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Pion-nucleon interaction in the P_{11} partial wave and the pion-nucleon vertex function

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We have studied the pion-nucleon interaction in the P_{11} channel considering its important role in the absorption and production of the pion by nuclei. Specifically, we emphasize that the total P_{11} t matrix (or amplitude) can be naturally decomposed into two parts: the direct (and dressed) pole part and the nonpole part. It is then easy to observe that individually these two parts can be large but tend nearly to cancel each other and produce small and negative phase shift values from threshold to $T_{\rm lab}^{\pi} \sim 170$ MeV and that at higher energies the nonpole term dominates giving positive phase shift values. We then try to find the parametrization of this partial wave t matrix within the framework of the Blankenbecler-Sugar reduction. As a by-product we have obtained the πNN vertex function (with one nucleon off mass shell) and the dressed nucleon propagator.

[NUCLEAR REACTIONS πN P_{11} channel, pole and nonpole decomposition of P_{11} t matrix, phase shift up to 300 MeV.]

I. INTRODUCTION

It is well-known that among several lowest πN partial waves the P_{11} ($J=I=\frac{1}{2}$) channel has a very unique behavior. Firstly, as far as the phase shift $[\delta(P_{11})]$ behavior goes, it is small and negative (repulsive) at low energies. Then it changes sign at $T_{\pi}^{\rm lab} \sim 170$ MeV and rises rapidly to pass $\delta(P_{11})=90°$ at $T_{\pi}^{\rm lab} \sim 530$ MeV: the Roper resonance. Secondly, it becomes inelastic rather quickly: in fact the inelasticity becomes noticeable already at $T_{\pi}^{\rm lab} \sim 350$ MeV. Consequently, the above-mentioned Roper resonance is highly inelastic as opposed to the well-known $P_{33}(\Delta)$ resonance which is almost elastic. Thirdly, it has a pole below the elastic threshold at $W=m$ ($W \equiv \sqrt{S}$, the πN c.m. energy and m the nucleon mass) called the *nucleon pole.* This is of course due to the fact that the pion and nucleon couple to the (positive energy) nucleon through this partial wave.

In order to explain these above-mentioned features, various theoretical attempts have been made in the "sixties" exploiting, e.g., single- or multi-channel N/D methods within the context of partial wave dispersion relations (Ref. 1 and references cited therein). The principal finding then was that the single-channel theories could explain the qualitative feature of this partial wave only when one introduced *a priori* the nucleon pole. Stated differently, one could dynamically generate the nucleon pole and at the same time reproduce the observed phase shift, provided that one introduced at least one inelastic channel. For a more recent dynamical approach the reader is referred to Wei and Banerjee.[2]

During the past several years interest in this partial wave has gradually been revived in connection with the pion-nucleus interactions. Let us briefly review its history in the following. For a calculation of π-nucleus amplitudes or optical potentials within the conventional multiple scattering formalism, one needs to use some off-shell-extended πN partial wave amplitudes. As a first step simple rank-one separable interactions were considered for this purpose.[3] The separable interactions are constructed for each partial wave to reproduce observed phase shifts. As for the P_{11} channel the above-mentioned separable models have difficulties in describing the sign change in the phase shift as well as in reproducing the nucleon pole. However, this drawback was not considered to be serious since the dominant contributions to the π-nucleus processes up to $T_\pi^{\rm lab} \sim 200$ MeV were believed to come from πN S waves and the P_{33} channel, while the P_{11} phase shift stays rather small there. For this reason the P_{11} channel was even disregarded altogether in most π-nucleus calculations although it was felt that the effect of π-absorption through the P_{11} channel could be sizable at low energies.

Later several experimental results were disclosed all indicating that even around the $P_{33}(\Delta)$ resonance region the genuine π-absorption cross sections are fairly large: typically as much as one-third of the total π-nucleus cross sections.[4] Theoretical explanation was then given in the isobar-doorway models,[5] where one finds a large Δ spreading width due to the coupling to the π-absorption channel. At any rate it seems almost established that in nuclei the pion couples to the nucleon through the P_{11} channel more strongly than could be inferred from its (on-shell) low energy phase shift. This then suggests that for use in π-nucleus problems, one should find a proper description of the input P_{11} amplitude *at least* around and below the πN elastic threshold.

The simplest P_{11} model that may meet the above requirement is the direct nucleon pole term or its unitarized version. In particular, the latter could give a qualitative account of the phase shift for $T_\pi^{\rm lab} \lesssim 30$ MeV with a suitable choice of the πNN vertex cutoff.[6] Thus one might think that for the low to medium energy π-nucleus processes this type of model should be sufficient, and is tempted to leave out the remaining contribution to the P_{11} channel [referred to as the nonpole part ($\equiv NP-P_{11}$), hereafter]. In fact almost all the calculations which include the effect of the P_{11} channel (in connection with the π absorption) employed this viewpoint.[7]

Now, is the remaining nonpole P_{11} contribution ($NP-P_{11}$) really unimportant compared with the direct pole part? In a recent perturbative calculation of $NN \rightarrow NN\pi$, VerWest has found[8] that the nucleon pole P_{11} contribution has become undesirably large and had to be suppressed by an artificial long range cutoff. This might indicate that the effect of $NP-P_{11}$ may be important. To see this more clearly, let us make a slightly detailed observation of the P_{11} amplitude.

Let f^P be the direct nucleon pole term for P_{11} obtained from, e.g., $PS-PS(\gamma_5)$ theory. Then we write ($f^{\rm exp}$ below is defined such that in the elastic region, $f^{\rm exp}=\sin\delta_{\rm exp}e^{i\delta}\exp$)

$$\mathrm{Re}f^{\rm exp}=f^P+\mathrm{Re}f_{NP}^{\rm exp}\ , \tag{1.1}$$

which defines the nonpole background $f_{NP}^{\rm exp}$ in terms of the experimental P_{11} amplitude $f^{\rm exp}$. As may be easily understood from Fig. 1, $\mathrm{Re}f_{NP}^{\rm exp}$ is of nearly the same magnitude as f^P up to $T_\pi^{\rm lab} \sim 200$ MeV, but with the opposite sign. At higher energies $|\mathrm{Re}f_{NP}^{\rm exp}| > |f^P|$ always holds. Furthermore, an important feature is, that while $\mathrm{Re}f^{\rm exp}$ is small up to $T_\pi^{\rm lab} \sim 200$ MeV, neither $|\mathrm{Re}f_{NP}^{\rm exp}|$ nor $|f^P|$ are small in the corresponding energy range even compared with, e.g., the dominant P_{33} amplitude [by putting a cutoff at each πNN vertex one could make $|f^P|$ smaller, which also makes $|\mathrm{Re}f_{NP}^{\rm exp}|$ smaller. However, the cutoff cannot be made arbitrarily strong in order to be consistent with, e.g., the peripheral one-pion exchange (OPE) contribution to the NN interaction or with the generation of the Δ resonance,[9] thus $|f^P|$ cannot be diminished, e.g., by an order of magnitude at the energies under consideration]. Therefore, except at very low energies, neither f^P nor its unitarized version (with or without the vertex cutoff) may be considered as good a representation of $f^{\rm exp}$.

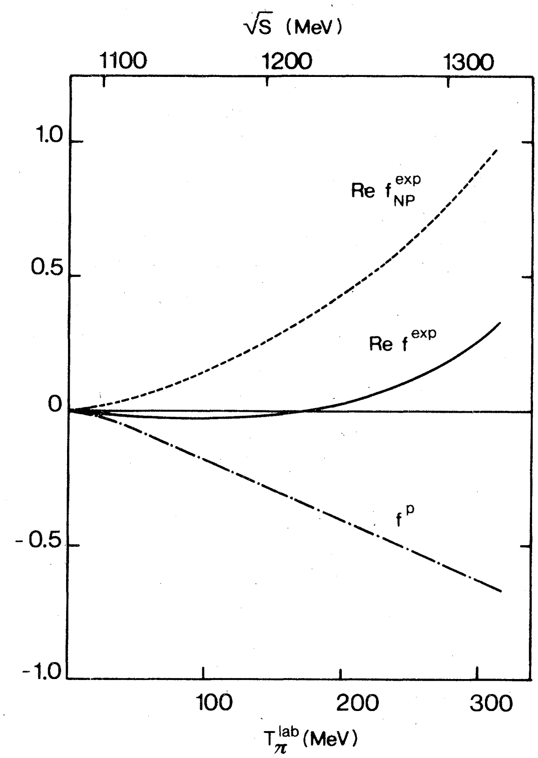

FIG. 1. The real part of various amplitudes for the πN P_{11} channel as defined in Eq. (1.1). The experimental amplitude is taken from Ref. 26 and is normalized such that in the πN elastic region it is expressed as $f^{\rm exp}=\sin\delta_{P_{11}}e^{i\delta_{P_{11}}}$, where $\delta_{P_{11}}$ is the real P_{11} phase shift.

From our observation above, we shall assume the following decomposition for the total P_{11} amplitude,

$$f_{\text{tot}} = f_P + f_{NP} \ , \tag{1.2}$$

where f_P is associated, in a certain way, with the direct nucleon pole term and f_{NP} is the $NP-P_{11}$ background (could be different from f_{NP}^{exp}), which will be given a more concrete meaning later. One might think that since both $|f_P|$ and $|f_{NP}|$ could be large, where $|f_{\text{tot}}|$ stays small (up to $T_\pi^{\text{lab}} \sim 200$ MeV), this decomposition is unnatural. However, as we shall see later, this in fact will turn out to be a very natural representation of f_{tot}. Incidentally, from the point of view of effective Lagrangian theories of πN interactions,[10] f_{NP} may be thought to obtain main contributions from the t-channel σ and ρ meson exchanges, while f_P is of course due to the nucleon pole. Thus the decomposition (1.2) is a natural one also in this respect.

Before going into a detailed study, it seems useful to refer to several recent works on the P_{11} interaction tailored for use in π-nucleus physics. Schwarz *et al.*[11] considered a modified rank-one separable interaction that admits the sign change in the phase shift. This interaction, however, is observed to have some undesirable off-shell behavior.[12] A rank-two separable interaction was adopted in Ref. 13 to fit $\delta(P_{11})$ and to reproduce the nucleon pole position but the details are not explained. Since there turns out to be a close relation between the two-potential type of approach (including rank-two separable interactions) and our present approach [although the former is not necessarily motivated by decomposition (1.2)], we shall discuss it in the Appendix. Yet another type of model is proposed by Ernst and Johnson,[14] which is an amalgam of a Chew-Low type of theory and the rank-one separable interaction that eventually becomes close to a two-potential model. It reproduces the observed phase shift rather well up to very high energies but has cutoff vertices whose range is unreasonably long as well as too small a value for the πN coupling constant. Lastly, we mention a K-matrix parametrization of the P-wave πN amplitude in terms of direct and crossed N, Δ, and N^* (1470) poles.[15]

The following section presents a study in the structure of the P_{11} amplitude which eventually gives an unambiguous and concrete form for the decomposition (1.2) with the help of unitarity (discontinuity)-analyticity considerations. Here one also finds the structure of the πNN vertex function and the dressed nucleon propagator. The latter half of the following section is devoted to finding the expression for the P_{11} amplitude (as well as the πNN vertex and the nucleon propagator) including the spin-isospin structure within the context of the Blankenbecler-Sugar reduction. In Sec. III an actual parametrization of the amplitude is performed by using experimental information up to $T_\pi^{\text{lab}} \lesssim 300$ MeV assuming that $NP-P_{11}$ is of rank-one separable. In our present parametrization we have left out the inclusion of inelasticity. As mentioned at the beginning of this section, the inelasticity in the πN P_{11} channel becomes non-negligible at $T_\pi^{\text{lab}} \sim 350$ MeV. However, since we are considering applications in π-nucleus interactions below this energy, the present result will be sufficient.

A. Structure of the P_{11} amplitude

We shall begin with giving a more concrete meaning to the decomposition implied in Eq. (1.2). Following Ref. 16 let us consider the t matrix for the direct nucleon pole term [Fig. 2(a)] and assume all possible radiative corrections to the πNN vertices as well as to the nucleon propagator (external particle legs are taken as on mass shell), thus we obtain the "dressed" direct nucleon pole term [Fig. 2(b)]. To simplify our discussion we assume that all the particles involved are of scalar-isoscalar nature and that the coupling is in S wave (although we shall retain expressions like the P_{11} amplitude, πNN P_{11} vertex, etc.). Let the dressed πNN vertex and the nucleon propagator be $h(S)$ and $d_N(S)$, respectively, where, as in Fig. 2(b), $S=Q^2$, $Q=p+q=p'+q'$. We stress that both functions depend only on S when momenta p and q (p' and q') are on the mass shell (recall that we are assuming S-wave vertices and scalar particles). Then clearly the dressed pole term reads

$$t_P(S) = h(S) d_N(S) h(S) \ , \tag{2.1}$$

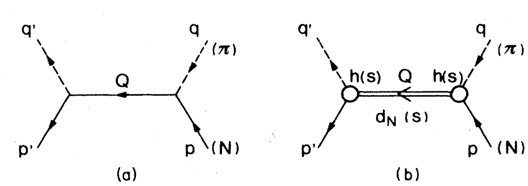

FIG. 2. (a) Usual (bare) direct nucleon pole contribution and (b) the same contribution when the πNN vertex and the nucleon propagator are dressed with radiative corrections.

and the total P_{11} t matrix may be written as

$$t_{\rm tot}(S)=t_P(S)+t_{NP}(S)\ . \tag{2.2}$$

Equation (2.2) is the concrete realization of decomposition (1.2) (in terms of t matrices). As may be clear from the above discussion, this decomposition is unambiguous and physically well defined. Next we would like to find some useful information with respect to the structure of $t_{NP}(S)$, etc. Using some general arguments it was shown some time ago[16] that not only $t_{\rm tot}(S)$ but also $NP-P_{11}$ part: $t_{NP}(S)$ do satisfy unitarity relations (not merely the elastic two-body unitarity). The reader is referred to a proof found in Ref. 16. Here we shall take a somewhat different path starting from the Taylor method of decomposing amplitudes in terms of irreducibilities.[17]

The method has been discussed by one of us[18] in the study of the coupled $\pi NN-NN$ problems, where in fact the equations for $t_{NP}(S)$, $h(S)$, etc., have been given. Therefore, we shall just briefly explain the result here. Implying the S dependence one may symbolically write

$$t_{NP}=t_{NP}^{(2)}+it_{NP}^{(2)}d_\pi d_N t_{NP}\ , \tag{2.3a}$$

$$h=u+iud_\pi d_N t_{NP}\ , \tag{2.3b}$$

$$d_N=d_N^{(0)}+d_N^{(0)}\Sigma^{(1)}d_N\ , \tag{2.3c}$$

$$\Sigma^{(1)}=\Sigma^{(2)}+iud_\pi d_N h\ . \tag{2.3d}$$

These equations were first derived by Mizutani,[17] Mizutani and Koltun[7] (using nonrelativistic Hamiltonian), and later by Afnan and Blankleider.[19] In the above expression (i) $t_{NP}^{(2)}$; the two-particle irreducible πN t matrix (may be regarded as a potential), (ii) $d_N(d_\pi)$; the dressed (or complete) nucleon (pion) propagator, (iii) u; the two-particle irreducible πNN vertex, (iv) $d_N^{(0)}$; bare nucleon propagator, and (v) $\Sigma^{(i)}(i=1,2)$; i-particle irreducible nucleon self-energy operator. As for the two-particle irreducible operators $t_{NP}^{(2)}$, u, and $\Sigma^{(2)}$, they are characterized by the S-cut structure in which the closest branch point is further than the one for the πN elastic unitarity: $S_b(\pi N{:}{\rm el})=(m+\mu)^2$, where m and μ are the masses of the nucleon and the pion, respectively. Incidentally, we note that our present amplitudes (or operators) are related to the corresponding ones in Ref. 18 by

$$d=-id\ \text{(Ref. 18)}\ ,$$

$$(t_{NP},h,\Sigma^{(1)})=i\times\ \text{(corresponding quantities in Ref. 18)}\ .$$

Next we study the discontinuity structure across the πN *elastic unitarity cut* (higher discontinuity structure is not considered here). Thus with

$$\Delta A(S)\equiv A(S^+)-A(S^-),\ S^\pm\equiv S\pm i\epsilon\ (S \text{ may imply } S^+ \text{ in the following})$$

we obtain from Eqs. (2.3)

$$\Delta t_{NP}(S)=-it_{NP}(S^-)\rho(S)t_{NP}(S^+)\ , \tag{2.4a}$$

$$\Delta h(S)=-ih(S^-)\rho(S)t_{NP}(S^+)\ , \tag{2.4b}$$

$$\begin{aligned}\Delta d_N(S)=&-id_N(S^-)h(S^-)\rho(S)h(S^+)d_N(S^+)\\&+[1+d_N(S^-)\Sigma^{(1)}(S^-)]\eta(S)\\&\times[1+\Sigma^{(1)}(S^+)d_N(S^+)]\ ,\end{aligned} \tag{2.4c}$$

$$\Delta\Sigma^{(1)}(S)=-ih(S^-)\rho(S)h(S^+)\ , \tag{2.4d}$$

where the one- and two-particle phase space factors are

$$\eta(S)\equiv-2\pi i\delta(S-m^2)\ , \tag{2.5a}$$

$$\begin{aligned}\rho(S)=&\int\frac{d^4p_\pi d^4p_N}{(2\pi)^4}\delta^4(Q-p_N-p_\pi)\\&\times[-2\pi\delta^{(+)}(p_N{}^2-m^2)][-2\pi\delta^{(+)}(p_\pi{}^2-\mu^2)]\\=&\frac{1}{(2\pi)^2}\frac{\tilde q}{4\sqrt{S}}\ \text{(in the } \pi N \text{ c.m. system)}\ ,\end{aligned} \tag{2.5b}$$

with

$$\tilde q^2\equiv\frac{[S-(m+\mu)^2][S-(m-\mu)^2]}{4S}\ \text{and}\ \delta^{(+)}(p^2-a^2)\equiv\theta(p_0)\delta(p^2-a^2)\ .$$

The proper renormalization of the nucleon self-energy requires

$$\Sigma^{(1)}(m^2)=0\ , \tag{2.6a}$$

$$\lim_{S\to m^2} d_N(S)\Sigma^{(1)}(S)=0\ , \tag{2.6b}$$

and this corresponds to a twice subtracted form for $d_N(S)$.

Taking this into account and noting that $t_{NP}(S)$, etc., are *real analytic* in S, Eqs. (2.4) become, for $S\le(m+2\mu)^2$

$$\mathrm{Im}t_{NP}(S)=-\tfrac{1}{2}\rho(S)\,|\,t_{NP}(S)\,|^2\ , \tag{2.4'a}$$

$$\mathrm{Im}h(S)=-\tfrac{1}{2}h(S^-)\rho(S)t_{NP}(S)\ , \tag{2.4'b}$$

$$\mathrm{Im}d_N(S)=-\pi\delta(S-m^2)-\tfrac{1}{2}\rho(S)\times|\,h(S)\,|^2\,|\,d_N(S)\,|^2\ , \tag{2.4'c}$$

$$\mathrm{Im}\Sigma^{(1)}(S)=-\tfrac{1}{2}\rho(S)\,|\,h(S)\,|^2\ . \tag{2.4'd}$$

With the condition in Eq. (2.6) the last two equations above are readily solved in terms of $h(S)$

$$\Sigma^{(1)}(S)=(S-m^2)^2\left[\frac{1}{2\pi}\int_{(m+\mu)^2}^{\infty}\frac{\rho(S')\,|\,h(S')\,|^2dS'}{(S'-S^+)(S'-m^2)^2}+\Phi(S)\right]\ , \tag{2.7a}$$

$$d_N(S)^{-1}=S-m^2+\Sigma^{(1)}(S)\ , \tag{2.7b}$$

where $\Phi(S)$ is an arbitrary (within our present study) real analytic function with possible (i) Castillejo-Dalitz-Dyson (CDD) poles below the elastic threshold and (ii) the lowest branch cut starting at $S=(m+2\mu)^2$ which takes care of the multiparticle discontinuity contributions to $d_N(S)$. Note that the nucleon wave function renormalization constant is given here as

$$Z_2{}^{-1}=\lim_{S\to\infty} d_N(S)(S-m^2)\ , \tag{2.8}$$

where

$$0\le Z_2\le 1\ .$$

As for Eq. (2.4'a) it is just the elastic unitarity relation. As mentioned earlier, it is possible to show that $t_{NP}(S)$ does also satisfy inelastic unitarity. Thus defining the scattering amplitude

$$f_{NP}(S)=-\frac{1}{32\pi^2\sqrt{S}}t_{NP}(S) \tag{2.9}$$

in the πN c.m. system, one could associate $f_{NP}(S)$ with its "phase shift" $\delta_{NP}(S)$ and "inelasticity" $\eta_{NP}(S)$ through

$$f_{NP}(S)=\frac{1}{2i\tilde{q}}[\eta_{NP}(S)e^{2i\delta_{NP}(S)}-1]\ . \tag{2.9'}$$

Of course $\eta_{NP}(S)=1$ for $S\le(m+2\mu)^2$. On the other hand Eq. (2.4'b) tells us that for $S\le(m+2\mu)^2$

$$h(S)\propto e^{i\delta_{NP}(S)}\ , \tag{2.10}$$

which implies the final state interaction of π and N contained in $h(S)$ goes through $NP-P_{11}$. It may be worth emphasizing that the final state interaction at the πNN vertex $h(S)$ is *not* through $t_{\rm tot}(S)$. In fact using Eqs. (2.1), (2.2), and (2.3b) one finds

$$h=u-iud_\pi d_N t_{\rm tot}-iud_\pi d_N h d_N h\ , \tag{2.11}$$

which has been obtained by Nutt and Shakin[20] as a nonlinear equation for the completely off-mass-shell πNN vertex h with u taken as the πNN coupling constant G (they included spin-isospin structure).

Defining

$$\hat{t}_P(S)=|\,h(S)\,|^2d_N(S)\ , \tag{2.12}$$

it is easy to show with Eqs. (2.7) that $\hat{t}_P(S)$ does satisfy elastic two-body unitarity for $S\le(m+2\mu)^2$, which means that the associated amplitude can be characterized by the real phase $\hat{\delta}_P(S)$ through

$$\hat{f}_P(S)=-\frac{1}{32\pi^2\sqrt{S}}\hat{t}_P(S)=\frac{1}{2i\tilde{q}}(e^{2i\hat{\delta}_P(S)}-1)\ , \tag{2.12'}$$

and the amplitude for the dressed nucleon pole term reads

$$f_P(S)=-\frac{1}{32\pi^2\sqrt{S}}t_P(S)=e^{2i\delta_{NP}(S)}\hat{f}_P(S) \tag{2.13}$$

in the πN elastic region. This suggests that the total amplitude $f_{\text{tot}}(S)[\equiv f_P(S)+f_{NP}(S)]$ or $t_{\text{tot}}(S)$ may be associated with some kind of two-potential scattering mechanism at least in the elastic region. This problem will be discussed in the Appendix where the reader will eventually find that it is in fact the case.

With Eqs. (2.9′), (2.12), and (2.13) it is trivial to show that

$$\delta_{\text{tot}}(S)=\hat{\delta}_P(S)+\delta_{NP}(S), \quad [S\le(m+2\mu)^2] , \tag{2.14}$$

where $\delta_{\text{tot}}(S)$ is the total (and observed) phase shift associated with $f_{\text{tot}}(S)$. Since $\hat{t}_P$ may be regarded as the unitarized nucleon pole term, it appears that our previous argument started from Eq. (1.1) can be translated into the one based upon Eq. (2.14). Actually, $\hat{\delta}_P(S)$ can be shown to be always negative provided that $d_N(S)$ has no zero below the πN elastic threshold.[21] The fact that $\delta_{\text{tot}}(S)$ stays small up to $T_\pi^{\text{lab}}\sim 200$ MeV indicates that in this energy region $\delta_{NP}(S)\sim-\hat{\delta}_P(S)$. However, we do not yet know the magnitude of $\delta_{NP}(S)$ or $\hat{\delta}_P(S)$ up to now. To obtain these quantities and thus $t_{\text{tot}}(S)$, $t_P(S)$, etc., several procedures have been used: (i) Within the elastic approximation $t_{NP}(S)$ is obtained from N/D calculation and the resulting $\delta_{NP}(S)$ is used in the form factor dispersion relation with Eq. (2.4′b) to find $h(S)$ [a constraint; $h(m^2)=G$, is imposed] and then $d_N(S)$ from Eqs. (2.7),[22] (ii) an extended coupled-channel Lee model is utilized to find the relevant quantities,[23] and (iii) the form factor dispersion relation is used to calculate the so-called incomplete πNN vertex $K(S)$ from $\delta_{\text{tot}}(S)$ and $\eta_{\text{tot}}(S)$ (experimental) first, and $K(S)$ is then used to calculate $d_N(S)$, and the combination of $K(S)$ and $d_N(S)$ leads to $h(S)$ and finally to $t_{NP}(S)$.[24] Since our present aim is to obtain the πN P_{11} channel amplitude [and so the πNN vertex $h(S)$] for practical use in the π-nucleus physics, we shall adopt a more simple-minded way: To parametrize $t_{NP}(S)$, assuming that it is rank-one separable and together with a simple parametrization of u [Eq. (2.3b)] we obtain $h(S)$, $d_N(S)$, and thus $t_{\text{tot}}(S)$. Then the parameters are adjusted to reproduce the experimental quantities [$\delta_{\text{tot}}(S)$, G, and the scattering volume] by the χ^2 procedure. In the next subsection we shall study the spin-isospin complications, combined with the Blankenbecler-Sugar reduction.

B. The P_{11} amplitude in the Blankenbecler-Sugar (Bb-S) reduction

Here in this subsection our main purpose is to introduce spin-isospin degrees and also to make our formalism manageable for practical use. One might thank that the following rather detailed derivation is not needed. However, in order to clearly see, for example, the nature of the πNN vertex (if it is pseudovector or pseudoscalar) we consider that the following argument should be useful. Yet, uninterested readers could skip this subsection up to Eq. (2.28). First, let us define various quantities used in what will follow.

Figure 3 shows the kinematical situation we are considering.

(i) Specifically, we write, in the πN c.m. system

$$p+q=p'+q'=(\sqrt{S},\vec{0})\equiv Q ,$$

$$p-q\equiv 2k, \quad p'-q'\equiv 2k' ,$$

in particular

$$\vec{p}=-\vec{q}=\vec{k}, \quad \vec{p}\,'=-\vec{q}\,'=\vec{k}\,' ,$$

and i, f, j: nucleon spin state; κ, η, ν: nucleon isospin state; a, b: pion isospin state.

(ii) Nucleon spinors are normalized as

$$\bar{u}_i(\vec{p})u_j(\vec{p})=2m\delta_{ij} , \tag{2.15a}$$

$$\sum_i \bar{u}_i(\vec{p})u_i(\vec{p})=\not{p}\pm m \; (\not{p}=\gamma_\mu p^\mu) , \tag{2.15b}$$

where the positive sign (negative sign) corresponds to the summation i over positive (negative) energy states. Equation (2.15) is satisfied with (e.g., for positive energy states):

$$u_i(\vec{p})=\sqrt{E_p+m}\begin{bmatrix} \xi_i \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E_p+m}\;\xi_i \end{bmatrix} \tag{2.16}$$

with $E_p=(\vec{p}^{\,2}+m^2)^{1/2}$ and ξ_i, the spin state $(\xi_i^+\xi_j=\delta_{ij})$.

(iii) Nucleon isospin states are designated as

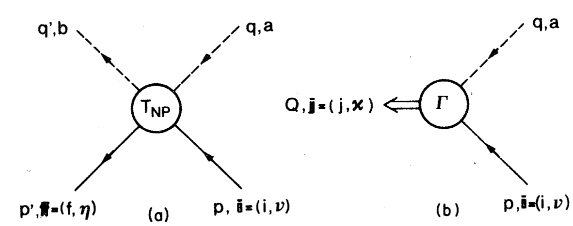

FIG. 3. Kinematics for (a) $T_{NP}(NP-P_{11})$ amplitude and (b) the πNN vertex with the nucleon (with momentum Q off-mass shell). See Sec. II B.

$\chi_\nu (\chi_\nu^+ \chi_\eta = \delta_{\nu\eta}, \sum_\nu \chi_\nu \chi_\nu^+ = 1)$.

Now the NP t matrix in the nucleon spin-isospin space is written as

$$\hat{T}_{NP}^{ba}(k',S,k) , \tag{2.17}$$

whose matrix element for on-mass-shell nucleons reads (in πN c.m.)

$$T_{NP}^{ba}(\vec{f},k',S,\vec{i},k) = \chi_\eta^+ \bar{u}_f(\vec{k}\,')\hat{T}_{NP}^{ba}(k',S,k)u_i(\vec{k}\,)\chi_\nu , \tag{2.18}$$

where $\vec{f} \equiv (f,\eta)$ and $\vec{i} \equiv (i,\nu)$, respectively. As for the πNN vertex [Fig. 3(b)] it consists of four independent scalar functions of three independent kinematical invariants in the case where all the particles are off-mass shell: $p^2 \neq m^2$. In most applications in π-nucleus problems where a Blankenbecler-Sugar (Bb-S) type of reduction (to be discussed later) is adopted, the nucleon with momentum p may be put on-mass-shell (however, the nucleon with momentum Q may not be close to its mass-shell after absorbing the pion momentum q). Then we have two independent scalar functions of two kinematical invariants which we choose to be $S(\equiv Q^2)$ and k^2. Thus we may write the πNN vertex function as (see Ref. 25)

$$\hat{\Gamma}_a(S,k^2)u_i(\vec{p}\,)\chi_\nu = G\left[\Gamma(S,k^2)\frac{m+\not{Q}}{2m} + \Gamma'(S,k^2)\frac{m-\not{Q}}{2m}\right]\tau_a\gamma_5 u_i(\vec{p}\,)\chi_\nu , \tag{2.19}$$

where G is the πN renormalized strong coupling constant, so that when all the particles are put on the mass shell (nucleon with momentum Q to be positive energy on-shell state)

$$\Gamma = 1$$

must be satisfied. In Eq. (2.19) function $\Gamma'(S,k^2)$ is accompanied by the projection operator for negative energy nucleon states. In forming the nucleon pole term, Γ' therefore contributes to the so-called Z graph in the πN S_{11} partial wave. Since our interest is in the P_{11} wave and in order to be consistent with a Bb-S reduction, we may set $\Gamma' = 0$. In fact, this appears to be a very good approximation (see Ref. 23). So we now have

$$\hat{\Gamma}_a(S,k^2)u_i(\vec{p}\,)\chi_\nu = G\Gamma(S,k^2)\frac{m+\not{Q}}{2m}\tau_a\gamma_5 u_i(\vec{p}\,)\chi_\nu . \tag{2.19$'$}$$

It may be worth mentioning here that Eq. (2.19′) can be rewritten as

$$\hat{\Gamma}_a(S,k^2)u_i(\vec{p}\,)\chi_\nu = \frac{G}{2m}\Gamma(S,k^2)\tau_a \not{q}\gamma_5 u_i(\vec{p}\,)\chi_\nu \tag{2.19$''$}$$

so that the resultant πNN coupling is of pseudoscalar-pseudovector (PS-PV) nature. Note that before making the above-mentioned approximation Eq. (2.19) presents a πNN coupling which is a combination of PS-PS and PS-PV whose ratio is a function of invariant scalars.

Now we want to find the expression corresponding to Eq. (2.3b). Clearly $\hat{\Gamma}_a(S,k^2)u_i(\vec{p}\,)\chi_\nu$ corresponds to $h(S)$, and we thus need the two-particle irreducible vertex which corresponds to $u(S)$. We introduce this as

$$\Gamma_a^{(2)}(S,k^2,p^2)u_i(\vec{p}\,)\chi_\nu = G\Gamma^{(2)}(S,k^2,p^2)\frac{m+\not{Q}}{2m}\tau_a\gamma_5 u_i(\vec{p}\,)\chi_\nu , \tag{2.20}$$

where for reasons to be clear soon we write the explicit dependence on p^2. With $\Gamma^{(2)}$ introduced above the expression for $\Gamma(S,k^2)$ is

$$\Gamma(S,k^2)\tau_a u_i(\vec{p}\,) = \Gamma^{(2)}(S,k^2,m^2)\tau_a u_i(\vec{p}\,) + i\sum_b \int \frac{d^4k'}{(2\pi)^4}\Gamma^{(2)}(S,k'^2,p'^2)\tau_b \frac{1}{(\not{p}'-m)}\frac{1}{[(Q/2-k')^2-\mu^2]}\hat{T}_{NP}^{ba}(k',S,k)u_i(\vec{p}\,) \tag{2.21}$$

with $p' = Q/2 + k'$, where the common factors have been eliminated from both sides of the equation. We note that up to this stage the dependence of $\Gamma^{(2)}$ on p^2 must be written out since Eq. (2.21) still contains $\Gamma^{(2)}$ whose two nucleons are still off-mass-shell in general.

Three more steps are to be taken before finding a handier expression for the vertex $\Gamma(S,k^2)$.

(1) Using a familiar decomposition into isosymmetric and antisymmetric part:

$$\hat{T}_{NP}^{ba}=\hat{T}_{NP}^{(+)}\delta_{ab}+\tfrac{1}{2}[\tau_b,\tau_a]\hat{T}_{NP}^{(-)} \qquad (2.22)$$

we can perform a summation over b in Eq. (2.21) and find

$$\sum_b \tau_b \hat{T}_{NP}^{ba}=(\hat{T}_{NP}^{(+)}+2\hat{T}_{NP}^{(-)})\tau_a$$
$$=\hat{T}_{NP}^{(I=1/2)}\tau_a\ , \qquad (2.23)$$

where $\hat{T}_{NP}^{(I=1/2)}$ is the t-matrix operator in the nucleon space projecting out the πN $I=\frac{1}{2}$ states. With this τ_a can be factored out from Eq. (2.21).

(2) The Bb-S reduction is applied to the propagators in Eq. (2.21). This preserves the πN elastic unitarity structure while disregarding the negative energy intermediate states and amounts to making the amplitude on-mass-shell but off energy shell. This is done by the replacement

$$\frac{1}{(\not{p}'-m)}\cdot\frac{1}{[(Q/2-k')^2-\mu^2]}\rightarrow -2\pi i\sum_\delta u_\delta(\vec{k}\,')\bar{u}_\delta(\vec{k}\,')\cdot G(S,\vec{k}\,')\delta[k_0'-\frac{(E_{k'}-\omega_{k'})}{2}]\ , \qquad (2.24)$$

where

$$G(S,\vec{k}\,')=\frac{(E_{k'}+\omega_{k'})}{2E_{k'}\omega_{k'}[S^+-(E_{k'}+\omega_{k'})^2]} \qquad (2.24')$$

with $\omega_{k'}=(\vec{k}\,'^2+\mu^2)^{1/2}$, $E_{k'}=(\vec{k}\,'^2+m^2)^{1/2}$. Thus with (1) and (2) Eq. (2.21) now reads

$$\Gamma(S,k^2)u_i(\vec{p}\,)=\Gamma^{(2)}(S,k^2,m^2)u_i(\vec{p}\,)$$
$$+\sum_\delta\int\frac{d^3k'}{(2\pi)^3}\Gamma^{(2)}(S,k'^2,m^2)u_\delta(\vec{k}\,')$$
$$\times\frac{(E_{k'}+\omega_{k'})}{2E_{k'}\omega_{k'}[S^+-(E_{k'}+\omega_{k'})^2]}T_{NP}^{I=1/2}(\delta,k',S,i,k)\ . \qquad (2.25)$$

In the above equation

$$T_{NP}^{I=1/2}(\delta,k',S,i,k)=\bar{u}_\delta(\vec{k}\,')\hat{T}_{NP}^{I=1/2}(k',S,k)u_i(\vec{p}\,)\ , \qquad (2.25')$$

(recall that $\vec{p}=\vec{k}$) and k_0' is put on its relative energy shell

$$k_0'=\frac{E_{k'}-\omega_{k'}}{2}\ .$$

To be consistent with the above relative energy shell procedure by the Bb-S reduction, one also takes k_0 to be on its relative energy shell. That is, one makes the following identification

$$\Gamma(S,k^2)=\Gamma(S,\vec{k}^{\,2})\ , \qquad (2.26a)$$

$$\Gamma^{(2)}(S,k^2,m^2)=\Gamma^{(2)}(S,\vec{k}^{\,2})$$

(suppressing the m^2 dependence) , (2.26b)

$$T_{NP}^{I=1/2}(\delta,k',S,i,k)=T_{NP}^{I=1/2}(\delta,\vec{k}\,',S,i,\vec{k}\,)\ . \qquad (2.26c)$$

So Eq. (2.25) reduces to the one involving essentially 3-(spatial) vectors. In an arbitrary reference frame $\vec{k}^{\,2}$, etc., may be regarded as the relativistically invariant scalar products of magic vectors.[9]

(3) One makes the *on-shell approximation* to the nucleon with momentum Q. This means that

$$\bar{u}_f(\vec{Q}\,)\frac{m+\not{Q}}{2m}\simeq\bar{u}_f(\vec{Q}\,)\ (\text{in }\pi N\text{ c.m. }\vec{Q}=0)\ , \qquad (2.27)$$

for a positive energy state f.

Of course (2.27) becomes an equality for on-shell Q. In the case of off-shell Q, the contribution lost due to the approximation (2.27) may be effectively contained in $\Gamma(S,\vec{k}^{\,2})$ in the course of its determination using experimental information.

Now taking into account the arguments above and using the expansions

$$T_{NP}^{J=1/2}(f,\vec{k}',S,i,\vec{k}) = \sum_{\substack{Jl\\ \mu\mu' M}} \langle \tfrac{1}{2} f l \mu' \mid JM \rangle Y_l^{\mu'}(\hat{k}') T_{NP(l)}^{J,I=1/2}(\mid \vec{k}' \mid, S, \mid \vec{k} \mid) Y_l^{\mu}(\hat{k})^* \langle \tfrac{1}{2} i l \mu \mid JM \rangle , \tag{2.28a}$$

$$\chi_\eta^{+} \bar{u}_f(\vec{Q}=\vec{0}) \tau_a \gamma_5 u_i(\vec{k}) \chi_\nu = \left[\frac{24\pi m}{E_k + m} \right]^{1/2} \mid \vec{k} \mid \langle \tfrac{1}{2} i 1 f - i \mid \tfrac{1}{2} f \rangle Y_1^{f-i}(\hat{k})^* \langle \tfrac{1}{2} \nu 1 a \mid \tfrac{1}{2} \eta \rangle , \tag{2.28b}$$

we finally find

$$\Gamma(S,\vec{k}^{\,2}) = \Gamma^{(2)}(S,\vec{k}^{\,2}) + \frac{\sqrt{E_K+m}}{\mid \vec{k} \mid} \int_0^\infty \frac{dk' k'^3}{(2\pi)^3} \frac{\Gamma^{(2)}(S,\vec{k}^{\,\prime 2})(E_{k'}+\omega_{k'}) T_{NP(l=1)}^{I=J=1/2}(\mid \vec{k}' \mid, S, \mid \vec{k} \mid)}{\sqrt{E_{k'}+m} \cdot 2E_{k'}\omega_{k'}[S^+ - (E_{k'}+\omega_{k'})^2]} , \tag{2.29}$$

with the normalization condition at the nucleon pole

$$\Gamma(S,\vec{k}^{\,2}) = 1 \text{ for } S = m^2, \ \ \vec{k}^{\,2} = -\mu^2 \left[1 - \frac{\mu^2}{4m^2} \right] .$$

Defining

$$h(S,\vec{k}^2) = \left[\frac{24\pi m}{E_k + m} \right]^{1/2} Gk\Gamma(S,\vec{k}^{\,2}) , \tag{2.30a}$$

$$R(S,\vec{k}^{\,2}) = \left[\frac{24\pi m}{E_k + m} \right]^{1/2} Gk\Gamma^{(2)}(S,\vec{k}^{\,2}) , \tag{2.30b}$$

we find the equation corresponding to Eq. (2.3b)

$$h(S,\vec{k}^{\,2}) = R(S,\vec{k}^{\,2}) + \int \frac{k'^2 dk'}{(2\pi)^3} \frac{(E_{k'}+\omega_{k'}) R(S,\vec{k}^{\,\prime 2}) T_{NP(l=1)}^{I=J=1/2}(\mid \vec{k}' \mid, S, \mid \vec{k} \mid)}{2E_{k'}\omega_{k'}[S - (E_{k'}+\omega_{k'})^2]} \tag{2.29'}$$

and the corresponding expressions for the nucleon self-energy, nucleon propagator, $t_P(\hat{t_P})$, etc., are just the same as in subsection II A. For example,

$$\Sigma^{(1)}(S) = (S - m^2)^2 \left[\frac{1}{2\pi} \int_{(m+\mu)^2}^{\infty} \frac{\rho(S') \mid h(S',\vec{k}^{\,\prime 2}) \mid^2}{(S' - S^+)(S' - m^2)^2} dS' + \Phi(S) \right] , \tag{2.7'a}$$

$$d_N(S)^{-1} = S - m^2 + \Sigma^{(1)}(S) , \tag{2.7'b}$$

whereas the *spinor* nucleon propagator reads $(\not{Q} + m) d_N(S), (S = Q^2)$. Also the angular momentum decomposed on-shell t matrices are written as (recall subsection II A),

$$t_{NP}^{\text{on shell}}(S) = T_{NP(l=1)}^{I=J=1/2}(\mid \vec{k} \mid, S, \mid \vec{k} \mid) , \tag{2.31a}$$

$$t_P^{\text{on shell}}(S) = h(S,\vec{k}^{\,2}) h(S,\vec{k}^{\,2}) d_N(S) , \tag{2.31b}$$

in the πN c.m., where

$$\vec{k}^{\,2} = \frac{[S - (m+\mu)^2][S - (m-\mu)^2]}{4S}$$

is the π (or N) c.m. on-shell momentum. A similar relation holds between $\vec{k}^{\,\prime 2}$ and S' in Eq. (2.7'). The definition of amplitudes and various phases are also the same as in Sec. II A.

III. DETERMINATION OF THE AMPLITUDE IN THE SEPARABLE APPROXIMATION

In order to eventually find a convenient parametrization of the P_{11} amplitude, we shall adopt the follow simplifying assumptions; (i) $NP - P_{11}$ t matrix is assumed as rank-one separable which may actually be a good approximation due to the Roper resonance. That is,

$$T_{NP(l=1)}^{I=J=1/2}(\mid \vec{q} \mid, S, \mid \vec{p} \mid) = g(\mid \vec{q} \mid) \tau_{NP}(S) g(\mid \vec{p} \mid) , \tag{3.1}$$

where

$$\tau_{NP}(S)^{-1} = \frac{1}{\lambda} - \frac{1}{(2\pi)^3} \times \int \frac{dk\, k^2(E_k+\omega_k) g(|\vec{k}|)^2}{2E_k\omega_k[S^+ - (E_k+\omega_k)^2]}$$

with $\lambda = \pm 1$ according to whether this NP interaction is repulsive or attractive. Here we take $\lambda = -1$ based upon the observation in Sec. I. (ii) We suppress the explicit S dependence of $\Gamma^{(2)}(S,\vec{k}^2)$ since the possible S variable singularity starts at $(m+2\mu)^2$, and may be considered as smooth function of S. (iii) We assume a monopole form for g and $\Gamma^{(2)}$:

$$g(|\vec{k}|) = \frac{\alpha|\vec{k}|}{\vec{k}^2+\beta^2}, \tag{3.2a}$$

$$\Gamma^{(2)}(S,\vec{k}^2) = \frac{\delta}{\vec{k}^2+\gamma^2}. \tag{3.2b}$$

A more complicated functional form may be taken, but at the present state of sophistication the above choice seems to be enough (an exponential type of form factors were also considered with the result of comparable fit to data. So we only mention the monopole results here). (iv) We take $\Phi(S)=0$ in Eq. (2.7′), namely, (i) no CDD poles are assumed, and (ii) no explicit account of the inelasticity will be made. This should be all right for applications for $T_\pi^{\rm lab} \lesssim 300$ MeV.

With the parametrizations (i) and (iii) mentioned above we are left with four parameters α, β, γ, and δ to be determined from the fit to experiments. The experimental data (scattering volume and phase shift in the πN P_{11} channel) are taken from the recent analysis by Koch and Pietarinen.[26] Then we have performed a χ^2 fit constrained by the condition $\Gamma(S,\vec{k}^2)=1$ at the nucleon pole $[S=m^2, \vec{k}^2 = -\mu^2(1-\mu^2/4m^2)]$. It may be worthwhile to remark in passing that we have also tried a fit with the P_{11} data by Rowe *et al.*[27] (which has a very small scattering volume) to find that a reasonable result was obtained only when the πNN coupling constant $f_{\pi NN}{}^2 = G^2/4\pi(\mu/2m)^2$ was set ≈ 0.06 or smaller for which the standard value is considered to be ≈ 0.080. Thus we did not use this set of data. We note that a similar tendency has been observed by Schwarz *et al.*[11] (see also Ref. 15).

We shall present a couple of typical results in the following which we call models A and B, respectively. The corresponding parameters: α, β, γ, and δ are found in Table I together with the nucleon wave function renormalization constant Z_2. Table II shows the pole and nonpole contributions to the scattering volume.

Also the corresponding $\delta_{\rm tot}$, $\hat{\delta}_p$, and δ_{NP} $(\delta_{\rm tot} = \hat{\delta}_p + \delta_{NP})$ are shown in Fig. 4 (recall the definition of the phases in Sec. II A). Both A and B show that while $\delta_{\rm tot}$ stays small up to $T_\pi^{\rm lab} \sim 200$ MeV the magnitudes of $\hat{\delta}_p$ and δ_{NP} are by no means small, which is what we have speculated in Sec. I in terms of Re$f^{\rm exp}$, etc. Thus we have demonstrated explicitly that in order to describe the P_{11} channel adequately in the intermediate energy region one must go beyond the simple (unitarized) nucleon pole model. A difference exists, however, between the two models. Model A better fits the scattering volume (a_{11}) and the lower energy part of $\delta_{\rm tot}$, while the situation is just the opposite with model B. This difference is caused solely by the fact that in model A a very strict constraint is imposed on the value a_{11} in the χ^2 fit. Knowing that the experimental determination of very low energy πN phase shifts is less accurate than at higher energies, we prefer model B, where the constraint on a_{11} is looser.

A very interesting feature is that while the model dependence of $\delta_{\rm tot}$ is rather small (as it must be from the fitting procedure), models A and B have produced considerably different $\hat{\delta}_p$ and δ_{NP}. In particular, δ_{NP} in model B shows a resonance behavior at $T_\pi^{\rm lab} \sim 250$ MeV ($\sqrt{S} \sim 1280$ MeV). This is then reflected in the behavior of $Z(S)$; $Z(S) = d_N(S)^{-1}$. $(S-m^2)^{-1}$, where $Z_2 = \lim_{S\to\infty} Z(S)$, and the πNN vertex with one nucleon off mass shell: $\Gamma(S,\vec{k}^2)$, where $\vec{k}$ is the on-shell πN c.m. momentum

TABLE I. Values of the parameters for fits A and B and the corresponding nucleon wave function renormalization constant.

	α (fm^{-1})	β (fm^{-1})	γ (fm^{-1})	δ (fm^{-2})	Z_2	a_{11} (μ^{-3})
A	116.25	2.5794	4.5265	7.6221	0.7740	−0.0821
B	196.60	3.9245	5.6207	6.4390	0.6341	−0.108

TABLE II. Values of scattering volume a_{11} and its decomposition into the pole and nonpole contributions (in unit of μ^{-3}). The experimental scattering volume is $a_{11}^{\mathrm{exp}} = -0.082\ \mu^{-3}$.

	pole	nonpole	total
A	-0.225	0.143	-0.082
B	-0.307	0.199	-0.108

$$\left[\vec{k}^2 = \frac{[S-(m+\mu)^2][S-(m-\mu)^2]}{4S} \right.$$

or

$$\left. \sqrt{S} = \sqrt{\vec{k}^2+m^2} + \sqrt{\vec{k}^2+\mu^2} \right] .$$

(See Fig. 5.) Namely, both $|Z(S)|$ and $|\Gamma(S,\vec{k}^2)|$ become considerably greater than unity (the value at the normalization point: $S=m^2$) and peak at around $\sqrt{S}=1170$ MeV ($T_\pi^{\mathrm{lab}} \sim 110$ MeV), while $\mathrm{Re}\Gamma(S,\vec{k}^2)$ passes zero at the resonance energy for the $NP-P_{11}$ amplitude and becomes negative [Fig. 5(b)]. On the other hand, model A results in smooth δ_{NP} and consequently smoother $|Z(S)|$ and $|\Gamma(S,\vec{k}^2)|$; they stay near unity over a wide range in S [Fig. 5(a)].

The resonance behavior in δ_{NP} is not so unusual. In fact Ida determined[22] t_{NP} in a N/D calculation to find that it resonates at $\sqrt{S} \sim 1230$ MeV ($T_\pi^{\mathrm{lab}} \sim 180$ MeV). Also in an extended Lee model it has been found[23] that by fitting $\delta_{\mathrm{tot}}(P_{11})$, t_{NP} has a resonance at even lower energy ($\sqrt{S} \sim 1\,115$ MeV). One could even find[24] a pole below the elastic threshold using a form factor dispersion relation combined with the P_{11} experimental information. We stress that evidently no such resonance (or pole) has been observed in the *physical* πN P_{11} partial wave. Thus one may guess that the possible $NP-P_{11}$ resonance (or pole) mentioned above is eventually canceled by t_p so that no remnant of the NP resonance exists in t_{tot}. This is in fact the case as proved in Ref. 16 (see also Refs. 23 and 27). There is an observation which states that this NP resonance may be the Roper resonance lowered in energy as the repulsive direct nucleon pole contribution is absent in t_{NP}.[23]

IV. DISCUSSION AND CONCLUSION

In the present paper we have studied the πN P_{11} amplitude based upon the observation that the total

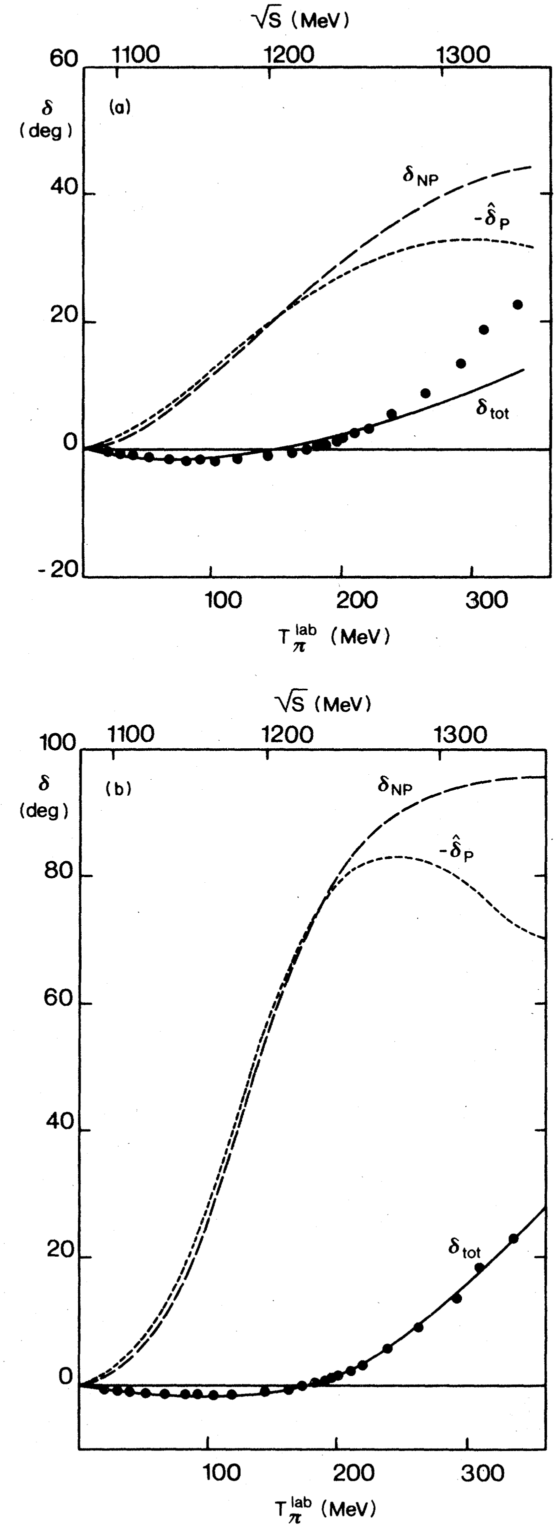

FIG. 4. Phase shift δ_{tot} and its pole ($\hat{\delta}_p$) and nonpole (δ_{NP}) components in (a) model A and (b) model B, respectively. Experimental points are from Koch and Pietarinen (Ref. 26).

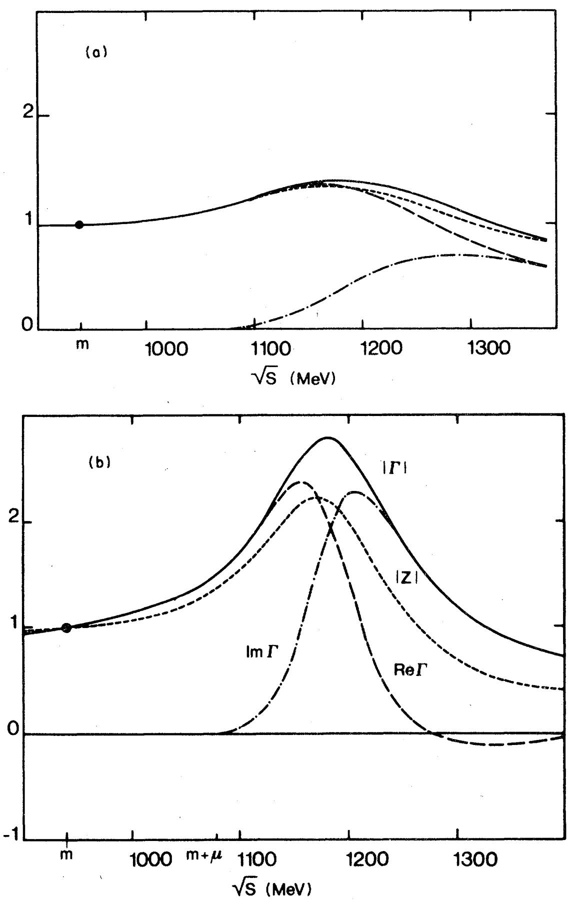

FIG. 5. Real and imaginary parts of the vertex Γ and the absolute value of Γ as well as the renormalization measure Z of the nucleon propagator for (a) model A and (b) model B, respectively. The solid line: $|\Gamma|$, dashed line: ReΓ, dot-dashed line: ImΓ, and dotted line: $|Z|$.

amplitude consists of the (repulsive) nucleon pole term and the (attractive) remainder: $NP-P_{11}$. Then we have shown that this decomposition is physically sound and can be made unambiguously. Based upon this observation we have determined the parametrization of the P_{11} wave amplitude with an application in π-nucleus physics in mind

We have obtained a couple of typical parametrizations (called model A and model B, respectively). Considering the quality of the experimental data at low energies, we think model B to be preferable. An interesting feature is that in model B the t matrix for the nonpole part ($NP-P_{11}$) exhibits a resonance behavior at $T_\pi^{\text{lab}} \sim 250$ MeV, which is then reflected in the strong energy dependence of the nucleon propagator and the πNN vertex function (with one nucleon off-shell). Considering some other previous studies, this behavior does not seem unrealistic. To see in more detail if this $NP-P_{11}$ resonance actually exists or not and, if so, to know the precise position of the resonance, requires a dynamical calculation as we have only used some analyticity structure of the amplitude to χ^2 fit the experimental data. Some simple model calculations in favor of such resonance are in fact available.[22,23]

As discussed somewhat in detail in Ref. 28, one may be able to observe the resonance in $NP-P_{11}$ in π-nucleus interactions if it ever exists, although it is not an observable in the physical πN scattering: Since t_p and t_{NP} appear separately in π-nucleus interactions, this possibility does exist. We emphasize in this connection that it is essential to know t_p and t_{NP} separately, but not t_{tot} alone if one aims at applying one's πN P_{11} model amplitude in π-nucleus physics.

From what we have discussed above, it should be interesting to apply the present models A and B in the calculation of elastic πd scattering, $\pi d \rightleftarrows NN$, and $NN \to NN$ to see which model seems more realistic or to see if the $NP-P_{11}$ resonance does exist or not. We are currently working on this aspect, and a preliminary result on the elastic πd case has been reported with the P_{11} model close to model A of the present work.[29] Of course our present model is still very simple and various improvements are due in order to be more definitive. Presumably the most important improvement to be made in the next step may be to incorporate the effect of inelasticity, which might play an important role for or against the possible $NP-P_{11}$ resonance.

Lastly, in applying the present P_{11} amplitude (or πNN vertex function) to π-nucleus problems, where the pion is rather highly off mass shell, we suggest that one should take the on-shell t_{NP}, h, etc., [on-shell h means $h(S,\vec{k}^2)$, where $\sqrt{S} = \sqrt{\vec{k}^2+m^2} + \sqrt{\vec{k}^2+\mu^2}$] and multiply some cutoff function simulating the off-shellness of the pion, as found in Ref. 20. The cutoff mass should be $\gtrsim 1$ GeV/c in order to be consistent with the peripheral NN phase shifts if one adopts a monopole type of cutoff function.

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APPENDIX

As may be guessed from our observation in Sec. II A, the t-matrix separation found in Eqs. (2.1) and (2.2) is closely associated with the well-known two-potential scattering formula. Here we shall explicitly show the relation (see also an appendix in Ref. 30).

Let us begin with specifying the potentials. We assume for brevity the S-wave πN interaction without spin-isospin degrees of freedom (which, however, we call the P_{11} channel just like in Sec. II A). The potentials are written as

$$V_1 \equiv V_1(p,S,q) \; , \tag{A1a}$$

$$V_2 \equiv \lambda(S) g(p) g(q) \; , \tag{A1b}$$

where $\lambda(S)^{-1} = S - m_0{}^2$ with m_0 the bare nucleon mass. Thus V_2 is considered to be the (bare) direct pole term with $g(p)$ the vertex function (supposed to be two-particle irreducible, i.e., no πN elastic cut in S). Writing the free πN propagator as G_0, one easily establishes the relation using the two-potential formula

$$T_{\text{tot}} = T_1 + T_2 \; , \tag{A2a}$$

$$T_1 = V_1 + V_1 G_0 T_1 \; , \tag{A2b}$$

$$T_2 = (1 + T_1 G_0) \widetilde{T}_2 (1 + G_0 T_1) \; , \tag{A2c}$$

$$\widetilde{T}_2 = V_2 + V_2 G \widetilde{T}_2 \; , \tag{A2d}$$

with

$$G = G_0 + G_0 V_1 G = G_0 + G_0 T_1 G_0 \; . \tag{A2e}$$

Noting (A1b) and (A2d), one finds

$$\widetilde{T}_2 = g(p) \tau_p g(q) \; , \tag{A3}$$

where

$$\tau_p(S) = [\lambda^{-1}(S) - \widetilde{\Sigma}(S)]^{-1} \tag{A4a}$$

with

$$\widetilde{\Sigma}(S) = g G(S) g \; , \tag{A4b}$$

may be identified as the dressed nucleon propagator $d_N(S)$. Then clearly $\widetilde{\Sigma}(S)$ is to be considered as the nucleon self-energy $\Sigma^{(1)}(S)$.

Now

$$h(S,q) = g(1 + G_0 T_1) \tag{A5}$$

may be regarded as the πNN vertex function and thus one can write

$$T_2 = h(S,p) d_N(S) h(S,q) \; , \tag{A6}$$

where in the totally on-shell situation h is just the function of S.

Making the following identifications

$$t_{NP}^{(2)} \equiv V_1, \quad d_N^{(0)} \equiv \lambda \; ,$$

$$t_{NP}^{(1)} \equiv T_1, \quad -i d_N d_\pi \equiv G_0 \; ,$$

$$u \equiv g, \quad t_P \equiv T_2 \; ,$$

one easily finds that Eqs. (A2)$-$(A6) do reproduce exactly Eqs. (2.3) with $\Sigma^{(2)} \equiv 0$, and Eqs. (2.1) and (2.2).

Thus, it appears that one could work on the P_{11} amplitude up to the end within the context of the two-potential scattering formula. However, the renormalization procedure becomes opaque; the procedure corresponding to the once subtraction is rather easy as it is essentially the elimination of the bare mass by requiring

$$d_N(m^2)^{-1} = m^2 - m_0{}^2 - \Sigma^{(1)}(m^2) = 0 \; ,$$

but the twice subtraction which further requires the vanishing of $\lim_{S \to m^2} d_N(S) \Sigma^{(1)}(S)$ [Eq. (2.6b)] is not trivial and the resultant physical picture is not clear. To go through this point the most transparent way is to exploit the discontinuity relations as we have used. Also, within the two-potential formula it is very difficult to see the πNN vertex structure (either PS or PV) as discussed in Sec. II B.

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