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Relativistic effects in the magnetic structure function of deuteron

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The deuteron magnetic structure function $B(q^2)$ is calculated in the one-photon exchange approximation using the recently described approach based on the Bethe-Salpeter equation. In this approach contributions to the deuteron electromagnetic form factors when the spectator or the struck nucleon is on-mass-shell appear with equal weight. Results obtained without any free parameters and without including any meson-exchange currents are in good agreement with the available data. Predictions are made for $B(q^2)$ up to $q^2=8$ (GeV/c)^2 and they show a minimum at $q^2 \simeq 2$ or 2.5 (GeV/c)^2 depending on the choice of the *NN* interaction. This is in sharp contrast with the exchange-current calculations which do not exhibit a minimum for any $q^2 \leq 8$ (GeV/c)^2.

[NUCLEAR REACTIONS ^{2}H(e,e), $q^2 \leq 8$ (GeV/c)^2, calculated $B(q^2)$, compared with experimental data.]

We recently calculated[1] the deuteron structure function, $A(q^2)$, and the tensor polarization of recoil deuterons in *ed* elastic scattering, $T(q^2)$, for $q^2 \leq 6$ (GeV/c)^2. Results for $A(q^2)$ were in good agreement with the experimental data, and predictions for $T(q^2)$ were significantly different from the standard results. Experimental data for $A(q^2)$ are now available up to $q^2=8$ (GeV/c)^2.[2] The last two points at $q^2=6$ and 8 (GeV/c)^2 are also in good agreement with our calculations. Experimental data for $B(q^2)$ on the other hand are available only up to $q^2=1$ (GeV/c)^2.[3–5] Large-angle electron scattering experiments are being planned to extend this information to higher q^2. In view of the possibility of measurement in the near future of $B(q^2)$ in the region $q^2>1$ (GeV/c)^2 [especially in the region $q^2 \sim 2$ (GeV/c)^2 where the experimental data are expected to throw new light on the importance of the exchange-current effects] we present here our predictions of the quantity $B(q^2)$ in the region $q^2 \leq 8$ (GeV/c)^2 based on the approach described in Ref. 1.

Calculation of deuteron electromagnetic form factors starts with the relativistic triangle diagram (Fig. 1). In this calculation one encounters integrals over deuteron momentum-space wave functions. In order to evaluate these wave functions it is necessary to make a Lorentz transformation from a frame in which the deuteron is moving (Fig. 1) to the frame in which it is at rest. In Ref. 6 this was done by taking the spectator nucleon in Fig. 1 on-mass-shell. In Ref. 1 it was shown that this amounts to assuming a rather slowly varying deuteron wave function. In reality the rapid variation of the momentum space deuteron wave function makes it necessary to treat the diagram in Fig. 1 more carefully. Indeed the two prescriptions, namely, taking the spectator or the struck N on-mass-shell lead to the evaluation of the deuteron wave functions at momenta which differ substantially from each other even at $q^2=2$ (GeV/c)^2.[1] Considering the sensitivity of the deuteron wave function these two prescriptions naturally give rise to quite different results for $A(q^2)$ as shown in Ref. 1. Relativistic calculation with spectator (struck) N on-mass-shell results in $A(q^2)$ which is lower (higher) than that obtained nonrelativistically, because these two prescriptions lead to *pn* relative momenta in the deuteron rest frame which are, respectively, higher and lower than the nonrelativistic value.[1]

In Ref. 7 it is further shown that if one treats the singularities of the deuteron vertex function in the framework of the (relativistic) Bethe-Salpeter equation using the ladder approximation then the deuteron form factors can be written in such a way that the contributions when the spectator or the struck N

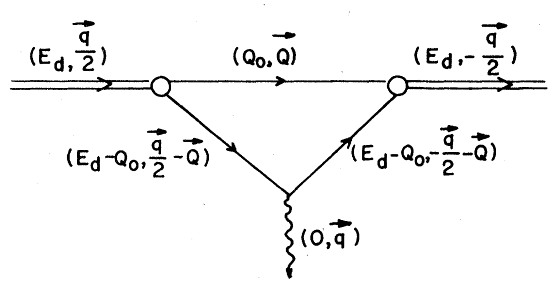

FIG. 1. Relativistic triangle diagram for the deuteron form factor in the Breit frame.

is on-mass-shell appear with equal weight. $A(q^2)$ calculated using the above method was in very good agreement with the experimental data.[1] In this paper we apply the above method to calculate the magnetic structure function $B(q^2)$ of deuteron.

Before we present the numerical results, some of the important equations are summarized below. The quantity $B(q^2)$ is obtained from *ed* elastic scattering data using

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}}\left(A(q^2)+B(q^2)\tan^2\frac{\theta}{2}\right) .$$

Following the notation of Ref. 8 we write

$$B(q^2)=4\eta(1+\eta)[C_E^1(q^2)G_E^S(q^2)+C_M^1(q^2)G_M^S(q^2)]^2/3 ,$$

where $\eta\equiv q^2/(4M_D^2)$, M_D is the deuteron mass, $G_{E,M}^S$ are nucleon isoscalar electromagnetic form factors, and

$$C_E^1(q^2)=3M_D(I_2+I_4)/(4M_N) ,$$

$$C_M^1(q^2)=M_D[I_1-(I_2-I_4)/2+I_3/\sqrt{2}]/M_N .$$

Here M_N is the nucleon mass and

$$I_1\equiv\int U\left(\left|\vec{Q}-\frac{\vec{q}}{4}\right|\right)U\left(\left|\vec{Q}+\frac{\vec{q}}{4}\right|\right)\frac{d\vec{Q}}{4\pi} ,$$

$$I_2\equiv\int W\left(\left|\vec{Q}-\frac{\vec{q}}{4}\right|\right)W\left(\left|\vec{Q}+\frac{\vec{q}}{4}\right|\right)P_2(Z_{+-})\frac{d\vec{Q}}{4\pi} ,$$

$$I_3\equiv\frac{1}{2}\int\left[U\left(\left|\vec{Q}-\frac{\vec{q}}{4}\right|\right)W\left(\left|\vec{Q}=\frac{\vec{q}}{4}\right|\right)P_2(Z_+)+U\left(\left|\vec{Q}+\frac{\vec{q}}{4}\right|\right)W\left(\left|\vec{Q}-\frac{\vec{q}}{4}\right|\right)P_2(Z_-)\right]\frac{d\vec{Q}}{4\pi} ,$$

and

$$I_4\equiv\frac{1}{2}\int W\left(\left|\vec{Q}-\frac{\vec{q}}{4}\right|\right)W\left(\left|\vec{Q}+\frac{\vec{q}}{4}\right|\right)(1-3Z_+Z_-Z_{+-})\frac{d\vec{Q}}{4\pi} .$$

In the above equations U and W are S and D state wave functions of the deuteron,

$$Z_\pm\equiv\cos\left(\vec{Q}\pm\frac{\vec{q}}{4},\vec{q}\right)$$

and

$$Z_\pm\equiv\cos\left(\vec{Q}+\frac{\vec{q}}{4},\vec{Q}-\frac{\vec{q}}{4}\right) .$$

The quantities $\vec{Q}\pm(\vec{q}/4)$ appearing in the integrals given above are in *pn* relative momenta at the right and left hand vertices in Fig. 1, and are to be evaluated in the appropriate deuteron rest frame after performing a Lorentz transformation. In the following the momentum of the spectator N in the deuteron rest frame (which equals the *pn* relative momentum in the deuteron rest frame) is denoted by $\vec{Q}'\equiv(Q_x,Q_y,Q_z')$. It can be shown that if the spectator N is on-mass-shell then

$$Q_z'=Q_z'\pm(1)\equiv Q_z\left(1+\frac{\vec{q}^{\,2}}{16M_N^2}\right)^{1/2}\pm\frac{q}{4}\left(1+\frac{\vec{Q}^{\,2}}{M_N^2}\right)^{1/2} ,$$

and if the struck N is on-mass-shell then

$$Q_z'=Q_z'\pm(2)\equiv\left(Q_z\pm\frac{q}{2}\right)\left(1+\frac{\vec{q}^{\,2}}{16M_N^2}\right)^{1/2}\mp\frac{q}{4}\left(1+\frac{\left(\frac{\vec{q}}{2}\pm\vec{Q}\right)^2}{M_N^2}\right)^{1/2} .$$

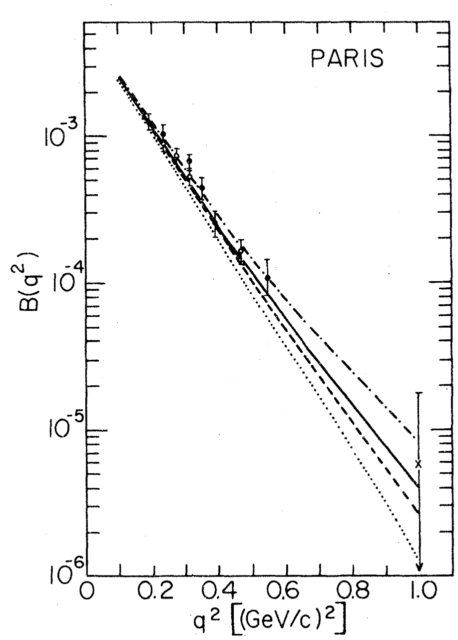

FIG. 2. Results of the present calculation compared with the experimental data of Refs. 3 (●), 4 (○), and 5 (×). Dashed curve corresponds to the nonrelativistic calculation, and dotted and dot-dashed curves to relativistic calculations with spectator and struck nucleons on-mass-shell, respectively. Solid curve also represents a relativistic calculation and is based on the theory described in Refs. 1 and 7. All curves are based on the Paris *NN* interaction (Ref. 9).

Here the upper (lower) signs correspond to the right (left) vertex and the z axis is taken along $\vec{q}$. In Ref. 7 it has been shown that the wave functions $U(\vec{Q} \pm \vec{q}/4)$ and $W(\vec{Q} \pm \vec{q}/4)$ appearing in the integrals given above should be replaced by

$$\tfrac{1}{2}[U(Q_x,Q_y,Q'_{z\pm}(1)) + U(Q_x,Q_y,Q'_{z\pm}(2))]$$

and

$$\tfrac{1}{2}[W(Q_x,Q_y,Q'_{z\pm}(1)) + W(Q_x,Q_y,Q'_{z\pm}(2))] \ ,$$

respectively. This result does not mean that the spectator or the struck N is really on-mass-shell during the scattering. It only tells us how to treat correctly the effects of the Lorentz transformation to the deuteron rest frame.[1] Lastly a factor $(M_D/E_D)^{1/2}$ should be included in the relativistic calculation of $B(q^2)$ to account for the relativistic normalization of the wave function. Now as can be seen from the expressions given above that though q^2 is rather large, the calculation of the structure functions is dependent largely upon the wave functions evaluated at pn relative momenta which are much smaller. So it is not unrealistic to use nonrelativistic wave functions in this calculation.

Results of our calculations are presented in Figs. 2–4. Figure 2 shows a comparison of calculated $B(q^2)$ with the experimental data which are available only up to $q^2=1$ (GeV/c)^2. Considering the large uncertainty in the point at $q^2=1$ (GeV/c)^2, the agreement of the solid curve with the data is good. Note also the large differences among the four curves even for $q^2 \leq 1$ (GeV/c)^2 as shown in Fig. 2. The results of Fig. 2 are extended to $q^2=8$ (GeV/c)^2 in Fig. 3. Both Figs. 2 and 3 are based on the Paris NN interaction.[9] Results obtained with the Reid soft-core[10] or the Hamada-Johnston hard-core (HJ)[11] NN interactions are qualitatively similar. In Fig. 4 we show only the final results for each of these three NN interactions. Note that both the Reid soft core and the Paris interactions predict a minimum in $B(q^2)$ around $q^2=2$ (GeV/c)^2 while the HJ interaction predicts a minimum at $q^2 \simeq 2.5$ (GeV/c)^2.

All the results presented in this paper make use of nucleon empirical dipole form factors with scaling, with $G_{En}=0$. The effect of other choices of nucleon form factors on $A(q^2)$ and $B(q^2)$ has been studied in Ref. 6. They used three different sets of the nucleon form factors other than the dipole form. They found that the results differed very little from those

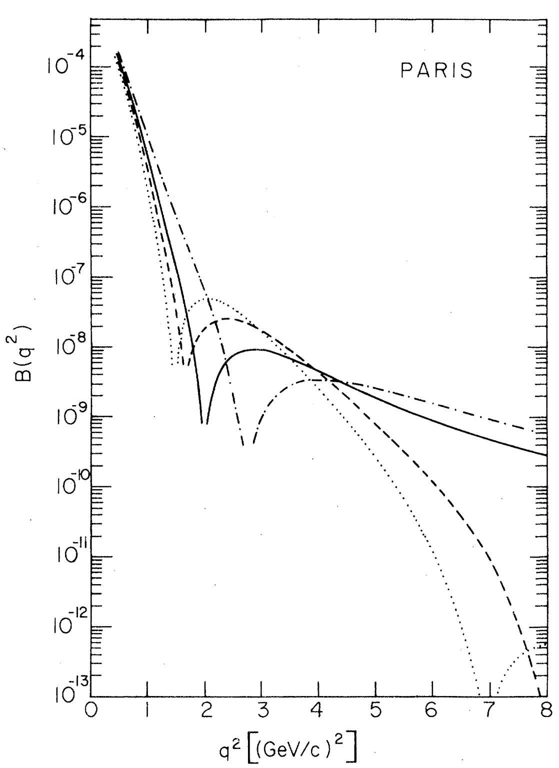

FIG. 3. Our prediction (solid curve) for the deuteron magnetic structure function $B(q^2)$. Description of the various curves as in Fig. 2. All curves are based on the Paris NN interaction (Ref. 9).

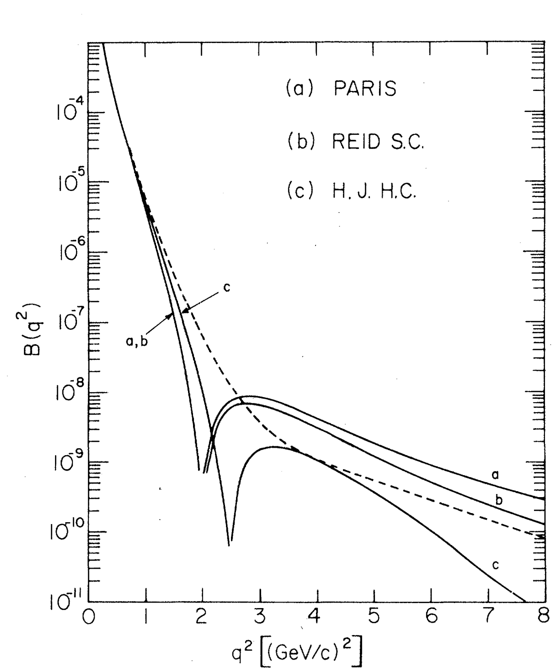

FIG. 4. Our prediction for the deuteron magnetic structure function $B(q^2)$ based on the (a) Paris (Ref. 9), (b) Reid soft-core (Ref. 10), and (c) Hamada-Johnston hard-core (Ref. 11) NN interaction. Note the minimum in $B(q^2)$ which is absent in the exchange-current calculation (dashed curve) of Ref. 13. In Ref. 13, however, relativistic kinematical effects are not included. Curve (a) is the same as the solid curve in Fig. 3.

obtained with the dipole form, with the exception of the Iachello, Jackson, and Lande (IJL) nucleon form factors.[12] However, IJL form factors contain an unreasonable[6] behavior of G_{En}. In particular the position of the minimum in $B(q^2)$ remained unchanged.

It is important to note that calculations which include the effects of meson exchange currents (see, e.g., Ref. 13) do not exhibit a minimum in $B(q^2)$ for any $q^2 \leq 8$ $(\mathrm{GeV}/c)^2$. Hence the planned measurements of $B(q^2)$ in this region would be of utmost importance to throw new light on this long-discussed problem.

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