πNN, $\pi N\Delta$, and $\pi\pi\rho$ form factors from quasi-two-body hadronic reactions

C. A. Dominguez*
Department of Physics, Texas A & M University, College Station, Texas 77843
(Received 23 September 1980)

Differential cross sections for $pp \to n\Delta^{++}$, $pp \to p\Delta^{+}$, and $\pi^+ p \to \rho^0 \Delta^{++}$ at high energies, $p_L \approx 3-16$ GeV/c, and small momentum transfers, $0 < |t| \lesssim 0.3$ GeV/c^2, have been analyzed using a Reggeized one pion exchange mechanism with form factors of the monopole and the dual model type. Results strongly confirm the dual model prediction that $F_{\pi NN}(t) = F_{\pi N\Delta}(t) = F_{\pi\pi\rho}(t)$ when the pion is the only virtual (off the mass shell) particle in each vertex and the form factors are all normalized to unity at $t = \mu_\pi^2$. No evidence has been found for non-one-pion exchange contributions in the kinematic region under consideration thus leading to a model independent determination of the range and asymptotic rate parameters of the three-point functions. The results are $\Lambda_\pi \approx 800-1000$ MeV and $\beta_\pi \approx 2.5-3$, in good agreement with earlier determinations from NN charge exchange scattering and pion photoproduction.

[NUCLEAR REACTIONS Quasi-two-body hadronic reactions at high energy; extraction of the πNN and other hadronic form factors at small momentum transfers; one pion exchange potential.]

I. INTRODUCTION

A knowledge of the off mass shell and analytic structure of the elastic and inelastic pionic form factors of the nucleon[1] is of great importance in nuclear physics.[2] Information on the range of the three-point functions is usually obtained by parametrizing them with monopole or dipole forms. For example, in the case of the πNX form factor (see Fig. 1), where X can be a nucleon, a $\Delta(1236)$, etc., if the pion is the only virtual particle in the vertex then the monopole reads

$$F_{\pi NX}(q^2) = \frac{\Lambda_{\pi NX}^2 - \mu_\pi^2}{\Lambda_{\pi NX}^2 - q^2}, \qquad (1)$$

where the range $\Lambda_{\pi NX}$ is normally treated as a free parameter to be determined independently. Although attractive from a computational point of view, Eq. (1) lacks physical motivation and leaves unanswered several fundamental questions. One of them is the fully off mass shell structure of the vertex, i.e., if $p_1^2 \neq M_N^2$, $p_2^2 \neq M_X^2$, and $q^2 \neq \mu_\pi^2$ corresponding to all three particles being virtual (off mass shell), the question is how should Eq. (1) be generalized. The dependence of $\Lambda_{\pi NX}$ on hadron X is also another important question which clearly cannot be answered unless Eq. (1) were to follow from a well defined dynamical model. Furthermore, the asymptotic behavior of Eq. (1) as $q^2 \to -\infty$ is most likely incorrect.[3,4]

In order to stress the importance of these issues let us consider as an example the iteration of the one-pion exchange (OPE) potential in the frame-

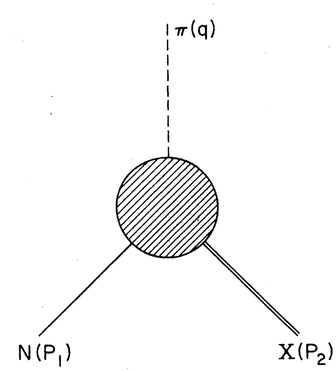

FIG. 1. The πNX vertex.

work of the Blankenbecler-Sugar equation. Traditionally, dynamical calculations have been carried out using the simple form Eq. (1) for the πNN form factor with no off mass shell nucleon dependence, despite the fact that after the first iteration all particles in the vertex become virtual. *A priori* there is no reason to expect off mass shell nucleon effects to be negligible and, therefore, this approach is basically incorrect. If this problem, as well as a whole variety of others, is to be treated correctly then it is necessary to go beyond the simple minded monopole form, Eq. (1), and formulate a dynamical model for hadronic three point functions starting from first principles. A major step forward in this direction has been taken recently with the proposal of a dual unitarizable model for fully off mass shell form factors.[5] In connection with the above example it has been shown in Ref. 5 that the off mass shell nature of the nucleons in the iterated OPE potential gives rise to some rather dramatic effects in NN phase shifts at intermediate energies.[5,6] The various physical assumptions needed to build the model, e.g., Lorentz invariance, analyticity, asymptotic power behavior, Regge trajectories, mass spectrum, etc., are fairly well established and have been discussed already in Ref. 5. The general expression of the form factor for a vertex made out of particles of four-momenta p_1, p_2, and p_3 reads

$$F_{123}(p_1^2,p_2^2,p_3^2) = \prod_{i=1}^{3}\Gamma(\beta_i-S_i)\frac{\Gamma[1-\alpha'(p_i^2-M_i^2)]}{\Gamma[\beta_i-S_i-\alpha'(p_i^2-M_i^2)]} . \tag{2}$$

In Eq. (2), F_{123} has been normalized to unity at the fully on mass shell point, S_i and M_i are the spin and mass of the ith particle, $\alpha' = 1/2M_\rho^2 \approx 0.83$ GeV^{-2} is the universal Regge slope, β_i are free parameters that govern the asymptotic behavior of F_{123} as $p_i^2 \to -\infty$, and the zero-width (nonunitary) approximation has been assumed for simplicity.[7] For $\beta_i = S_i + 1$, Eq. (2) reduces to the one-particle approximation or no vertex structure. Equation (2) exhibits the factorization property of the model which implies that once a free parameter β has been somehow determined for a given hadron in a particular vertex it should have the same value in any other vertex where that hadron participates.

This prediction has been tested recently by analyzing separately NN charge exchange scattering[3] and pion photoproduction[4] at high energies and small momentum transfers. In the former reaction the OPE amplitudes contain two powers of $F_{\pi NN}(q^2,M^2,M^2)$ while the latter involves the product $F_{\pi NN}(q^2,M^2,M^2)F_{\pi\pi\gamma}(q^2,\mu_\pi^2,0)$ (see Fig. 2). Since the only virtual particle in all these vertices is the pion, factorization implies that $F_{\pi\pi\gamma}(q^2,\mu_\pi^2,0)$ should be identical to $F_{\pi NN}(q^2,M^2,M^2)$ when both form factors are normalized to unity at the fully on shell point, a prediction that has been confirmed by the above mentioned analyses. Furthermore, the value of β_π obtained from those fits, i.e., $\beta_\pi \simeq 2.5-3$, implies that for small pion four momentum squared Eq. (2) can be approximated very well by Eq.(1) with $\Lambda_\pi \simeq 800-1000$ MeV. Hence, the dual model provides the physical justification for the monopole form factor in the restricted region $0 \lesssim -q^2 \lesssim \mu_\pi^2$; as $-q^2 \to \infty$, however, Eq. (2) predicts that $F_{\pi NN}(q^2, M^2, M^2) \sim (-q^2)^{1-\beta}$ in agreement with the constituent interchange quark model[8] prediction. It must be emphasized, though, that this type of analysis does not test directly the full off shell structure of Eq. (2) and, therefore, factorization is tested only in the limited sense described above, i.e., the pionic part of the form factor is the same in different vertices that involve one virtual pion.

In this paper, I wish to discuss some additional tests of the dual model that involve the form factors $F_{\pi NN}(q^2,M^2,M^2)$, $F_{\pi N\Delta}(q^2,M^2,M_\Delta^2)$, and $F_{\pi\pi\rho}(q^2,\mu_\pi^2,M_\rho^2)$ as they appear in the quasi-two-body reactions $pp \to n\Delta^{++}$, $pp \to p\Delta^+$, and

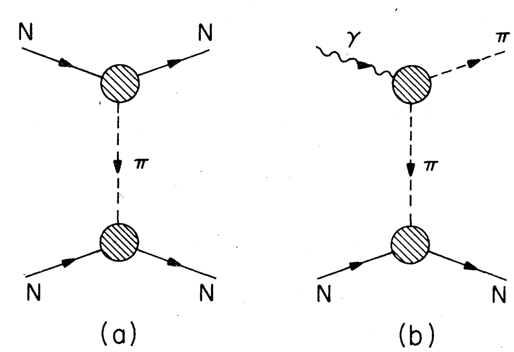

FIG. 2. One-pion exchange diagrams for (a) NN scattering and (b) charged pion photoproduction on nucleons. Factorization implies that all vertex functions are identical to $F_{\pi NN}(t)$ when normalized to unity at $t = \mu_\pi^2$.

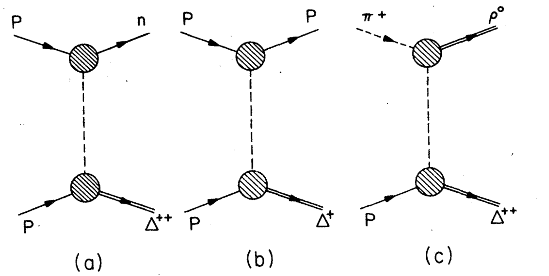

FIG. 3. One-pion exchange diagrams for the three quasi-two-body reactions studied here. (a) $pp \to n\Delta^{++}$, (b) $pp \to p\Delta^+$, and (c) $\pi^+ p \to \rho^0 \Delta^{++}$.

$\pi^+ p \to \rho^0 \Delta^{++}$ (see Fig. 3). According to factorization all these three vertex functions should be identical[9] (if they are all normalized to unity at the respective on mass shell points). There are some indications from a dispersion theory analysis[10] that if $F_{\pi NN}$ and $F_{\pi N\Delta}$ are both parametrized by monopoles, then $\Lambda_{\pi NN} \simeq \Lambda_{\pi N\Delta}$ in agreement with the dual model prediction. The analysis to be presented here, however, is essentially model independent since it is based on the Reggeized OPE parametrization of high energy scattering amplitudes. At small momentum transfers, $0 < |t| \lesssim 0.3$ (GeV/c)^2, the diagrams in Fig. 3 are expected to account for basically all of the cross sections, in which case the only unknowns in the problem are the form factors.

Differential cross sections for the three reactions in the energy ranges 3–16 GeV and momentum transfers $|t| < 0.3$ (GeV/c)^2 have been fitted with the following results. First, no evidence has been found for non-OPE contributions in the kinematic range under consideration, thus leading to a model independent analysis. Second, the results are in agreement with factorization, i.e., $\Lambda_{\pi NN} = \Lambda_{\pi N\Delta} = \Lambda_{\pi\pi\rho} \equiv \Lambda_\pi$ and $\beta_{\pi NN} = \beta_{\pi N\Delta} = \beta_{\pi\pi\rho} \equiv \beta_\pi$, when only one pion is off the mass shell. The values obtained from the fits, i.e., $\Lambda_\pi \simeq 800-1000$ MeV and $\beta_\pi \simeq 2.5-3$, are in good agreement with the results of earlier analyses of NN charge exchange scattering and pion photoproduction.

II. FORMALISM

The three quasi-two-body reactions to be studied here are (a) $pp \to n\Delta^{++}$, (b) $pp \to p\Delta^+$, and (c) $\pi^+ p \to \rho^0 \Delta^{++}$. The dominant OPE diagrams are illustrated in Fig. 3 and it should be recalled that at high energies the pion propagator must be properly Reggeized.[11] Reactions (a) and (b) are clearly related by isospin but since there are independent data for them they will be treated separately. The calculation of differential cross sections has been discussed in the literature[12] and the results are

$$\frac{d\sigma}{dt}(pp \to n\Delta^{++}) = \frac{\pi}{24M^2 p_L^2} \left[\frac{g_{\pi^+ np}^2}{4\pi}\right] F_{\pi NN}^2(t) \times \left[\frac{g_{\pi^+ p\Delta^{++}}^2}{4\pi}\right] F_{\pi N\Delta}^2(t) \frac{t}{\mu_\pi^2 M_\Delta^2} [t-(M_\Delta+M)^2]^2 [t-(M_\Delta-M)^2] R^2(t) , \tag{3}$$

$$\frac{d\sigma}{dt}(\pi^+ p \to \rho^0 \Delta^{++}) = \frac{\pi}{6\mu_\pi^2 m_\rho^2 M_\Delta^2} \frac{1}{[s-(\mu_\pi+M)^2][s-(\mu_\pi-M)^2]} \left[\frac{g_{\rho^0\pi^+\pi^+}^2}{4\pi}\right] F_{\pi\pi\rho}^2(t) \left[\frac{g_{\pi^+ p\Delta^{++}}^2}{4\pi}\right] F_{\pi N\Delta}^2(t) \times [(\mu_\pi+m_\rho)^2 - t][(m_\rho-\mu_\pi)^2 - t][(M_\Delta-M)^2 - t][(M_\Delta+M)^2 - t]^2 R^2(t) . \tag{4}$$

The cross section for $pp \to p\Delta^+$ has the same mathematical form as Eq. (3) except for an obvious change in coupling constants. In Eqs. (3) and (4) the form factors are defined with the pion pole removed and are normalized to unity at $t = \mu_\pi^2$, P_L is the laboratory three-momentum of the incident proton, s the square of the center of mass energy in the direct channel, and the Reggeized pion propagator $R(t)$ is given by

$$R(t) = \pi\alpha'[1+2\alpha_\pi(t)] \frac{1+e^{-i\pi\alpha_\pi(t)}}{2\sin \pi\alpha_\pi(t)} \times \frac{1}{\sqrt{\pi}} \frac{\Gamma[\frac{1}{2}+\alpha_\pi(t)]}{\Gamma[1+\alpha_\pi(t)]} \left[\frac{s-u}{2s_0}\right]^{\alpha_\pi(t)} . \tag{5}$$

In Eq. (5) the pion Regge trajectory $\alpha_\pi(t)$ is

$$\alpha_\pi(t)=\alpha'(t-\mu_\pi^2)\ , \qquad (6)$$

where

$$\alpha'=\frac{1}{2M_\rho^2}\approx 0.83\ \text{GeV}^{-2}$$

and u is the standard Mandelstam variable, viz., $s+t+u=\sum_{i=1}^4 M_i^2$. For small values of t, Eq. (5) can be approximated by

$$R(t)\underset{t\to 0}{\approx}\left[\frac{1}{t-\mu_\pi^2}\right]\left[\frac{s-u}{2s_0}\right]^{\alpha_\pi(t)}\ , \qquad (7)$$

which shows explicitly the effects of Reggeization of the pion propagator. The various coupling constants have the values[13]

$$\frac{g_{\pi^0 pp}^2}{4\pi}=14.28;\quad \frac{g_{\pi^+ np}^2}{4\pi}=2\left[\frac{g_{\pi^0 pp}^2}{4\pi}\right],$$

$$\frac{g_{\pi^+ p\Delta^{++}}^2}{4\pi}\approx 0.3-0.5,\quad \frac{g_{\pi^0 p\Delta^+}^2}{4\pi}=\frac{2}{3}\left[\frac{g_{\pi^+ p\Delta^{++}}^2}{4\pi}\right]\ , \qquad (8)$$

$$\frac{g_{\rho^0\pi^+\pi^+}^2}{4\pi}\approx 3\ .$$

The form factors will be parametrized by monopole formulas, i.e.,

$$F(t)=\frac{\Lambda^2-\mu_\pi^2}{\Lambda^2-t}\ , \qquad (9)$$

or by the dual model, i.e.,

$$F(t)=\Gamma(\beta)\frac{\Gamma[1-\alpha_\pi(t)]}{\Gamma[\beta-\alpha_\pi(t)]}\ , \qquad (10)$$

where Λ or β are the parameters to be determined by the fits discussed in the next section.

III. FITS

For energies above the resonance region but below intersecting storage rings (ISR) values,[14] i.e, $2\lesssim P_L\lesssim 20$ GeV/c, and small t, the three reactions considered here are expected to be dominated by one-pion exchange. The presence of additional contributions such as, e.g., other Regge poles (ρ, A_2, etc.), Regge cuts, and pole-cut interferences, can be easily tested by modifying the Reggeized OPE formulas, (3) and (4), accordingly and rerunning the fits to the data. In the kinematic region studied here, i.e., $P_L\simeq 3-16$ (GeV/c) and $|t|<0.3$ (GeV/c)^2, no evidence has been found for non-OPE contributions and, therefore, the following analysis will be based entirely on Eqs. (3) and (4). The Regge parameters that determine the pion Regge trajectory are known independently and thus, *a priori,* the only unknowns are the two form factors. However, the $\pi N\Delta$ coupling constant is known less accurately than the πNN coupling constant so that it might be convenient to consider the former as a free parameter together with the form factor ranges Λ or the asymptotic rates β. An additional advantage of this procedure is that by allowing the coupling constant to float we can study the compatibility of the various sets of data measured at different energies and at different laboratories. This is important because it is known that the data on the reactions under consideration[15] may suffer from non-negligible normalization uncertainties.

The data sets used in the analysis are the following.[15] For $pp\to n\Delta^{++}$, differential cross sections $d\sigma/dt$ at $P_L=2.8$, 6.6, 8.1, and 10.0 GeV/c; for $pp\to p\Delta^+$, $d\sigma/dt$ at $P_L=4.55$, 6.06, 7.88, and 9.9 GeV/c; for $\pi^+p\to\rho^0\Delta^{++}$, $(\rho_{00}d\sigma/dt')$ at $P_L=5.45$ and 16 GeV/c and $(\rho_{00}d\sigma/dt)$ at $P_L=7.1$, where $t'=t-t_{\min}$ and ρ_{00} is the spin density matrix element which asymptotically isolates the unnatural spin parity exchange contribution to the helicity zero state. For each reaction, fits to the cross sections have been performed separately at each energy in order to determine the magnitude of the normalization uncertainties. If the data on a given reaction at different energies was compatible as indicated by the fitted value of the $\pi N\Delta$ coupling constant then a simultaneous fit to all the data was carried out. This turned out to be the case basically for all three reactions where the various results for $g_{\pi N\Delta}^2/4\pi$ were in fair agreement with one another as well as with the experimental range given in Eq. (8). In any case, it was found that the form factor parameters were highly insensitive to changes in normalization.

The results of the fits are shown in Table I and Figs. 4–7 and they can be summarized as follows.

TABLE I. Form factor parameters from Reggeized OPE fits to differential cross sections.

Reaction	Form factor	Λ_π (MeV)	β_π	$\chi_F{}^2$
$pp \to n\Delta^{++}$	Monopole	788 ± 20		1.3
	Dual model		3.7 ± 0.3	1.3
$pp \to p\Delta^{+}$	Monopole	807 ± 37		1
	Dual model		3.4 ± 0.4	1
$\pi^+ p \to \rho^0\Delta^{++}$	Monopole	$850-1000$		$1-2$
	Dual model		$2.5-3.1$	$1-2$

First, the prediction of the dual model, that $F_{\pi NN}$ should be equal to $F_{\pi N\Delta}$ and to $F_{\pi\pi\rho}$ when only the pion is virtual and all form factors are normalized to unity at the pole, has been amply confirmed. In fact, using the monopole or the dual model with different initial parameters for the πNN, $\pi N\Delta$, and $\pi\pi\rho$ vertices one obtains as a result of the fits that $\Lambda_{\pi NN}=\Lambda_{\pi N\Delta}=\Lambda_{\pi\pi\rho}\equiv\Lambda_\pi$ and $\beta_{\pi NN}=\beta_{\pi N\Delta}=\beta_{\pi\pi\rho}\equiv\beta_\pi$. This result actually holds with a higher confidence level than what the standard deviations or the values of the chi squared per degree of freedom, $\chi_F{}^2$, in Table I seem to indicate. Second, the results obtained for Λ_π and β_π are in good agreement with previous determinations from NN charge exchange scattering and charged pion photoproduction. Last but not least, it should be pointed out that the presence of the form factors has been unequivocally established by attempting to fit the data with $F_{\pi NN}(t)=1$, in which case the resulting $\chi_F{}^2$ increased by as much as 450, with typical values in the range $\chi_F{}^2 \simeq 30-100$.

IV. CONCLUSIONS

Additional evidence has been presented here supporting the prediction of the dual model that the

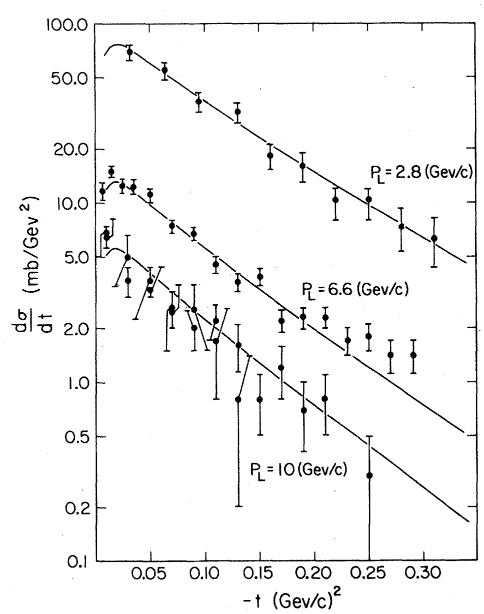

FIG. 4. Experimental data on $pp \to n\Delta^{++}$ at $P_L=2.8$, 6.6, 8.1, and 10.0 GeV/c. Solid curves are the predictions of Eq. (3) with the form factor parameters of Table I.

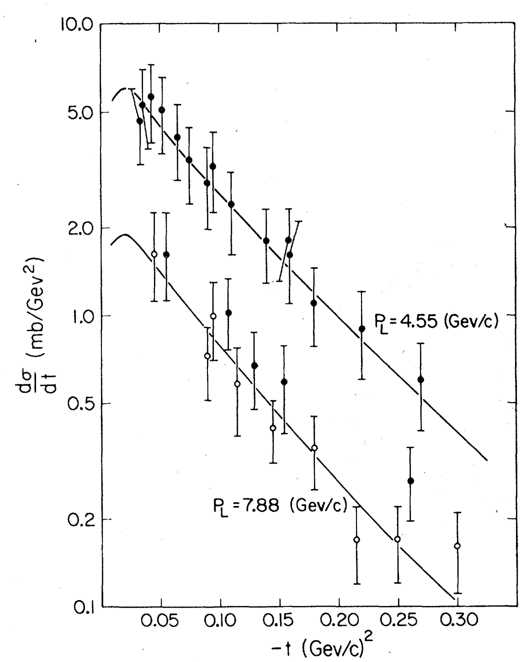

FIG. 5. Experimental data on $pp \to p\Delta^+$ at $P_L=4.55$, 6.06, and 7.88 GeV/c. The $P_L=9.9$ GeV/c data is not shown for clarity but it has been used in the fits. Solid curves are the predictions of Eq. (3) with the form factor parameters of Table I.

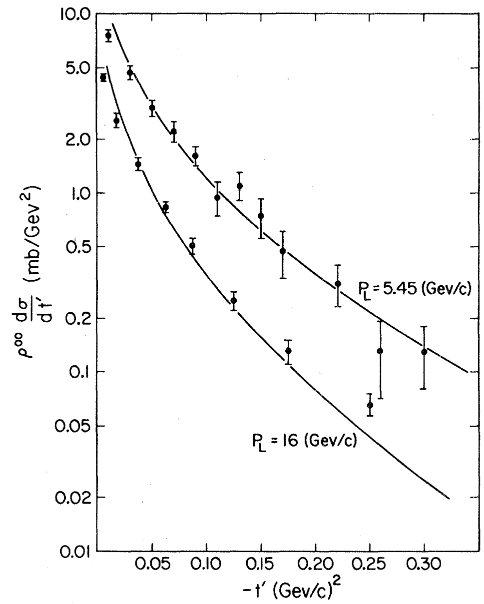

FIG. 6. Experimental data on $\pi^+ p \to \rho^0 \Delta^{++}$ at $P_L = 5.45$ and 16 GeV/c. Solid curves are the predictions of Eq.(4) with the form factor parameters of Table I.

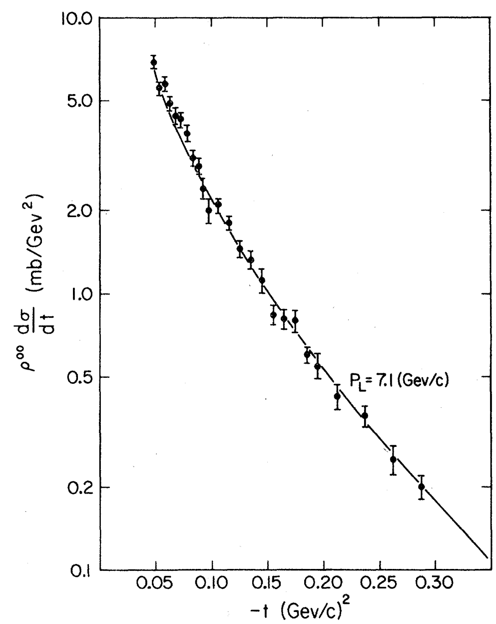

FIG. 7. Experimental data on $\pi^+ p \to \rho^0 \Delta^{++}$ at $P_L = 7.1$ GeV/c. Solid curve is the prediction of Eq. (4) with the form factor parameters of Table I.

pionic piece of any vertex function, with only one pion off the mass shell, is universal. Although this universality (which follows from factorization) is much more general, the very nature of the present analysis restricts the test to pionic vertices with two particles on the mass shell. Although the tests have been performed in the zero-width approximation, no major changes are expected from a unitarized version of the model since unitarity corrections have been shown to be minimal in the space-like region.[5,7]

At the same time, new independent determinations of the range and asymptotic rate parameters of the πNN vertex function have been performed with the result that $\Lambda_\pi \simeq 800-1000$ MeV and $\beta_\pi \simeq 2.5-3$. This agrees with earlier determinations[3,4] based on the same method i.e., Reggeized OPE parametrization of scattering amplitudes at high energies and small momentum transfers, as well as with the other extractions.[16].

ACKNOWLEDGMENTS

The author wishes to thank Jörg Hüfner for valuable comments, and the Aspen Center for Physics where this research was completed. This work was supported in part by the U. S. Department of Energy Contract No. DOE-AS05-76-ER05223.

---

*On leave of absence from Centro de Investigacion y de Estudios Avanzados del IPN, Mexico.

[1]One can refer to the πNN vertex as elastic and to the πNX, where X is any hadron other than the nucleon, as inelastic.

[2]G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976); J. J. de Swart and M. M. Nagels, Fortschr. Phys. 28, 215 (1978).

[3]C. A. Dominguez and B. J. VerWest, Phys. Rev. Lett. 89B, 333 (1980).

[4]C. A. Dominguez and R. B. Clark, Phys. Rev. C 21, 1944 (1980).

[5]R. A. Bryan, C. A. Dominguez, and B. J. VerWest, Phys. Rev. C 22, 160 (1980). For earlier work, see C. A. Dominguez, Phys. Rev. D 17, 1252 (1973); 16, 2320 (1977).

[6]For a different approach to this problem, see M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Orsay Report

No. IPNO/TH 78-46 (1978); R. Vinh Mau, in *Proceedings of the Eighth International Conference on High Energy Physics and Nuclear Structure, Vancouver, 1979,* edited by D. F. Measday and A. W. Thomas (North-Holland, Amsterdam, 1980).

[7]In applications where the form factor is needed in the spacelike region, or well below the first pole in the timelike region, the zero-width approximation works well. For details on the unitarization prescriptions see Ref. 5.

[8]D. Sivers, S. J. Brodsky, and R. Blankenbecler, Phys. Rep. 23C, 1 (1976).

[9]The form factor $F_{\pi\pi\rho}(q^2,\mu_\pi^2,M_\rho^2)$ should not be confused with $F_{\pi\pi\rho}(\mu_\pi^2\ \mu_\pi^2,q^2)$. The former is a function of the pion four momentum squared and exhibits pseudoscalar meson poles while the latter depends on the rho-meson four momentum squared and thus has vector meson poles. According to factorization the former factor should be identical to $F_{\pi NN}(q^2,M^{2,}M^2)$ and, therefore, should have a different analytic structure than $F_{\pi\pi\rho}(\mu_\pi^2,\mu_\pi^2,q^2)$. That this is so, in the context of the dual model discussed here, may be seen by looking at the electromagnetic form factor of the pion. The lastest data up to $q^2=-10$ (GeV/c)^2, C. J. Bebek *et al.* Phys. Rev. D 17, 1963 (1978), requires that $F_{\pi\pi\rho}\times(\mu_\pi^2,\mu_\pi^2,q^2)$ have a very mild dependence on q^2.

[10]M. Dilling and M. Brack, J. Phys. G 5, 223 (1979).

[11]P. D. B. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press, Cambridge, 1977). For a recent review on the status of Reggeized pion exchange, see L. M. Jones, University of Illinois Report No. ILL-(TH)-79-7, 1979.

[12]See, e.g., B. Haber *et al.,* Phys. Rev. 168, 1773 (1968).

[13]M. M. Nagels *et al.,* Nucl. Phys. B147, 189 (1979).

[14]At extremely high energies, ρ and A_2 exchanges seem to take over pion exchange. See K. J. M. Moriarty and H. Navelet, Phys. Lett. 71B, 208 (1977).

[15]Data on $pp\rightarrow n\Delta^{++}$ taken from T. C. Bacon *et al.,* Phys. Rev. 162, 1320 (1967); Z. M. Ma *et al.,* Phys Rev. Lett. 23, 342 (1969); J. Ginestet *et al.,* Nucl. Phys. B13, 283 (1969); H. C. Dehne *et al.,* Nuovo Cimento 53A, 232 (1968). Data on $pp\rightarrow p\Delta^+$ taken from I. M. Blair *et. al.,* Nuovo Cimento 63A, 529 (1969); R. M. Edelstein *et al.,* Phys. Rev. D 5, 1073 (1972). Data on $\pi^+p\rightarrow\rho^0\Delta^{++}$ taken from I. J. Bloodworth *et al.,* Nucl. Phys. B35, 79 (1971); S. U. Chung *et al.,* Phys. Rev. D 12, 693 (1975); R. Honecker *et al., Nucl. Phys.* B106, 365 (1976).

[16]For other calculations see Ref. 10 and J. W. Durso, A. D. Jackson, and B. J. VerWest, Nucl. Phys. A282, 40 (1977); D. J. Ernst and M. B. Johnson, Phys. Rev. C 17, 247 (1978).