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Comment on analytical evaluation of the Coulomb distortion

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The simple treatment of Coulomb distortion in intermediate energy hadron-nucleus scattering developed in the analytic approximation of Amado, Dedonder, and Lenz is implemented. In three examples we compare the impact of the Coulomb distortion calculated with their analytic formula with that calculated by the standard numerical method. These calculations are compared with the data for 800 MeV p-^{208}Pb and 291 MeV $\pi^{\pm}$-^{208}Pb elastic scattering. We find that the analytic approximation accurately reproduces the effect of the Coulomb distortion as calculated numerically while illuminating the essentially simple physics involved.

NUCLEAR REACTIONS Analytic approximation for Coulomb distortion in intermediate energy hadron-nucleus scattering. Example calculations for 800 MeV p-^{208}Pb and 291 MeV $\pi^{\pm}$-^{208}Pb elastic scattering.

It is well known that in scattering charged particles from large nuclei at intermediate energies one cannot ignore the Coulomb contribution. It is equally well known that the infinite range of the Coulomb interaction introduces technical problems in the numerical evaluation of such contributions.[1] In fact, the nuclear Coulomb field is screened by atomic electrons, but it is clearly impractical, as well as unnecessary, to integrate out to atomic scales in steps small compared to nuclear scales. The usual prescription for circumventing this problem is to add and subtract a point Coulomb contribution.[2,3] The point Coulomb scattering amplitude is known analytically, and the difference between the nuclear (Coulomb) and point potential phase shifts are short ranged. The divergent phase factors out and the answer lies in the delicate cancellation between the point and short-range nuclear Coulomb scattering amplitudes. The numerical difficulty is surmounted, but an essentially simple process is obscured. The point of this paper is to show how the Coulomb scattering effects can be quantitatively understood from a simple analytic viewpoint.

In a recent paper by Amado, Dedonder, and Lenz[4] (hereafter referred to as ADL) as asymptotic analytic approximation to the eikonal scattering integral was developed using the method of stationary phase. In addition to the leading asymptotic corrections, an approximate treatment of the Coulomb contribution is also presented. In ADL, comparison of their analytic expression is made only to the numerically evaluated eikonal integral; they do not show the effect of their Coulomb correction nor compare with data. We find correcting this omission worthy of comment because the results are remarkable considering the simplicity of their treatment. Although this method, based on asymptotic approximations, fails for low momentum transfer q, it succeeds quite well in the large q region, where numerical methods experience the most difficulty. In this paper we wish to emphasize the insights and simplicity offered by the analytic methods, and, therefore we sacrifice some quantitative accuracy in absolute scale in order to avoid details unrelated to treating the Coulomb distortion. Specifically, throughout this paper we use a simple Fermi distribution for the nuclear density and find minor scale discrepancies with the data that could otherwise be corrected by using a more complicated functional form. Similarly, for the relatively low energy pion example we should include the higher order (non-Coulomb) asymptotic corrections to the ADL formula to achieve proper scaling *vis à vis* the exact calculations. We exclude these higher order corrections to emphasize the impact of the Coulomb distortion alone. We find that the impact of the Coulomb distortion treated by the analytic method of ADL successfully duplicates that of the exact result and, given the simplicity of the analytic treatment, offers considerable insight into the mechanism involved.

We begin with a review of the standard and ADL approaches to including Coulomb distortion. We then will present and discuss our results for 800 MeV p-^{208}Pb and 291 MeV $\pi^{\pm}$-^{208}Pb elastic scattering.

We wish to describe the elastic scattering of a charged hadron from a nucleus (A,Z) at intermediate energies. The eikonal description is known to account accurately for a wide body of data from such processes; so, for completeness, we begin with a brief review of the standard approach in the eikonal formalism. The eikonal scattering amplitude is written

$$\mathscr{F}(q)=ik\int_0^\infty db\, bJ_0(qb)(1-e^{i\chi(b)})\ , \tag{1}$$

where k is the incident momentum and χ is the eikonal phase which for charged projectiles is the sum of hadronic and Coulombic contributions.

$$\begin{aligned}\chi&=\chi_N+\chi_c\ ,\\ \chi_N&=i\gamma\int_{-\infty}^{\infty} dz\,\rho(b,z)\ ,\end{aligned} \tag{2}$$

$$\chi_c=-\frac{1}{\hbar v}\int_{-\infty}^{\infty} dz\, V_c(b,z)\ , \tag{3}$$

where we have assumed a short range first order optical description for the hadronic contribution with $\gamma=\frac{1}{2}\sigma_T(1-ir)$ with σ_T the total hadron-nucleon cross section, r the real to imaginary ratio of the fundamental forward amplitude, and $\rho(r)$ the nuclear density normalized to A nucleons.

The Coulomb potential for a spherical nucleus is given by

$$\left[\frac{A}{4\pi Z}\right]V_x(r)=\frac{1}{r}\int_0^r ds\, s^2\rho(s)+\int_r^\infty ds\, s\rho(s)\ . \tag{4}$$

In the applications which follow, ρ will be given by a Fermi distribution $\rho=\rho_0[1+e^{(r-c)/\beta}]^{-1}$. One can see in (3) and (4) that the Coulomb phase will be logarithmically divergent. As mentioned earlier the usual remedy is to add and subtract a point Coulomb contribution, which gives for the eikonal treatment considered here[2]

$$\mathscr{F}(q)=ike^{-2i\eta\ln 2kR}\left[\frac{2i\eta}{q^2}e^{-2i\eta\ln(q/2k)+2i\varphi(\eta)}+\int_0^\infty db\, b\,(kb)^{2i\eta}J_0(qb)(1-e^{i\chi_N+i\tilde{\chi}_c})\right]\ , \tag{5}$$

where $\eta=Ze^2/v$, $\varphi(\eta)=\arg\{\Gamma(1+i\eta)\}$, R is a large screening distance which influences only the unobserved overall phase, and

$$\tilde{\chi}_c(b)=2\eta\frac{4\pi}{A}\int_b^\infty dr\, r\rho(r)\left[r\ln\left[\frac{\sqrt{r^2-b^2}+r}{b}\right]-\sqrt{r^2-b^2}\right]\ . \tag{6}$$

The first term is the point Coulomb contribution while the second is the point-Coulomb-distorted nuclear contribution. The resulting Coulomb phase $\tilde{\chi}_c$ is short ranged, and the divergent phase factors out entirely.

One can evaluate (5) numerically and find that as the momentum transfer increases both terms of (5) will be dominated by the $1/q^2$ point-Coulomb behavior, but in combination these terms largely cancel, leaving the desired result. The numerical cancellation is delicate, but the technical problem of calculation is solved. Yet how are we to understand this process which is revealed only in the near cancellation of two complex numbers?

As an alternative and complimentary approach, one can evaluate the scattering amplitude approximately, but analytically as in ADL. Thereby one can gain insight into the physics by examining the function's analytic behavior. The ADL asymptotic approximation has been shown to preserve all the essential properties of the full eikonal amplitude.[4] For asymptotic momentum transfers (compared with the inverse radius) the extremely rapid oscillations of the Bessel function permit an accurate application of the stationary phase approximation. The stationary phase point b_s is determined solely by the hadronic eikonal phase (2); the Coulomb phase is slowly varying due to the long range of the Coulomb potential. By approximating the eikonal integral in this manner, the very property that encumbers numerical evaluations of the Coulomb piece renders the solution simple here—

we need only evaluate the Coulomb phase at the stationary point. In ADL the Coulomb phase is actually evaluated at the singular point of the density, $b_0 = c + i\pi\beta$, which they show largely determines the stationary point for asymptotic q. If we let R be some large screening length, the Coulomb eikonal phase evaluated at b_0 to leading order in β/c is given by [4]

$$i\chi_c(b_0) = 2i\eta \ln(2R/b_0) + O[(\beta/c)^{7/2}] . \quad (7)$$

The leading term is just that expected by considering the nuclear charge to be pointlike. Since the Coulomb phase need only be evaluated at b_0 and nearly all the nuclear charge is within this radius, the nuclear charge appears as if purely pointlike. The only nuclear structure dependence is carried in $b_0 = c + i\pi\beta$. The imaginary part of (7) which contains the logarithmically divergent phase simply factors out. The real part, which we call ρ_c, influences the cross section and is given by

$$\rho_c = \mathrm{Re}[i\chi_c(b_0)] \simeq -2\eta \arctan\frac{\pi\beta}{c} . \quad (8)$$

With this phase inserted in the analytic amplitude the resulting analytic approximation with Coulomb distortion is given by [ADL Eq. (46)]

$$\mathscr{F}_{\mathrm{ADL}}(q) = A(q) e^{-\pi\beta q}[e^{i\theta(q)+\rho_h+\rho_c} + e^{-i\theta(q)-\rho_h-\rho_c}] , \quad (9)$$

where $A(q)$ is a slowly varying function of q which sets the scale of the scattering

$$A(q) \simeq \frac{2\pi}{\gamma} \frac{\alpha^{1/3} c^{2/3}}{\sqrt{3} q^{4/3}} \exp[-1.46\gamma\rho_0\sqrt{2\pi\beta c} + \tfrac{3}{2}(qc\alpha^2)^{1/3}\cos\frac{\pi}{6}] , \quad (10)$$

$$\rho_h \simeq -\tfrac{3}{2}(qc)^{1/3}\sin\frac{\pi}{6}\,\mathrm{Im}\alpha^{2/3} , \quad (11)$$

where $\alpha = 2\pi\beta\gamma\rho_0$ is a complex dimensionless hadronic strength parameter and $\theta(q) = qc\{1 + O[(qc)^{-2/3}]\}$. It is clear from the form of (9) that the familiar diffractive character of intermediate energy scattering is given by the interference of the two phases. Physically this separation is equivalent to the elementary diffraction slit example, whereby the slit is conceptually divided into two symmetric interfering parts. In the absence of Coulomb the influence of ρ_h is to make the two interfering amplitudes differ in magnitude so the destructive interference will not be complete, and the amplitude would have exact zeroes corresponding to infinitely deep cross section minima. The effect of ρ_h is to fill these minima in addition to shifting the overall scale. Since the Coulomb contribution ρ_c enters like ρ_h, it follows that its impact will be primarily to influence the minimum filling and shift the scale as well. Whether ρ_c makes the minimum deeper or shallower depends on its relative sign with respect to the hadronic minimum filling contribution ρ_h. For large nuclei $\mathrm{Arg}(b_0) \sim \pi\beta/c$ can be quite small and the sign of ρ_h, Eq. (11), is determined by $\mathrm{Im}(\gamma) = -\frac{1}{2}\sigma_T r$. For 800 MeV protons scattering from ^{208}Pb r is very small (-0.18), so $\rho_c >> \rho_h$ and the Coulomb term alone dominates the minimum filling. For the 291 MeV $\pi^{\pm}$-^{208}Pb applications $|r| \simeq 1$, $\rho_h \simeq \rho_c$, and the two can interfere. Thus the Coulomb term will deepen the minima of one while further filling the minima of the other.

We first examine 800 MeV p-^{208}Pb elastic scattering. In Fig. 1 the cross sections (with and without Coulomb distortion) given by the numerically evaluated full eikonal amplitude, Eq. (5), are shown along with the cross sections (with and without Coulomb distortion) given by the ADL analytic amplitude, Eq. (9). The data[5] are also presented. The ^{208}Pb density parameters used are $c = 6.6$ fm and $\beta = 0.63$ fm. The nucleon-nucleon strength parameter used is $\gamma = 2.1\ (1 + i.18)$ fm^2. We see from Fig. 1 that beyond the second maximum the ADL analytic amplitude with or without Coulomb distortion accurately reproduces the corresponding exact results (since the ADL amplitude is an asymptotic approximation, comparisons for smaller momentum transfers are not meaningful). Both the scale change and minimum filling of the exact result, when Coulomb is included, are duplicated by the ADL formula. Beyond the second maximum the shapes of both theoretical curves with Coulomb agree well with the data. The small scale differences between the data and theoretical curves can be eliminated by using a different functional form for the density.

Next we consider 291 MeV $\pi^{\pm}$-^{208}Pb elastic scattering. In Fig. 2 we compare the exact eikonal calculation with and without Coulomb to the ADL analytic result with and without Coulomb and the data.[6] The nuclear parameters are unchanged. The pion nucleon strengths, however, are $\gamma_{\pi^+} = 2.26(1 + i.84)$ fm^2 and $\gamma_{\pi^-} = 2.65(1. + i1.0)$ fm^2. Unlike the earlier proton example the pion-nucleon strengths have a significant imaginary component. Since it is precisely this component

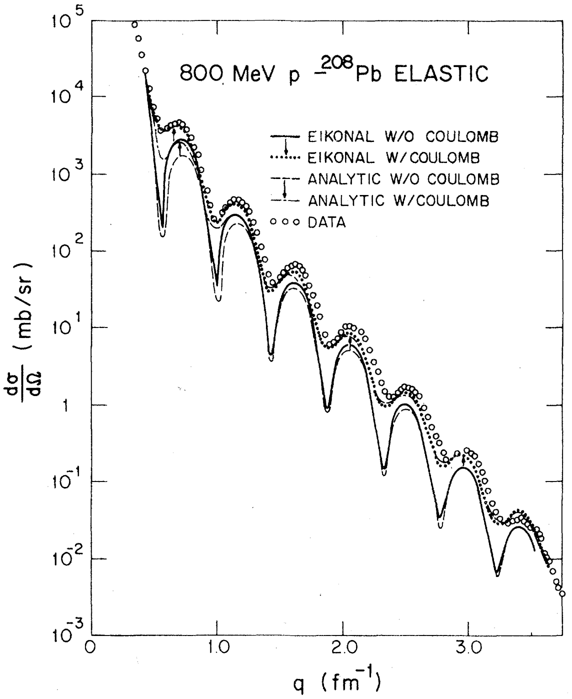

FIG. 1. Theoretical calculations of 800 MeV p-^{208}Pb elastic scattering cross section are compared with the data (Ref. 5). The solid curve uses the exact eikonal amplitude but without Coulomb distortion [Eq. (5) with $\eta=0$]. The dashed curve uses the analytic approximation of ADL also without Coulomb [Eq. (40) of Ref. 4]. The dotted curve uses the exact eikonal calculation with Coulomb distortion included [Eq. (5)], and the dash-dotted curve uses the analytic approximation of ADL with Coulomb distortion included [Eq. (9) or Eq. (46) of Ref. 4]. The arrows indicate the impact of including the Coulomb distortion. The needed parameters are given in the text.

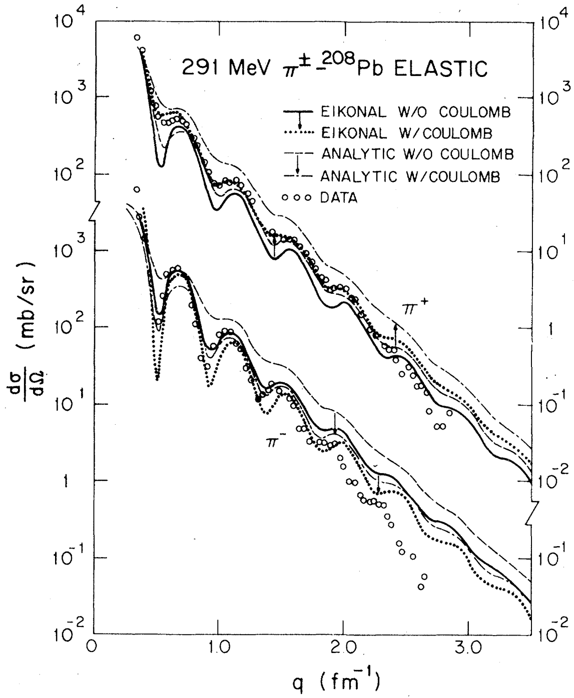

FIG. 2. Theoretical calculations of 291 MeV $\pi^{\pm}$-^{208}Pb elastic scattering cross sections are compared with each other and the data (Ref 6). The key is the same as in Fig. 1.

which determines the hadronic minimum filling contribution, the impact of including the Coulomb distortion will have dramatically different effects for π^+ vs π^-. In the case of 291 MeV π^--^{208}Pb elastic scattering shown in Fig. 2, the Coulomb distortion largely cancels the hadronic minimum filling term. Thus the minima are deeper when Coulomb effects are included as indicated by the exact calculations in Fig. 2. Although the scale of the ADL and exact calculations differ, the *impact* of including Coulomb is properly reproduced by the ADL analytic amplitude. In the π^+-^{208}Pb case the Coulomb term reverses sign, while the hadronic term [Im(γ)] does not; thus the hadronic and Coulomb terms combine constructively. Again the ADL approximation reproduces the effect of Coulomb distortion with reasonable accuracy. The scale discrepancies between the analytic and exact results can be largely corrected by including the higher order terms in the asymptotic expansion. These corrections, however, have no bearing on the main result of this paper—the impact of the Coulomb distortion is accurately duplicated by the simple analytic term of ADL. The success of this simple treatment stems from the very property that makes the numerical approach difficult— namely, the long range of the Coulomb force. This property causes the Coulomb distorting phase to vary slowly permitting one to approximate it well by its value at the singular point b_0. Finally, for large nuclei ($\beta/c << 1$) only the pointlike part of the Coulomb phase matters, with the nuclear structure entering only in the location of the singularity. This picture offers considerable insight into the physics of Coulomb distortion in hadron-nucleus scattering due to its simplicity and accuracy.

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