vec{V})]\delta[E(\vec{p}_{2f}-M\vec{V})-E(\vec{p}-M\vec{V})] \\ &\qquad\quad \times t_1(\vec{p}_{1f}M\vec{V}\leftarrow-\vec{p}-M\vec{V})t_2(\vec{p}_{2f}-M\vec{V}\leftarrow\vec{p}-M\vec{V})\,\} \ .\end{aligned} \quad (4)$$

The first term is spurious because it violates overall energy conservation, the second and third terms constitute the usual impulse approximation, and the fourth term is the double scattering term that can generate effective D-wave resonances from nucleon-alpha P-wave scattering, so the full result of a calculation is given by the first and second order terms of the multiple scattering approximation. When the infinite mass scattering center is replaced by the alpha of mass $4M$ the S matrix no longer rigorously factors, even with the np interaction ignored, but we obtain an approximate formula by making the replacements

$$\begin{aligned}&\vec{p}_i-M\vec{V}\to\vec{p}_i-\vec{p}_{\alpha i}/4 \ , \\ &\vec{p}_f-M\vec{V}\to\vec{p}_f-\vec{p}_{\alpha f}/4 \ , \\ &E(\vec{p}-\vec{p}_\alpha/4)=2/5M(\vec{p}-\vec{p}_\alpha/4)^2 \ ,\end{aligned} \quad (5)$$

which follows from the assumption that the on-shell T matrices are functions of the relative velocities of the colliding particles.

For alpha energies of experimental interest the final integral in Eq. (4) [after incorporating the changes in Eq. (5)] can be simplified. The deuteron wave function decreases rapidly for momentum components larger than $p_B=(ME_B)^{1/2}$. Since $p_\alpha/4=(ME_\alpha/2)^{1/2}$ the momentum p inside the T-matrix elements can be approximated by zero and both T-matrix elements removed from under the integral. In order to remove one energy delta function outside the integral the other one is evaluated by retaining linear terms in p and carrying out the angular integration. The energy delta function outside the integral has for its argument $E(\vec{p}_{1f}-\vec{p}_{\alpha f}/4)+E(\vec{p}_{2f}-\vec{p}_{\alpha f}/4)-2E(-\vec{p}_{\alpha f}/4)$, consistent with our approximation. Associating each of the energy delta functions explicit or implicit in Eq. (4) with the correct energy conserving delta function for the breakup process our estimate for the T matrix becomes[15]

$$\begin{aligned}\langle \vec{p}_{1f},\vec{p}_{2f},\vec{p}_{\alpha} | T | \Psi,\vec{p}_{\alpha i}\rangle &= \psi_d(\vec{p}_{2f})t_1(\vec{p}_{1f}-\vec{p}_{\alpha f}/4\leftarrow -\vec{p}_{2f}-\vec{p}_{\alpha i}/4)\\ &+\psi_d(\vec{p}_{1f})t_2(\vec{p}_{2f}-\vec{p}_{\alpha f}/4\leftarrow -\vec{p}_{1f}-\vec{p}_{\alpha i}/4)-(2\pi)^2 i5M/4\cdot 4/\vec{p}_{\alpha i}\\ &\times t_1(\vec{p}_{1f}-\vec{p}_{\alpha f}/4\leftarrow -\vec{p}_{\alpha i}/4)t_2(\vec{p}_{2f}-\vec{p}_{\alpha f}/4\leftarrow -\vec{p}_{\alpha i}/4)\int_{p_0} dp\, p\psi_d(p) \ ,\end{aligned} \tag{6}$$

where

$$p_0=2\,|(\vec{p}_{2f}-\vec{p}_{\alpha f}/4)^2-p_{\alpha i}{}^2/16\,|/p_{\alpha i} \ .$$

In our application the region of three body phase space in which we are interested is that where the nucleons have essentially equal momenta. Then the T matrix can be further simplified to

$$\begin{aligned}\langle \vec{p}_{1f},\vec{p}_{2f},\vec{p}_{\alpha f} | T | \Psi_i,\vec{p}_{\alpha i}\rangle &= \psi_d(\vec{p}_{2f})[t_1(\vec{P}_f\leftarrow\vec{P}_i)+t_2(\vec{P}_f\leftarrow\vec{P}_i)\\ &-iX(2\pi)^2MP_f t_1(\vec{P}_f\leftarrow\vec{P}_i)t_2(\vec{P}_f\leftarrow\vec{P}_i)] \ ,\end{aligned} \tag{7}$$

where $\vec{P}_i$ and $\vec{P}_f$ are the initial and final combinatins of particle momenta proportional to the relative velocities,

$$\vec{P}_i=\vec{p}_i-\vec{p}_{\alpha i}/4, \ \vec{P}_f=\vec{p}_f-\vec{p}_{\alpha f}/4 \ , \tag{8}$$

and X is a dimensionless parameter which varies as $\psi_d(\vec{p}_f)^{-1}$.

At this point it is convenient to replace the T-matrix elements by scattering amplitudes, the relation between the two being

$$t(\vec{p}_f\vec{p}_i)=-1/(2\pi)^2 5/4MF(\Omega_{fi}) \ . \tag{9}$$

Since we are dealing with spin $\frac{1}{2}$ nucleons scattering from a spinless target the spin dependent proton and neutron amplitudes ($1\rightarrow p, 2\rightarrow n$) are of the form[16]

$$F_{p,n}=f_{p,n}(\theta)+ig_{p,n}(\theta)\sigma\cdot\hat{n} \ . \tag{10}$$

The combinations of amplitudes which appear implicitly in Eq. (7) are

$$\begin{aligned}F_1+F_2&=(f_p+f_n)+i/2(g_p+g_n)\Sigma\cdot\hat{n}\\ &+i/2(g_p-g_n)\Delta\cdot\hat{n}\end{aligned} \tag{11a}$$

and

$$\begin{aligned}F_1F_2&=f_pf_n-g_pg_n(\vec{\sigma}_p\cdot\hat{n})(\vec{\sigma}_n\cdot\hat{n})\\ &+i/2(f_ng_p+f_pg_n)\Sigma\cdot\hat{n}\\ &+i/2(f_ng_p-f_pg_n)\vec{\Delta}\cdot\hat{n} \ .\end{aligned} \tag{11b}$$

The two combinations of spin vectors are

$$\vec{\Sigma}=\vec{\sigma}_p+\vec{\sigma}_n, \ \vec{\Delta}=\vec{\sigma}_p-\vec{\sigma}_n \ . \tag{12}$$

The $\vec{\Delta}$ that appears in both the first order and second order terms induces the transition between the triplet and singlet deuteron. It is an isovector spin-flip term that is nonzero because of the differing nuclear amplitudes for protons and neutrons scattering from ^{4}He.

Our formalism enables us to calculate the amplitudes for breakup of the deuteron into either a triplet or singlet free nucleon pair with zero relative momentum. In the standard way we approximately take into account rescattering by introducing enhancement factors S and T for the singlet or triplet final state. The density matrix ρ_f for the process can then be written as

$$\begin{aligned}\rho_f&=\left[\frac{1-\vec{\sigma}_p\cdot\vec{\sigma}_n}{4}S+\frac{3+\vec{\sigma}_p\cdot\vec{\sigma}_n}{4}T\right]M\rho_i\tilde{M}\\ &\times\left[\frac{1-\vec{\sigma}_p\cdot\sigma_n}{4}S^*+\frac{3+\vec{\sigma}_p\cdot\sigma_n}{4}T^*\right] \ ,\end{aligned} \tag{13}$$

where the initial density matrix is

$$\rho_i=(3+\vec{\sigma}_p\cdot\vec{\sigma}_n)/12 \ . \tag{14}$$

S are T are the enhancement factors,[17,18] typically expressed in terms of the singlet (triplet) scattering lengths and effective ranges. The scattering matrix M is given by

$$M = \psi_d(p_f)\{ F_p + F_n + i5\chi P_f/4\, F_p F_n \}$$
$$\equiv f_A + i g_A \Sigma \cdot \hat{n}/2 + f_B(\vec{\sigma}_p \cdot \hat{n} \vec{\sigma}_n \cdot \hat{n})$$
$$+ i g_B \vec{\Delta} \cdot \hat{n}/2 \ . \qquad (15)$$

The new amplitudes in Eq. (15) are obtained directly from Eqs. (7), (9), and (11). Both first and second order terms appear in all coefficients except f_B which is directly proportional to χ.

The triply differential breakup cross section is directly proportional to the trace of ρ_f;

$$d^3\sigma/d\Omega_\alpha d\Omega_p dE_p$$
$$\propto |T|^2\{ |f_A|^2 + |f_B|^2 + 2/3[|g_A|^2 + \mathrm{Re}(f_A f_B^*)]\} + 1/3 |S|^2 |g_B|^2 . \qquad (16)$$

III. COMPARISON TO EXPERIMENT

Rausch *et al.*[4] have presented an analysis of breakup data for incoming alpha of $E_\alpha = 23.7$ MeV near $E_{np} = 0$ employing a formula slightly more complicated than that implied by Eq. (16) namely,

$$d^3\sigma/d\Omega_\alpha d\Omega_p dE_p = C_T{}^2 |T|^2 + C_S{}^2 |S|^2 + C_{\mathrm{He}}{}^2 |\mathrm{He}|^2 , \qquad (17)$$

where the C's are constants and $|\mathrm{He}|^2$ is a final state enhancement factor for the ^{5}He ground state. Since the peak in the proton spectrum due to the last term is far removed from the kinematic region of E_p for which $E_{np} = 0$ the term when treated additively as in Eq. (17) contributes a slowly varying background. The authors state that the experimental value of $C_S{}^2/C_T{}^2$ is little changed if the analysis is carried out without the third term but with the $|\mathrm{He}|^2$ final state function included multiplicatively in the first two. A similar analysis of their new breakup experiment performed with incoming alphas of 12.87 MeV has been presented by Bruno *et al.*[5] Once again, the three terms were treated additively. At this point, we should remark that our Eq. (16) predicts a value for $C_S{}^2/C_T{}^2$ given by

$$C_S{}^2/C_T{}^2 = \frac{\frac{1}{3}|g_B|^2}{|f_A|^2 + |f_B|^2 + \frac{2}{3}[|g_A|^2 + \mathrm{Re}(f_A f_B^*)]} . \qquad (18)$$

It is more consistent with including scattering from ^{4}He multiplicatively than additively, but the multiplicative analysis of Rausch *et al.* is still not directly comparable. Both proton and neutron resonances are included in Eq. (18) and the angular functions having their origin in nucleon-alpha scattering are different for the numerator as compared to the denominator. In addition, both differ from the free nucleon-alpha amplitude, even for an isolated $P_{3/2}$ resonance.

As the initial step in comparing to experiment we have evaluated Eq. (18) using a zero range deuteron wave function. The neutron and proton nuclear phase shifts were taken from the work of Stammbach and Walter[19] and only the nuclear amplitudes were included. The Coulomb factor $e^{2i\sigma_l}$ multiplying each partial wave in the p-alpha amplitude was set equal to one. We make the argument that the neutron and proton accumulate a common Coulomb phase distortion in the initial d and final

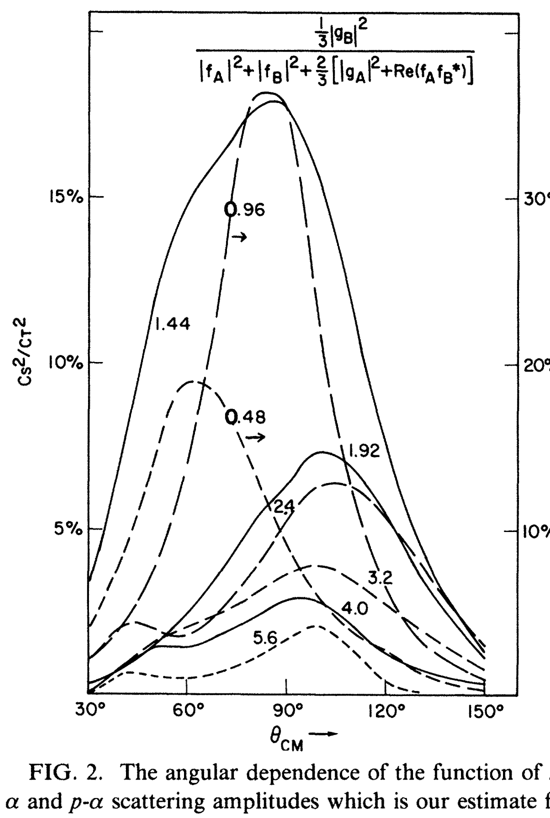

FIG. 2. The angular dependence of the function of n-α and p-α scattering amplitudes which is our estimate for the ratio of singlet to triplet strength around $E_{np} = 0$ in the $\alpha + d$ breakup reaction. The curves are calculated from Eq. (18) using a zero range deuteron model. The numbers on each curve refer to the nucleon alpha internal energies at which the phase shifts were taken from Ref. 19; only nuclear phase shifts have been included in the amplitudes. For $E_{np} = 0$ twice the listed numbers equals the excitation energy in ^{6}Li relative to the three body threshold.

d^* so that only phase differences coming from the interaction region are significant. The energy at which the scattering amplitudes were evaluated corresponds to the final nucleon-alpha relative energy. (We note that the d^*-alpha relative energy is simply twice this quantity.) The resulting curves labeled by the nucleon-alpha relative energies are shown in Fig. 2. The peak percentages rise from less than 2% at 5.6 MeV (11.2 MeV d^*-alpha energy) to about 35% for 0.96 MeV (1.92 MeV d^*-alpha energy).

In comparing to experiment, the five body center of mass scattering angle has been calculated in two different ways: firstly by assuming the nucleon struck is moving with a momentum $-\vec{P}_{1f}$, secondly, by assuming that the nucleon is at rest. In Table I a comparison of theory and experiment is made. In both instances the experimental ratio was obtained by a minimum χ^2 procedure using Eq. (17) to fit the three body breakup cross section near $E_{np} = 0$. In both cases qualitative agreement is obtained as far as the magnitude of the effect is concerned but the angular dependence is wrong at the lower energy.

The discrepancy in the angular distribution at low energies seems to be due to an over emphasis of the negative parity amplitudes in both the triplet to singlet and triplet to triplet breakup reactions. In (d,d^*) the partial wave amplitudes contributing are 1^-, 2^+, 3^-, etc. The 1^- contributes a $\sin\theta$, the 2^+ a $\sin\theta\cos\theta$ angular dependence. The numerator of Eq. (18) which is proportional to the (d,d^*) differential cross section is plotted for different nucleon-alpha relative energies in Fig. 3. One sees a systemtic pattern of 1^--2^+ interference throughout the energy range studied. Our calculated angular distribution is in fact strongly backward peaked at $E_{N-\alpha} = 1.44$ MeV, the relevant energy for the experiment of Ref. 5. However, the angular distribution for the (d,d') breakup (not shown) is also strongly peaked in the backward direction, much more than revealed[20] in elastic $^2H(\alpha,\alpha)^2H$ angular distribution at comparable incident alpha energies. In taking the ratio of the required cross sections to form $C_S{}^2/C_T{}^2$ this backward peaking in the denominator pushes the peak in the ratio forward to 90°, as plotted in Fig. 2.

The experiment reported in Ref. 5 was carried out at an alpha energy corresponding to an excitation energy in ^{6}Li on the high side of the $2^+(5.36, T = 1)$ resonance. Their results (see Table I) imply that the differential cross section for d^* formation is almost entirely $(\sin\theta\cos\theta)^2$ which is to be expected if there is significant isospin mixing between the $2^+(4.31, T = 0)$ and the $2^+(5.36, T = 1)$ states. Assuming that the angular dependence of the triplet breakup is the same as that of elastic (d,d) at the same incident energy the angular dependence of $C_S{}^2/C_T^2)$ is understandable, as is also the null result for d^* formation of Ref. 1 since $(\theta_{N\alpha}^{\text{c.m.}} \simeq 90°)$. We consider the problem of isospin mixing within the context of an intermediate cou-

TABLE I. Calculated values of $C_S{}^2/C_T{}^2$ (in percent) from Eq. (18) of the text. The two N-α c.m. scattering angles for each experimental condition are calculated (i) assuming the struck nucleon is at rest in the deuteron and (ii) assuming the struck nucleon has a momentum equal and opposite to the final laboratory momentum of the other nucleon. The first angle corresponds to the production angle for a d^* of zero binding energy. The calculation has been done using phase shifts from Ref. 19, $E_{N\alpha}$(lab) = 4.0 and 1.8 MeV, respectively. Only nuclear phase shifts have been included.

E_α (MeV)	Relative N-α energy (MeV)	$\theta_{\text{c.m.}}$	Theoretical	Experimental
23.7	3.2	73°	2.6	3.9[a]
		93°	3.8	
23.7	3.2	84°	3.2	4.5[b]
		104°	3.8	
12.87	1.44	77°	17.3	$2.3 \pm .8$[c]
		93°	17.0	
12.87	1.44	138°	3.0	19 ± 3[d]
		147°	1.7	

[a]Reference 4, Fig. 6, $\theta_p = 47°$, $\theta_\alpha = 20°$, $E_p = 3.2$ MeV.
[b]Reference 4, Fig. 6, $\theta_p = 43°$, $\theta_\alpha = 22°$, $E_p = 4.1$ MeV.
[c]Reference 5, $\theta_p = 39°$, $\theta_\alpha = 17.5°$, $0 \leq E_{pn} \leq 0.59$ MeV.
[d]Reference 5, $\theta_p = 17°$, $\theta_\alpha = 17.5°$, $0 \leq E_{pn} \leq 0.6$ MeV.

pling model of ^{6}Li in the next section and find that the shell model predicts minimal isospin mixing between the two adjacent 2^+ states, leaving open the problem of satisfactorily explaining the data.

IV. RELATION TO SHELL MODEL

As we have seen, our model which ignores the residual np interaction, predicts large isospin mixing in the 1^- and 2^+ states. Since the 2^+ partial wave is strongly influenced by discrete resonances which are reasonably well described by the shell model we have reexamined the question of isospin mixing in the intermediate coupling model. The mixing occurs because of the differences in the single particle energies of the spin-orbit split P states of the proton and neutron. One sees this by writing the sum of the single particle energies of the np pair as

$$E(1,2)=\frac{1}{2(2l+1)}\sum_{i=1,2}\{[(l+1)\Sigma^+ + l\Sigma^-]+[\Sigma^+ - \Sigma^-]\vec{L}_i\cdot\vec{\sigma}_i+[(l+1)\Delta^+ + l\Delta^-]\tau_{iz} + [\Delta^+ - \Delta^-]\vec{\sigma}_i\cdot\vec{L}_i\tau_{iz}\}\;, \tag{19}$$

where

$$\Sigma^{\pm}=\epsilon_l^{\pm}(p)+\epsilon_l^{\pm}(n),\quad \Delta^{\pm}=\epsilon_l^{\pm}(p)-\epsilon_l^{\pm}(n)\;.$$

The ($\pm$) signs refer to $j=l\pm\frac{1}{2}$ states, respectively.

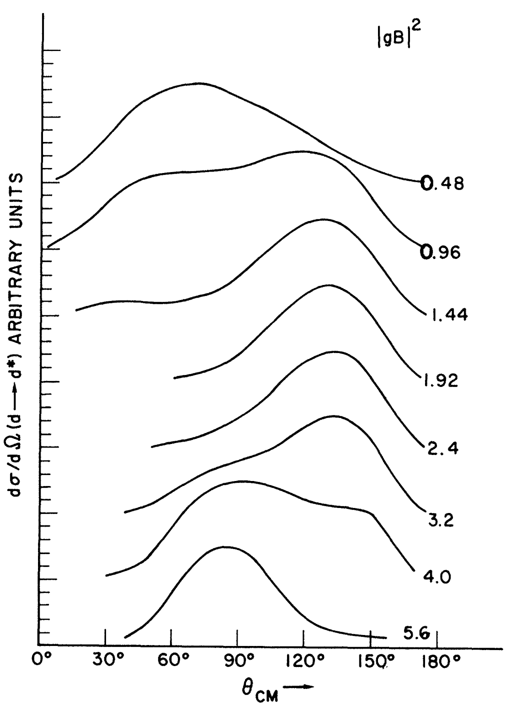

FIG. 3. The function $|g_B|^2$ in arbitrary units, our estimate of the angular distribution for the production of singlet deuterons with zero binding energy. The cross section at these energies is principally due to the sum of 1^- and 2^+ amplitudes with $\sin\theta$ and $\sin\theta\cdot\cos\theta$ distributions, respectively.

As far as 2^+ states are concerned, there are two antisymmetric j-j states ($T=1$), $|\frac{3}{2},\frac{3}{2}\rangle$ and $1/\sqrt{2}\{|\frac{1}{2},\frac{3}{2}\rangle - |\frac{3}{2},\frac{1}{2}\rangle\}$, but only one symmetric j-j state ($T=0$), $1/\sqrt{2}\{|\frac{1}{2},\frac{3}{2}\rangle + |\frac{3}{2},\frac{1}{2}\rangle\}$ where the first entry in each ket refers to the proton. The last term in Eq. (19) couples the two $(\frac{1}{2},\frac{3}{2})$ states of different isospin.

Barker[21] has carried out an intermediate coupling calculation of the $A=6$ low-lying spectrum. He used experimental energies to fix shell model matrix elements and obtained as his output energy predictions of unobserved levels as well as detailed wave functions for all levels. However, his results for the 2^+ spectrum may not be completely reliable since he obtained abnormally large odd-parity matrix elements for the np pair. In addition, his prediction for the structure of the 2^+ (5.36, $T=1$) is not in satisfactory agreement with the inelastic electron form factor obtained by Neuhausen and Hutcheon.[22] We have carried out a simplified calculation which determines the 2^+ wave functions by adjusting the effective even parity np matrix elements to reproduce the 4.31 and 5.36 MeV energy levels and by setting $\langle {}^3P_2|V|{}^3P_2\rangle=0$. The unperturbed particle-particle states of pure isospin are obtained from the isoscalar part of Eq. (19). Using the resonance energies of Stammbach and Walter[19] which parametrize the proton and neutron p-wave phase shifts the unperturbed energies are

$$(\tfrac{3}{2},\tfrac{3}{2}T=1);\quad E(1,2)=3.03\text{ MeV}\;,$$
$$(\tfrac{1}{2},\tfrac{3}{2}T=0,1);\quad E(1,2)=8.76\text{ MeV}\;. \tag{20}$$

The energies are taken relative to the alpha + n + p threshold which is at an excitation energy of 3.70 MeV in ^{6}Li. The sizes of the two nonzero matrix elements

$$\langle {}^3D_2 | V | {}^3D_2 \rangle = -8.10 \text{ MeV} ,$$
$$\langle {}^1D_2 | V | {}^1D_2 \rangle = -2.94 \text{ MeV} , \tag{21}$$

are very reasonable. Their introduction has the effect of lowering the $|\frac{1}{2},\frac{3}{2}T=0\rangle$ to 0.61 MeV (4.31 excitation in ^{6}Li) and slightly lowering the $|\frac{3}{2},\frac{3}{2}T=1\rangle$ to 1.66 MeV (5.36 excitation in ^{6}Li).

The last term in Eq. (19) couples the two $(\frac{1}{2},\frac{3}{2})$ states of different isospin but cannot couple the $(\frac{1}{2},\frac{3}{2},T=0)$ to the $(\frac{3}{2},\frac{3}{2},T=1)$. While the isovector spin-orbit matrix element is appreciable,

$$\frac{(\Delta^+ - \Delta^-)}{2(2l+1)} \left\langle \frac{1}{2},\frac{3}{2}T=1 \left| \sum_{i=1,2} \vec{\sigma}_i \cdot \vec{\mathrm{L}}_i \tau_{iz} \right| \frac{1}{2},\frac{3}{2}T=0 \right\rangle = 0.29 \text{ MeV} , \tag{22}$$

very little mixing between the 4.31 and 5.36 MeV states is predicted because the latter state remains about 93% $(\frac{3}{2},\frac{3}{2})$ which cannot be coupled to the former state directly by the perturbation. In the *LS* basis the three 2^+ states are predicted to be

$$|4.31\rangle = |{}^3D_2\rangle ,$$
$$|5.36\rangle = 0.769\,|{}^1D_2\rangle + 0.639\,|{}^3P_2\rangle , \tag{23}$$
$$|10.88\rangle = 0.639\,|{}^1D_2\rangle - 0.769\,|{}^3P_2\rangle ,$$

and the 5.36 state is in good agreement with the (e,e') analysis. The isospin mixture in the lower two states can be parametrized by

$$|4.31,2^+\rangle = (1-\alpha^2-\beta^2)^{1/2}\,|4.31\rangle - \alpha\,|5.36\rangle - \beta\,|10.88\rangle , \tag{24}$$
$$|5.36,2^+\rangle = (1-\alpha^2)^{1/2}\,|5.36\rangle + \alpha\,|4.31\rangle .$$

Using first order perturbation theory the matrix element of Eq. (22) and the states of Eq. (23) are consistent with

$$\alpha = 0.071, \quad \beta = 0.0425 . \tag{25}$$

The isospin admixture, of the order of 0.5%, is consistent with the small upper limits placed on the deuteron decay width of the ^{6}Li $(2^+,5.36)$ found by a number of researchers[23–25] who formed the state through the ^{7}Li(^{3}He,α)^{6}Li* reaction or through ^{6}Li $(p,p\alpha)^2$H.[26] However, it should be noted that one group[27] has reported a positive result for the detection of the deuteron decay of the 5.36 MeV state and Noble[28] has proposed isospin impurities of the order of 5% in the observed 2^+ states in order to explain isospin breaking in ^{12}C(d,α)^{10}B(0^+,$T=1$). He pointed out that the excitation function for the latter reaction seems to favor the formation of ^{6}Li* as an intermediate step. In light of the observation of d^* formation[4,5] and our prediction of minimal mixing of isospin states in the 2^+ states his first suggested mechanism,[11] in so far as it is independent, may well be the dominant one in the ^{12}C(d,α)^{10}B(0^+,$T=1$) reaction.

V. DISCUSSION

We have shown that our scattering model satisfactorily describes the relative strengths of the ^{2}H(α,α)^{2}H* and ^{2}H(α,α)^{2}H′ components of the breakup cross section near $E_{np}=0$. The relevant amplitude for d^* formation, $g_B = \psi_d(p_f)\{g_p - g_n + i\chi(f_n g_p - f_p g_n)\}$ [from Eq. (15) and earlier equations], is large at low energies because of the difference in the p-α and n-α spin-flip amplitudes dominated by p-wave resonances. While the first order term in g_B by itself predicts large values of $C_S{}^2/C_T{}^2$ of the observed order of magnitude the second order term dominates numerically because of the low energy at which the resonances occur (in contrast to ^{2}H(π,πp)n, for example).

Earlier in the paper it was stressed that $\psi_d(p_f)\chi$ is a constant with respect to the angle of production of the d^* while $\psi_d(p_f)$ itself falls off rapidly at backward angles. The importance of the second order term is coroborated by the results of several experimental groups[5,29] in fitting the shape of the ^{5}He peak seen in ^{2}H(α,αp)n. These authors find that the spectrum is best fit by omitting a $\psi_d(p_f)$ factor. In addition we find that the d^* and d' angular distributions take on more reasonable shapes with the inclusion of the term.

There seems to be a theoretical contradiction between the small amount of isospin mixing in the 2^+ states that comes out of our simple shell model calculation and the large value of C_S^2/C_T^2 obtained in our scattering model which has a large contribution from the 2^+ partial wave. This situation is mirrored by the apparent experimental contradictions between the (e,e') and the ^{7}Li(^{3}He,^{4}He)^{6}Li(5.36,$T=1$) results on one hand and the ^{2}H(α,αp)n on the other. We stress that our shell model calculation would normally be thought to be directly relevant to a d to d^* transition due to isospin mixing in a compound state. The usual approach of reaction theories describing isospin mixing effects in nuclear reactions at excitation energies where isolated resonances occur is to ascribe the mixing to Coulomb coupling between localized compound states. For example, the two level formulas exhibited by Wildermuth and Tang[30] in connection to this topic have the property that the d to d^* coupling goes to zero as the coupling matrix between the compound states goes to zero.

In our opinion, the large measured values of C_S^2/C_T^2 in spite of the small estimated value of mixing in the compound 2^+ states can be taken as empirical evidence for the direct nature of the $^2H(\alpha,\alpha)^2H^*$ reaction. The direct nature of the scattering model appears in an examination of the energy behavior of the 1^- and 2^+ partial waves at an energy where the effect of the $\frac{3}{2}^-$ state of ^3He can be cleanly isolated in g_B. Both partial waves are large and dominated by the $\frac{3}{2}^-$ state affecting g_n. This is reminiscent of the "psuedoresonances"[31] which occur in a large number of partial waves in pion-deuteron elastic scattering, all of which are due to the fundamental pion-nucleon resonance. Within the context of R-matrix theory isospin mixing could quite likely be included in the compound states if channel radii much larger than those routinely used in data fitting were introduced. A radius sufficiently large so that no interaction exists between the deuteron and alpha would certainly be of the order of 8 fm. This would be large enough to contain the interaction regions of the important $^5He + p$ and $^5Li + n$ clusters. We would anticipate that in such a large volume isospin mixing could occur; here we have in mind the concept of "threshold states" introduced by Baz.[32]

The connection between our scattering model and the concept of threshold states can be seen clearly in the expression given above for g_B. This amplitude attains its maximum when $g_n >> g_p$, this condition being satisfied when the n-energy corresponds to the ^5He ground state. It is appropriate to describe this situation in terms of the dominance of the $^5He + p$ cluster over the $^5Li + n$ cluster, a situation implying large isospin mixing. We note that the isospin nonconservation could even appear in analyses of $d + \alpha$ elastic scattering. It is possible that better R-matrix fits could be obtained to the experimental phase shifts if the reduced widths for the $^5He + p$ and $^5Li + n$ channels are allowed to be different.

There may still be some question about the identification of the d^* in the breakup experiments. The experimental determination of C_S^2/C_T^2 depends on the interpretation that the small peak seen around $E_{np} = 0$ is due to the d^* state. The reaction amplitudes for a reaction going from a channel spin one (d) to a channel spin zero (d^*) pair of fragments are particularly simple because of the restrictions $J = L_{d^*} = L_d$. Then the analyzing tensor elements iT_{11} and T_{21} are zero and, for example, the reaction cross section vanishes if the initial deuterons are vector polarized along the normal to the scattering plane.[33] In this case the d^* bump should disappear if it is indeed due to singlet dueteron production.

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