Charge-symmetry breaking in the *n-p* interaction

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A simplified formalism of evaluating the isospin-violating amplitude of the neutron-proton scattering in terms of the bar phase shifts is presented. The singlet-triplet mixing angles are evaluated for the following effects: electromagnetic contributions, $\omega\rho^0$ mixing, and neutron-proton mass difference effect in the charged π and ρ one-meson exchanges. The possible difference in the polarization of neutrons and protons is estimated for four lab energies: 200, 325, 500, and 750 MeV.

[NUCLEAR REACTIONS *n-p* scattering $E=25-750$ MeV; singlet-triplet transitions, charge symmetry breaking.]

I. INTRODUCTION

The possibility of mixing isospin states in the *n-p* interaction was indicated recently in several works, where some estimates of the effects were given.[1–5] It seems that in order to well incorporate the efforts a simplified formalism or approach is needed. The main purpose of this work is the development of such a formalism and the indication of the main sources of isospin mixing.

In Ref. 4 it was proposed to use the following parametrizaton of the transition between the singlet ($L=J$, $S=0$) and uncoupled triplet states ($L=J$, $S=1$) which are states of different isospin

$$\langle J1\,|\,T(J)\,|\,J0\rangle = -\tfrac{1}{2}\sin 2\bar{\gamma}_j \times \exp(i\bar{\delta}_J + i\bar{\delta}_{JJ}), \qquad (1)$$

where $\bar{\delta}_J$ and δ_{JJ} are the bar singlet and uncoupled triplet phase shifts, respectively; $\bar{\gamma}_J$ is the new bar mixing angle parameter of the singlet-triplet transition. The scattering matrix was described with the aid of six amplitudes, and it included the isospin-violating amplitude $f(\theta)$ in the form $f(\theta)[(\vec{\sigma}_n - \vec{\sigma}_p)\cdot\vec{n}]$, where $\vec{\sigma}_n, \vec{\sigma}_p$ are the neutron and proton spin operators, respectively, and $\vec{n}$ is a unit vector normal to the scattering plane. A simple relation was found between this amplitude and the helicity amplitudes

$$f(\theta) = i[\phi_5(\theta) + \phi_6(\theta)]. \qquad (2)$$

It was also found that the amplitude $f(\theta)$ is simply related to the matrix elements of Eq. (1) in the following way:

$$f(\theta) = \frac{i}{2k}\sum_{J=1}^{\infty}(2J+1)\sin 2\bar{\gamma}_J \times \exp[i(\bar{\delta}_J + \bar{\delta}_{JJ})]d^J_{10}(\theta), \qquad (3)$$

where k is the c.m. momentum and d^J_{10} are the Wigner functions.

II. ELECTROMAGNETIC CONTRIBUTIONS

The main contribution to the isospin-violating amplitude comes from electromagnetic origin, namely from the interaction of the magnetic moment of the neutron with the current of the proton. Before giving estimates of this effect, let us make a digression to the treatment of electromagnetic effects in the *p-p* interaction. There, the total phase shift δ is given by

$$\delta = \delta_N + \delta_{NC} + \delta_{EM}, \qquad (4)$$

where δ_N is the nuclear phase shift calculated without electromagnetic effects, δ_{EM} is the Coulomb phase shift, and δ_{NC} is the contribution of the interference of these two processes. Above approximately 20 MeV the following approxiations seem to work quite well:

$$\delta \simeq \delta_N + \delta^B_{EM} + \delta_{NC}, \quad (5)$$

where δ^B_{EM} is the electromagnetic phase shift in the first Born approximation and the interference effect is included. In the following we shall use the approximation of Eq. (5) for including the electromagnetic effects. In Ref. 4 we found a very simple expression for the electromagnetic phase shifts in the first Born approximation

$$\bar{\gamma}_J^{EM} = \eta \nu_n k^2 / \{M^2[J(J+1)]^{1/2}\}, \quad (6)$$

where η is the Coulomb parameter [$\eta = e^2 M/(2k)$], ν_n is the absolute value of the anomalous magnetic moment of the neutron ($\nu_n = 1.913\,148$), k is the c.m. momentum, and M is the nucleon mass.

Expansion (3) is divergent because of the infinite range of the electromagnetic interaction. One can replace it by a convergent expansion by subtracting and adding the resummed electromagnetic part. We will do it in the following way. First we make the replacement $\sin 2\bar{\gamma}_J \simeq 2\bar{\gamma}_J$, as $\bar{\gamma}_J$ is very small. Next we use the approxiation (5) and rewrite Eq. (3) in the following way:

$$f(\theta) = \frac{i}{k} \sum_{j=i}^{\infty} (2J+1) \left[\left[\bar{\gamma}_J^N + \bar{\gamma}_J^{EM} + \bar{\gamma}_J^{NC}\right] \exp\left[i\bar{\delta}_J^N + \bar{\delta}_{JJ}^N\right] - \bar{\gamma}_J^{EM}\right] d^J_{10}(\theta) - \frac{i\eta\nu_n k^2 \sin\theta}{M(1-\cos\theta)}, \quad (7)$$

where the last term on the rhs of Eq. (7) is the resummed electromagnetic contribution. The phases $\bar{\delta}_J$ and $\bar{\delta}_{JJ}$ are almost unaffected by the electromagnetic interaction; therefore, their values can be taken directly from phase-shift analyses.

III. NUCLEAR CONTRIBUTIONS

So far the approximations leading to Eq. (7) seem to be quite accurate; therefore, Eqs. (7) and (6) may serve as a simple starting point for phenomenological and theoretical analysis of the isospin-violating amplitude. Next comes the problem of the evaluation of the $\bar{\gamma}_J^N$ phases. As we shall see later, further simplifications are also possible. At present it is rather impossible to make an accurate evaluation of the $\bar{\gamma}_J^N$. Expressions in the first Born approximation will be given here for some of the possible processes. This is done in order to have estimates on the order of magnitude of the possible effects. The expressions were derived using Eq. (2) and the techniques of Ref. 6. We list the effects with the corresponding results:

(1) Neutron proton mass difference in the charged one-pion-exchange (OPE) diagram

$$\bar{\gamma}_J = (-1)^J \frac{g^2 k}{16\pi} \frac{\Delta M}{M^2} \left[A_J(x_\pi) - A_J(x_\Lambda)\right], \quad (8)$$

where $g^2/4\pi \simeq 14.5$ is the pseudoscalar coupling constant,

$$\Delta M/M^2 = 2(W_n - W_p)(W_n + W_p)/[(E_n + E_p)W_n W_p], \quad (9)$$

where $W_n = E_n + M_n$, $W_p = E_p + M_p$, $E_n = (K^2 + M_n^2)^{1/2}$, $E_p = (k^2 + M_p^2)^{1/2}$, M_n is the mass of the neutron, M_p the mass of the proton, k is the c.m. momentum,

$$x = 1 + \tfrac{1}{2} m_\pi / k^2, \quad (10)$$

m_π is the mass of the π^+ meson,

$$A_J(x) = \left[\frac{J}{J+1}\right]^{1/2} [xQ_J(x) - Q_{J-1}(x)], \quad (11)$$

and $Q_J(x)$ are the Legendre functions of the second kind. In Eq. (8) we have subtracted from the pion contribution a similar contribution corresponding to a mass Λ. This was obtained assuming a form factor squared of the form

$$\frac{\Lambda^2 - m_\pi^2}{\vec{q}^2 + \Lambda^2}.$$

In our calculation we have used $\Lambda = 600$ MeV.

(2) ω,ρ° mixing

$$\bar{\gamma}_J = -\frac{f_\rho g_\omega \langle \omega | H_{em} | \rho^0 \rangle k}{4\pi M(m_\omega{}^2 - m_\rho{}^2)} [A_J(x_\rho) - A_J(x_\omega)], \quad (12)$$

where f_ρ is the magnetic coupling of the ρ meson in the NN vertex, m_ω and m_ρ are the ω and ρ meson masses, respectively, and $\langle \omega | H_{em} | \rho^0 \rangle$ is the matrix element of the electromagnetic interaction which mixes ω and ρ^0. In our calculation we have used: $g_\rho{}^2/4\pi = 0.6$, $f_\rho/g_\rho = 3.7$; $g_\omega{}^2/4\pi = 5.4$, $\langle \omega | H_{em} | \rho^0 \rangle = -6\times 10^{-3}$ GeV^2, $m_\rho = 767$ MeV, $m_\omega = 783$ MeV; x_ρ and x_ω are given according to Eq. (10) with the ρ and ω masses, respectively.

(3) Neutron proton mass difference in the charged one ρ meson exchange diagram

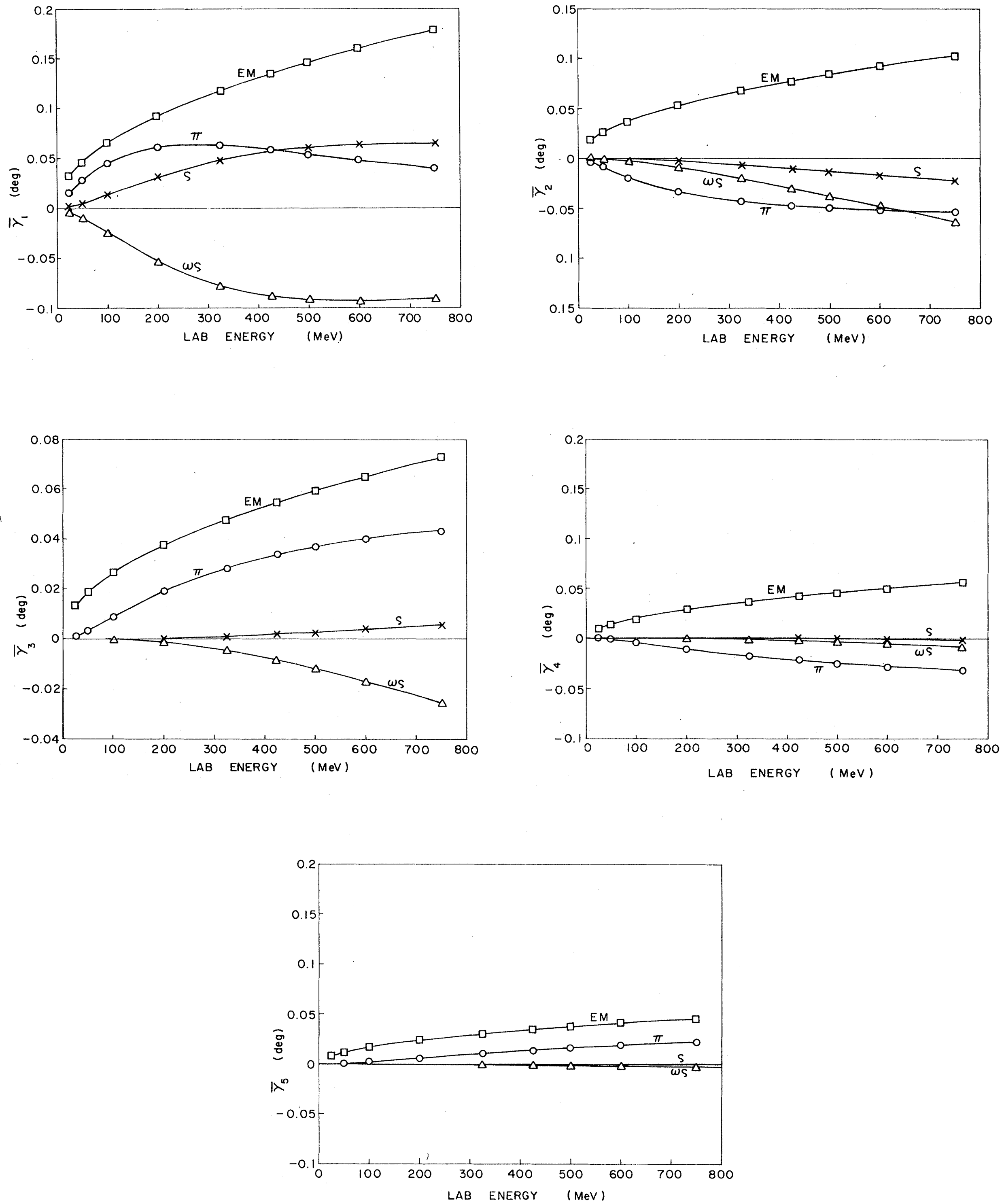

FIG. 1. The mixing angles $\bar{\gamma}_J$ in degrees as functions of the laboratory energy. The indices EM, $\omega\rho$, π, and ρ refer to the electromagnetic contribution, $\omega\rho^0$ mixing contribution, neutron-proton mass difference contribution in the charged pion exchange, and to the neutron proton mass difference contribution in the charged ρ exchange, respectively.

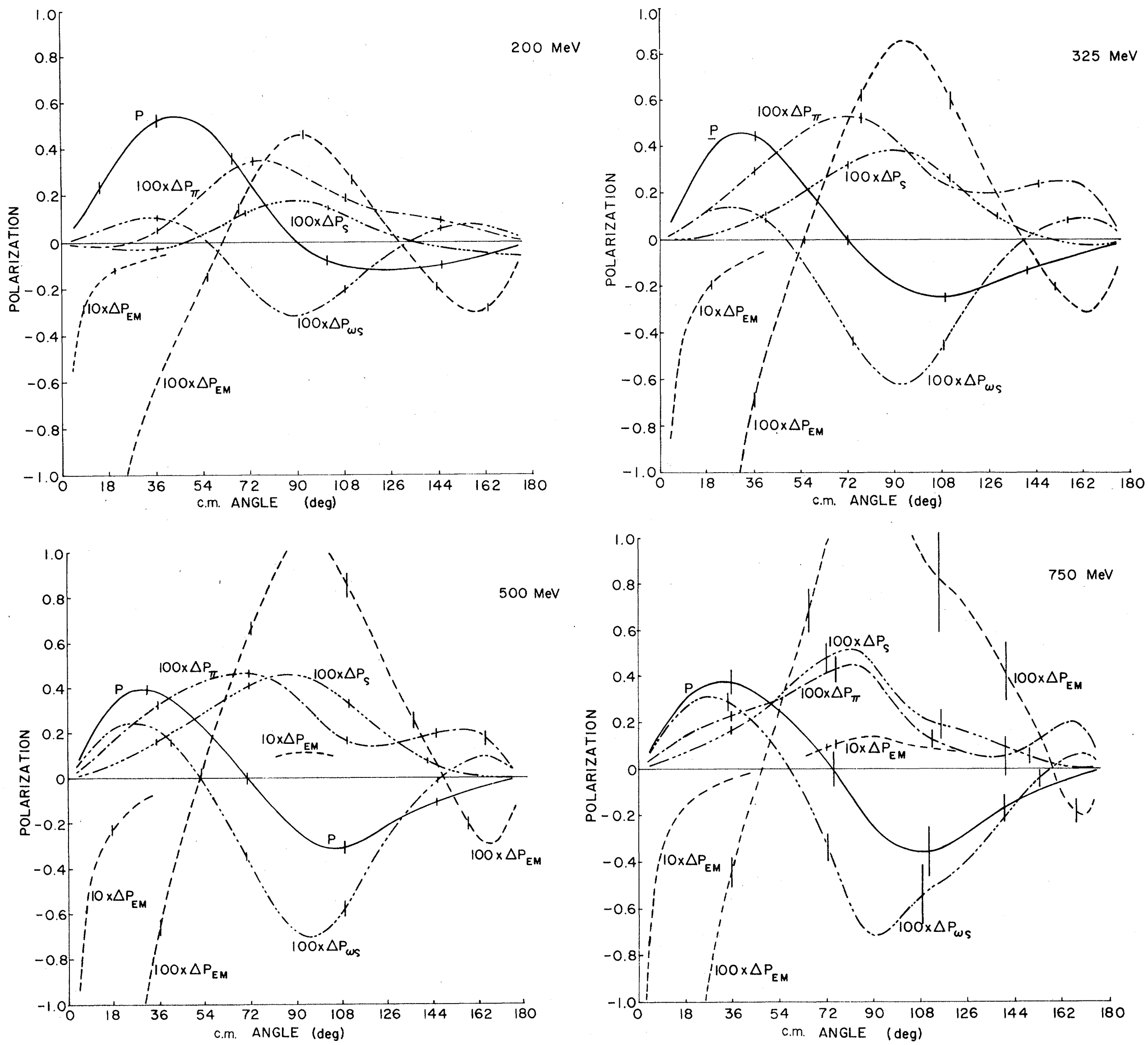

FIG. 2. Polarizations as functions of the c.m. scattering angle. P denotes the neutron and proton polarization without isospin-violating effects. ΔP is the neutron-proton polarization difference. The indices EM, $\omega\rho$, π, and ρ have the same meaning as in Fig. 1. The bars on the curves denote the uncertainties calculated from the errors of the phase-shift analysis of Arndt and VerWest (Ref. 6).

$$\bar{\gamma}_J = (-1)^J \frac{g_\rho{}^2 k}{16\pi} \frac{\Delta M}{M^2} \left[1 + \frac{f_\rho}{g_\rho} \frac{3W_n W_p - k^2}{M(W_n + W_p)} + \left[\frac{f_\rho}{g_\rho} \right]^2 \frac{W_n + W_p}{2M} \right] A_J(x_\rho)$$

$$\simeq (-1)^J \frac{g_\rho{}^2}{16\pi} \frac{\Delta M}{M^2} \left[1 + 3\frac{f_\rho}{g_\rho} + 2\left[\frac{f_\rho}{g_\rho} \right]^2 \right] A_J(x_\rho). \quad (13)$$

In Fig. 1 we display the energy dependence (up to 750 MeV laboratory energy) of the resulting mixing angle $\bar{\gamma}_J$ for the above three processes as well as the electromagnetic $\bar{\gamma}_J^{\mathrm{EM}}$. As we can see, for $J > 3$ the electromagnetic and OPE contributions dominate. In the N-N interaction the mixing angles $\bar{\epsilon}_J$ of the coupled triplet states for $J > 3$ are dominated by the OPE. We can expect also such a dominance for $\bar{\gamma}_J^N$ for $J > 3$. Hence, we can make a further simplification and approximate $\bar{\gamma}_J^N$ by the first Born approximation of the OPE contribution. Thus we are left with three undetermined mixing angles $\bar{\gamma}_1^N$, $\bar{\gamma}_2^N$, and $\bar{\gamma}_3^N$, which should be responsible for the main features of the isospin-violating amplitude $f(\theta)$ of Eq. (7).

In Fig. 2 we display the polarization difference of the neutrons and protons resulting from the charge symmetry nonconservation. In these figures only the electromagnetic contribution is expected to be quantitatively well described, while the other contributions should be treated as indicating the order of magnitude of the effect. For this reason we do not give a total polarization difference.

IV. SUMMARY

In conclusion it seems that a simplified treatment of the isospin-violating amplitude $f(\theta)$ is quite plausible in terms of Eqs. (7) and (6). For $J>3$, Eq. (8) of the OPE contribution can be used as a good first approximation. The process thus should be described with the aid of the three parameters $\bar{\gamma}_1^N$, $\bar{\gamma}_2^N$, $\bar{\gamma}_3^N$. In this work we give an estimate of their order of magnitude and correspondingly the expected magnitude of the difference in the polarization of the neutrons and protons.

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