Shell effects on the determination of neutron densities from hadron scattering

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(Received 1 June 1981)

The influence of spin-unsaturated subshells on the coupling between hadronic probes and the target nucleons is studied for the case of elastic scattering. Particular attention is focused on the case of pion scattering from nuclei where the $\vec{\sigma}\cdot\hat{n}$ term in the pion-nucleon t matrix gives rise to a surface correction term to the π -nucleus optical potential. A closed form expression for this term is given and its effects on a few cases of π-nucleus scattering are estimated. Similar estimates are made for proton-nucleus scattering at 800 MeV bombarding energy.

[NUCLEAR REACTIONS Pion-nucleus and proton-nucleus elastic scattering. Estimates of shell effects on determination of neutron densities.]

One of the major activities in intermediate energy physics over the past several years has been focused on extracting the neutron and proton composition[1] of both ground-state densities and transition densities. Although electron scattering is now providing detailed information about the proton (or more precisely the charged) component of the density, it provides little or no direct information about neutron densities. To learn about neutron densities, complementary studies of hadron scattering must be made. Because of uncertainties in the strong interaction, the interpretation of hadron scattering experiments is not nearly as transparent as for electron scattering. Here we focus attention on ground-state densities; most of the present considerations also have counterparts in inelastic scattering.

For nuclei with $N=Z$ one expects[2] very small differences between r_n and r_p, the rms radii of neutrons and protons, respectively. Somewhat larger differences are expected for $N>Z$ nuclei since the excess neutrons occupy different shell model states than those of the protons. Considering only even-even nuclei for simplicity, ^{18}O, ^{48} Ca, ^{90}Zr, ^{116} Sn, and ^{208}Pb would appear good candidates for measuring r_n or r_n-r_p, and such measurements have been reported.[1] A common characteristic of each of the above nuclei (and many others with $N>Z$ as well) is that a significant fraction of the neutron excess is believed to occupy spin-unsaturated subshells (SUS) in which the $j=l+\frac{1}{2}$ orbit is largely occupied and the $j=l-\frac{1}{2}$ orbit is largely empty. The effects of these SUS on the scattering of electrons have been discussed by Bertozzi *et al.*[3] and are non-negligible. Here we investigate the analogous structure effect on the scattering of pions and protons which arises from the $\vec{\sigma}\cdot\hat{n}$ part of the projectile-nucleon coupling where $\vec{\sigma}$ is the spin of the target nucleon. Just as the $\vec{\sigma}\cdot\hat{n}$ part of the projectile-nucleon coupling gives rise to a correction to the spin-independent part of the projectile-nucleus coupling, the terms in the t matrix bilinear in the spin operators[5] acting on the target nucleons and projectile can contribute to the projectile-nucleus spin-orbit potential, and this has been discussed previously.[6] Here we focus on corrections to the optical model potential arising from the $\vec{\sigma}\cdot\hat{n}$ term in the t matrix, since it is present even for projectiles with spin = 0.

For definiteness, we consider the elastic scattering of pions. Although it has been stated[4] that the $\vec{\sigma}_N\cdot\hat{n}$ term is small for pion scattering from heavy nuclei at low energies, we explore here its effects on the delicate issue of extracting neutron rms radii. The pion-nucleon (π-N) t matrix in the π-N c.m. system is given by

$$t_{\pi N}=t_0+it_1\vec{\sigma}_N\cdot\hat{n},\ \ \hat{n}=\frac{\vec{k}\times\vec{k}'}{|\vec{k}\times\vec{k}'|}\ , \quad (1)$$

where $\vec{k}(\vec{k}')$ is the initial (final) momentum of the pion in the π-N cm. When only p waves are important in t_1 it is convenient to write

$$t_{\pi N}=t_0+i\bar{t}_1\vec{\sigma}_N\cdot\frac{(\vec{q}\times\vec{k})}{k^2}\ , \quad (2)$$

where

$$\bar{t}_1\equiv\frac{t_1k^2}{|\vec{k}\times\vec{k}'|},\ \ \vec{q}\equiv\vec{k}-\vec{k}'\ , \quad (3)$$

and $\bar{t}_1$ is independent of angle. The vector $\vec{k}$ may to a good approximation be expressed as[7]

$$\vec{k}=\alpha_\pi\vec{k}_\pi-\alpha_N\vec{k}_N\ , \quad (4)$$

where

$$\alpha_\pi=\frac{E_N}{M}\simeq\frac{E_N}{E_N+E_\pi},\ \ \alpha_N=\frac{M-E_N}{M}\simeq\frac{E_\pi}{E_N+E_\pi}\ , \quad (5)$$

and $\vec{k}_\pi(\vec{k}_N)$ is the momentum of the pion (target nucleon) in the π-nucleus c.m. system, $E_N(E_\pi)$ is the total energy of the nucleon (pion) in the lab frame, and $M^2=E^2-\vec{P}^2$, the square of the four-momentum. Terms of order $1/A$ have been dropped. The t_0 term gives rise to the usual π-nucleus optical potential; the $\bar{t}_1$ term may, at each bombarding energy, be regarded as the momentum-space matrix element of the operator

$$t_1(\vec{r})=\frac{\bar{t}_1}{k^2}\int\frac{d\vec{q}}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}}(\vec{q}\times\nabla_r)\cdot\vec{\sigma}_N\ ,$$
$$\vec{r}=\vec{r}_\pi-\vec{r}_N\ . \quad (6)$$

In a folding-model context this term contributes to the π-nucleus optical-model potential a term

$$\Delta U(r_\pi)=\eta\frac{\bar{t}_1}{k^2}\int\frac{d\vec{q}}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}_\pi}\times\left\langle\psi_0\left|\sum_N e^{i\vec{q}\cdot\vec{r}_N}(\vec{q}\times\nabla_r)\cdot\vec{\sigma}_N\right|\psi_0\right\rangle , \quad (7)$$

where ψ_0 is the ground state wave function of the nucleus assumed to have zero total angular momentum, and $\eta=(\epsilon_\pi\epsilon_N/E_\pi E_N)$, where ϵ_i is the total energy of particle i in the π-N system; η converts the t matrix to the lab system. Parity and angular momentum considerations forbid the participationof ∇_π and lead to ($\vec{r}_\pi\to\vec{r}$):

$$\Delta U(r)=\alpha_N\eta\frac{\bar{t}_1}{k^2}\frac{1}{2\pi^2}\int_0^\infty q^4dq\,j_0(qr)\left\langle\psi_0\left|\sum_N\frac{j_1(qr_N)}{qr_N}\vec{L}_N\cdot\vec{\sigma}\right|\psi_0\right\rangle , \quad (8)$$

which displays explicitly the sensitivity to the *correlation* of the spin and orbital motion of the target nucleons; j_n is a spherical Bessel function. [The contribution of t_1 to Eq. (8) is a special case of the term 4a from Eq. (5) of Ref. 8]. Apart from its (suppressed) isospin dependence, the nuclear matrix element in Eq. (8) is proportional to that given by Bertozzi *et al.*[3] for corrections to the charge form factor. It is also proportional to a finite-q version of the model-dependent $M1$ sum rule of Kurath.[9] The integration over q may be done, giving

$$\Delta U(r)=\frac{\alpha_N\eta}{4\pi k^2}\sum_{nl}\bar{t}_1[lN_{l+(1/2)}-(l+1)N_{l-(1/2)}]\times\frac{1}{r^2}\frac{d}{dr}[ru_{nl}{}^2(r)]\ , \quad (9)$$

where

$$\int_o^\infty u_{nl}^2r^2dr=1\ , \quad (10)$$

N_j is the ground state occupation number of the level (nlj), and the sum includes occupied proton and neutron orbitals with the appropriate isospin average of $\bar{t}_1$. Using the relationship between the scattering amplitude f and the t matrix

$$t=-2\pi(\hbar c)^2f\frac{\epsilon_\pi+\epsilon_N}{\epsilon_\pi\epsilon_N},\ \ \bar{f}_1=\frac{f_1k^2}{|\vec{k}\times\vec{k}'|} \quad (11)$$

gives

$$\Delta U(r) = -(\hbar c)^2 \frac{(\epsilon_\pi+\epsilon_N)}{2(E_\pi+E_N)E_N} \sum_{nl} \frac{\bar{f}_1}{k^2}[lN_{l+1/2}-(l+1)N_{l-1/2}]\frac{1}{r^2}\frac{d}{dr}[ru_{nl}^2(r)] , \tag{12}$$

a correction to the optical potential which typically changes sign near the nuclear surface.

The reduced scattering amplitude $\bar{f}_1$ is a matrix in isospin space[10]

$$\bar{f}_1 = c + d\,\vec{t}_\pi\cdot\vec{\tau}_N , \tag{13a}$$

where

$$c = \tfrac{1}{3}[\alpha_{13}-\alpha_{11}+2\alpha_{33}-2\alpha_{31}] ;$$

$$d = \tfrac{1}{3}[\alpha_{11}-\alpha_{13}+\alpha_{33}-\alpha_{31}] , \tag{13b}$$

$$\alpha_{2T,2J} = \frac{e^{i\delta}\sin\delta}{k} , \quad \delta=\delta_{2T,2J} , \tag{13c}$$

and $\vec{t}_\pi(\vec{\tau}_N/2)$ is the isospin operator of the pion (nucleon). The spin-independent part of the π-N amplitude may similarly be written as

$$f_0 = a + b\,\vec{t}_\pi\cdot\vec{\tau}_N . \tag{14}$$

It is this part of $f_{\pi N}$ which is usually assumed[10] responsible for the π-nucleus optical potential. Contributions from the c term in Eq. (13) clearly modify the isoscalar part of the π nucleus optical potential; the d term in Eq. (13) modifies the isovector part of the optical potential and should give rise to differences between π^+ and π^- scattering beyond that implied by the b term in Eq. (14). The (π^+,π^0) reaction[10] should be even more sensitive to ΔU associated with the d term, since in this case only the neutron excess is sampled. To include single charge exchange (SCX) we write the π-nucleus optical potential as

$$U = U_0 + U_1\,\vec{t}_\pi\cdot\vec{T}, \quad U_0 = \frac{U_- + U_+}{2}, \quad U_1 = \frac{U_- - U_+}{N-Z} , \tag{15}$$

where $\vec{T}$ is the isospin operator of the nucleus and $U_-(U_+)$ is the optical potential for π^- (π^+) elastic scattering.

Near the (3,3) resonance where measurements are often made:

$$c = 2d = \frac{2e^{i\delta}\sin\delta}{3k}, \quad \delta=\delta(3,3) . \tag{16}$$

If the SUS are all neutrons as in ^{48}Ca, pion scattering near the (3,3) resonance gives

$$\bar{f}_1 \rightarrow \begin{cases} 3d & \text{for } U_- \\ d & \text{for } U_+ \\ 2d & \text{for } U_0 \\ \dfrac{2d}{N-Z} & \text{for } U_1 \end{cases} \tag{17}$$

in Eq. (12). The *relatively* large isovector $\vec{\sigma}\cdot\hat{n}$ term for π-N scattering is in sharp contrast to that in N-N scattering at intermediate energies.[11]

An estimate of the size of the effect of ΔU on the extraction of mean square radii may be made using the local Laplacian model[10] for U. If the moments of U are defined by

$$J_n = 4\pi \int_0^\infty r^{2+n} dr\, U(r) , \tag{18}$$

then $\langle r^2\rangle = J_2/J_0$. From Eq. (9) it is seen that ΔU contributes nothing to J_0 but changes J_2 by

$$\delta J_2 = \frac{4\pi(\hbar c)^2}{(E_\pi+E_N)} \frac{k_\pi}{k}\gamma\frac{\bar{f}_1}{k^2}, \quad \gamma \equiv \left\langle \sum_N \vec{L}_N\cdot\vec{\sigma}_N \right\rangle , \tag{19}$$

where $\gamma \rightarrow l(N-Z)$ if the *entire* neutron excess is in a single $j=l+\frac{1}{2}$ subshell. Near the (3,3) resonance[10] with $\gamma = l(N-Z)$,

$$J_2^\pm + \delta J_2^\pm = \frac{-8\pi(\hbar c)^2}{E_\pi}\left[\frac{k_\pi}{k}\right] d\left\{N\left[\langle r^2\rangle_n' - \frac{E_\pi}{E_N+E_\pi}\frac{l}{k^2}\frac{(N-Z)}{N}\right](1\mp\tfrac{1}{2}) + Z\langle r^2\rangle_p'(1\pm\tfrac{1}{2})\right\} \tag{20a}$$

and

$$J_0^\pm = \frac{-8\pi(\hbar c)^2}{E_\pi}\left[\frac{k_\pi}{k}\right] d[N(1\mp\tfrac{1}{2}) + Z(1\pm\tfrac{1}{2})] , \tag{20b}$$

where

$$\langle r^2\rangle_{n,p}' = \langle r^2\rangle_{n,p} + \frac{6}{k_\pi^2} , \tag{21}$$

$\langle r^2\rangle_n$ is the mean square radius of the point neutron distribution, and $J^+(J^-)$ is for $\pi^+(\pi^-)$ scattering. For *both* π^+ and π^- scattering, the lowest order effect of ΔU is to change the *effective* mean square radius of the neutrons by

$$\delta\langle r^2\rangle_n = \frac{-E_\pi}{E_N+E_\pi}\frac{l}{k^2}\frac{(N-Z)}{N} \underset{(E_\pi=300\text{ MeV})}{\rightarrow} -0.22l\left[\frac{N-Z}{N}\right]\text{fm}^2 \tag{22}$$

so that in the Laplacian model an extracted value of $\langle r^2\rangle_n$ will be smaller than the true value of $\langle r^2\rangle_n$ by $|\delta\langle r^2\rangle_n|$. Near the (3,3) resonance $\delta r_n \simeq -0.026$ and -0.020 fm for ^{48}Ca and ^{90}Zr, respectively. Equation (22) may be compared with the corresponding change in the mean square radius of the *charge* distribution due to SUS contributions from Ref. 3,

$$\delta\langle r^2\rangle_{\text{ch}} = \frac{(N-Z)}{Z}l\mu_N\left[\frac{\hbar}{m_N c}\right]^2, \tag{23}$$

$$\mu_N = -1.91,$$

which gives $\delta\langle r^2\rangle_{\text{ch}}^{1/2} = -0.014$ fm for ^{48}Ca and $\delta\langle r^2\rangle_{\text{ch}}^{1/2} = -0.01$ fm for ^{90}Zr. The case of ^{208}Pb is of interest since both the $1h_{11/2}$ proton and $1i_{13/2}$ neutron shells are presumably full but their spin-orbit partners are empty. The contributions of SUS from protons and neutrons to $\langle r^2\rangle_{\text{ch}}$ nearly cancel,[3] while the analogous contributions to $\pi^\pm$ scattering from Eq. (22) change the apparent rms radii by $\delta r_n \simeq -0.013$ fm and $\delta r_p \simeq -0.015$ fm near the (3,3) resonance.

Although the estimated changes in the rms radii due to ΔU are small, they are comparable to analogous electromagnetic corrections arising from the same shell effects and to other sources of error quoted[1] in the determination of $\langle r^2\rangle_n$. Changes in $\langle r^2\rangle_n$ are, however, much harder to detect (than changes in $\langle r^2\rangle_p$), but estimates using Eq. (22) suggest a limit to the determination of $\langle r^2\rangle_n$ without explicit consideration of shell effects. (δr_n may be as large as 20% of presently quoted values of $r_n - r_p$).

For single charge exchange scattering the relative importance of SUS is enhanced, since in Eq. (22) $(N-Z)/N \rightarrow 1$ for nuclei like ^{48}Ca and ^{90}Zr, where the rms radii of the neutron excess are decreased by 0.07 and 0.09 fm, respectively. This correction should roughly unaffect SCX measurements at 0°, since δJ_0, the change in the forward scattering ($q=0$) t matrix in Born approximation, vanishes.

The effects of SUS can be more important away from the (3,3) resonance. For example, the large predicted[10] cancellation between s- and p- wave contributions near $T_\pi = 80$ MeV for SCX scattering increases the sensitivity of the overall process to SUS corrections.

Similar corrections should be present for the scattering of other hadrons. For proton scattering the size of these effects may be estimated within the impulse approximation,[5,12] where the Fourier transform $(\tilde{U})$ of the central part of the optical potential is given by

$$\left[\frac{A}{A-1}\right]\tilde{U}(q) = t(q)\rho(q) = ZJ_0^{pp}\left[1-\frac{q^2}{6}\left[\langle r^2\rangle_p + \frac{J_2^{pp}}{J_0^{pp}}\right]\right] + NJ_0^{pn}\left[1-\frac{q^2}{6}\left[\langle r^2\rangle_n + \frac{J_2^{pn}}{J_0^{pn}}\right]\right] + \cdots \tag{24}$$

and J_m^{pp} and J_m^{pn} are the moments of the central parts of the effective pp and pn interactions as defined by Eq. (18). δJ_2 may be evaluated for proton scattering using Eq. (19) with $k_\pi(E_\pi)$ replaced by $k_p(E_p)$. In the notation of Ref. 12,

$$\bar{f}_1 \simeq \left[\frac{\hbar}{2Mc}\right]\frac{i\theta_{pj}}{4\pi}k^2(1-i\alpha_{sj}), \tag{25}$$

where j labels n or p. For the scattering of 800 MeV protons (θ_p and α_s from Ref. 13, solution 2, ^{48}Ca, Table I) Eqs. (24) and (25) lead to

	^{48}Ca	^{208}Pb	
Re	$\delta r_n = 0.04$ (0.05)	$\delta r_n = 0.02$ (0.02),	$\delta r_p = -0.06$ (-0.35)
Im	$\delta r_n = -0.02$ (0.005)	$\delta r_n = -0.01$ (0.003),	$\delta r_p = -0.01$ ($+0.003$),

where the $\delta r = \delta\langle r^2\rangle^{1/2}$ are in fm. A negative value of δr_n, for example, implies that conventional analyses of scattering data which do not explicitly include SUS effects should lead to values of r_n which are too small by $|\delta r_n|$. The relatively large changes in Reδr are due to the very small ReJ_0 at 800 MeV; their significance is unclear. The changes in Imδr are smaller and within the uncertainties of the currently quoted[12,13] differences between proton and neutron rms radii and isotopic differences in neutron radii. They are also comparable to those changes for pion scattering estimated above. The numbers in parentheses correspond to more recent unpublished values of θ_{pj}, α_{sj}, and J_0^{pp} provided by Ray.[12] SUS corrections arising from the two-body electromagnetic interaction[14] were estimated to be small.

The importance of the SUS term for other projectiles (p) may be estimated by using

$$\frac{\bar{f}_1}{k_2} = \frac{-1}{6\pi\hbar^2 c^2}\left[\frac{\epsilon_p \epsilon_N}{\epsilon_p + \epsilon_N}\right] J_2(v_{l\sigma}) , \qquad (26)$$

where $J_2(v_{l\sigma})$ is the moment of r^2 for that part of projectile-nucleon interaction multiplying $\vec{1}\cdot\vec{\sigma}$.

In summary, the effects of SUS on the extraction of $r_n - r_p$ (or isotopic differences in r_n) are found to be relatively small but comparable to several other currently quoted sources of error. The effects of SUS on single charge exchange reactions are relatively more important providing[15] approximately 20% of the (π^+,π^0) cross section near $T_\pi = 50$ MeV.

The authors are indebted to L. Ray for helpful suggestions. This work was supported in part by the NSF.

[1]J. Arvieux, Nucl. Phys. A335, 353 (1980) and references cited therein; G. W. Hoffmann *et al.*, Phys. Rev. C 21, 1488 (1980) and references cited therein; S. G. Iverson *et al.*, Phys. Lett. 82B, 51 (1979); L. Ray, Nucl. Phys. 443 (1980).

[2]J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972).

[3]W. Bertozzi, J. Friar, J. Heisenberg, and J. W. Negele, Phys. Lett. 41B, 408 (1972).

[4]G. E. Brown, B. K. Jennings, and V. I. Rostokin, Phys. Rep. 50C, 227 (1978).

[5]A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).

[6]W. G. Love, Phys. Rev. C 20, 1638 (1979).

[7]R. Hagedorn, *Relativistic Kinematics* (Benjamin, New York, 1964), p. 33.

[8]W. G. Love, Nucl. Phys. A192, 49 (1972).

[9]Dieter Kurath, Phys. Rev. 130, 1525 (1963).

[10]D. S. Koltun, in *Advances in Nuclear Physics,* edited by M. Baranger and E. Vogt (Plenum, New York, 1969), Vol. 3, p. 71; G. A. Miller and J. E. Spencer, Ann. Phys. (N.Y.) 100, 562 (1976).

[11]W. G. Love and M. A. Franey, Phys. Rev. C 24, 1073 (1981).

[12]L. Ray, Phys. Rev. C 20, 1857 (1979); 19, 1855 (1979); and private communication.

[13]L. Ray *et al.*, Phys. Rev. C 23, 828 (1981); G. W. Hoffmann, in Proceedings of the LAMPF Workshop on Nuclear Structure with Intermediate Energy Probes, Los Alamos Scientific Laboratory Report LA-8303C, p. 99.

[14]C. Lechanoine, F. Lehar, F. Perrot, and P. Winternitz, Nuovo Cimento 56A, 201 (1980); G. W. Hoffmann *et al.*, Phys. Rev. C (to be published).

[15]B. A. Sparrow, Phys. Lett. 58B, 309 (1975).