Quantum-mechanical and classical-limit coupled-channels calculations for very heavy systems

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Exact quantum-mechanical coupled-channels and classical-limit calculations are compared for the case of a very-heavy-ion reaction exciting strongly collective states via Coulomb and complex nuclear potentials. The calculations are in excellent agreement for transitions which are likely to be experimentally important and provide further support for the classical localization picture in very-heavy-ion collisions.

[NUCLEAR REACTIONS HI coupled-channels inelastic scattering. Comparison of quantal and classical-limit theories.]

With the new generation of very-heavy-ion accelerators, it has been possible to measure inelastic excitation probabilities[1] which show Coulomb-nuclear interference (CNI) for reactions exciting high-spin states ($I \sim 20\hbar$). A detailed theoretical understanding of these reactions is important because very-heavy-ion inelastic scattering and transfer experiments are expected to be a continued source of information on nuclear moments and localization[2] in nuclear collisions.

The data from these collisions have been analyzed with the classical limit S-matrix formalism[1,3] (CLSM) developed by Miller and Marcus.[4] However, the recent introduction of new mathematical techniques[5] has made large, fully quantum-mechanical, coupled-channels calculations of very heavy systems feasible. For the first time, it is possible to study in detail the validity of classical-limit approximations for reactions exciting strongly coupled collective states via a complex potential.

We present here the preliminary results of a theoretical comparison of quantal and CLSM calculations for the inelastic scattering reaction $^{40}\mathrm{Ar} + {}^{160}\mathrm{Gd}$ at $E_{\mathrm{lab}} = 120-185$ MeV, exciting the ground-state rotational band of ^{160}Gd up to the 12^+ state by multiple quadrupole excitation. This energy range spans regions of pure Coulomb excitation, strong Coulomb-nuclear interference, and strong absorptive damping.

Most of the high-spin experimental data available so far are for almost back-angle scattering. Therefore, we restrict ourselves to the problem of head-on collisions for which the classical motion is confined to a plane and the total angular momentum J is zero.

The two degrees of freedom employed in a classical description are the radial separation of the center of masses r and χ, the angle between the symmetry axis of the rotor and the line joining the centers of the target and projectile. Assuming only quadrupole excitation, the classical Hamiltonian H was parametrized in the form

$$H = \frac{P_r^2}{2m} + P_\chi^2 \left[\frac{1}{2mr^2} + \frac{1}{2\mathscr{I}} \right] + \frac{Z_p Z_T e^2}{r} + \frac{Z_p e Q_0^{(2)}}{2r^3} P_2(\cos\chi)$$
$$- (V + iW) f(r) - (V + iW)\sqrt{5/4\pi} \frac{\partial f(r)}{\partial r} P_2(\cos\chi)\delta_2^N, \quad (1)$$

where P_r and P_χ are the classical radial and angular momenta, respectively, m is the reduced mass of the system, $Z_p e$ and $Z_T e$ are the projectile and target charges, $Q_0^{(2)}$ is the quadrupole moment of the target, $P_\lambda(\cos\chi)$ is a Legendre polynomial, and $f(r)$ is a Woods-Saxon radial form for the nuclear potential. The moment of inertia $\mathscr{I}$ was taken from the first excited 2^+ state of the rotor. We used the realistic parameters $Q_0^{(2)} = 6.168\ e$ b, $V=15$ MeV, $W=10$ MeV, and $\delta_2^N = 1.2 \times 160^{1/3} \times \beta_2^N$ fm, where for the nuclear deformation parameter β_2^N we took the value 0.2. The radius parameter R and diffuseness a in the Woods-Saxon potential were taken to be $R=1.2(40^{1/3}+160^{1/3})$ fm and $a=0.5$ fm. The higher excitation energies were assumed to be given by the deduced value of $\mathscr{I}$ and the static rotor model.

By solving the Hamiltonian classical equations of motion for P_χ and back-angle scattering, it is readily deduced for the energy range of interest that the 8^+ is the highest classically allowed excited state. To reach states of higher spin, it is necessary to analytically continue the dynamical motion into the classically forbidden domain using standard Airy function techniques.[6]

The details of the CLSM have been given elsewhere.[3,4,6] Briefly, one assumes the scattering eigenstate can be approximated by a generalized multidimensional WKB wave function. The amplitude of this wave function is constructed to conserve probability flux. The multidimensional phase, containing all the dynamics of the problem satisfies the *classical* Hamilton-Jacobi equation constructed from Eq. (1). The overlap integral of this wave function with the asymptotic WKB wave function for the appropriate exit channel defines the S-matrix element of interest.

In the partial-wave configuration-space representation the usual quantal technique is to expand the scattering eigenstate in terms of a subset of channel eigenstates. For a given total angular momentum and parity (J,π) the unknown radial functions in this expansion are determined by solving a set of coupled differential equations subject to the usual boundary condition.[5] The S-matrix elements of interest are then determined from this boundary condition. We used two quantum-mechanical calculations to compare with the $J=0$ classical trajectory, one in which the partial wave sum was truncated to $J=0$ only and a full quantal calculation in which we included all J values up to a maximum of $J=2400$.

For the CLSM calculations and the $J=0$ quantal calculations, the probability for inelastic excitation $P_I(180°)$ is defined through the square modulus of the appropriate S-matrix element. We defined a probability $P_I(\theta)$ for the full quantal calculations by using the semiclassical relation

$$\sigma_{0^+,I^+}(\theta)=P_I(\theta)[\sigma_{0^+}^{\text{opt}}(\theta)\sigma_{I^+}^{\text{opt}}(\theta)]^{1/2}, \tag{2}$$

where $\sigma_{0^+,I^+}(\theta)$ is the inelastic differential cross section and $\sigma_{I^+}^{\text{opt}}(\theta)$and $\sigma_{0^+}^{\text{opt}}(\theta)$ are the optical model differential cross sections for $W=0$ and the appropriate channel energy, all evaluated for $\theta=180°$.

In Fig. 1 we compare calculations of the excitation probabilities $P_I(180°)$ as a function of laboratory energy using the $J=0$ classical and quantum methods, as well as the full quantum technique. For $E_{\text{lab}} < 160$ MeV, the Coulomb potential completely dominates. For $E_{\text{lab}} \geq 185$ MeV nuclear absorption strongly attenuates the excitation probabilities. In both quantum calculations the model space was truncated at the 16^+ state, assuring convergence for transitions to the 12^+ state. No equivalent truncation problem exists in the classical formalism. For every J value >15, it was necessary to solve a set of 81 coupled radial equations in the quantum-mechanical approach.

Figure 1 shows the excellent agreement between the CLSM and $J=0$ quantum-mechanical calculations across the whole energy range and up to the 12^+ state, i.e., well inside the classically forbidden region. The close agreement between the two quantal calculations for reactions exciting spins up to the 8^+ state indicates the monotonic and classical nature of the deflection function, as discussed by Glendenning.[7] For higher spin states, classically forbidden at back angles, one observes an increasing systematic difference across the whole energy range between the $J=0$ and full quantum-mechanical results. We believe the small differences between the $J=0$ and full quantal calculations for the $2^+$and 4^+ excitations are due to small numerical rounding errors in the full partial wave sums in Eq. (2). This problem disappears for higher spin states because fewer J values contribute to these transitions.

These results have several important implications.

(1) They confirm the accuracy of the classical-limit formalism for the $J=0$ S-matrix elements in a domain where one expects the radial and collective degrees of freedom may be described with classical dynamical equations. This is true even when

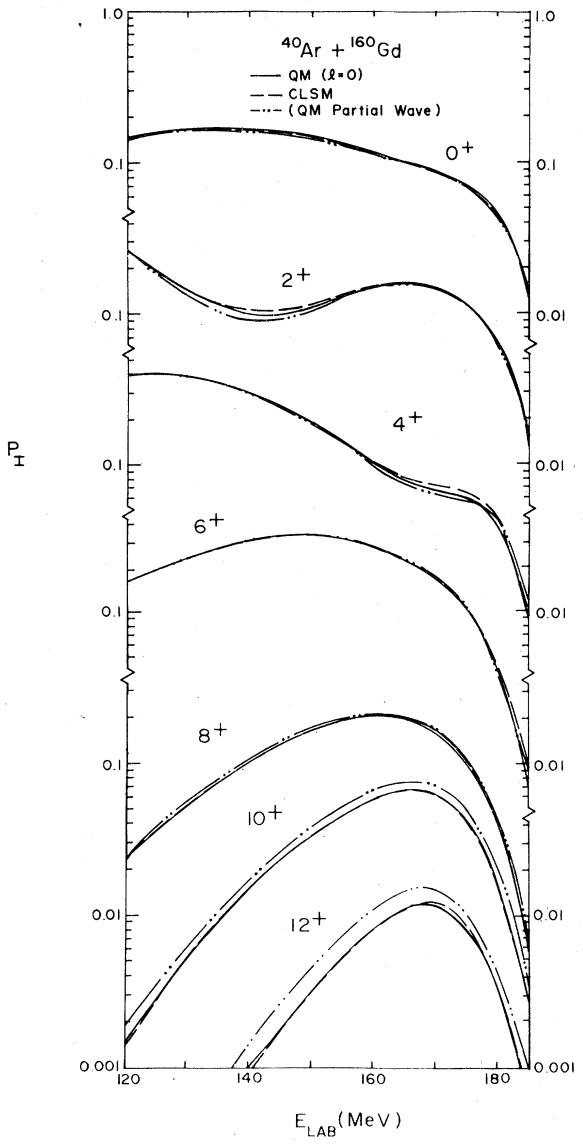

FIG. 1. Excitation probabilities calculated with the potential parameters described in the text. The solid line represents the quantal excitation probabilities calculated with $J=0$ only. The dashed line is the $J=0$ CLSM results. The dotted-dashed line is the result of the full quantal calculation summed over many partial waves.

the complex nuclear potential, a concept alien to classical particle physics, dominates the reaction mechanism. The excellent agreement between the quantal and classical-limit calculations for $J=0$ in the classically forbidden region implies the accuracy of the analytical continuation method used.

(2) For the classically-allowed transitions, the close agreement between all three calculations for the excitation probabilities shows that just a single $J=0$ classical-limit or quantal calculation is sufficient to analyze such back-angle experimental data. In fact, for $J=0$, the angular momentum selection rules reduce the quantal calculation to the solution of only one set of nine coupled equations for this problem. Hence, the quantal technique is a practical alternative to classical-limit calculations.

(3) For low-spin states, the small difference between the quantal $J=0$ calculation and the full quantal calculation summed over many partial waves implies a monotonic and classical nature for the deflection function.[7] This lends further evidence for the concept of radial and surface angular localization[2] in very-heavy-ion collisions.

(4) The differences between the $J=0$ quantal calculations and the full quantal calculations for $P_{10^+}(180°)$ and $P_{12^+}(180°)$ show that quantum interference effects are becoming important in the partial wave sums of Eq. (2). Thus, although an excited rotational state with very large angular momenta tends to behave classically, the reaction mechanism to reach that state can be classically forbidden, in which case many quantum paths may contribute to the excitation probability. Further calculations are in progress for reactions between heavier systems to test this.

It should be noted that the nuclear potential used in these calculations does not give rise to a pocket inside the Coulomb barrier. This is a property of many heavy-ion optical potentials found in the literature. For potentials with pockets, it may be necessary to add quantum-mechanical tunneling explicitly to the classical prescription.

Calculations are in progress for other systems, including vibrators, exploring a range of asymptotic wavelengths and studying the extent of radial and angular localization in nuclear collisions. Studies of the validity of stationary phase techniques for evaluating the WKB S-matrix integral and the quantum-mechanical scattering amplitude summed over many partial waves are also in progress.

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