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Relativistically corrected impulse approximation in the presence of a local central potential

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An impulse approximation with relativistic corrections is derived to incorporate the fact that each constituent nucleon of the target nucleus is moving in an average local central potential generated by the other nucleons. The constraint imposed by current conservation is described. An application of the resultant impulse approximation to reanalyze symmetry tests in the $A=12$ nuclei induces a few subtle changes but does not upset the standard interpretation of these experimental results.

[NUCLEAR REACTIONS Impulse approximation with relativistic corrections, derived in the presence of a local central potential, constrained by current conservation; symmetry tests in the $A=12$ nuclei.]

I. INTRODUCTION

A microscopic treatment of electroweak interactions in nuclei often begins with the invocation of the impulse approximation (IA),[1] in which the interaction of the probe particle (such as γ, e, μ, and ν) with the whole nucleus is approximated by a simple sum of the "elementary" interactions with the constituent nucleons. In addition to the one-body operator given by the IA, meson-exchange currents (MEC),[2] which give rise primarily to two-body operators, are needed to complete the theoretical description but generally expected to be of less importance. In view of the extensive literature[1,2] in developing this "standard" theoretical framework, most of us tend to conclude that there remains, if any, little room for further improvements. However, a careful thinker can easily find many reasons for dissatisfaction, viz.:

(1) The one-body IA operators have been derived as if the constituent nucleons in the target nucleus were "free." In reality, however, each constituent nucleon moves in a potential generated by the other constituents (nucleons and mesons).

(2) Although conservation of the vector current (CVC) and partial conservation of the axial current (PCAC) hold at the *nucleon* level, these symmetry principles are, in general, violated by the resultant *nuclear* IA currents.[3] To restore CVC and PCAC to the IA, interactions among the constituent nucleons and the MEC should be formulated on the same footing. This is *not* practical in the shell-model description of the nucleus.

(3) Relativistic corrections become increasingly important in medium energy electron elastic and inelastic scattering as well as in detailed investigations of weak interactions in nuclei. So far, only relativistic corrections in elastic electron-deuteron scattering have been investigated extensively.[4,5]

(4) The Foldy-Wouthuysen (FW) transformation[6] is generally used in the derivation of the nonrelativistic expression of the IA and MEC operators. It is well known that the nonrelativistic operators obtained in this manner are unique only up to certain unitary transformations. In addition, this method is *not* applicable in the presence of a local central potential.

The main purpose of this paper is to derive a relativistically corrected impulse approximation (RCIA) in the presence of a local central potential. Instead of the FW transformation, in which an operator is considered separately from the initial and final wave functions, the reduction procedure described in a previous paper[7] is used to generate a *unique* nonrelativistic representation of a given matrix element. A modification of the RCIA to incorporate current conservation is suggested along the same line as in our studies of parity violation in electron-deuteron scattering.[8] The formulation of the MEC in the presence of a central potential is beyond the scope of this paper.

At the moment, it is not yet possible to elucidate all major consequences of the RCIA since the Dirac phenomenology,[9,10] in which the constituent nucleon of a nucleus is described by a Dirac equation with potentials given by the meson fields generated by the other nucleons, awaits more quantitative investigations. We reconsider the test of fundamental symmetries in the $A=12$ nuclei[11,12] to illustrate effects caused by the presence of a central potential but leave any quantitative determination of relativistic corrections in, e.g., electron scattering for future publications. However, an interested reader is urged to look into any reaction for which the effects described in this paper are relevant.

II. FORMULATION

We define the matrix elements of the polar vector and axial vector currents $V_\lambda(x)$ and $A_\lambda(x)$ between any two on-shell nucleon states of definite four-momenta $p^{(i)}$ and $p^{(f)}$,

$$\langle N(p^{(f)})\,|\,V_\lambda(0)\,|\,N(p^{(i)})\rangle = iu^\dagger(p^{(f)})\gamma_4\left[\gamma_\lambda f_V(q^2)+\frac{\sigma_{\lambda\eta}q_\eta}{2m_N}f_M(q^2)+\frac{i2m_Nq_\lambda}{m_\pi{}^2}f_S(q^2)\right]u(p^{(i)})$$
$$\equiv iu^\dagger(p^{(f)})\hat{\Gamma}_\lambda^{(V)}(q,\mathscr{P})u(p^{(i)})\,, \tag{1a}$$

$$\langle N(p^{(f)})\,|\,A_\lambda(0)\,|\,N(p^{(i)})\rangle = iu^\dagger(p^{(f)})\gamma_4\left[\gamma_\lambda\gamma_5 f_A(q^2)-\frac{i2m_Nq_\lambda\gamma_5}{m_\pi{}^2}f_P(q^2)+\frac{\sigma_{\lambda\eta}q_\eta\gamma_5}{2m_N}f_E(q^2)\right]u(p^{(i)})$$
$$\equiv iu^\dagger(p^{(f)})\hat{\Gamma}_\lambda^{(A)}(q,\mathscr{P})u(p^{(i)})\,, \tag{1b}$$

where $q_\lambda\equiv(p^{(i)}-p^{(f)})_\lambda$, $\mathscr{P}_\lambda\equiv(p^{(i)}+p^{(f)})_\lambda$, $\gamma_\lambda^\dagger=\gamma_\lambda$, $\gamma_\lambda\gamma_\eta+\gamma_\eta\gamma_\lambda=2\delta_{\lambda\eta}$, $\sigma_{\lambda\eta}=(\gamma_\lambda\gamma_\eta-\gamma_\eta\gamma_\lambda)/(2i)$, and $m_N\equiv(m_p+m_n)/2\simeq m_p\simeq m_n$. The subscripts V, M, S, A, P, and E stand for vector, magnetism, scalar, axial vector, pseudoscalar, and electricity, respectively. We suppress all isospin indices which can easily be introduced whenever necessary.

Using the polar vector current as an illustrative example, we write the matrix element of this current between two nuclear states $N_i(p^{(i)})$ and $N_f(p^{(f)})$, after carrying out the integration over the c.m. coordinate,

$$\langle N_f(p^{(f)})\,|\,V_\lambda(0)\,|\,N_i(p^{(i)})\rangle=\int\left[\prod^{A-1}d^3r^{(a)}\right]\tilde{\Phi}_f^\dagger(\ldots,\vec{r}^{(a)},\ldots)\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}i\hat{\Theta}_\lambda^{(a)}(q;\vec{\nabla}_r)\tilde{\Phi}_i(\ldots,\vec{r}^{(a)},\ldots)$$
$$=\int\left[\prod_{a=1}^{A}d^3r^{(a)}\right]\delta^{(3)}\left[\sum_{a=1}^{A}\vec{r}^{(a)}\Big/A\right]\tilde{\Phi}_f^\dagger(\ldots,\vec{r}^{(a)},\ldots)$$
$$\times\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}i\hat{\Theta}_\lambda^{(a)}(q,\vec{\nabla}_r)\tilde{\Phi}_i(\ldots,\vec{r}^{(a)},\ldots)$$
$$\equiv\left\langle\Psi_f\left|\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}i\hat{\Theta}_\lambda^{(a)}(q,\vec{\nabla}_r)\right|\Psi_i\right\rangle. \tag{2}$$

Here, $\tilde{\Phi}_i$ ($\tilde{\Phi}_f$) is the initial (final) *internal* wave function defined in the reference frame moving with the entire nucleus and $\vec{r}^{(a)}$ is the spatial coordinate of the ath nucleon expressed relative to the c.m. coordinate $\vec{R}=\sum_{a=1}^{A}\vec{x}^{(a)}/A$ with $\vec{x}^{(a)}$ being the ath nucleon coordinate defined in the same reference frame as for the operator $\hat{\Theta}_\lambda$. The nonrelativistic operator $\hat{\Theta}_\lambda$ can be obtained from the relativistic expression $\hat{\Gamma}_\lambda^{(V)}$ of Eq. (1a) by the reduction procedure which has been discussed in a previous paper[7] and is to be employed in our derivation of the RCIA.

To take into account relativistic corrections up to the order $p^2/m_N{}^2$, with p the magnitude of some characteristic three-momentum, we need to specialize our derivation to a particular reference frame since terms of order $p^2/m_N{}^2$ are known to be frame dependent. Following the existing literature,[4,5,3] we choose to work with the Breit frame, in which the three-momenta of the initial and final nuclei are treated symmetrically, i.e., $\vec{p}^{(i)}=-\vec{p}^{(f)}=\vec{q}/2$.

Let us now assume that, in the rest frame of the *initial* nucleus, a constituent nucleon is described by the Dirac equation

$$\left\{\gamma_\mu\left[\frac{\partial}{\partial x'_\mu}-i\hat{e}A_\mu(x')-iU_\mu(x')\right]+[V(x')+i\gamma_5\tilde{V}(x')]+m_N\right\}\psi(x')=0\ . \tag{3}$$

Here, $\psi(x')$ is a *normalized* Dirac wave function of binding energy B and $\hat{e}$ is the electric charge of the given nucleon. We consider only the four-vector, scalar, and pseudoscalar potentials $U_\mu(x')$, $V(x')$, and $\tilde{V}(x')$, which represent the average meson fields generated by the other nucleons.[9,10] The electromagnetic potential $A_\mu(x')$ is included since the formulation of the IA in the presence of a strong electromagnetic field (internal or external) might also be of some theoretical interest. In this paper, all the potentials are assumed to be real, central (i.e., functions of $\vec{r}'\equiv\vec{x}'-\vec{R}'$), and local (i.e., independent of $\partial/\partial x'_\mu$). Therefore, we write in the rest frame of the *initial* nucleus

$$\psi(x')=\begin{bmatrix}1-\hat{K}(\vec{r}',\vec{R}')\\ \hat{H}(\vec{r}',\vec{R}')\end{bmatrix}\phi(x')\zeta\ . \tag{4}$$

Here, $\vec{R}'\equiv\sum_{a=1}^{A}\vec{x}'^{(a)}/A$, ζ is a two-component Pauli spinor normalized to unity, and the spatial wave function $\phi(\vec{x}')$ is normalized such that $\int d^3x'\phi^*(\vec{x}')\phi(\vec{x}')=1$. The operators $\hat{K}$ and $\hat{H}$ are given by

$$\hat{K}(\vec{r}',\vec{R}')\equiv\frac{1}{8m_N{}^2}\vec{\sigma}\cdot\left[\frac{\vec{\nabla}'_x}{i}-\vec{U}(\vec{r}')-\hat{e}\vec{A}(\vec{r}')\right]G(\vec{r}')\vec{\sigma}\cdot\left[\frac{\vec{\nabla}'_x}{i}-\vec{U}(\vec{r}')-\hat{e}\vec{A}(\vec{r}')\right]\ , \tag{5a}$$

$$\hat{H}(\vec{r}',\vec{R}')\equiv\frac{1}{2m_N}F(\vec{r}')\vec{\sigma}\cdot\left[\frac{\vec{\nabla}'_x}{i}-\vec{U}(\vec{r}')-e\vec{A}(\vec{r}')\right]\ , \tag{5b}$$

$$F(\vec{r}')\equiv 2m_N[2m_N-B+V(\vec{r}')-U_0(\vec{r}')-\hat{e}\Phi(\vec{r}')]^{-1}\ ,$$

$$G(\vec{r}')\equiv[F(\vec{r}')]^2\ , \tag{5c}$$

$$\Phi(\vec{r}')\equiv -iA_4(\vec{r}')\ .$$

The normalization condition is satisfied, viz.,

$$\int d^3x'\psi^\dagger(\vec{x}')\psi(\vec{x}')=1+O(p^3/m_N{}^3)\ . \tag{6}$$

To obtain the nonrelativistic operator $\hat{\Theta}_\lambda$, we need to rewrite $\psi(\vec{x}')$ as a function of the variables $\vec{x}$ and $\vec{s}$ which are defined in the Breit frame, $\vec{p}^{(i)}=-\vec{p}^{(f)}=\vec{q}/2$. We note that[5,7]

$$\begin{aligned}\vec{x}'&=\vec{x}-(\vec{x}\cdot\vec{q})\vec{q}/(8A^2m_N{}^2)\\ &\quad-\vec{q}t/(2Am_N)+O(p^3/m_N{}^3)\ ,\\ \vec{s}'&=\vec{s}-(\vec{p}\times\vec{q})\times\vec{s}/(4Am_N{}^2)+O(p^3/m_N{}^3)\ .\end{aligned} \tag{7}$$

Equation (7) implies

$$\begin{aligned}\zeta&\to\left[1+\frac{i}{4Am_N{}^2}(\vec{p}\times\vec{q})\cdot\vec{s}\right]\zeta\ ,\\ \vec{r}'&=\vec{r}-(\vec{r}\cdot\vec{q})\vec{q}/(8A^2m_N{}^2)\ ,\\ \vec{R}'&=\vec{R}-\vec{q}t/(2Am_N)\ .\end{aligned} \tag{8}$$

In view of Ref. 7, the phase factor generated by $\delta\vec{R}=-\vec{q}t/(2Am_N)$ from the *initial* state as well as that due to the *final* state contributes only to the δ function $\delta(E_i-E_f-q_0)$ (energy conservation). Noting also that corrections to the operators $\hat{K}$ and $\hat{H}$ are higher order in p/m_N, we obtain

$$\psi(\vec{x}\,')=\left[\frac{1-\hat{K}(\vec{r},\vec{R})+\hat{S}(\vec{r},\vec{R})}{\hat{H}(\vec{r},\vec{R})}\right]\phi(\vec{x})\zeta$$

$$\equiv\hat{P}\phi(\vec{x})\zeta\ , \tag{9a}$$

with

$$\hat{S}(\vec{r},\vec{R})\equiv-\frac{i}{8A^2m_N{}^2}(\vec{q}\cdot\vec{r})\left[\vec{q}\cdot\frac{\vec{\nabla}_x}{i}\right]+\frac{i}{8Am_N{}^2}\left[\frac{\vec{\nabla}_x}{i}\times\vec{q}\right]\cdot\vec{\sigma}\ . \tag{9b}$$

The procedure applies identically to the reduction of the four-component Dirac wave function in the *final* state. This yields

$$\hat{\mathscr{O}}^\dagger\equiv e^{i\vec{q}\cdot\vec{x}}\hat{P}^\dagger e^{-i\vec{q}\cdot\vec{x}}\ . \tag{9c}$$

The adjoint operator $\hat{P}^\dagger$ can easily be obtained from Eqs. (9a) and (9b) by taking its Hermitian conjugate. (Note, in particular, that $\vec{\nabla}_x/i$ is self-adjoint.)

We are now ready to apply the reduction procedure of Ref. 7 to obtain the nonrelativistic operator $\hat{\Theta}_\lambda(q,\vec{\nabla}_r)$ from the relativistic expressions given by Eq. (1a) or (1b). This consists of two steps, viz.: (1) one calculates the operator

$$\hat{\Omega}_\lambda(q;\vec{\nabla}_x)\equiv\hat{\mathscr{O}}^\dagger\hat{\Gamma}_\lambda(q,\mathscr{O})\hat{P}\ , \tag{10a}$$

and (2) one then obtains the operator $\hat{\Theta}_\lambda(q,\vec{\nabla}_r)$ from $\hat{\Omega}_\lambda(q;\vec{\nabla}_x)$ by the substitution rule

$$\vec{\nabla}_x\to\zeta\vec{\nabla}_r+i\vec{q}/(2A)\ ,\ \ \zeta\equiv1-\frac{1}{A}\ . \tag{10b}$$

This substitution arises from the extraction of the c.m. coordinate in the Breit frame. The resultant RCIA for both the polar vector current $V_\lambda(x)$ and the axial vector current $A_\lambda(x)$ are recorded immediately below. We note that these final results are unique. Furthermore, evaluation of terms of order $p^2/m_N{}^2$ given by our RCIA is meaningful only in the Breit frame.

For the polar vector current $V_\lambda(x)$, we have

$$\begin{aligned}\langle N_f(p^{(f)})\,|\,\vec{V}(0)\,|\,N_i(p^{(i)})\rangle\,|_{\text{RCIA}}
&=\Bigg\langle\Psi_f\Bigg|\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}\Bigg[f_V(q^2)\Bigg\{\frac{F(\vec{r}^{(a)})}{2m_N}\Bigg[2\zeta\frac{\vec{\nabla}^{(a)}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r}^{(a)})-2\hat{e}\vec{A}(\vec{r}^{(a)})+i\vec{q}\times\vec{\sigma}^{(a)}\Bigg]\\
&\qquad-\frac{i}{2m_N}\{[\vec{\nabla}F(\vec{r}^{(a)})]+i\vec{\sigma}^{(a)}\times[\vec{\nabla}F(\vec{r}^{(a)})]\}\Bigg\}\\
&\qquad+f_M(q^2)\Bigg[\frac{i}{2m_N}\vec{q}\times\vec{\sigma}^{(a)}+\frac{iq_0}{4m_N{}^2}\vec{\Delta}_M^{(a)}\Bigg]-f_S(q^2)\frac{2m_N\vec{q}}{m_\pi{}^2}[\,1+O(p^2/m_N{}^2)]\Bigg]\Bigg|\Psi_i\Bigg\rangle\ ,\end{aligned} \tag{11a}$$

$$\langle N_f(p^{(f)})\,|\,V_4(0)\,|\,N_i(p^{(i)})\rangle\,|_{\text{RCIA}}=\Bigg\langle\Psi_f\Bigg|\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}i\Bigg\{f_V(q^2)\Bigg[1+\frac{\delta_V^{(a)}}{8m_N{}^2}\Bigg]+f_M(q^2)\frac{\delta_M^{(a)}}{4m_N{}^2}-f_S(q^2)\frac{2m_Nq_0}{m_\pi{}^2}[1+O(p^2/m_N{}^2)]\Bigg\}\Bigg|\Psi_i\Bigg\rangle\ . \tag{11b}$$

Here $\vec{\Delta}_M$, δ_V, and δ_M are defined by

$$\begin{aligned}\vec{\Delta}_M&=F(\vec{r})\left\{\vec{\sigma}\times\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right]+i\vec{q}\right\}-\{[\vec{\nabla}F(\vec{r})]+i\vec{\sigma}\times[\vec{\nabla}F(\vec{r})]\}\ ,\\
\delta_V&=-(i\vec{q}\cdot\vec{r})\vec{q}^{\,2}/A^2+(2i\zeta/A)\vec{\sigma}\times\frac{\vec{\nabla}}{i}\cdot\vec{q}-G(\vec{r})\left\{\vec{q}^{\,2}+2i\vec{\sigma}\cdot\vec{q}\times\left[\zeta\frac{\vec{\nabla}}{i}-\vec{U}(\vec{r})-\hat{e}\vec{A}(\vec{r})\right]\right\}\\
&\quad+\{\,-i[\vec{\nabla}G(\vec{r})]\cdot\vec{q}+\vec{\sigma}\cdot[\vec{\nabla}G(\vec{r})]\times\vec{q}\,\}\ ,\end{aligned} \tag{11c}$$

$$\delta_M=-F(\vec{r})\left\{\vec{q}^2+2i\vec{\sigma}\cdot\vec{q}\times\left[\zeta\frac{\vec{\nabla}}{i}-\vec{U}(\vec{r})-\hat{e}\vec{A}(\vec{r})\right]\right\}+\{-i\vec{q}\cdot[\vec{\nabla}F(\vec{r})]+\vec{\sigma}\cdot[\vec{\nabla}F(\vec{r})]\times\vec{q}\}\ .$$

For the axial vector current $A_\lambda(x)$, we have

$$\langle N_f(p^{(f)})\,|\,\vec{A}(0)\,|\,N_i(p^{(i)})\rangle\,|_{\mathrm{RCIA}}$$
$$=\left\langle\Psi_f\left|\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}\left\{f_A(q^2)\left[-\vec{\sigma}^{(a)}\left[1+\frac{\delta_A^{(a)}}{8m_N{}^2}\right]-\frac{1}{4m_N{}^2}\vec{\Delta}_1^{(a)}-\frac{G(\vec{r}^{(a)})}{4m_N{}^2}\vec{\Delta}_2^{(a)}+\frac{1}{8m_N{}^2}\vec{\Delta}_3^{(a)}\right]\right.\right.$$
$$\left.\left.-f_P(q^2)\frac{\vec{q}}{m_\pi{}^2}\{F(\vec{r}^{(a)})\vec{\sigma}^{(a)}\cdot\vec{q}+i\vec{\sigma}^{(a)}\cdot[\vec{\nabla}F(\vec{r}^{(a)})]\}-f_E(q^2)\frac{\vec{\Delta}_E^{(a)}}{4m_N{}^2}\right\}\right|\Psi_i\right\rangle, \quad (12a)$$

$$\langle N_f(p^{(f)})\,|\,A_4(0)\,|\,N_i(p^{(i)})\rangle\,|_{\mathrm{RCIA}}$$
$$=\left\langle\Psi_f\left|\sum_{a=1}^{A}e^{-i\vec{q}\cdot\vec{r}^{(a)}}i\left\{f_A(q^2)\left\{-\frac{F(\vec{r}^{(a)})}{2m_N}\vec{\sigma}^{(a)}\cdot\left[2\zeta\frac{\vec{\nabla}^{(a)}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r}^{(a)})-2\hat{e}\vec{A}(\vec{r}^{(a)})\right]\right.\right.\right.$$
$$\left.+\frac{i}{2m_N}\vec{\sigma}^{(a)}\cdot[\vec{\nabla}F(\vec{r}^{(a)})]\right\}$$
$$\left.\left.-f_P(q^2)\frac{q_0}{m_\pi{}^2}\{F(\vec{r}^{(a)})\vec{\sigma}^{(a)}\cdot\vec{q}+i\vec{\sigma}^{(a)}\cdot[\vec{\nabla}F(\vec{r}^{(a)})]\}+f_E(q^2)\frac{\vec{\sigma}^{(a)}\cdot\vec{q}}{2m_N}\right\}\right|\Psi_i\right\rangle, \quad (12b)$$

with

$$\delta_A=-(i\vec{q}\cdot\vec{r})\vec{q}^2/A^2-G(\vec{r})\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right]^2$$
$$+\left\{i\vec{q}\cdot[\vec{\nabla}G(\vec{r})]+2i[\vec{\nabla}G(\vec{r})]\cdot\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right]\right\},$$
$$\vec{\Delta}_1=(\zeta i/A)\frac{\vec{\nabla}}{i}\times\vec{q}\ ,$$
$$\vec{\Delta}_2=\left[\zeta\frac{\vec{\nabla}}{i}-\tfrac{1}{2}(1+\zeta)\vec{q}-\vec{U}(\vec{r})-\hat{e}\vec{A}(\vec{r})\right]\vec{\sigma}\cdot\left[\zeta\frac{\vec{\nabla}}{i}+\frac{\vec{q}}{2A}-\vec{U}(\vec{r})-\hat{e}\vec{A}(\vec{r})\right]$$
$$+\vec{\sigma}\cdot\left[\zeta\frac{\vec{\nabla}}{i}-\tfrac{1}{2}(1+\zeta)\vec{q}-\vec{U}(\vec{r})-\hat{e}\vec{A}(\vec{r})\right]\left[\zeta\frac{\vec{\nabla}}{i}+\frac{\vec{q}}{2A}-\vec{U}(\vec{r})-e\vec{A}(\vec{r})\right]$$
$$+2\{\vec{\nabla}\times[\vec{U}(\vec{r})+\hat{e}\vec{A}(\vec{r})]\}+i\vec{q}\times\left[\zeta\frac{\vec{\nabla}}{i}-\vec{U}(\vec{r})-\hat{e}\vec{A}(\vec{r})\right], \quad (12c)$$
$$\vec{\Delta}_3=2i\vec{\sigma}\cdot[\vec{\nabla}G(\vec{r})]\vec{q}+[\vec{\nabla}G(\vec{r})]\times\left[4\zeta\frac{\vec{\nabla}}{i}-(2\zeta-1)\vec{q}-4\vec{U}(\vec{r})-4\hat{e}\vec{A}(\vec{r})\right]$$
$$+i\vec{\sigma}\cdot\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right][\vec{\nabla}G(\vec{r})]+i\vec{\sigma}\cdot[\vec{\nabla}G(\vec{r})]\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right],$$
$$\vec{\Delta}_E=F(\vec{r})\left\{\vec{q}\cdot\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right]\vec{\sigma}-(\vec{\sigma}\cdot\vec{q})\left[2\zeta\frac{\vec{\nabla}}{i}-\zeta\vec{q}-2\vec{U}(\vec{r})-2\hat{e}\vec{A}(\vec{r})\right]\right\}$$
$$+([\vec{\nabla}F(\vec{r})]\times\vec{q}+i(\vec{\sigma}\cdot\vec{q})[\vec{\nabla}F(\vec{r})]-i\{\vec{q}\cdot[\vec{\nabla}F(\vec{r})]\}\vec{\sigma})\ .$$

The conventional IA is reproduced if (i) terms of order p^2/m_N^2 are neglected from Eqs. (11) and (12); (ii) the deviation of $F(\vec{r})$ and of $G(\vec{r})$ from unity is neglected; and (iii) the vector potentials $\vec{U}(\vec{r})$ and $\vec{A}(\vec{r})$ are chosen to vanish identically. However, a minor difference from the conventional IA should be noted, viz., the extraction of the c.m. coordinates gives rise to a correction factor $\zeta = 1 - 1/A$ which appears in our RCIA but not in the conventional IA.

For a *conserved* polar vector current $V_\lambda(x)$, such as the electromagnetic current or charged weak polar vector current, the *nuclear* polar vector current generated by the conventional IA or by the RCIA is in general at variance with current conservation (CC). *Assuming* that the time component of the polar vector current is reliably given by the RCIA, we follow Ref. 8 to defined a new IA scheme:

$$\{ V_4(\vec{x}) \}_{\mathrm{RCIA/CC}} \equiv \{ V_4(\vec{x}) \}_{\mathrm{RCIA}} , \tag{13a}$$

$$\int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \vec{\epsilon}_0^{\,*} \cdot \{ \vec{V}(\vec{x}) \}_{\mathrm{RCIA/CC}} \equiv \frac{i}{|\vec{q}|} \int d^3x\, e^{-i\vec{q}\cdot\vec{x}} [\hat{H}, \{ V_4(\vec{x}) \}_{\mathrm{RCIA}}] , \tag{13b}$$

$$\begin{aligned} \int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \vec{\epsilon}_\pm^{\,*} \cdot \{ \vec{V}(\vec{x}) \}_{\mathrm{RCIA/CC}} &\equiv \int d^3x\, e^{-i\vec{q}\cdot\vec{x}} \vec{\epsilon}_\pm^{\,*} \cdot \{ \vec{V}(\vec{x}) \}_{\mathrm{RCIA}} \\ &\quad + \int d^3x \sum_L \left[\frac{2\pi(2L+1)}{L(L+1)} \right]^{1/2} (-i)^L \frac{1}{|\vec{q}|} (\vec{\nabla}\cdot \{ \vec{V}(\vec{x}) \}_{\mathrm{RCIA}} + [\hat{H}, \{ V_4(\vec{x}) \}_{\mathrm{RCIA}}]) \left[1 + x\frac{d}{dx} \right] \\ &\quad \times j_L(|\vec{q}|x) Y_{L,\mp 1}(\hat{x}) . \end{aligned} \tag{13c}$$

Here, $\hat{H}$ is the Hamiltonian operator, $\vec{\epsilon}_0 \equiv \hat{z}' = \vec{q}/|\vec{q}|$, and $\vec{\epsilon}_\pm = \mp(\hat{x}' \pm i\hat{y}')/\sqrt{2}$ with $\hat{x}', \hat{y}', \hat{z}'$ three orthogonal unit vectors. To obtain Eq. (13b), we use $\vec{\epsilon}_0 e^{-i\vec{q}\cdot\vec{x}} = (i/|\vec{q}|)\vec{\nabla} e^{-i\vec{q}\cdot\vec{x}}$ and invoke the continuity equation after integration by parts. The derivation of Eq. (13c) has been given elsewhere.[13] We refer to the scheme defined by Eqs. (13a)–(13c) as the "relativistically corrected impulse approximation constrained by current conservation" (RCIA/CC).

There is an important difference between the RCIA/CC and the *conserved* IA of Ref. 7. In Ref. 7, an off-shell form factor proportional to q_λ/q^2 is introduced to restore CC to the IA. Further studies indicate that CC is restored at the expense of analyticity. In the RCIA/CC, Eqs. (13b) and (13c) are implemented *simultaneously* to ensure both CC and analyticity.[8] This result illustrates why the RCIA/CC is more attractive than the conserved IA proposed in Ref. 7.

For a partially conserved axial vector current, such as charged weak axial vector current, similar considerations are not so useful for two major reasons, viz., (1) it is difficult to assess which component of the axial vector current $A_\lambda(x)$ is determined more reliably by the RCIA, and (2) the operator characterizing the divergence of the axial vector current (e.g., the pion source current) is, in general, more uncertain than the current itself. Therefore, we do not modify the RCIA for a partially conserved axial current although the *nuclear* axial vector current generated by the RCIA might be at variance with PCAC.

To utilize the RCIA of Eqs. (11) and (12) or the RCIA/CC of Eqs. (13) and (11) in investigations of electroweak interactions in nuclei, one must know how to obtain the input functions $F(\vec{r})$, $G(\vec{r})$, $\vec{U}(\vec{r})$, and $\vec{A}(\vec{r})$, or equivalently, the potentials $U_\mu(\vec{r})$, $A_\mu(\vec{r})$, and $V(\vec{r})$. According to the interpretation that the potentials $U_\mu(\vec{r})$ and $V(\vec{r})$ represent the "average" meson fields generated by the other constituent nucleons, one can, in principle, generate $U_\mu(\vec{r})$ and $V(\vec{r})$ in a self-consistent manner (e.g., via the self-consistent Hartree approximation).[9,10] By the same token, one may study effects caused by the electromagnetic potential $A_\mu(\vec{r})$ in the case of heavy nuclei (large Z) or in the presence of a strong external electromagnetic field.

In general, one expects from Eq. (5c) that either of $\vec{\nabla}F(\vec{r})$, and $\vec{\nabla}G(\vec{r})$ is of order p^2/m_N. (This is, of course, subject to explicit quantitative justification.) Accordingly, terms proportional to either of $\vec{\nabla}F(\vec{r})$, and $\vec{\nabla}G(\vec{r})$ can be neglected from $\vec{\Delta}_M$, δ_V, δ_M, δ_A, $\vec{\Delta}_2$, $\vec{\Delta}_3$, and $\vec{\Delta}_E$ in Eqs. (11c) and (12c). Other such terms which appear in Eqs. (11a), (12a), and (12b) represent effects of order p^2/m_N^2 and so enter a reliable determination of relativistic corrections.

To date, electron scattering off light nuclei[4,5] may provide us with an opportunity to investigate relativistic corrections in a quantitative fashion. This task is a difficult but important one, since we need to locate the kinematic regime for which not only the (nonrelativistic) expansion in powers of p/m_N remains valid but effects of order $p^2/m_N{}^2$ become sizable. The difficulty arises partly from the subtraction of the contribution due to meson-exchange currents (MEC),[2] of which the formulation at medium energies must be scrutinized as well. Nevertheless, a careful formulation of both relativistic corrections and the MEC at medium energies has already become indispensible for understanding experimental data on electron scattering off light nuclei. Since it is not the scope of this paper to discuss the MEC, we conclude this section only by appending two additional comments on relativistic corrections, viz.:

(1) It is particularly important to obtain reliably $U_\mu(\vec{r})$, $A_\mu(\vec{r})$, and $V(\vec{r})$ via a self-consistent approach, since a detailed knowledge of $F(\vec{r})$, $\vec{U}(\vec{r})$, and $\vec{A}(\vec{r})$ is required in a quantitative treatment of relativistic corrections.

(2) One must apply the RCIA/CC rather than the RCIA to analyses of electromagnetic interactions in nuclei. We refer the reader to Ref. 8 for an illustrative example of how current conservation can be implemented explicitly in practical calculations.

III. TEST OF FUNDAMENTAL SYMMETRIES IN THE $A=12$ NUCLEI—REVISITED

So far, we have observed that inclusion of a static local central potential induces several subtle changes to the conventional IA. The application of the RCIA or the RCIA/CC to electromagnetic processes in nuclei is expected to yield predictions different slightly from the conventional IA results, since terms of order p/m_N are already modified by the presence of a central potential. In our opinion, effects caused by the presence of a central potential are at least as important as corrections due to meson-exchange currents (MEC). Quantitative treatment of these modifications, including terms of order $p^2/m_N{}^2$, in electron scattering off light nuclei is in progress and will only be reported in future publications. In what follows, we reanalyze experimental tests of fundamental symmetries in the $A=12$ nuclei[11,12] in order to assess importance of the modifications to terms of order p/m_N.

Following the results obtained by Noble,[10] we assume $\vec{U}(\vec{r})\cong 0$, $A_\mu(\vec{r})\cong 0$, $F(\vec{r})=F(|\vec{r}|)$, and $U_0(r=0)-V(r=0)\approx 750$ MeV. (To be more careful, one should generate these average central potentials in a self-consistent manner.) In addition, we assume no second-class currents, $f_S(q^2)=0$ and $f_E(q^2)=0$. (These two form factors have been included in our RCIA only for the sake of generality.) We define, for the ^{12}B(g.s.) $\rightarrow$ ^{12}C(g.s.) transition,[11]

$$\langle {}^{12}\mathrm{C}(p^{(f)})\,|\,V_\lambda(0)\,|\,{}^{12}\mathrm{B}(p^{(i)},\xi)\rangle=\sqrt{2}\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa\frac{q_\rho}{2m_N}\frac{\mathcal{O}_\eta}{2M}F_M(q^2)\,, \tag{14a}$$

$$\langle {}^{12}\mathrm{C}(p^{(f)})\,|\,A_\lambda(0)\,|\,{}^{12}\mathrm{B}(p^{(i)},\xi)\rangle=\sqrt{2}\left[\xi_\lambda F_A(q^2)+q_\lambda\frac{q\cdot\xi}{m_\pi^2}F_P(q^2)-\frac{\mathcal{O}_\lambda}{2M}\frac{q\cdot\xi}{2m_N}F_E(q^2)\right]. \tag{14b}$$

Using a method similar to that described by Delorme,[14] we derive the RCIA predictions on these nuclear form factors, viz.,

$$\sqrt{2}F_M(q^2)=-f_V(q^2)\zeta\frac{\sqrt{2}m_N}{|\vec{q}|}[\nabla/m_N]_F^{11}+f_V(q^2)\frac{1}{\sqrt{3}}\left\{[\sigma]_F^{01}-\frac{1}{\sqrt{2}}[\sigma]_F^{21}\right\}+f_M(q^2)\frac{1}{\sqrt{3}}\left\{[\sigma]_0^{01}-\frac{1}{\sqrt{2}}[\sigma]_0^{21}\right\}, \tag{15a}$$

$$\sqrt{2}F_A(q^2)=f_A(q^2)\frac{1}{\sqrt{3}}\left\{[\sigma]_0^{01}-\frac{1}{\sqrt{2}}[\sigma]_0^{21}\right\}, \tag{15b}$$

$$\sqrt{2}F_P(q^2)=f_P(q^2)\frac{1}{\sqrt{3}}\{[\sigma]_F^{01}+\sqrt{2}[\sigma]_F^{21}\}+f_A(q^2)\frac{m_\pi{}^2}{|\vec{q}|^2}(3/2)^{1/2}[\sigma]_F^{21}\,, \tag{15c}$$

$$\sqrt{2}F_E(q^2)=f_A(q^2)\frac{\zeta}{\sqrt{3}}\{[\sigma]_F^{01}+\sqrt{2}[\sigma]_F^{21}\}+f_A(q^2)\frac{2m_N\Delta}{|\vec{q}|^2}(3/2)^{1/2}[\sigma]_0^{21}+f_A(q^2)\zeta\frac{2m_N}{|\vec{q}|}[\vec{\sigma}\cdot\vec{\nabla}/m_N]_F^{11}\,, \tag{15d}$$

where $\Delta \equiv M(^{12}\text{B}) - M(^{12}\text{C}) = 13.881$ MeV. For a tensor operator of rank j, i.e.,

$$\mathcal{O} = \{ \mathcal{O}_{jm} \} , \quad \mathcal{O}_{jm} \equiv \left[\frac{4\pi}{2j+1} \right]^{1/2} ||\mathcal{O}|| Y_{jm}(\hat{\mathcal{O}}) , \tag{16a}$$

we define, for the $N_i \to N_f$ transition,

$$\left\langle N_f(p^{(f)};J_f M_f) \left| \sum_{a=1}^{A} \tau_+^{(a)} j_L(|\vec{q}\,| r^{(a)}) Y_{LM}(\hat{r}^{(a)}) F(r^{(a)}) \mathcal{O}_{jm} \right| N_i(p^{(i)};J_i M_i) \right\rangle$$
$$\equiv \sum_J \langle LM;jm \,|\, J,M+m \rangle \langle J_i M_i; J, M+m \,|\, J_f M_f \rangle (4\pi)^{-1/2} [\mathcal{O}]_F^{LJ} . \tag{16b}$$

The nuclear matrix element $[\mathcal{O}]_0^{LJ}$ is defined similar to Eq. (16b) except that $F(r^{(a)})$ is set to unity. Equations (15a)–(15d) can be contrasted with the conventional IA results reported in previous publications.[15,11]

As an estimate, we take[10]

$$[\mathcal{O}]_F^{LJ} / [\mathcal{O}]_0^{LJ} = 1.13 , \quad \mathcal{O} = \vec{\sigma}, \ \vec{\nabla}, \ \vec{\sigma}\cdot\vec{\nabla} . \tag{17}$$

The small uncertainty in this enhancement factor is not likely to be of any numerical significance. We obtain, for $0 \lesssim q^2 \lesssim m_\mu^2$,

$$\frac{F_M(q^2)}{F_A(q^2)} = 3.90x - 0.03 , \tag{18a}$$

$$\frac{F_E(q^2)}{F_A(q^2)} = 1.04 \left[1 + \frac{q^2 \delta}{m_\pi^2} \right] + \frac{2 m_N \Delta}{m_\pi^2} \delta + 1.04(y-1) \left[1 + \frac{1}{3} \frac{q^2 \delta}{m_\pi^2} + \frac{q^2 \tilde{\delta}}{m_\pi^2} \right] , \tag{18b}$$

$$\frac{F_P(q^2)}{F_A(q^2)} = -\frac{1.15}{1 + q^2/m_\pi^2} \left[1 + \frac{q^2 \delta}{m_\pi^2} \right] + 1.13\delta , \tag{18c}$$

with

$$x - 1 \equiv -\lim_{|\vec{q}|\to 0} \frac{\sqrt{6}}{4.71} \frac{m_N}{|\vec{q}\,|} \frac{[\nabla/m_N]_0^{11}}{[\sigma]_0^{01}} , \tag{18d}$$

$$y - 1 \equiv \lim_{|\vec{q}|\to 0} \frac{2\sqrt{3} m_N}{|\vec{q}\,|} \frac{[\vec{\sigma}\cdot\vec{\nabla}/m_N]_0^{11}}{[\sigma]_0^{01}} , \tag{18e}$$

$$\delta \equiv \lim_{|\vec{q}|\to 0} \frac{3}{\sqrt{2}} \frac{m_\pi^2}{|\vec{q}\,|^2} \frac{[\sigma]_0^{21}}{[\sigma]_0^{01}} . \tag{18f}$$

Here the factor $q^2 \tilde{\delta}/m_\pi^2$ in Eq. (18b) signifies the difference between the q^2 dependence of $(m_N/|\vec{q}\,|)[\sigma\cdot\nabla/m_N]_0^{11}$ and that of $[\sigma]_0^{01}$. Averaging the results obtained by Morita *et al.*[16] for the three configurations of the Cohen-Kurath model,[17] we have

$$x = 0.975, \quad y = 3.61, \quad \delta = -0.282 . \tag{19}$$

Equations (18) and (19) yield

$$\frac{F_M(0)}{F_A(0)} = 3.77 , \quad \frac{F_E(0)}{F_A(0)} = 3.37 , \quad \frac{F_P(0.74 m_\mu^2)}{F_A(0.74 m_\mu^2)} = -1.02 . \tag{20}$$

If effects caused by the central potentials are neglected from Eqs. (15a)–(15d), we obtain, instead of Eq. (20),

$$\frac{F_M(0)}{F_A(0)} = 3.68 , \quad \frac{F_E(0)}{F_A(0)} = 2.92 , \quad \frac{F_P(0.74 m_\mu^2)}{F_A(0.74 m_\mu^2)} = -0.91 . \tag{21}$$

Experimentally, the asymmetry measurements in $^{12}\text{B} \to {}^{12}\text{C}\, e^- \bar{\nu}_e$ and $^{12}\text{N} \to {}^{12}\text{C}\, e^+ \nu_e$ yield[18,11]

$$\frac{F_M(0)}{F_A(0)} = 4.07 \pm 0.44 , \quad \frac{F_E(0)}{F_A(0)} = 3.67 \pm 0.44 , \tag{22a}$$

while the measured value of average polarization in $\mu^- \, ^{12}\text{C} \to \nu_\mu \, ^{12}\text{B}$ leads to[12]

$$\frac{F_P(0.74 m_\mu^2)}{F_A(0.74 m_\mu^2)} = -1.03 \pm 0.14 . \tag{22b}$$

On the other hand, the observed β and γ decay rates[19] of ^{12}B and its isospin analog ^{12}C* yield, via an application of CVC,[11]

$$\left.\frac{F_M(0)}{F_A(0)}\right|_{\text{CVC}} = 3.86 \pm 0.12 \,. \qquad (22c)$$

The RCIA predictions, Eq. (20), agree surprisingly well with the experimental results, Eqs. (22a)–(22c). Such agreement substantiates the standard theoretical picture, namely, the absence of second-class axial currents, CVC, and PCAC, provided that the MEC contributions to the various covariant form factors are negligible for $0 \leq q^2 \lesssim m_\mu^2$. The conclusion that the standard picture has been verified experimentally can also be reached by a model-independent analysis.[11,12,18] Thus, unless corrections to the impulse approximation schemes are important, one may conclude that the existing data already favor the RCIA over the conventional IA.

A sizable MEC contribution to the time component of the axial current has been suggested[20] so that $F_E(0)/F_A(0)$ may differ considerably from its IA value, Eqs. (20) or (21). However, a recent calculation[21] indicates that, if Cohen-Kurath wave functions are used, the core polarization effect leads to a modification of $F_E(0)/F_A(0)$ which is about the same size as the MEC contribution but *opposite* in sign. On the other hand, there has not been any systematic calculation of the MEC contributions to $F_A(q^2)$, $F_M(q^2)$, and $F_P(q^2)$ for small q^2. A sizable MEC corrections to $F_A(0)$ or to $F_M(0)$ is unlikely in the Cohen-Kurath model[17] since the observed β and γ decay rates of ^{12}B and its isospin analog ^{12}C* are reproduced well by the IA. (Here the core polarization effect does not contribute.[21]) Thus, it is generally believed that corrections to the impulse approximation schemes are negligible in the Cohen-Kurath model. Nevertheless, it seems premature to rule out the possibility of resolving the discrepancy between the conventional IA predictions, Eq. (21), and the experimental results, Eqs. (22a)–(22c), by including the MEC effects. In any event, it is fair to conclude that the application of the RCIA to reanalyze symmetry tests in the $A = 12$ nuclei does not upset the standard interpretation of these experimental results. Furthermore, the possibility to discriminate between the RCIA and the conventional IA shall be greatly enhanced both (1) if the MEC and core polarization effects in the Cohen-Kurath model can be determined reliably, and (2) if the experimental errors appearing in Eqs. (22a)–(22c) can be reduced by, e.g., a factor of 2.

IV. CONCLUDING REMARKS

We have derived an impulse approximation with relativistic corrections, which incorporates explicitly the fact that each constituent nucleon of the target nucleus is moving in an average local central potential generated by the other nucleons. An application of the resultant impulse approximation to reanalyze symmetry tests in the $A = 12$ nuclei induces a few subtle changes but does not upset the standard interpretation[11,12,18] of these experimental results. However, implications of this impulse approximation in other problems such as electron scattering off nuclei remain to be unraveled.

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