Semimicroscopic cluster-model analysis of ^{7}Li charge form factor

Z. El-Itaoui and Y. C. Tang
School of Physics, University of Minnesota, Minneapolis, Minnesota 55455

H. Kanada
Department of Physics, Niigata University, Niigata, Japan
and School of Physics, University of Minnesota, Minneapolis, Minnesota 55455

Q. K. K. Liu
Bereich Physik, Hahn-Meitner-Institut für Kernforschung, Berlin-West, Germany
(Received 13 July 1981)

The charge form factor of ^{7}Li is calculated with a semimicroscopic method in which the Pauli principle is taken fully into account but the intercluster relative motion is parametrized in a way guided by cluster-model considerations. The results show that there is a large degree of $t+\alpha$ clustering in the ground state of ^{7}Li and that specific distortion effects are fairly important in this system.

[NUCLEAR STRUCTURE ^{7}Li; charge form factor with semimicroscopic approach.]

In a previous publication,[1] hereafter referred to as KLT, the ^{7}Li charge form factor in the q^2 region from 0 to 7 fm^{-2} was calculated with a single-channel $t+\alpha$ resonating-group wave function. As has been emphasized there, the important characteristics of this calculation are: (i) a totally antisymmetric seven-nucleon wave function is used, (ii) the center-of-mass motion is correctly considered, and (iii) no adjustable parameters are involved. The results obtained were generally satisfactory; in particular, the deduced values for the rms charge radius and spectroscopic quadrupole moment agreed quite well with empirically determined values.

The resonating-group wave function used has the form[2]

$$\psi_M = \mathscr{A}\left\{\phi_\alpha\phi_t\left[\frac{1}{R}f_{Jl}(R)\mathscr{Y}^M_{Jls}\right]Z(\vec{R}_{\text{c.m.}})\right\}, \quad (1)$$

where ϕ_α and ϕ_t describe the internal spatial structures of the α and t clusters; they are assumed to be represented by single-Gaussian functions characterized by width parameters α_A and α_B, respectively. Using this wave function, one can readily formulate the form-factor problem by employing the complex-generator-coordinate technique[3] and then compute numerical results at various q^2 values.

Even though this microscopic way to calculate the charge form factor is a desirable procedure, we should point out that there is one difficulty involved. This difficulty is connected with the fact that one must first obtain the relative-motion function f_{Jl} [see Eq. (1)] for which it is necessary to derive the rather complicated kinetic-energy and potential-energy kernels present in the resonating-group formulation. For the light $t+\alpha$ system, the derivation of such kernel functions is comparatively simple; however, it can become quite lengthy, even with the complex-generator-coordinate technique, if one considers heavier systems, with the constituent clusters described by flexible internal functions. In this brief report, our purpose is to demonstrate a semimicroscopic approach, in which one employs the microscopic seven-nucleon wave function of Eq. (1), but parametrizes the relative-motion function in a way guided by cluster-model considerations. Using this approach which involves the determination of some parameter values from fitting empirical form-factor data, one obviously simplifies the calculation to a large extent by circumventing the difficulty mentioned above. However, as will be shown below, one can still obtain much useful information concerning the structure of the seven-nucleon system.[5]

The function $f_{Jl}(R)$ is parametrized as

$$f_{Jl}(R)=(R^4+CR^6)\exp(-\tfrac{6}{7}\gamma R^2)\ . \qquad (2)$$

To choose this particular parametrization which contains adjustable quantities C and γ, we have used the consideration that, in the oscillator cluster model,[4,6] the relative motion between the t and α clusters must consist of at least three quanta of oscillation to comply with the Pauli principle. With this function, the calculational procedure is identical to that given in KLT, and one can readily compute the $C0$ and $C2$ contributions of the proton and neutron distributions to the charge form factor $F_{\rm ch}$.

Two quantities will receive special emphasis in this study. These are the rms charge radius $R_{\rm ch}$ and the spectroscopic quadrupole moment Q. For $R_{\rm ch}$, the empirical value is 2.35 ± 0.10 fm determined by a model-independent analysis[7] of experimental data at low momentum transfers of less than about 1 fm^{-1}, or 2.39 ± 0.03 fm determined by an oscillator shell-model analysis[8] of the higher q^2 data. An appropriate average to be used in our investigation is, therefore, 2.37 fm. For Q, the present experimental values[9,10] are -3.4 ± 0.6 and -4.1 ± 0.6 fm^2; these can be combined to yield an average value of -3.7 ± 0.4 fm^2.

In our present investigation, the following cases have been studied:

(i) Oscillator shell model. By setting $\alpha_A=\alpha_B=\gamma$ and $C=0$, it is well known[4] that, with a proper choice of $Z(\vec{R}_{\rm c.m.})$, the wave function ψ_M reduces to an oscillator shell-model function of the highest spatial symmetry and the configuration $(1s)^4(1p)^3$. The single parameter γ can then be adjusted to yield the desired value of 2.37 fm for $R_{\rm ch}$. The result is $\gamma=0.32$ fm^{-2}, which in turn fixes[11] the spectroscopic quadrupole moment as -1.87 fm^2. This latter value is a factor of 2 smaller than the experimental result.[12]

Calculated values of $F_{\rm ch}{}^2$ for q^2 up to 7 fm^{-2} are shown by curve (a) in Fig. 1. As is seen, the agreement between calculation and experiment[8] is rather poor. This shows that the oscillator shell model in the lowest $(1s)^4(1p)^3$ configuration cannot properly explain the observed features and a more flexible model must be sought.

(ii) Cluster model without specific distortion. When specific distortion effects[4] are not taken into account, one chooses the width parameters α_A and α_B to reproduce empirical values of the rms matter radii of free α and t nuclei, i.e.,

$$\alpha_A=0.514\ \text{fm}^{-2}\ ,\quad \alpha_B=0.378\ \text{fm}^{-2}\ . \qquad (3)$$

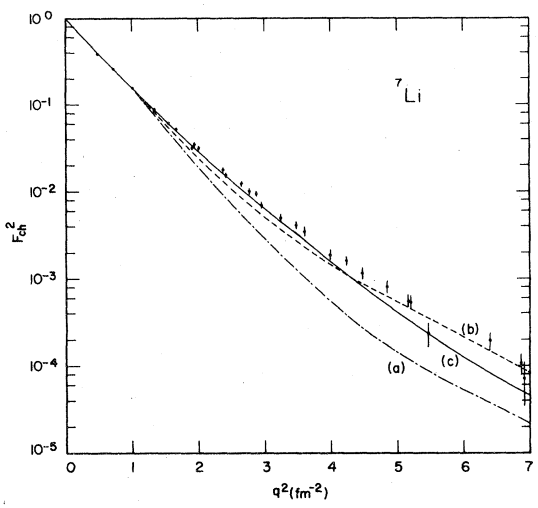

FIG. 1. Comparison of calculated and empirical results for $F_{\rm ch}{}^2$. Curve (a): oscillator shell-model calculation. Curve (b): cluster-model calculation without specific distortion, with $(C,\gamma)=(0,\ 0.227\ \text{fm}^{-2})$. Curve (c): cluster-model calculation without specific distortion, with $(C,\gamma)=(-0.0715\ \text{fm}^{-2}\ ,\ 0.25\ \text{fm}^{-2})$. Empirical data shown are those of Ref. 8.

In our initial study with the cluster model, we set $C=0$ and adjust γ to yield $R_{\rm ch}$ of 2.37 fm. This results in $\gamma=0.227$ fm^{-2} which is substantially smaller than either α_A or α_B. As has been discussed previously,[13] this is a strong indication that there is a large degree of $t+\alpha$ clustering in the ground state of ^{7}Li.

The value of Q in the $C=0$ case is -3.21 fm^2, which differs by about 15% from the experimental result. For $F_{\rm ch}{}^2$ [curve (b) in Fig. 1] the calculated values agree reasonably well with experiment in the low- and high-q^2 regions, but are too small for q^2 between 2 and 5 fm^{-2}.

Next, we briefly examined the situation where C is not set as zero. For a (C,γ) combination of $(-0.0715\ \text{fm}^{-2}\ ,\ 0.25\ \text{fm}^{-2})$ designed to given $R_{\rm ch}$ of 2.37 fm, the value of Q turns out, somewhat surprisingly, to be almost unchanged at -3.20 fm^2.[14] The form-factor behavior, as depicted by curve (c) in Fig. 1, is, however, improved for $q^2\lesssim4$ fm^{-2}, although there is now a poorer agreement with experiment in the higher q^2 region.

(iii) Cluster model with specific distortion. To further improve the result, one must take into account specific distortion effects which have been found to be important in the neighboring $d+\alpha$ system[15,16] As has been shown there, a proper consideration of such effects would require the performance of a rather complicated calculation by em-

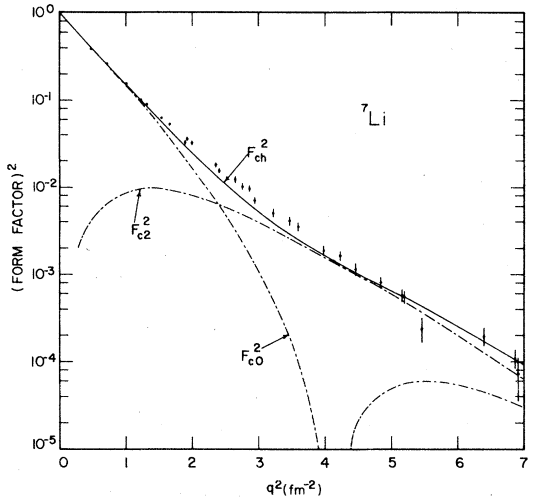

FIG. 2. Comparison of calculated and empirical results for F_{ch}^2. The calculated curves are obtained using the cluster model with specific distortion. The parameters are $C=0$ and $\gamma=0.21$ fm^{-2}.

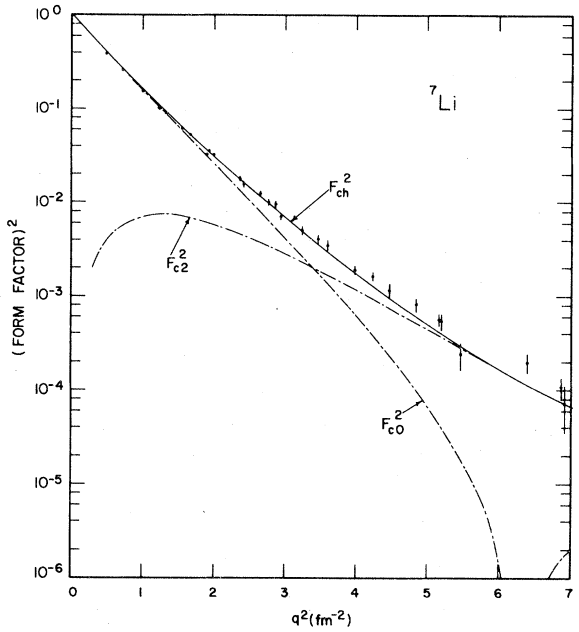

FIG. 3. Comparison of calculated and empirical results for F_{ch}^2. The calculated curves are obtained using the cluster model with specific distortion. The parameters are $C=-0.067$ fm^{-2} and $\gamma=0.225$ fm^{-2}.

ploying a seven-nucleon trial wave function which is more flexible than the one used here. In this study, we have not done this; instead, we have simply adopted a comparatively crude procedure previously suggested.[17] In this procedure, one uses ψ_M of Eq. (1) but variationally determines the optimum value of the width parameter α_B. Thus, based on the considerations given in Ref. 17, one now takes α_B to be 0.47 fm^{-2}, which is somewhat larger than the width parameter appropriate for a free triton.

Again, we first consider the case with $C=0$. To obtain $R_{ch}=2.37$ fm, one finds that γ should be set as 0.21 fm^{-2}. This leads to a value for Q equal to -3.43 fm^2, which agrees reasonably with the measured result. Calculated values for F_{ch}^2, together with contributions from $C0$ and $C2$ interactions, are shown in Fig. 2. Here one notes that the agreement with experiment is fairly satisfactory. The only discrepancy occurs in the intermediate q^2 region between about 2 and 4 fm^{-2}, where the calculated curve lies somewhat below the experimental points.

Learning from the experience described above in (ii), one might hope to improve the form-factor behavior in the intermediate q^2 region by allowing C to be nonzero. This was found to be indeed so. Using a (C,γ) combination of (-0.067 fm^{-2}, 0.225 fm^{-2}) which yields the same values for R_{ch} and Q as in the $C=0$ case, we obtain results for F_{ch}^2, F_{C0}^2, and F_{C2}^2 which are shown in Fig. 3. As is seen, the agreement with experiment is now quite good. In the low-q^2 region, F_{C0}^2 dominates over F_{C2}^2, while the reverse is true in the high-q^2 region. For q^2 between 5 and 7 fm^{-2}, it is noted that the form factor is accounted for almost completely by F_{C2}^2.

In conclusion, we have shown that even a simple, semimicroscopic examination can yield valuable information concerning the property of the system under investigation. For the ^{7}Li case, it is found that there is convincing evidence for strong clustering in the ground state and that specific distortion effects are fairly important.

Comparing with the calculation reported in KLT, it should be noted that the present calculation is much easier to perform. Essentially, only kernels not much harder to derive than the norm kernel need to be evaluated, while for the calculation of KLT one must first obtain the relative-motion wave function by carrying out a full resonating-group study involving the evaluation of complicated kinetic-energy and potential-energy kernels. Therefore, it seems evident that the present semimicroscopic approach may be quite useful and can be employed to compute form fac-

tors and other electromagnetic properties even in heavier systems where complete resonating-group calculations would be rather difficult.

This research was supported in part by the U.S. Department of Energy under Contract No. DOE/DE-AC02-79 ER 10364.

[1]H. Kanada, Q. K. K. Liu, and Y. C. Tang, Phys. Rev. C 22, 813 (1980).

[2]The notation used here is the same as that adopted in KLT.

[3]Y. C. Tang, M. LeMere, and D. R. Thompson, Phys. Rep. 47, 167 (1978).

[4]K. Wildermuth and Y. C. Tang, *A Unified Theory of the Nucleus* (Vieweg, Braunschweig, 1977).

[5]It should be mentioned that the procedure adopted here is not new. Similar attempts have been undertaken by Neudatchin and his collaborators [see V. G. Neudatchin and Yu F. Smirnov, Prog. Nucl. Phys. 10, 273 (1969)] and by Bergstrom [J. C. Bergstrom, Nucl. Phys. A327, 458 (1979)], directed mainly towards the calculation of the charge form factor of ^{6}Li.

[6]Y. C. Tang, K. Wildermuth, and L. D. Pearlstein, Phys. Rev. 123, 548 (1961).

[7]F. A. Bumiller, F. R. Buskirk, J. N. Dyer, and W. A. Monson, Phys. Rev. C 5, 391 (1972).

[8]L. R. Suelzle, M. R. Yearian, and H. Crannell, Phys. Rev. 162, 992 (1967).

[9]P. Egelhof, R. Böttger, W. Dreves, K. H. Möbius, F. Roesel, E. Steffens, G. Tungate, P. Zupranski, and D. Fick, contributed abstract to Proceedings of the International Conference on Nuclear Physics, Berkeley, California, 1980, LBL Report No. LBL-11118, 1980, p. 105.

[10]H. Orth, H. Ackermann, and E. W. Otten, Z. Phys. A 273, 221 (1975).

[11]J. H. van der Merwe, Phys. Rev. 131, 2181 (1963).

[12]The result cannot be much improved even if one includes less symmetric $(1s)^4(1p)^3$ configurations in the calculation (see Ref. 11).

[13]Y. C. Tang, K. Wildermuth, and L. D. Pearlstein, Nucl. Phys. 32, 504 (1962).

[14]The combination of $(C,\gamma)=(0.03\ \text{fm}^{-2}, 0.25\ \text{fm}^{-2})$ yields similarly R_{ch} and Q equal to 2.37 fm and $-3.21\ \text{fm}^2$, respectively.

[15]D. R. Thompson and Y. C. Tang, Phys. Rev. C 8, 1649 (1973).

[16]H. Kanada, T. Kaneko, H. Nishioka, and S. Saito, Prog. Theor. Phys. 63, 842 (1980).

[17]R. E. Brown and Y. C. Tang, Phys. Rev. 176, 1235 (1968).