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Relativistic many-body theory

L. S. Celenza and C. M. Shakin
Department of Physics and Institute for Nuclear Theory,
Brooklyn College of the City University of New York,
Brooklyn, New York 11210
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Self-consistent field theories have had extensive application in the study of nonrelativistic systems. In the past decade there has been some interest in the use of relativistic self-consistent-field techniques for the study of strongly interacting systems such as finite nuclei and infinite nuclear matter. In nonrelativistic theories the standard analysis involves the determination of a new representation for the Hamiltonian in which the residual interaction admixes no one-particle, one-hole states into the self-consistent ground-state wave function. In lowest-order perturbation theory the next admixtures are of the two-particle, two-hole type. We demonstrate that the relativistic theory can be formulated in a similar fashion. In the relativistic theory the new representation is determined such that the residual interaction does not admix particle-hole states however, here "hole" refers to *all* occupied states including the *negative energy* states. Again the corrections to the theory involve the introduction of two-particle two-hole states, where "hole" is understood in the more general sense. In the relativistic theory the specification of the new representation requires the solution of a Dirac-Hartree, Dirac-Hartree-Fock, or Dirac-Brueckner-Hartree-Fock equation, the choice depending upon the nature of the physical system. Once the new representation is found, our techniques allow us to exhibit a useful form for the Hamiltonian of the relativistic system in which the residual interaction between the relativistic quasiparticles is given explicitly. The theory given here is readily extended to the study of finite systems where it provides the basis for a relativistic shell model of nuclear structure.

[NUCLEAR STRUCTURE Derivation of relativistic Hartree-Fock and Brueckner-Hartree Fock theories; interaction between relativistic quasiparticles; dynamics of saturation in a relativistic model.]

I. INTRODUCTION

In the past decade there has been much interest in studying relativistic models of nuclear structure.[1-23] In these models one can relate the self-consistent field in a nucleus, or in nuclear matter, to specific aspects of the meson field which mediates the nucleon-nucleon interaction. (One may attempt to limit consideration to the σ and ω fields as in the Walecka model,[5] or consider the full set of mesons which play a role in the one-boson-exchange model of nuclear forces.[24]) The relativistic models of nuclear structure are able to provide a satisfactory description of many nuclear properties, however, often this is accomplished via the introduction of phenomenological parameters. Since these parameters are small in number, two in the case of the Walecka model of nuclear matter, there is a true simplicity in the specification of these models. These models may be used to calculate the binding energy, spin-orbit splittings[11,12] and electromagnetic form factors of finite nuclei. In addition, if one admits several further parameters into the theory one can provide a good description of nucleon-nucleus scattering at intermediate energies.[20,21] Various other applications are possible, for example one can study nuclear collective motion, the equation of state at high density, anomalous forms of nuclear matter, modification of electromagnetic, and weak interactions in a relativistic medium,[25] etc.

In recent works we have shown how one may in-

clude the effects of short-range correlations in a relativistic theory of nuclear matter.[26–28] This new theory has the significant advantage of being able to explain the saturation and binding energy of nuclear matter without the introduction of phenomenological parameters. In our work we used a Green's function technique to obtain an expression for the energy of the relativistic system.[27] In this paper we wish to analyze the relativistic many-body problem using more standard techniques involving the reorganization of the Hamiltonian.[5] The use of these techniques allows us to write the Hamiltonian as a relativistic Hartree-Fock (or Brueckner Hartree-Fock) Hamiltonian plus a residual interaction. Further, we are able to unify the nonrelativistic self-consistent-field theories with the (self-consistent) relativistic field theories since essentially the same techniques may be used to introduce both formalisms. Indeed, the relativistic theory is readily seen to reduce to the nonrelativistic theory if the admixture of negative-energy states into the wave function may be neglected. The somewhat surprising fact is that these admixtures are very important in explaining nuclear properties. (For example, they play an essential role in explaining the saturation and binding energy of nuclear matter.[28])

Our paper is organized as follows: In Sec. II we introduce a unitary transformation in the space of the negative and positive energy free-particle spinors and a corresponding transformation of the operators of the free field. These transformations leave the Dirac field, $\Psi(\vec{x},0)$ invariant. We also introduce a modified vacuum state.[5] We specify the contractions of the new field operators with respect to the new vacuum state and study the Hamiltonian of the Dirac field in Sec. III. In Sec. IV we require that the unitary transformation be chosen so that we may write the Hamiltonian as a diagonal form [the relativistic Hartree-Fock (HF) Hamiltonian] plus a residual interaction. This requires that the new spinors satisfy a relativistic self-consistent-field equation. The residual interaction is such that no one-particle, one-hole states are admixed into the wave function when this interaction is treated in perturbation theory. (When speaking of the relativistic theory, "hole states" include all those states below the Fermi level, that is, states of positive and negative energy.) We note that inspection of the residual interaction immediately provides the form of the interaction between relativistic quasiparticles. The extension of this analysis to the study of finite systems and the construction of a relativistic shell model of nuclear structure is straightforward. Finally, in Sec. V we discuss the σ plus ω model of nuclear structure in order to provide insight into the saturation mechanism in a relativistic theory.

II. TRANSFORMATION OF THE HAMILTONIAN

For simplicity we begin with a Hamiltonian describing a nucleon field interacting with a scalar meson field. (The generalization to include other fields describing, for example, the interaction with the π, ρ, and ω mesons introduces no new complexity.[27]) We have

$$H=H_{\text{Dirac}}+H_{\sigma}+H_{\text{int}}\ , \tag{2.1}$$

where

$$H_{\text{Dirac}}=\int d\vec{x}:\bar{\Psi}(\vec{x})(\vec{\gamma}\cdot\vec{p}+m)\bar{\Psi}(\vec{x}):\ ,$$

$$H_{\sigma}=\tfrac{1}{2}\int d\vec{x}:[\Pi^2(\vec{x})+|\vec{\nabla}\phi|^2+\mu^2\phi^2(\vec{x})]:\ , \tag{2.2}$$

$$H_{\text{int}}=g\int :\bar{\Psi}(\vec{x})\Psi(\vec{x})\phi(\vec{x}):\ . \tag{2.3}$$

It was shown in some detail in a previous work,[27] that if we make the static approximation for the meson field, we have $H_{\sigma}=-\frac{1}{2}H_{\text{int}}$ and may write,

$$H=H_{\text{Dirac}}+\tfrac{1}{2}H_{\text{int}}\ ,$$

$$\text{(static approximation)}\ . \tag{2.4}$$

We will use this approximation throughout this work, and therefore, the Hamiltonian of Eq. (2.4) will be the starting point of our analysis.

It is useful to expand the field $\Psi(\vec{x})\equiv\Psi(\vec{x},t=0)$ in terms of free-field creation and destruction operators,

$$\Psi(\vec{x})=\sum_{\vec{p},s}\left[\frac{m}{VE(\vec{p})}\right]^{1/2}[u(\vec{p},s)b(\vec{p},s)e^{i\vec{p}\cdot\vec{x}}+v(\vec{p},s)d^{\dagger}(\vec{p},s)e^{-i\vec{p}\cdot\vec{x}}]\ , \tag{2.5}$$

where we have used the notation of Bjorken and Drell.[29] Here V is the quantization volume. The vacuum state for the noninteracting fields is defined to have the property,

$$d(\vec{p},s)\,|\,\text{vac}\rangle=b(\vec{p},s)\,|\,\text{vac}\rangle=0\ . \tag{2.6}$$

It is now useful to rewrite Eq. (2.5) as

$$\Psi(\vec{x})=\sum_{\vec{p},s}\left[\frac{m}{VE(\vec{p})}\right]^{1/2}[u(\vec{p},s)b(\vec{p},s) +w(\vec{p},s)d^{\dagger}(-\vec{p},-s)]e^{i\vec{p}\cdot\vec{x}} \tag{2.7}$$

$$=\sum_{\vec{p},s}\left[\frac{m}{VE(\vec{p})}\right]^{1/2}U(\vec{p},s)\cdot\Phi(\vec{p},s)e^{i\vec{p}\cdot\vec{x}}\ , \tag{2.8}$$

where we have put $w(\vec{p},s)\equiv v(-\vec{p},-s)$. In Eq. (2.8) we have introduced a scalar product between the arrays:

$$U(\vec{p},s)\equiv\begin{bmatrix}u(\vec{p},s)\\ w(\vec{p},s)\end{bmatrix}\ , \tag{2.9}$$

and

$$\Phi(\vec{p},s)\equiv\begin{bmatrix}b(\vec{p},s)\\ d^{\dagger}(-\vec{p},-s)\end{bmatrix}\ . \tag{2.10}$$

We can distinguish between the two members of the arrays by an additional index, for example, we can put $U_1(\vec{p},s)=u(\vec{p},s)$ and $U_2(\vec{p},s)=w(\vec{p},s)$, etc. We will also make use of the relation

$$\bar{\Psi}(\vec{x})=\sum_{\vec{p},s}\left[\frac{m}{VE(\vec{p})}\right]^{1/2}\Phi^{\dagger}(\vec{p},s)\cdot \times\bar{U}(\vec{p},s)e^{-i\vec{p}\cdot\vec{x}}\ . \tag{2.11}$$

The use of this notation facilitates the introduction of a unitary transformation of the spinors and operators that leaves the field $\Psi(\vec{x})$ invariant. In analogy to Eqs. (2.9) and (2.10), we introduce the arrays:

$$F(\vec{p},s)\equiv\begin{bmatrix}f(\vec{p},s)\\ h(\vec{p},s)\end{bmatrix}\ , \tag{2.12}$$

$$\Theta(\vec{p},s)\equiv\begin{bmatrix}B(\vec{p},s)\\ D^{\dagger}(-\vec{p},-s)\end{bmatrix}\ , \tag{2.13}$$

and we require that

$$U(\vec{p},s)\cdot\Phi(\vec{p},s)=F(\vec{p},s)\cdot\Theta(\vec{p},s)\ . \tag{2.14}$$

To this end we define

$$f(\vec{p},s)=e^{i\theta_{\vec{p}}(i\vec{\gamma}\cdot\hat{p})}u(\vec{p},s)\ , \tag{2.15}$$

$$=(\cos\theta_{\vec{p}}-\vec{\gamma}\cdot\hat{p}\sin\theta_{\vec{p}})u(\vec{p},s)\ , \tag{2.16}$$

$$h(\vec{p},s)=e^{i\theta_{\vec{p}}(i\vec{\gamma}\cdot\hat{p})}w(\vec{p},s)\ , \tag{2.17}$$

$$=(\cos\theta_{\vec{p}}-\vec{\gamma}\cdot\hat{p}\sin\theta_{\vec{p}})w(\vec{p},s)\ , \tag{2.18}$$

or equivalently,

$$F(\vec{p},s)=e^{i\theta_{\vec{p}}(i\vec{\gamma}\cdot\hat{p})}U(\vec{p},s)\ . \tag{2.19}$$

We remark that as $w(\vec{p},s)=\gamma^5\gamma^0u(\vec{p},s)$, we also have $h(\vec{p},s)=\gamma^5\gamma^0f(\vec{p},s)$. We also note that, while $u^{\dagger}(\vec{p},s')u(\vec{p},s)=\delta_{ss'}E(\vec{p})/m$ and $\bar{u}(\vec{p},s')u(\vec{p},s)=\delta_{ss'}$, we have

$$f^{\dagger}(\vec{p},s')f(\vec{p},s)=h^{\dagger}(\vec{p},s')h(\vec{p},s)=\delta_{ss'}E(\vec{p})/m\ , \tag{2.20}$$

$$\bar{f}(\vec{p},s)f(\vec{p},s)=\left[1-2\sin^2\theta_{\vec{p}}-2\frac{|\vec{p}|}{m}\cos\theta_{\vec{p}}\sin\theta_{\vec{p}}\right]\ , \tag{2.21}$$

and

$$\bar{h}(\vec{p},s)h(\vec{p},s)=\bar{f}(\vec{p},s)\gamma^5\gamma^0\gamma^5\gamma^0f(\vec{p},s)\ , \tag{2.22}$$

$$=-\bar{f}(\vec{p},s)f(\vec{p},s)\ . \tag{2.23}$$

It will also be useful to note the relations:

$$f(\vec{p},s)=\alpha(\vec{p})u(\vec{p},s) +\beta(\vec{p})\sum_{s'}\langle s'\mid\vec{\sigma}\cdot\hat{p}\mid s\rangle w(\vec{p},s') \tag{2.24}$$

and

$$h(\vec{p},s)=\alpha(\vec{p})w(\vec{p},s) -\beta(\vec{p})\sum_{s'}\langle s'\mid\vec{\sigma}\cdot\hat{p}\mid s\rangle u(\vec{p},s')\ , \tag{2.25}$$

where $\alpha(\vec{p})\equiv\cos\theta_{\vec{p}}$ and $\beta(\vec{p})\equiv\sin\theta_{\vec{p}}$. These relations are particularly simple for states of definite helicity. For example, if $\vec{p}$ is taken along the quantization axis we have

$$f(\vec{p},s)=\alpha(\vec{p})u(\vec{p},s) +\beta(\vec{p})(-1)^{1/2-s}w(\vec{p},s)\ , \tag{2.26}$$

and

$$h(\vec{p},s)=\alpha(\vec{p})w(\vec{p},s) -\beta(\vec{p})(-1)^{1/2-s}u(\vec{p},s)\ . \tag{2.27}$$

Now we may use the relations

$$[m/E(\vec{p})]f^{\dagger}(\vec{p},s)u(\vec{p},s)=\alpha(\vec{p})\delta_{ss'}\ , \tag{2.28}$$

$$[m/E(\vec{p})]f^{\dagger}(\vec{p},s)w(\vec{p},s') = \beta(\vec{p})\langle s \mid \vec{\sigma}\cdot\hat{p} \mid s'\rangle , \tag{2.29}$$

$$[m/E(\vec{p})]h^{\dagger}(\vec{p},s)u(\vec{p},s') = -\beta(\vec{p})\langle s \mid \vec{\sigma}\cdot\hat{p} \mid s'\rangle , \tag{2.30}$$

$$[m/E(\vec{p})]h^{\dagger}(\vec{p},s)w(\vec{p},s') = \delta_{ss'}\alpha(\vec{p}) , \tag{2.31}$$

and

$$f^{\dagger}(\vec{p},s)h(\vec{p},s') = h^{\dagger}(\vec{p},s)f(\vec{p},s') = 0 , \tag{2.32}$$

to show that Eq. (2.14) implies

$$B(\vec{p},s) = \alpha(\vec{p})b(\vec{p},s) + \beta(\vec{p})\sum_{s'}\langle s \mid \vec{\sigma}\cdot\hat{p} \mid s'\rangle d^{\dagger}(-\vec{p},-s') , \tag{2.33}$$

and

$$D^{\dagger}(-\vec{p},-s) = \alpha(\vec{p})d^{\dagger}(-\vec{p},-s) - \beta(\vec{p})\sum_{s'}\langle s \mid \vec{\sigma}\cdot\hat{p} \mid s'\rangle b(\vec{p},s') . \tag{2.34}$$

Again, these relations are particularly simple for states of definite helicity.

We remark that the new operators $\Theta_{\lambda}(\vec{p},s)$ satisfy the same commutation relations as the $\Phi_{\lambda}(\vec{p},s)$:

$$\{\Theta_{\lambda}^{\dagger}(\vec{p},s),\Theta_{\lambda'}(\vec{p}',s)\} = \delta_{\lambda\lambda'}\delta_{\vec{p}\vec{p}'}\delta_{ss'} , \tag{2.35}$$

$$\{\Theta_{\lambda}(\vec{p},s),\Theta_{\lambda'}(\vec{p}',s')\} = 0 , \tag{2.36}$$

and

$$\{\Theta_{\lambda}^{\dagger}(\vec{p},s),\Theta_{\lambda'}^{\dagger}(\vec{p}',s')\} = 0 . \tag{2.37}$$

Finally we introduce a new vacuum state, $|\overline{\text{vac}}\rangle$, which depends on $\theta_{\vec{p}}$. This state has the property:

$$D(\vec{p},s)\,|\,\overline{\text{vac}}\rangle = B(\vec{p},s)\,|\,\overline{\text{vac}}\rangle = 0 . \tag{2.38}$$

We may consider either an infinite system or a finite system, however for simplicity we will consider nuclear matter in this work. The transformations discussed above are those appropriate to the study of nuclear matter.

III. THE DIRAC HAMILTONIAN

In this section we consider the form of the Dirac Hamiltonian in the new basis. We have

$$H_{\text{Dirac}} = \sum_{\vec{p}s_1s_2\lambda\lambda'} :\Theta_{\lambda}^{\dagger}(\vec{p},s_1)\Theta_{\lambda'}(\vec{p},s_2): \times\bar{F}_{\lambda}(\vec{p},s_1)(\vec{\gamma}\cdot\vec{p}+m)F_{\lambda'}(\vec{p},s_2) , \tag{3.1}$$

where the normal product is defined with respect to the vacuum state $|\overline{\text{vac}}\rangle$. This is the same prescription as that used by Walecka[5] and, in part, *this normal ordering prescription serves to define a model problem.* Other procedures may be considered more satisfactory. For example, Chin has given a treatment of the relativistic Hartree problem using a renormalizable field theory.[9] In that theory various mass counter terms are added to the Lagrangian and these terms give rise to density dependent effects when one calculates the energy of a many-body system. However, the calculations with the renormalized Hartree theory give results that are not much different from the model used in this work, which is based on the normal-ordering prescription given above.

Clearly, it would be desirable to have a fundamental theory which would avoid the necessity of somewhat arbitrary calculational prescriptions. However, in formulating a model to describe interacting nucleons and mesons one is not dealing with fundamental fields. The model of Chin represents an attempt to achieve a degree of mathematical consistency in a relativistic Hartree theory, but in that work one is still using local fields to describe composite objects such as nucleons and mesons.

With these reservations in mind we return to the evaluation of matrix elements of H_{Dirac}. Recalling Eq. (2.38), we note that the only nonvanishing contraction with respect to the ground state of baryon number N is

$$B_{\lambda}^{\dagger}(\vec{p},s)B_{\lambda'}(\vec{p}',s') = \langle N \mid B_{\lambda}^{\dagger}(\vec{p},s)B_{\lambda'}(\vec{p},s') \mid N\rangle = \delta_{\lambda,1}\delta_{\lambda',1}\theta(k_F - |\,\vec{p}\,|)\delta_{ss'}\delta_{\vec{p}\vec{p}'} . \tag{3.2}$$

Here we introduce an index k_F, which is related to the density as $\rho = 2k_F^3/(3\pi^2)$. In writing Eq. (3.2) we have assumed that our system does not exhibit pairing in the ground state and that the new quasiparticles can be put into one-to-one correspondance with the quasiparticles of the noninteracting system.

We have,

$$\langle N \mid H_{\text{Dirac}} \mid N \rangle = \sum_{\vec{p},s} \bar{f}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m)f(\vec{p},s) \tag{3.3}$$

$$= \sum_{s} \int \frac{d\vec{p}}{(2\pi)^3} \frac{m}{E(\vec{p})} f^{\dagger}(\vec{p},s)(\vec{\alpha}\cdot\vec{p}+\gamma^0 m)f(\vec{p},s) \tag{3.4}$$

$$= \sum_{s} \int \frac{d\vec{p}}{(2\pi)^3} E(\vec{p})[1-2\beta^2(\vec{p})] \, . \tag{3.5}$$

In these equations we are suppressing explicit reference to isospin, for simplicity of notation. [We note that Eq. (3.5) has been given previously and was obtained using a Green's function technique.[26–28]]

For future reference we now write out Eq. (3.1) in detail:

$$\begin{aligned} H_{\text{Dirac}} = \sum_{\vec{p}ss'} [& B^{\dagger}(\vec{p},s)\bar{f}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m)f(\vec{p},s')B(\vec{p},s') \\ & -D^{\dagger}(-\vec{p},-s)\bar{h}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m)h(\vec{p},s')D(-\vec{p},-s') \\ & +B^{\dagger}(\vec{p},s)\bar{f}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m)h(\vec{p},s')D(-\vec{p},-s') \\ & +D^{\dagger}(-\vec{p},-s)\bar{h}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m)f(\vec{p},s')B(\vec{p},s')] \, . \end{aligned} \tag{3.6}$$

Our program now involves the study of the interaction Hamiltonian with the aim of writing the total Hamiltonian as a constant ($E_{\text{RHF}} = \langle N \mid H \mid N \rangle$), a part diagonal in the quasiparticle operators, and a residual interaction, the latter in a normal-ordered form, where the ordering is with respect to the ground state, $\mid N \rangle$.

IV. THE RELATIVISTIC HARTREE-FOCK HAMILTONIAN AND THE RESIDUAL INTERACTION

We consider Eq. (2.3) and introduce an expression for the field $\phi(\vec{x})$ in terms of its source. We again work in the static limit for the meson field.[27] Thus we can write

$$\begin{aligned} H_{\text{int}} = \tfrac{1}{2} : \sum_{p,\lambda,s} & \Theta^{\dagger}_{\lambda_1}(\vec{p}_1,s_1)\Theta_{\lambda_2}(\vec{p}_2,s_2)\Theta^{\dagger}_{\lambda_3}(\vec{p}_3,s_3)\Theta_{\lambda_4}(\vec{p}_4,s_4) : \\ & \times \langle \bar{F}_{\lambda_1}(\vec{p}_1,s_1)\bar{F}_{\lambda_3}(\vec{p}_3,s_3) \mid \hat{V} \mid F_{\lambda_2}(\vec{p}_2,s_2)F_{\lambda_4}(\vec{p}_4,s_4)\rangle \, , \end{aligned} \tag{4.1}$$

where we have incorporated factors of $\{m/[VE(\vec{p})]\}^{1/2}$ in the definition of the interaction $\hat{V}$. Again we define the normal ordering of the interaction with respect to the new vacuum $\mid \overline{\text{vac}} \rangle$. The same comments made after Eq. (3.1) pertain here. This normal ordering prescription is part of the definition of our *model* problem.

Let us now think of the normal product in Eq. (4.1) written out in terms of various contractions which arise when one defines a *new* normal product taken with respect to the ground state of baryon number N, $\mid N \rangle$. That is, we are introducing particle and hole states with respect to the Fermi level of the interacting system. Thus

$$
\begin{aligned}
&:\Theta^{\dagger}_{\lambda_1}(\vec{p}_1,s_1)\Theta_{\lambda_2}(\vec{p}_2,s_2)\Theta^{\dagger}_{\lambda_3}(\vec{p}_3,s_3)\Theta_{\lambda_4}(\vec{p}_4,s_4):\\
&\quad=\Theta^{\dagger\cdot}_{\lambda_1}(\vec{p}_1,s_1)\Theta^{\cdot}_{\lambda_2}(\vec{p}_2,s_2)\Theta^{\dagger\cdot\cdot}_{\lambda_3}(\vec{p}_3,s_3)\Theta^{\cdot\cdot}_{\lambda_4}(\vec{p}_4,s_4)\\
&\qquad+\Theta^{\dagger\cdot}_{\lambda_1}(\vec{p}_1,s_1)\Theta^{\cdot\cdot}_{\lambda_2}(\vec{p}_2,s_2)\Theta^{\dagger\cdot\cdot}_{\lambda_3}(\vec{p}_3,s_3)\Theta^{\cdot}_{\lambda_4}(\vec{p}_4,s_4)\\
&\qquad+N[:\Theta^{\dagger}_{\lambda_1}(\vec{p}_1,s_1)\Theta_{\lambda_2}(\vec{p}_2,s_2):\Theta^{\dagger\cdot}_{\lambda_3}(\vec{p}_3,s_3)\Theta^{\cdot}_{\lambda_4}(\vec{p}_4,s_4)]\\
&\qquad+N[\Theta^{\dagger\cdot}_{\lambda_1}(\vec{p}_1,s_1)\theta^{\cdot}_{\lambda_2}(\vec{p}_2,s_2):\Theta^{\dagger}_{\lambda_3}(\vec{p}_3s_3)\Theta_{\lambda_4}(\vec{p}_4,s_4):]\\
&\qquad+N[\Theta^{\dagger\cdot}_{\lambda_1}(\vec{p}_1,s_1):\Theta_{\lambda_2}(\vec{p}_2,s_{2)}\Theta^{\dagger}_{\lambda_3}(\vec{p}_3,s_3):\Theta^{\cdot}_{\lambda}(\vec{p}_4,s_4)]\\
&\qquad+N[:\Theta^{\dagger}_{\lambda_1}(\vec{p}_1,s_1)\Theta^{\cdot}_{\lambda_2}(\vec{p}_2,s_2)\Theta^{\dagger\cdot}_{\lambda_3}(\vec{p}_3,s_3)\Theta_{\lambda_4}(\vec{p}_4,s_4):]\\
&\qquad+N[:\Theta^{\dagger}_{\lambda_1}(\vec{p}_1,s_1)\Theta_{\lambda_2}(\vec{p}_2,s_2)\Theta^{\dagger}_{\lambda_3}(\vec{p}_3,s_3)\Theta_{\lambda_4}(\vec{p}_4,s_4):]\ .
\end{aligned}
\tag{4.2}
$$

Since the normal product of any operator product yields zero expectation value in the state $|N\rangle$, we can calculate the energy from the fully contracted terms. We obtain

$$
\begin{aligned}
E_{\mathrm{RHF}}&=\langle N|H|N\rangle\\
&=\sum_s\int\frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}\bar{f}(\vec{p},s)(\vec{\gamma}\cdot\vec{p}+m)f(\vec{p},s)\\
&\quad+\tfrac{1}{2}\sum_{ss'}\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{m}{E(\vec{p})}\frac{m}{E(\vec{q})}\langle\bar{f}(\vec{p},s)\bar{f}(\vec{q},s')|V(1-P_{12})|f(\vec{p},s)f(\vec{q},s')\rangle\ ,
\end{aligned}
\tag{4.3}
$$

where we have now written the $[m/E(\vec{p})]$ factors explicitly. (This result was obtained previously using a Green's function technique.[27]) Of course, we have not as yet provided a prescription for determining $\alpha(\vec{p})=\cos\theta_{\vec{p}}$. The prescription for the choice of the spinors $f(\vec{p},s)$ and $h(\vec{p},s)$ is obtained from the study of the singly-contracted terms of Eq. (4.2). As usual it is useful to write these terms of H_{int} together with H_{Dirac}. Let us define

$$
H_1=H_{\mathrm{Dirac}}+\tfrac{1}{2}H^{\mathrm{sc}}_{\mathrm{int}}\ ,
\tag{4.4}
$$

where the superscript sc refers to the extraction of only the singly-contracted terms of H_{int}.

We have

$$
H_1=\sum_{\vec{p}ss'\lambda\lambda'}:\Theta^{\dagger}_{\lambda}(\vec{p},s)\bar{F}_{\lambda}(\vec{p},s)[\vec{\gamma}\cdot\vec{p}+m+\Sigma(\vec{p})]F_{\lambda'}(\vec{p},s')\Theta_{\lambda'}(\vec{p},s'):\ ,
\tag{4.5}
$$

where

$$
\Sigma(\vec{p})=\sum_{\vec{p}s''}\langle\vec{p},\bar{f}(\vec{p}',s'')|\hat{V}(1-P_{12})|\vec{p},f(\vec{p}',s'')\rangle
\tag{4.6}
$$

or alternatively,

$$
\bar{F}_{\lambda}(\vec{p},s)\Sigma(\vec{p})F_{\lambda'}(\vec{p},s')=\sum_{\vec{p}'s''}\langle\bar{F}_{\lambda}(\vec{p},s)\bar{F}_1(\vec{p}',s'')|\hat{V}(1-P_{12})|F_{\lambda'}(\vec{p},s')F_1(\vec{p}',s'')\rangle\ .
\tag{4.7}
$$

We now require that $F_{\lambda}(\vec{p},s)$ be a solution of the Dirac equation,

$$
[\vec{\gamma}\cdot\vec{p}+m+\Sigma(\vec{p})]F_{\lambda}(\vec{p},s)=\gamma^0\epsilon_{\lambda}(\vec{p})F_{\lambda}(\vec{p},s)\ ,
\tag{4.8}
$$

or

$$
[\vec{\gamma}\cdot\vec{p}+m+\Sigma(\vec{p})]f(\vec{p},s)=\gamma^0\epsilon(\vec{p})f(\vec{p},s)\ ,
\tag{4.9}
$$

$$
[\vec{\gamma}\cdot\vec{p}+m+\Sigma(\vec{p})]h(\vec{p},s)=-\gamma^0\bar{\epsilon}(\vec{p})h(\vec{p},s)\ ,
\tag{4.10}
$$

where we have put $\epsilon_1(\vec{p})=\epsilon(\vec{p})$ and $\epsilon_2(\vec{p})=-\bar{\epsilon}(\vec{p})$ with $\bar{\epsilon}(\vec{p})>0$.

Thus we have

$$H_1=\sum_{\vec{p},s}[\epsilon(\vec{p})B^\dagger(\vec{p},s)B(\vec{p},s)+\bar{\epsilon}(\vec{p})D^\dagger(\vec{p},s)D(\vec{p},s)]\ . \tag{4.11}$$

Inspection of Eq. (3.6) clarifies these manipulations somewhat since we see that, for example,

$$\bar{h}(\vec{p}s,)[\vec{\gamma}\cdot\vec{p}+m+\Sigma(\vec{p})]f(\vec{p},s')=\bar{h}(\vec{p},s)\gamma^0\epsilon(\vec{p})f(\vec{p},s')$$
$$=\epsilon(\vec{p})h^\dagger(\vec{p},s)f(\vec{p},s')=0\ . \tag{4.12}$$

Thus by combining the singly-contracted terms from $H_{\rm int}$ with $H_{\rm Dirac}$ we bring H_1 into diagonal form.

We now have

$$H=E_{\rm RHF}+\sum_{\vec{p},s}\{\epsilon(\vec{p})N[B^\dagger(\vec{p},s)B(\vec{p},s)]+\bar{\epsilon}(\vec{p})D^\dagger(\vec{p},s)D(\vec{p},s)\}$$
$$+\tfrac{1}{4}\sum_{\vec{p},s,\lambda}N[:\Theta^\dagger_{\lambda_1}(\vec{p}_1,s_1)\Theta_{\lambda_2}(\vec{p}_2,s_2)\Theta^\dagger(\vec{p}_3,s_3)\Theta(\vec{p}_4,s_4):]$$
$$\times\langle\bar{F}_{\lambda_1}(\vec{p}_1,s_1)\bar{F}_{\lambda_3}(\vec{p}_3,s_3)\,|\,\hat{V}(1-P_{12})\,|\,F_{\lambda_2}(\vec{p}_2,s_2)F_{\lambda_4}(\vec{p}_4,s_4)\rangle\ . \tag{4.13}$$

Note that we have written $N[B^\dagger(\vec{p},s)B(\vec{p},s)]$ in the second term of Eq. (4.13) since the contracted term has been removed to construct the first term, $E_{\rm RHF}=\langle N\,|\,H\,|\,N\rangle$.

As usual, it is useful to introduce creation and destruction operators defined with respect to the Fermi surface:

$$C^\dagger(\vec{p},s)=B^\dagger(\vec{p},s),\quad |\,\vec{p}\,|>k_F\ ; \tag{4.14}$$

$$C(\vec{p},s)=B^\dagger(\vec{p},s),\quad |\,\vec{p}\,|<k_F\ ; \tag{4.15}$$

$$C(\vec{p},s)\,|\,N\rangle=0,\quad \text{all } \vec{p} \tag{4.16}$$

so that

$$\sum_{\vec{p},s}\epsilon(\vec{p})N[B^\dagger(\vec{p},s)B(\vec{p},s)]$$
$$=\sum_{\vec{p},s}[\epsilon(\vec{p})C^\dagger(\vec{p},s)C(\vec{p},s)\theta(|\,\vec{p}\,|-k_F)-\epsilon(\vec{p})C^\dagger(p,s)C(\vec{p},s)\theta(k_F-|\,\vec{p}\,|)]\ .$$

We see that the symbol N: : in Eq. (4.13) can be considered as a prescription to move all $C(\vec{p},s)$ and $D(\vec{p},s)$ to the right in any operator product. Using the notation $\mathcal{N}$ for this generalized normal product we have

$$H=E_{\rm RHF}+\sum_{\vec{p},s}[\,\epsilon(\vec{p})C^\dagger(\vec{p},s)C(\vec{p},s)\theta(|\,\vec{p}\,|-k_F)$$
$$-\epsilon(\vec{p})C^\dagger(\vec{p},s)C(\vec{p},s)\theta(k_F-|\,\vec{p}\,|)+\bar{\epsilon}(\vec{p})D^\dagger(\vec{p},s)D(\vec{p},s)]$$
$$+\tfrac{1}{4}\mathcal{N}\sum_{\vec{p},\lambda,s}[\Theta^\dagger_{\lambda_1}(\vec{p}_1,s_1)\Theta_{\lambda_2}(\vec{p}_2,s_2)\Theta^\dagger_{\lambda_3}(\vec{p}_3,s_3)\Theta_{\lambda_4}(\vec{p}_4,s_4)]$$
$$\times\langle\bar{F}_{\lambda_1}(\vec{p}_1,s_1)\bar{F}_{\lambda_3}(\vec{p}_3,s_3)\,|\,\hat{V}(1-P_{12})\,|\,F_{\lambda_2}(\vec{p}_2,s_2)F_{\lambda_4}(\vec{p}_4,s_4)\rangle\ . \tag{4.17}$$

If we discard the residual interaction for the moment we can define

$$H_{\rm RHF}=E_{\rm RHF}+\sum_{\vec{p},s}\{\,\epsilon(\vec{p})C^\dagger(\vec{p},s)C(\vec{p},s)\theta(|\,\vec{p}\,|-k_F)$$
$$-\epsilon(\vec{p})C^\dagger(\vec{p},s)C(\vec{p},s)\theta(k_F-|\,\vec{p}\,|)+\bar{\epsilon}(\vec{p})D^\dagger(\vec{p},s)D(\vec{p},s)\}\ . \tag{4.18}$$

Thus

$$H_{\mathrm{RHF}}\,|\,N\rangle = E_{\mathrm{RHF}}\,|\,N\rangle\ , \tag{4.19}$$

with

$$|\,N\rangle = \prod_{|\,\vec{\mathrm{p}}\,|\,\le k_F} B^{\dagger}(\vec{\mathrm{p}},s)\,|\,\overline{\mathrm{vac}}\rangle\ . \tag{4.20}$$

If the residual interaction is now included in perturbation theory, we see that the admixtures to the wave function are of the two-particle, two-hole type, where by "hole" we include the states excited by $D^{\dagger}(\vec{\mathrm{p}},s)$ as well as those excited by $C^{\dagger}(\vec{\mathrm{p}},s)$. This result is typical of self-consistent field theories except it is now extended to include *all* states below the Fermi level, *including the negative-energy states.* Of course, in the infinite system, one-particle, one-hole states of the usual type, that is those created by two $C^{\dagger}$'s, are excluded by consideration of momentum conservation.

We have shown elsewhere[26–28] that the inclusion of negative-energy states makes a major change in the saturation curve of nuclear matter and that a theory with $\theta_{\vec{\mathrm{p}}}=0$ is inadequate. Once one goes to the new representation defined by the solution of Eqs. (4.9) and (4.10) the corrections to the theory involve the introduction of two-particle, two-hole pairs. More precisely, we can classify the holes as holes in the positive-energy sea, h, and holes in the negative-energy sea, $\bar{\mathrm{h}}$. Therefore, we can note that the corrections to the theory involve the admixture of (2p, 2h), (2p, 2$\bar{\mathrm{h}}$), and (2p,h,$\bar{\mathrm{h}}$) states. The matrix elements for the admixtures of such states are significantly smaller than those that are included in the passage to the new representation. Thus it is not unreasonable to neglect the terms involving D and $D^{\dagger}$ at this point. In this approximation one has

$$\begin{aligned}\tilde{H} = E_{\mathrm{RHF}} &+ \sum_{\vec{\mathrm{p}},s}\left[\epsilon(\vec{\mathrm{p}})C^{\dagger}(\vec{\mathrm{p}},s)C(\vec{\mathrm{p}},s)\theta(\,|\,\vec{\mathrm{p}}\,|\,-k_F)-\epsilon(\vec{\mathrm{p}})C^{\dagger}(\vec{\mathrm{p}},s)C(\vec{\mathrm{p}},s)\theta(k_F-|\,\vec{\mathrm{p}}\,|\,)\right]\\ &+\tfrac{1}{4}\sum_{\vec{\mathrm{p}},\lambda,s} N[B^{\dagger}(\vec{\mathrm{p}}_1,s_1)B(\vec{\mathrm{p}}_2,s_2)B^{\dagger}(\vec{\mathrm{p}}_3,s_3)B(\vec{\mathrm{p}}_4,s_4)]\\ &\times\langle\bar{f}(\vec{\mathrm{p}}_1,s_1)\bar{f}(\vec{\mathrm{p}}_3,s_3)\,|\,\hat{V}(1-P_{12})\,|\,f(\vec{\mathrm{p}}_2,s_2)f(\vec{\mathrm{p}}_4,s_4)\rangle\ ,\end{aligned} \tag{4.21}$$

which can be rewritten in a more conventional form,

$$\begin{aligned}\tilde{H} = E_{\mathrm{RHF}} &+ \sum_{\vec{\mathrm{p}},s} C^{\dagger}(\vec{\mathrm{p}},s)C(\vec{\mathrm{p}},s)\left[\epsilon(\vec{\mathrm{p}})\theta(\,|\,\vec{\mathrm{p}}\,|\,-k_F)-\epsilon(\vec{\mathrm{p}})\theta(k_F-|\,\vec{\mathrm{p}}\,|\,)\right]\\ &+\tfrac{1}{4}\sum_{\vec{\mathrm{p}},\lambda,s} N[B^{\dagger}(\vec{\mathrm{p}}_1,s_1)B^{\dagger}(\vec{\mathrm{p}}_2,s_2)B(\vec{\mathrm{p}}_3,s_3)B(\vec{\mathrm{p}}_4,s_4)]\\ &\times\langle\bar{\phi}(\vec{\mathrm{p}}_1,s_1)\bar{\phi}(\vec{\mathrm{p}}_2,s_2)\,|\,V(1-P_{12})\,|\,\phi(\vec{\mathrm{p}}_4,s_4)\phi(\vec{\mathrm{p}}_3,s_3)\rangle\ ,\end{aligned} \tag{4.22}$$

where we have introduced

$$\phi(\vec{\mathrm{p}},s)\equiv\left[\frac{m}{VE(\vec{\mathrm{p}})}\right]^{1/2} f(\vec{\mathrm{p}},s)\ . \tag{4.23}$$

In a previous work we showed how to include short-range correlations in a theory of this type.[26–28] We do not repeat that analysis here except to note that the interaction V may be replaced by a reaction matrix, G, which represents the scattering amplitude in the medium. Thus we may replace the two-body matrix element in Eq. (4.22) by

$$\langle\vec{\mathrm{p}}_1s_1,\vec{\mathrm{p}}_2s_2\,|\,G[\epsilon(\vec{\mathrm{p}}_1)+\epsilon(\vec{\mathrm{p}}_2)]\,|\,\vec{\mathrm{p}}_4s_4,\vec{\mathrm{p}}_3s_3\rangle \equiv\langle\bar{\phi}(\vec{\mathrm{p}}_1,s_1)\bar{\phi}(\vec{\mathrm{p}}_2,s_2)\,|\,G(1-P_{12})\,|\,\phi(\vec{\mathrm{p}}_4,s_4)\phi(\vec{\mathrm{p}}_3,s_3)\rangle\ . \tag{4.24}$$

V. A VARIATIONAL PRINCIPLE AND THE SATURATION MECHANISM

In this section we rederive the equations determining the wave functions, $f(\vec{p},s)$, from a variational principle in a manner similar to that used in nonrelativistic problems. The elimination of dangerous diagrams (the diagonalization of H_1) is equivalent to this technique. Further we discuss the saturation mechanism in the relativistic theory using a simple model. Here we are able to use the insight obtained from quite complete numerical calculations we have performed previously.[26–28] This discussion, although equivalent in many aspects to that of Walecka,[5] is focused on the new terms introduced into the expression for the energy when negative-energy states are included in the $f(\vec{p},s)$. In this way we are able to isolate those terms that have their origin in the passage from a nonrelativistic to a relativistic description.

A. Variational principle

The wave functions $f(\vec{p},s)$ are to be normalized as in Eq. (2.20):

$$\frac{m}{E(\vec{p})}\bar{f}(\vec{p},s)\gamma^0 f(\vec{p},s')=\delta_{ss'} . \tag{5.1}$$

We may consider Eq. (5.1) as a constraint equation for each value of $\vec{p}$ and s. (We are again suppressing reference to isospin.) We now modify H by incorporating these constraints using Lagrange multipliers, $\epsilon(\vec{p})$. Thus we study

$$H'=H-\sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}\epsilon(\vec{p})\bar{f}(\vec{p},s)\gamma^0 f(\vec{p},s):B^\dagger(\vec{p},s)B(\vec{p},s): \tag{5.2}$$

The expectation value of H' in the new ground state $|N\rangle$ is

$$\langle N|H'|N\rangle=E_{\rm RHF}-\sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}\epsilon(\vec{p})\theta(k_F-|\vec{p}|)\bar{f}(\vec{p},s)\gamma^0 f(\vec{p},s) , \tag{5.3}$$

where $E_{\rm RHF}$ is given by Eq. (4.3). Variation of Eq. (5.3) with respect to $\bar{f}(\vec{p},s)$ yields the equation of motion for $f(\vec{p},s)$ given in Eq. (4.9). We quote here one other result which is quite similar to that obtained in a nonrelativistic calculation. When the eigenenergies and eigenfunctions, $\epsilon(\vec{p})$, and $f(\vec{p},s)$, have been determined, we may write

$$E_{\rm RHF}=\tfrac{1}{2}\sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}\bar{f}(\vec{p},s)[\vec{\gamma}\cdot\vec{p}+m+\gamma^0\epsilon(\vec{p})]f(\vec{p},s) , \tag{5.4}$$

where we have eliminated the potential terms using the equation of motion obeyed by $f(\vec{p},s)$.

B. Saturation in the σ and ω model

The σ and ω model of Walecka[5] is quite useful in understanding the dynamics of saturation on a qualitative basis. We are limited to qualitative considerations since our calculations have shown that (a) Short-range correlations play an essential role in modifying the values used for the σ-nucleon and ω-nucleon coupling constants from their free-space values to the values of the effective coupling constants of the σ-ω model. (b) The pion-exchange contribution to the nucleon self-energy is approximately $+30$ MeV in a Hartree-Fock calculation. If one includes correlations the (correlated) one-pion-exchange contribution the nucleon self-energy can vary from zero to -50 MeV depending upon the strength of the π-N tensor force. (c) If the self-energy operator for the nucleon is written as a sum of scalar and vector parts, i.e., $\Sigma(p)=A(p)+\gamma^0 B(p)$, the values of A and B are not directly related to σ and ω exchange. For example, about 30% of the value of A can be ascribed to effects of ω exchange. (Note that A is about -400 MeV, while σ exchange alone gives a value of about -250 MeV for A.[27])

Our numerical calculations show that the parameter $\beta(\vec{p})$ is well approximated by a linear function of $|\vec{p}|$ for $|\vec{p}|\leq k_F$. That is $\beta(\vec{p})=\lambda|\vec{p}|/m$ with $\lambda\simeq 0.4$ for $k_F=1.36$ fm^{-1}.

In the following we shall ignore the π and ρ mesons and analyze the phenomenological σ and ω model in a manner different from that of Walecka[5] in order to focus more clearly on the physical phenomena that are responsible for saturation.

(Indeed Walecka's treatment is exact but somewhat more complicated.) Let us write E_{RHF}, incorporating the constraint $\alpha^2+\beta^2=1$,

$$E_{\mathrm{RHF}}^{\sigma,\omega}=\sum_s \int \frac{d\vec{p}}{(2\pi)^3}E(\vec{p})[1-2\beta^2(\vec{p})]$$

$$-\frac{1}{2}\frac{g_\sigma{}^2}{m_\sigma{}^2}\left\{\sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}[1-2\beta^2(\vec{p})-2\alpha(\vec{p})\beta(\vec{p})\,|\,\vec{p}\,|\,/m]\right\}^2+\frac{1}{2}\frac{g_\omega{}^2}{m_\omega{}^2}\left[\sum_s \int \frac{d\vec{p}}{(2\pi)^3}\right]^2 . \tag{5.5}$$

Here we are working in the Hartree approximation. We see that the use of self-consistent wave functions leaves the ω vertex unaffected, but changes the σ vertex considerably. Recall Eqs. (2.20) and (2.21),

$$\bar{f}(\vec{p},s)\gamma^0 f(\vec{p},s)=\bar{u}(\vec{p},s)\gamma^0 u(\vec{p},s)=E(\vec{p})/m\ ,$$

while

$$\bar{f}(\vec{p},s)f(\vec{p},s)=\bar{u}(\vec{p},s)u(\vec{p},s)[1-2\beta^2(\vec{p})-2\alpha(\vec{p})\beta(\vec{p})\,|\,\vec{p}\,|\,/m]\ .$$

If we now use Eq. (5.5) we find that the binding energy per nucleon is given by $(m-\epsilon^{\sigma,\omega})$ with

$$\epsilon^{\sigma,\omega}\equiv\frac{3}{k_F{}^3}\int_0^{k_F}p^2dp\,E(\vec{p})\left[1-2\lambda^2\left[\frac{p}{m}\right]^2\right]$$

$$-\frac{1}{2}\frac{g_\sigma{}^2}{m_\sigma{}^2}\rho_0\left[\frac{3}{k_F{}^3}\int_0^{k_F}p^2dp\frac{m}{E(\vec{p})}\left\{1-2[\lambda^2+\lambda\sqrt{1-\lambda^2(p/m)^2}]\left[\frac{p}{m}\right]^2\right\}\right]^2+\frac{1}{2}\frac{g_\omega{}^2}{m_\omega{}^2}\rho_0\ . \tag{5.6}$$

Upon expanding the integrands to order $(p/m)^2$ and to order λ^2 we find,

$$\epsilon^{\sigma,\omega}-m\simeq\frac{3}{5}\left[\frac{k_F{}^2}{2m}\right](1-4\lambda^2)-\frac{1}{2}\frac{g_\sigma{}^2}{m_\sigma{}^2}\rho_0\left[1-\frac{3}{5}\left[\frac{k_F}{m}\right]^2-\frac{12}{5}\left[\frac{k_F}{m}\right]^2\lambda(\lambda+1)\right]+\frac{1}{2}\frac{g_\omega{}^2}{m_\omega{}^2}\rho_0\ . \tag{5.7}$$

In this approximate treatment we see that there is a significant decrease in the kinetic energy in the relativistic theory accompanied by a reduced attraction which has its origin in the weakening of the σ vertex.

In our Hartree analysis, which has only single parameter λ, the variational principle is particularly simple,

$$\frac{\partial\epsilon^{\sigma,\omega}}{\partial\lambda}=0\ . \tag{5.8}$$

From Eq. (5.8) we find,

$$\lambda=\frac{1}{2}\frac{\frac{g_\sigma{}^2}{m_\sigma{}^2}\frac{\rho_0}{m}}{\left[1-\left[\frac{g_\sigma{}^2\rho_0}{m_\sigma{}^2 m}\right]\right]}$$

$$\equiv\frac{1}{2}\frac{K}{1-K}\ . \tag{5.9}$$

At this value of λ we have,

$$\epsilon^{\sigma,\omega}-m \simeq \tfrac{3}{5}k_F{}^2\left[1+\frac{K^2}{2(1-K)}\right] - \frac{1}{2}\frac{g_\sigma{}^2}{m_\sigma{}^2}\rho_0\left[1-\frac{3}{2}\left[\frac{k_F}{m}\right]^2\right] + \frac{1}{2}\frac{g_\omega{}^2}{m_\omega{}^2}\rho_0 \,. \tag{5.10}$$

In Eq. (5.10) we have combined the kinetic-energy modification with the potential-energy modication in a single term. It is clear that the relativistic system will saturate at a smaller density than the nonrelativistic system[26–28] in a model of this type.

One could go on at this point to obtain the effective coupling constants which will reproduce the generally accepted values for the binding energy and density of nuclear matter, however, that approach has already been carried through by Walecka and his collaborators.

VI. DISCUSSION

In this work we have shown how one can start with a relativistic model Hamiltonian and use standard techniques of many-body theory to derive a relativistic Hartree-Fock (or Brueckner-Hartree-Fock) theory of nuclear structure. In previous works we have compared the saturation curves for nuclear matter obtained from the nonrelativistic and relativistic theories and have demonstrated the necessity of using the relativistic formalism at densities appropriate to the study of nuclear matter. (The nonrelativistic and relativistic formalisms give similar results only at low densities, say $k_F < 1.2$ fm^{-1}.) The Hamiltonian presented in Eq. (4.22) can be easily generalized to provide a description of finite nuclei. The study of the finite system introduces no new concepts into this analysis, only the technical problem of solving the Dirac equation with the self-energy operator calculated from the reaction matrix appropriate to the finite system. It is also clear that the negative-energy parts of the spinors $\phi(\vec{p},s)$ of Eq. (4.23) introduce a density dependence into the effective interaction of the quasiparticles that is significantly stronger than that which would be obtained if one chose to put $\theta_{\vec{p}}=0$.[30]

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