Reactive content of the Klein-Gordon optical potential

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The reactive content of the pion-nucleus optical potential is explicated in the Klein-Gordon and relativistic Schrödinger theories. It is proven that even though the solution of the Klein-Gordon equation introduces intermediate multipion states, there is no inelastic contribution of these states to the scattering amplitudes. This is shown to come about from an intricate cancellation between diagrams which contain different numbers of pions at specific intermediate times. Approximation schemes which expand the optical potential of the relativistic Schrödinger equation in terms of a fixed number of pions present at a given time are shown not to maintain this cancellation whenever truncated at any finite order of perturbation theory.

[NUCLEAR REACTIONS Reactive content of relativistic Schrödinger and Klein-Gordon equations are compared. Expansion in terms of fixed numbers of intermediate meson states is shown to lead to pathological results.]

Pion-nucleus scattering can be described theoretically in terms of the solution of the Klein-Gordon equation[1] or of the relativistic Schrödinger equation.[2] Theoretical justification for the use of the relativistic Schrödinger equation is afforded by relativistic potential theory.[3] This approach is convenient because traditional multiple scattering[4] potential theory can be carried over to pion physics. In such theories pion creation and annihilation is not a requirement for Lorentz invariance, and intermediate states correspond to single particle pionic states. This approach ignores the fundamental field theoretic understanding of the pion and its interactions. The argument in favor of the Klein-Gordon equation is that it builds into the equation of motion the propagation of the pion field both forward and backward in time, in accordance with the requirements of modern quantum field theory.[5] As such, the Klein-Gordon theory is unavoidably a theory having multiple numbers of pions present during time intervals of sufficiently short duration. Numerical studies have shown that these differences in principle may lead to large differences in practice as well.[6]

There exists a point of view midway between these two philosophies, which would argue that the Klein-Gordon and relativistic Schrödinger theories can be made identical provided one is willing to calculate additional terms. A method of connecting the two approaches is discussed by Cammarata and Banerjee,[7] but another systematic method exists, namely to expand in terms of a maximum number of pions allowed in any given time interval, the fixed pion number expansion[8] (FPNE). The FPNE has been used both in the meson-nucleus scattering problem[9a] and also in the meson-few body problem.[9b] One of the consequences of our work is that the truncation of the FPNE at any finite order of perturbation theory is incorrect *in principle* as a method for approximating the solution of the n-body problem.

Since each of these approaches treats the multipion intermediate states differently, it is of interest to study the reactive content of the relativistic equations for pion-nucleus scattering. A question basic to this study is whether the pions produced virtually in intermediate states implied by propagation of pions backward in time can ever become real, for example, when the initial pion energy becomes sufficiently large. The answer turns out to be different depending upon whether the question is addressed within the framework of the Klein-Gordon equation or the FPNE approach. The contradiction is resolved by a detailed study of higher order terms in perturbation theory which shows that the spurious production of pions at one order of the FPNE theory is canceled by terms of higher order containing a larger number of pions. Thus, the only way to completely assure the absence of spurious pion production in the FPNE theory is to retain terms containing arbitrary numbers of pions in intermediate states.

Exact results explicating the reactive content of the Klein-Gordon and relativistic Schrödinger theory in an optical potential framework can be found using unitarity arguments. The use of unitarity arguments to explicate the reactive content of the optical potential has received much attention lately.[10–13] For the purposes of the present work we borrow a critical formula from Ref. 10, in which the necessary conditions for the validity of the impulse approximation for the optical potential were first discussed in relation to inelastic scattering.

We begin by reviewing the transformation which enables the solution of the Klein-Gordon equation to be obtained as the solution of an equation of the relativistic Schrödinger form. We define the optical potential for the Schrödinger equation by U_{RS},

$$T(\omega)=U_{RS}(\omega)+U_{RS}(\omega)\frac{1}{\omega+i\eta-h_0}T(\omega)\ , \qquad (1)$$

where we use an abstract operator notation and h_0 is the kinetic energy operator for the pion. The operator $U_{RS}(\omega)$ is the relativistic Schrödinger equation optical potential and may be identified diagrammatically in the scheme of Ref. 7 as the sum of all diagrams which cannot be broken into two pieces by cutting a *forward* going pion line. The corresponding operator is identified diagrammatically in the FPNE by a different criterion, which is discussed below. One can further define an additional proper self energy $\Sigma(\omega)$ which is the sum of all diagrams which cannot be broken into two pieces by cutting *either* a forward going or backward going pion line. The relation between $U_{RS}(\omega)$ and Σ is pictured in Fig. 1 and is given

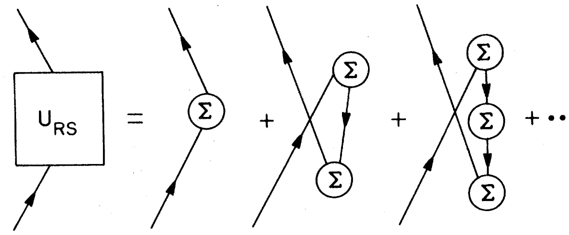

FIG. 1. The relation between the self-energy $U_{RS}(\omega)$ (the optical potential appropriate for the relativistic Schrödinger equation) and the self energy $\Sigma(\omega)$ (the optical potential that occurs in a Klein-Gordon equation) as given in Ref. 7. The graphs distinguish between the forward and backward going pions but do not correspond to strict time ordering of blobs.

by[7]

$$U_{RS}(\omega)=\Sigma(\omega)-\Sigma(\omega)\frac{1}{\omega+h_0}U_{RS}(\omega)\ . \qquad (2)$$

The propagators in the Feynman diagrams corresponding to nucleons have not been shown explicitly; they are included as part of the "blobs." Inserting Eq. (2) into Eq. (1) gives

$$T(\omega)=\Sigma(\omega)+\Sigma(\omega)\frac{2h_0}{\omega^2+i\eta-h_0{}^2}T(\omega)\ , \qquad (3)$$

which in terms of M and U_{KG} defined by

$$M(\omega)\equiv(2h_0)^{1/2}T(\omega)(2h_0)^{1/2}\ ,$$

$$U_{KG}(\omega)\equiv(2h_0)^{1/2}\Sigma(\omega)(2h_0)^{1/2}\ , \qquad (4)$$

becomes

$$M(\omega)=U_{KG}(\omega)+U_{KG}(\omega)\frac{1}{\omega^2+i\eta-h_0}M(\omega)\ . \qquad (5)$$

This is the Klein-Gordon equation, and U_{KG} is the Klein-Gordon optical potential. These results permit us to use unitarity arguments derived in potential theory[10,11] to obtain the reactive content of the Klein-Gordon equation and to study approximations used in the FPNE of the relativistic Schrödinger equation.

The unitarity relation for the Schrödinger equation, Eq. (1), is[10]

$$T(\omega)-T^\dagger(\omega)=T^\dagger(\omega)[G^{(+)}(\omega)-G^{(-)}(\omega)]T(\omega)+\Omega^\dagger(\omega)[U_{RS}(\omega)-U_{RS}^\dagger(\omega)]\Omega(\omega)\ , \qquad (6)$$

with

$$G^{(\pm)}(\omega)\equiv(\omega\pm i\eta-h_0)^{-1}\ ,$$
$$\Omega(\omega)\equiv1+G^{(+)}(\omega)T(\omega)\ . \qquad (7)$$

The first term on the right hand side of Eq. (6) is proportional to the total elastic cross section while the second term is proportional to the total inelastic cross section. The identification of this second term as the total inelastic cross section enables one to extract[10–13] from a model of U the implicit model of inelastic scattering which it contains.

The same algebra that derives Eq. (6) from Eq. (1) can be used to derive a relationship between $\mathrm{Im}\,U_{\mathrm{RS}}(\omega)$ and $\mathrm{Im}\Sigma(\omega)$ from Eq. (2). The result is

$$U_{\mathrm{RS}}(\omega)-U_{\mathrm{RS}}^{\dagger}(\omega)=\tilde{\Omega}^{\dagger}(\omega)\left[\Sigma(\omega)-\Sigma^{\dagger}(\omega)\right]\tilde{\Omega}(\omega)\,, \tag{8}$$

with

$$\tilde{\Omega}(\omega)\equiv 1-\frac{1}{\omega+h_0}U_{\mathrm{RS}}(\omega)\,. \tag{9}$$

The term proportional to the difference in Green's functions in Eq. (6) vanishes for this case because $(\omega+h_0)^{-1}$ is real for physical $(\omega>m_\pi)$ values of the energy. The important aspect of Eq. (8) is that $\mathrm{Im}\,U_{\mathrm{RS}}(\omega)$ is proportional to $\mathrm{Im}\Sigma(\omega)$: as long as one is using Eq. (2) the physical nature of the reactive content of $\Sigma(\omega)$ is the same as the reactive content of $U_{\mathrm{RS}}(\omega)$. Substituting Eq. (8) into Eq. (6) we find the result

$$T(\omega)-T^{\dagger}(\omega)=T^{\dagger}(\omega)[G^{(+)}(\omega)-G^{(-)}(\omega)]T(\omega)\;+\Omega^{\dagger}(\omega)\tilde{\Omega}^{\dagger}(\omega)\left[\Sigma(\omega)-\Sigma^{\dagger}(\omega)\right]\tilde{\Omega}(\omega)\Omega(\omega)\,. \tag{10}$$

This can be further simplified if we notice

$$\tilde{\Omega}(\omega)\Omega(\omega)=\Omega_{\mathrm{KG}}(\omega)=1+\frac{2h_0}{\omega^2+i\eta-h_0{}^2}T(\omega)\,, \tag{11}$$

which gives the desired result

$$T(\omega)-T^{\dagger}(\omega)=T^{\dagger}(\omega)[G^{(+)}(\omega)-G^{(-)}(\omega)]T(\omega)\;+\Omega_{\mathrm{KG}}^{\dagger}(\omega)\left[\Sigma(\omega)-\Sigma^{\dagger}(\omega)\right]\Omega_{\mathrm{KG}}(\omega)\,. \tag{12}$$

This equation can, of course, be derived directly from Eq. (3) or Eq. (5). We have found it instructive to present an alternate derivation which explicitly demostrated the proportionality between $\mathrm{Im}\,U_{\mathrm{RS}}(\omega)$ and $\mathrm{Im}\Sigma(\omega)$ given in Eq. (8).

The technique for extracting the model for inelastic scattering which is implicitly contained in a model of the optical potential is best illustrated by a specific example. The simplest model is the impulse approximation for $\Sigma(\omega)$ which requires simply the folding of the free pion-nucleon amplitude with a target density. The corresponding approximation for the Schrödinger equation would require one to generate $U_{\mathrm{RS}}(\omega)$ from $\Sigma(\omega)$ via Eq. (2). However, the usual impulse approximation for $U_{\mathrm{RS}}(\omega)$ neglects the backward going pions in Fig. 1 or Eq. (2) and approximates $U_{\mathrm{RS}}(\omega)$ simply by $\Sigma(\omega)$. The arguments of Refs. 10 and 11 state that the model of inelastic scattering implicit in this approximation to $U_{\mathrm{RS}}(\omega)$ is a distorted wave impulse approximation model for quasielastic scattering in which only[13] the incident pion is distorted. From Eq. (12) it follows immediately that the use of the impulse approximation for $\Sigma(\omega)$ in the Klein-Gordon equation contains an implicit model for inelastic scattering which also is a distorted wave impluse approximation model for quasielastic scattering. The distorted waves are now, however, distorted by the Klein-Gordon distortion operator, Eq. (11), rather than the Schrödinger distortion operator. Thus, even though the Klein-Gordon

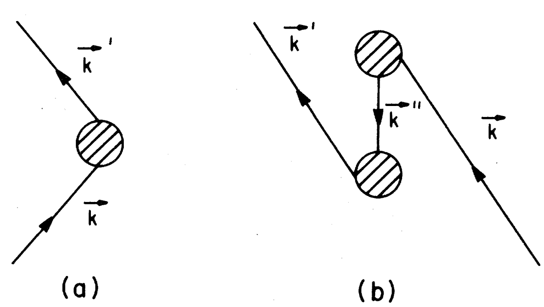

FIG. 2. Some low-order terms in the optical. For $U_{\mathrm{RS}}^{\mathrm{FPNE}}(\omega)$ the nucleon propagators are not shown explicitly. The term pictured in (b) would occur as an iterate of the term in (a) if one were using the Klein-Gordon equation, while it would be part of optical potential $U_{\mathrm{RS}}^{\mathrm{FRNE}}(\omega)$ in the relativistic Schrödinger equation approach.

equation introduces intermediate multipion states, it does *not* introduce any implicit contributions from pion production.

This specific application demonstrates a general conclusion which follows from Eq. (12): even though in iterating the Klein-Gordon equation multipion intermediate states are present, the unitarity relations do *not* introduce any contributions from pion production other than that which is explicitly introduced into $\Sigma(\omega)$ itself.

Consider now the reactive content of the FPNE theory of the optical potential. The self-energy Σ is defined as before. However, in contrast to the theory of Ref. 7 the optical potential for the relativistic Schrödinger equation is defined by the condition that no *time interval* contain only forward propagating pions. We call this operator $U_{\rm RS}^{\rm FPNE}(\omega)$. The leading terms in the optical potential are shown in Fig. 2. The value of these diagrams is

$$\langle \vec{k}' \,|\, U_{\rm RS}^{\rm FPNE}(\omega) \,|\, \vec{k} \rangle = \langle \vec{k}' \,|\, \Sigma \,|\, \vec{k} \rangle + \int \frac{d\vec{k}''}{(2\pi)^3} \frac{\langle \vec{k}' \,|\, \Sigma \,|\, \vec{k}'' \rangle \langle \vec{k}'' \,|\, \Sigma \,|\, \vec{k} \rangle}{\omega - \omega_k' - \omega_k'' - \omega_k + i\eta} , \tag{13}$$

where

$$\omega_k \equiv (k^2 + m_\pi{}^2)^{1/2} . \tag{14}$$

and where for the purposes of illustration we assume that Σ has no energy dependence. The second term in Eq. (13) contains three pions present in the time interval shown in Fig. 2. It follows from Eq. (13) that

$$\mathrm{Im} \langle \vec{k}' \,|\, U_{\rm RS}^{\rm FPNE}(\omega) \,|\, \vec{k} \rangle = -\pi \int \frac{d\vec{k}''}{(2\pi)^3} \Big\langle \vec{k}' \Big| \sum \Big| \vec{k}'' \Big\rangle \delta(\omega - \omega_k' - \omega_k'' - \omega_k) \Big\langle \vec{k}'' \Big| \sum \Big| \vec{k} \Big\rangle \tag{15}$$

which is *not* zero provided $\omega > 3m_\pi$. Using Eq. (15) in Eq. (6) implies that the FPNE includes actual pion production which correponds to virtual mesons propagating backward in time in the Klein-Gordon equation.

The only possible explanation for this apparent contradiction is that there is an intricate cancellation among many diagrams such that the total result contains no contribution from pion production. We shall demonstrate this cancellation explicitly for the diagrams pictured in Fig. 3. The diagrams are strictly time ordered. Figure 3(a) shows a contribution to the scattering amplitude in the FPNE generated when the relativistic Schrödinger equation is solved. In this figure the pion interacts with the nucleus over the interval t_2-t_1 through contributions to $U_{\rm RS}$ shown in Fig. 2(b), propagates with the relativistic Schrödinger propagator until time t_3 and then interacts with the nucleus over a second time interval t_4-t_3 through the contribution shown in Fig. 2(b). The final interaction is with the term in Fig. 2(a). Because of the requirement that the number of pions remain fixed during any interval of time, $t_3 > t_2$ and $t_3 > t_4$ always. The terms shown in Fig. 3(b) and 3(c) are also legitimate contributions to the scattering amplitude, but because these diagrams have pieces with as many as five pions present during any given time interval they would contain pieces corresponding to separate higher order corrections to the optical potential according to the assumption of the FPNE. What we want to show now is that the imaginary piece of the optical potential which occurs over the interval t_4-t_3 in Fig. 3(a) is identically canceled by similar terms coming from contributions of higher order in the FPNE in Figs. 3(b) and 3(c).

In order to deal with the solution of the time-independent equation of motion, we will consider the Fourier-transform of the time-dependent terms in Fig. 3, and write out the contributions of the energy denominators. In these equations $\omega_1 = \omega_6 = \omega_0$. Because the figures correspond to scattering amplitudes, there is no contribution from the incident and final legs. Figure 3(a) gives

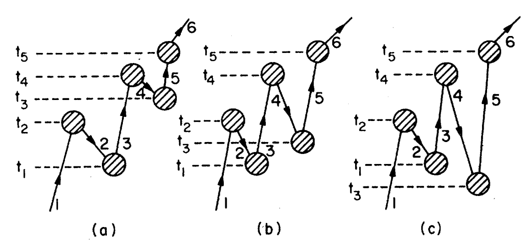

FIG. 3. Some higher order diagrams which contain three and five pion intermediate states. The diagrams are strictly time ordered as required in the FPNE approach.

$$\left[\frac{1}{-\omega_2-\omega_3}\right]\frac{1}{\omega_0-\omega_3+i\eta}\left[\frac{1}{\omega_0-\omega_3-\omega_4-\omega_5+i\eta}\right]\frac{1}{\omega_0-\omega_5+i\eta}\,. \tag{16}$$

Likewise Fig. 3(b) gives

$$\left[\frac{1}{-\omega_2-\omega_3}\right]\frac{1}{-\omega_2-\omega_3-\omega_4-\omega_5}\left[\frac{1}{\omega_0-\omega_3-\omega_4-\omega_5+i\eta}\right]\frac{1}{\omega_0-\omega_5+i\eta}\,, \tag{17}$$

and Fig. 3(c)

$$\left[\frac{1}{-\omega_4-\omega_5}\right]\frac{1}{-\omega_2-\omega_3-\omega_4-\omega_5}\left[\frac{1}{\omega_0-\omega_3-\omega_4-\omega_5+i\eta}\right]\frac{1}{\omega_0-\omega_5+i\eta}\,. \tag{18}$$

The bracketed terms in Eqs. (16)–(18) correspond to separate contributions to the optical potential. Note that the last two energy denominators in Eqs. (16)–(18) are common, so we consider

$$\frac{-1}{\omega_2+\omega_3}\frac{1}{\omega_0-\omega_3+i\eta}+\frac{1}{\omega_2+\omega_3}\frac{1}{\omega_2+\omega_3+\omega_4+\omega_5}+\frac{1}{\omega_4+\omega_5}\frac{1}{\omega_2+\omega_3+\omega_4+\omega_5}$$
$$=\frac{1}{\omega_2+\omega_3}\frac{1}{\omega_4+\omega_5}\frac{\omega_0-\omega_3-\omega_4-\omega_5}{\omega_0-\omega_3}\,. \tag{19}$$

Thus,

$$\text{Eq. (16)}+\text{Eq. (17)}+\text{Eq. (18)}=\frac{1}{\omega_2+\omega_3}\frac{1}{\omega_4+\omega_5}\frac{1}{\omega_0-\omega_3+i\eta}\frac{1}{\omega_0-\omega_5+i\eta}\,, \tag{20}$$

and one sees that there is no singular multipion propagator in the sum of all the diagrams. An alternative and simpler proof of this cancellation of the singularity would follow directly from considering the time dependence of these diagrams. Relations among energy denominators similar to those we have used here were exploited in the early work on nuclear matter theory.[14] That this cancellation of all singularities corresponding to multipion intermediate states must occur was proven in general by our earlier general arguments leading to Eq. (12).

In summary, we have found that the reactive content of an optical potential in a Klein-Gordon equation is completely analagous to the reactive content of the optical potential used in a Schrödinger equation. The only change is the replacement in the inelastic model of a Schrödinger distorted wave for the incident pion by a Klein-Gordon distorted wave. This result follows even though the use of the Klein-Gordon equation includes intermediate states with many pions present. The result comes about through an intricate cancellation among many diagrams.

Finally, we remark that one might want to utilize the relativistic Schrödinger theory without the annoying, spurious particle production by correcting the theory with Eq. (2). This would not amount to an expansion in terms of the number of particles present at a given time and could, therefore, be arranged to circumvent the difficulties we have pointed out in attempts to construct a fixed pion number expansion. However, one would have introduced the superfluous and numerically difficult step of first using Eq. (2) to generate $U_{\mathrm{RS}}(\omega)$ from a model for $\Sigma(\omega)$ and then generating the scattering from $U_{\mathrm{RS}}(\omega)$.

We believe that the best solution to these difficulties is to work with the Klein-Gordon equation from the outset, especially since this equation is easily solved in momentum space as a modification of PIPIT[15] or in coordinate space with, for example, PIRK.[16] There is no worry about the cancellation of spurious pion production terms as we have shown that the Klein-Gordon equation handles this matter automatically.

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