Influence of the statistical distribution of level parameters on compound nuclear cross sections for strongly overlapping resonances

H. M. Hofmann and T. Mertelmeier
Institut für Theoretische Physik der Universität Erlangen-Nürnberg, Germany

Hans A. Weidenmüller
Max-Planck-Institut für Kernphysik, Heidelberg, Germany

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We examine numerically the influence of the distribution of the poles and residues of the K matrix on the elastic enhancement factor W in the Hauser-Feshbach formula. For a distribution of pole parameters consistent with the results of random matrix theory, our results strongly suggest a value of $W=2.0$, in the limit of strongly overlapping resonances.

[NUCLEAR REACTIONS Influence of statistical distribution of level parameters on compound nucleus reaction cross section for strong absorption.]

I. INTRODUCTION

Recently, Moldauer[1] examined numerically the influence of the distribution of the poles and residues of the K matrix on the values of compound-nucleus cross sections for strongly overlapping resonances. He found that the elastic enhancement factor W depends on both these distributions. This opens the exciting possibility of investigating experimentally, albeit indirectly, these distribution functions in a domain of excitation energies where such tests have so far not been possible. In view of the importance of this possibility, we have extended Moldauer's numerical calculations. While our numerical findings corroborate his results for equally chosen distributions, our wider sample of distributions leads us to somewhat different conclusions.

We write the S matrix in the form[1,2]

$$S=(1+iK)(1-iK)^{-1}\,, \tag{1.1}$$

where the real and symmetric K matrix is given by

$$K_{ab}=\delta_{ab}K_a^0+\sum_\mu \gamma_\mu^a\gamma_\mu^b/(E_\mu-E)\,. \tag{1.2}$$

Channels are labeled a, b, ... and levels μ, ν, ... The γ_μ^a and the E_μ are uncorrelated random variables. We neglect direct reactions by choosing the constant background matrix K diagonal, and by taking γ_μ^a and γ_ν^b uncorrelated for $a\neq b$. Moreover, the γ_μ^a and γ_ν^b are also uncorrelated for $\mu\neq\nu$, and the distribution function for γ_μ^a is independent of the label μ. Average cross sections are generated in the manner described in Refs. 2 and 3: Values of the E_μ and γ_μ^a are drawn from a random-number generator, taking into account the assumed distribution function for either variable. The S matrix is calculated from Eqs. (1.2) and (1.1) and, by repeating this process, the ensemble average of $|S_{ab}|^2$ can be calculated. In our work, we have set $K_a^0=0$. The statistical assumptions just mentioned imply that for $a\neq b$, the ensemble average of S_{ab} vanishes. Denoting the fluctuating part of S by $S^{\rm fl}=S-\langle S\rangle$, we obtain the elastic enhancement factor W of a set of N_0 equivalent channels a, b (that is, channels with equal transmission coefficients) from the formulas

$$W_a=\langle |S_{aa}^{\rm fl}|^2\rangle \Big/ \left[\sum_{b\neq a}\langle |S_{ab}^{\rm fl}|^2\rangle/(N_0-1)\right]\,, \tag{1.3a}$$

$$W=N_0{}^{-1}\sum_a W_a$$

or

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$$W=\left[\sum_a \langle |S_{aa}^{\mathrm{fl}}|^2\rangle/N_0\right]\Big/\left[\sum_{b<c}\langle |S_{ab}^{\mathrm{fl}}|^2\rangle/\binom{N_0}{2}\right]. \tag{1.3b}$$

The results of both expressions for W agree within the statistical error. It seems, however, that the method of Eq. (1.3b) underestimates the error of W. This is concluded by inspecting the distribution of W values.

What is a suitable choice for the distribution functions for the E_μ and for the γ_μ^a? We expect that we should use the results of random-matrix theory of spectra as a guide, testing the sensitivity of W against deviations from these results. Random-matrix models predict the distribution of eigenvalues and eigenvectors of a random Hamiltonian. Using either the R-matrix theory,[4] or the shell-model approach to nuclear reactions,[5] we can identify the poles E_μ of the K matrix (1.2) with the eigenvalues, the reduced partial widths γ_μ^a with matrix elements of the eigenfunctions, of a random Hamiltonian. The predictions of random matrix models fall into two classes, those concerning *mean values* (for instance, the mean level density and its inverse, the mean level spacing, as functions of excitation energy), and those concerning *fluctuations* about the mean (for instance, the local fluctuations of the nearest-neighbor spacings of energy levels about the mean spacing). In the present context we are interested only in the fluctuations. While different random-matrix models (GOE, TBRE, etc.) predict[6–9] different mean values, it now appears that all available models predict the *same* distribution functions for the fluctuations of energy levels about the mean. This situation simplifies our task, and enhances confidence in the applicability of these latter predictions to actual nuclei. These predictions are consistent with the statements made following Eq. (1.2). Moreover, the Gaussian orthogonal ensemble yields for the γ_μ^a a Gaussian distribution centered at zero. There exists numerical evidence[16] that the embedded Gaussian ensemble does not yield such a distribution. The local fluctuations of the eigenvalues E_μ are, unfortunately, not completely known analytically. This poses a major difficulty: How can we study deviations from the predictions of random-matrix theory when these predictions are not fully available? Perhaps the simplest way out of this difficulty would consist of diagonalizing a random Hamiltonian, thereby generating a set of eigenvalues $\{E_\mu\}$, the distribution of which is completely consistent with a random-matrix model, and in studying the deviations of W that occur when this distribution is changed. Such an investigation is presently under way.[10] We have not followed this route. Instead, we have attempted to construct a distribution for the E_μ which is consistent with the essential features predicted by random-matrix theory, and to identify those aspects of the distribution which are essential for the value of W.

In order to accomplish this goal, we investigate separately in Secs. II and III the influence on W of the distribution of the E_μ and of the γ_μ^a, respectively. In Sec. II, we assume a Gaussian distribution centered at zero for the γ_μ^a, consistent with the GOE random-matrix model and investigate the influence of various distribution functions for the E_μ on the value of W. Since a number of cases of this sort were studied already by Moldauer,[1] we focus our attention in this section on identifying the prediction for W made by random-matrix theory. Using a distribution for the E_μ consistent (in its effect upon W) with this prediction, we investigate in Sec. III the consequences of modifying the distribution of the γ_μ^a. Section IV contains our conclusions.

II. THE DISTRIBUTION OF THE E_μ AND ITS INFLUENCE UPON W

We take all penetration factors to be constant, neglect the existence of channel thresholds in the energy interval of interest, and are thus led to a situation where the zero of energy is arbitrary, and where the quantity of statistical interest is the distribution of level spacings $E_\mu - E_\nu$. This distribution is gauged in units of the mean level spacing d which we set equal to unity.

For a pair of nearest neighbors, the spacing distribution is approximated very closely by the Wigner distribution $g_W(x)=(\pi/2)x\exp(-\pi x^2/4)$ with $\langle x\rangle_0=1$ and with variance $\sigma_0=[(4-\pi)/\pi]^{1/2}\cong 0.52$. For kth nearest neighbors with $k\geq 1$, it is known[9] that the mean values obey $\langle x\rangle_k=(k+1)$, and that the variances σ_k increase slowly with k, assuming, for instance, the value 0.80 for $k=5$. For $k\to\infty$, we have[11] $\sigma_k{}^2\to\sigma_0{}^2+2\pi^{-2}\ln(k+1)$. These statements severely constrain the distribution of the E_μ in a perhaps unexpected manner. Indeed, intuitively one might expect that a physically meaningful way of constructing a sequence of E_μ, consistent with random-matrix theory, would consist of drawing a

sequence of nearest-neighbor spacings from the Wigner distribution, and in identifying the differences $E_{\nu+1}-E_\nu$, $\nu=1,2,\ldots,$ with the spacings so chosen.[1] It is straightforward to show, however, that with this procedure one obtains $\langle x\rangle_k=k+1$ and $\sigma_k=\sigma_0(k+1)^{1/2}$. The variances increase with k much more strongly than logarithmically as predicted by random-matrix models. In other words, the spectra obtained from a set of randomly chosen Hamiltonians are much stiffer than those generated from the procedure just mentioned. For large k, the kth nearest-neighbor spacing of a random-matrix differs from that of a picket-fence model by a fluctuation which is only of order $[\ln(k+1)]^{1/2}$, not of order $(k+1)^{1/2}$.

Figure 1 shows values of σ_k generated with various prescriptions, plotted versus k. The procedure described in the preceding paragraph leads to the curve labeled Wi, which stands for Wigner. More generally, given any nearest-neighbor spacing distribution $g(x)$ with $\langle x\rangle_0=1$ and $\sigma_0=\alpha$, one finds that $\langle x\rangle_k=(k+1)$ and $\sigma_k=\alpha(k+1)^{1/2}$, irrespective of the higher moments of $g(x)$. For many simple, analytically given distribution functions $\langle x\rangle$ and σ_0 are determined by the same parameter,

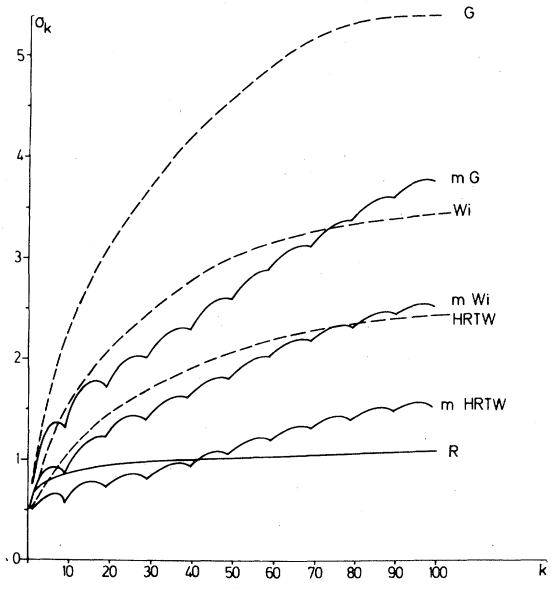

FIG. 1. The variance σ_k of kth nearest-neighbor spacing distributions for various model distribution as described in the text. The reader will observe that even for large k, σ_k does not attain its theoretical value $\sigma_0(k+1)^{1/2}$. This is because by fixing the total length of the interval (Ref. 3), we have imposed the condition that $\sigma_k=0$ for $k=200$.

however, so that σ_0 can be varied, with $\langle x\rangle_0$ fixed only by changing the form of the distribution. The curve labeled G shows σ_k for a Gaussian distribution (confined to $x>0$) with $\langle x\rangle_0=1$. (For the Gaussian, $\sigma_0=[(\pi-2)/2]^{1/2}$.) The curve labeled HRTW shows σ_k for the choice of spacings described in Refs. 2 and 3. Any of these curves lies well above the curve labeled R which displays the analytical result of random-matrix theory.[11]

In order to produce kth nearest-neighbor distributions which are closer to the results of random-matrix theory than the curves G, Wi, and HRTW, we have used the following procedure. For each of the three prescriptions G, Wi, and HRTW, we have taken adjacent groups of 11 eigenvalues, and we have scaled the interval I_{10} between the first and the last eigenvalue by a scaling factor β. This factor was chosen in such a way that βI_{10} was equal to $10+\gamma$, where γ is a random number drawn from a Gaussian distribution with mean value zero and with variance $\delta=0.5$. In this way one obviously generates a distribution for which the σ_k grow asymptotically like $[(k+1)/10]^{1/2}\delta$ instead of like $(k+1)^{1/2}\sigma_0$. The resulting curves are labeled m Wi, m G, m HRTW in Fig. 1, where m stands for modified. We have convinced ourselves numerically that the nearest-neighbor spacing distributions generated by the HRTW and the m HRTW procedures are statistically indistinguishable, although both differ somewhat from a Wigner distribution. This reflects the fact that we do not have a procedure for generating an eigenvalue distribution which would be totally consistent with the results of random-matrix theory.

Using the six eigenvalue distributions Wi, G, HRTW, m Wi, m G, and m HRTW generated in this manner, and a Gaussian distribution for the γ_μ^a, we have calculated a series of elastic enhancement factors W, for a varying number of open channels in the strong absorption limit $T_a\cong 1$. The results are displayed in Figs. 2 and 3. Figure 2 contains in addition the results of a picket-fence model for the E_μ. Comparing these two figures with each other and with Fig. 1, we are led to the conclusion that W is sensitive mainly to the increase of σ_k with k and not to other details of the distribution, and that W approaches 2.0 as the asymptotic value of σ_k decreases. Our results strongly suggest, in fact, that $W=2.0$ for a distribution of eigenvalues fully consistent with a random-matrix model.

Is it possible to understand this trend of the calculations? While we cannot offer an analytical

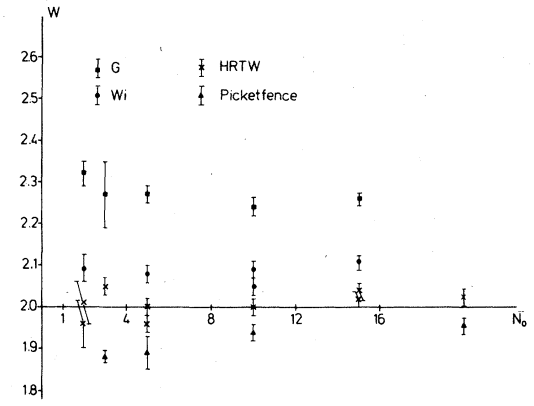

FIG. 2. The elastic enhancement factor W versus N_0, the number of equivalent channels ($T_c=1$ in all channels), for various eigenvalue distributions as explained in the text.

proof, we do have a plausibility argument which is based on the techniques of Ref. 12. Let us label the eigenvalues E_μ of a random Hamiltonian consecutively such that $E_\mu > E_\nu$, for $\mu > \nu$, and let $P_\mu(x)$ be the probability distribution for $x = E_\mu$ (irrespective of the values of the E_ν for $\mu > \nu$), and let $P_{\mu\nu}(x,y)$ be the joint probability distribution for $x = E_\mu$ and $y = E_\nu$ (irrespective of the values of E_τ for $\tau \neq \mu,\nu$). Let us further make the plausible assumption that the variance σ of $P_\mu(E_\mu)$ is independent of μ. The fact that σ_k becomes virtually independent[11] of k for large k suggests that, for sufficiently large k, we have

$$P_{\mu\ \mu+k}(x,y) \cong P_\mu(x) P_{\mu+k}(y) \ . \tag{2.1}$$

Indeed, if this factorization property holds, then σ_k is independent of k. The actual dependence of σ_k upon k leads us to expect that the factorization (2.1) of $P_{\mu\ \mu+k}(x,y)$ holds to a good degree of accuracy already for $k=4$ or so. To support this claim, we recall[9] the k dependence of σ_k for small k: $\sigma_0=0.53$, $\sigma_1=0.65$, $\sigma_2=0.71$, $\sigma_3=0.75$, $\sigma_4=0.78$, and $\sigma_5=0.80$. From then on, the increase is very slow, with $\sigma_{100} \cong 1.10$.

Applying the technique of Ref. 12, one is led to consider products of sums of the type $\Sigma = \sum_\mu |E - E_\mu + (i/2)\tilde{\Gamma}|^{-2}$, where $\tilde{\Gamma}$ is independent of μ. In the notation of Ref. 12, we have $\tilde{\Gamma} = 2\sum_c \bar{\gamma}_c{}^2/(1+x_c)$. As an example, let us focus attention on the evaluation of Σ^2. The procedure of Ref. 12 consists of using a picket-fence model for the E_μ. The correct procedure would be to evaluate the expression

$$\langle \Sigma^2 \rangle = \sum_{\mu,\nu} \int dx \int dy\, P_{\mu\nu}(x,y) |E - x + \frac{i}{2}\tilde{\Gamma}|^{-2} |E - y + \frac{i}{2}\tilde{\Gamma}|^{-2} \ . \tag{2.2}$$

Using the factorization assumption (2.1), one finds immediately that $\langle \Sigma^2 \rangle \simeq \langle \Sigma \rangle^2$ and $\langle \Sigma \rangle \simeq \sum_\mu |E - \langle E_\mu \rangle + (i/2)\tilde{\Gamma}|^{-2}$ if the width $\tilde{\Gamma}$ is large compared to the width σ of the function $P_\mu(x)$, i.e., compared to the average level spacing. We see that two assumptions—applicability of the factorization (2.1) and $\tilde{\Gamma} >> d$—are sufficient to yield the same result as the picket-fence model. Both assumptions are, however, fulfilled to a very good approximation under the condition of applicability of the technique of Ref. 12. Indeed, the condition is that the number $N = (\Gamma/d) \leq \tilde{\Gamma}/d$ of overlapping resonances fulfills $N >> 1$. Here, $\Gamma = d\sum_c T_c/(2\pi)$ is the correlation length of Ericson fluctuations, T_c the transmission factor in channel c, and the inequality $\tilde{\Gamma} \geq \Gamma$ follows from the definition.[12] Let us compare N with n, the number chosen such that for $k > n$, approximation (2.1) is valid. If $N >> n$, then the number of terms typically contributing to sums of the form (2.2) is very large compared to the number of terms for which the approximation (2.1) does not hold, and the error made in using (2.1) to evaluate (2.2) is of the order $n/N << 1$. (In view of the values of σ_k cited above, we believe that $n \lesssim 4$.) For the second

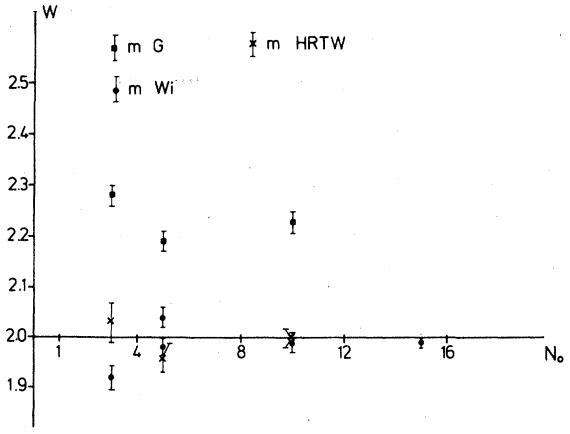

FIG. 3. Same as Fig. 2 for the modified set of distributions.

assumption, the argument rests again on the inequality $N >> 1$. Similar arguments apply to the evaluation of Σ^α, $\alpha > 2$.

In summary, we have advanced a plausibility argument to explain why a picket-fence model yields a good approximation to the results of a random-matrix model in the limit $N >> 1$, in keeping with the numerical results presented in Figs. 2 and 3, and why for decreasing σ_k, k large, we approach the limit $W=2$ which is the result of a picket-fence model for large N. We are aware, of course, that our arguments are heuristic and do not constitute a proof. We also recall that in the cases evaluated numerically, N does not exceed the value $N=N_0/(2\pi)=20/(2\pi)$, which is not large compared to $n=4$. However, the fact that $\tilde{\Gamma} > \Gamma$ is probably of considerable help. In the examples investigated numerically where all transmission coefficients are unity, we have $\tilde{\Gamma}=2\Gamma$ for the evaluation of Σ^2 and find an even bigger factor for products of the form $\sum_\mu [E-E_\mu+(i/2)\Gamma_0]^{-1}$, with $\Gamma_0=4\Gamma$ (see Ref. 12).

We conclude this section with a few further observations. As explained above, the modified eigenvalue distributions were generated by scaling βI_{10} to the value $10+\gamma$, where γ had a variance of $\delta=0.5$. Since $\sigma_{11}\cong 0.88$ from random-matrix theory, the use of a larger value might have appeared more sensible. We have also investigated such cases with the result that W always was somewhat larger than found for $\delta=0.5$. This is in keeping with the conclusions drawn above. We also noted that the modified Gaussian distribution produces values for W which differ from $W=2.0$ more than for any of the other modified distributions. Moreover, even for $\delta=0$, we found in the case of the modified Gaussian distribution the result $W=2.1$. This points to the fact that σ_k is not the only determining factor for W. It appears that a wider distribution of spacings—as present in the Gaussian—also affects W. We finally remark that the picket-fence (PF) model approaches $W=2.0$ from below, for $N=20$, we have $W_{\mathrm{PF}}=1.95$.

III. THE DISTRIBUTION OF THE γ_μ^a AND ITS INFLUENCE UPON W

For isolated resonances, the value of W is given by $W=\mu_4/(\mu_2)^2$, where $\mu_K=\langle(\gamma_\mu^a)^k\rangle$ is the kth moment of the γ_μ^a. For the Gaussian distribution one has $W=3$. For strongly overlapping resonances, the dependence of W on the moments of the distribution is not known analytically. Moldauer[1] found that W depends on the γ_μ^a distribution. In the cases he investigated, there were none in which the eigenvalue distribution came close to that of a random matrix model. Moreover, by changing the 4th moment μ_4 Moldauer also affected all higher moments in an unknown fashion. For these reasons, we have calculated a number of cases where we changed the μ_K in a systematic way, keeping the eigenvalue distribution fixed and reasonably close to that of a random-matrix model. For the reasons given in Sec. II, we took for the eigenvalues a picket-fence model.

The most convenient procedure of testing the dependence of W on the μ_K would consist of changing one μ_K, keeping the others fixed. However, aside from all practical considerations, the following theoretical difficulty arises. The moments μ_K of any probability distribution have to fulfill the inequalities[13]

$$\begin{vmatrix} \mu_0 & \mu_1 & \cdots & \mu_l \\ \mu_1 & \mu_2 & \cdots & \mu_{l+1} \\ \vdots & & & \vdots \\ \mu_l & \mu_{l+1} & \cdots & \mu_{2l} \end{vmatrix} > 0, \quad l=0,1,2,\ldots . \tag{3.1}$$

It is easy to verify that if one starts with a given distribution (for instance, a normal distribution), already the determinants with small l give very strict bounds on the possible change of a single low moment. We therefore abandoned this idea and proceeded instead as follows. Starting from a normal distribution $f(x)$, we modified $f(x)$ for $|x|>E$ by "stretching" it. We define

$$g(y)dy=f(x)dx \tag{3.2}$$

to preserve normalization and choose

$$\begin{aligned} &y=x, \quad \text{for } |x|\le E, \\ &y=x+c(|x|-E)\cdot x, \quad \text{for } |x|>E . \end{aligned} \tag{3.3}$$

Since $dy/dx>0$ for $c>0$ we obtain an authentic probability distribution $g(y)$. For sufficiently large values of E, this procedure leaves μ_2 practically unchanged. This is the case for $E\cong 2$ and $c\cong 2$. By varying E and c, one can generate a set of distribution functions $g(y)$, the moments of which can be calculated. Note that $g(y)$ is discontinuous at $|y|=E$.

In Table I, we give the values of W obtained for various choices of the parameters E and c, and for $N_0=20$, 30, or 40 equivalent channels with $T_c=1$.

TABLE I. Dependence of W on the moments μ_k of the distribution function for the γ_μ^a. The number N_0 denotes the number of equivalent channels with $T_c=1$. The other symbols are explained in the text.

Case no.	N_0	E	c	$\mu_4/3(\mu_2)^2$ theor.	$\mu_4/3(\mu_2)^2$ num.	$\mu_6/15(\mu_2)^3$ theor.	$\mu_6/15(\mu_2)^3$ num.	$\mu_8/105(\mu_2)^4$ theor.	$\mu_8/105(\mu_2)^4$ num.	W
1	20	Gauss		1.00	1.00	1.00	1.00	1.00	0.98	1.95±0.02
2	20	2.12	1.77	1.04	1.03	1.64	1.60	11.56	10.65	1.96±0.02
3	20	1.92	1.77	1.13	1.11	3.05	2.93	35.79	33.32	2.00±0.02
4	20	2.17	2.48	1.05	1.10	2.33	5.47	38.67	207.96	2.02±0.02
5	20	2.09	2.13	1.07	1.09	2.35	3.67	30.95	67.94	2.01±0.02
6	20	1.83	1.77	1.20	1.19	4.25	3.80	56.93	38.57	2.06±0.02
7	30	Gauss		1.00	1.00	1.00	1.00	1.00	1.00	2.00±0.02
8	30	2.20	1.77	1.02	1.02	1.38	1.67	7.13	19.50	2.00±0.02
9	30	1.98	1.77	1.09	1.09	2.49	2.47	26.03	23.46	2.03±0.02
10	30	1.83	1.77	1.20	1.20	4.25	3.80	56.93	35.86	2.12±0.02
11	40	Gauss		1.00	1.00	1.00	1.00	1.00	1.00	1.99±0.02
12	40	2.17	1.77	1.03	1.03	1.45	1.47	8.36	6.89	2.00±0.02
13	40	2.09	2.13	1.07	1.06	2.35	2.27	30.95	26.47	2.00±0.02
14	40	2.09	2.13	1.07	1.07	2.35	2.73	30.95	40.00	2.00±0.02
15	40	2.17	2.48	1.05	1.07	2.33	3.80	38.67	118.38	1.99±0.02
16	40	1.83	1.77	1.20	1.21	4.25	4.27	56.93	45.78	2.13±0.02

The first line of each group belonging to the same N_0 corresponds to the Gaussian distribution. For the ratios $\mu_{2K}/[(2k-1)!!(\mu_2)^k]$ we give two values. The theoretical value (theor) corresponds to $g(y)$ as determined from Eqs. (3.2) and (3.3) for the given values of E and c. The actual value (num) corresponds to the ratio of moments actually sampled in the calculations. The difference in the two columns is caused by the fact that with increasing k, μ_{2k} is increasingly determined by large values of y. These are produced comparatively infrequently in the random-number generator. The difference between the two columns may thus be taken as some indication of the statistical error in determining the moments.

Table I shows that the statistical accuracy with which we calculate W is about 0.02. The largest observed deviations from $W=2.0$ are only 0.13 or about six times the statistical error. This limits the conclusions we can draw.

Comparing cases with similar values of $\mu_4/[3(\mu_2)^2]$ but different higher moments (cases 3, 4, and 5 or cases 13, 14, and 15), we conclude that the influence of all higher moments but the fourth is very small. This is particularly demonstrated by cases 4 and 15. There appears to be a slight tendency for equal values of $\mu_4/[3(\mu_2)^2]$ to have decreasing influence on W with increasing N_0. This is suggested by the comparison of cases 3, 4, and 5 with case 9 and cases 13, 14, and 15. On the other hand, cases 6, 10, and 16 do not display this trend. We have to keep in mind, however, that our statistics for the moments are limited.

The facts unambiguously established are that W depends mainly on μ_4 and very little on the higher moments, and that W increases with increasing μ_4.

IV. CONCLUSIONS

Our numerical results confirm Moldauer's observation[1] that even in the strong-absorption limit ($N_0 >> 1$, all $T_c \cong 1$) the elastic enhancement factor does depend on both the distributions of the poles and residues of the K matrix. Moreover, our results are consistent with those of Ref. 1. For strongly overlapping resonances, a precise measurement of W might yield information on the distribution of the pole parameters which is not accessible in any other way.

Keeping the distributions of the residues fixed and Gaussian, we have found that W decreases with increasing stiffness of the spectrum. We found the largest values of W for a Gaussian distribution of nearest-neighbor spacings (which leads to the largest values of the variances σ_k among all the distributions investigated), and the smallest value for the picket-fence model. By modifying the dis-

tribution in the manner indicated, i.e., by forcing the spectrum to become stiffer, we observed, furthermore, that other features of the distribution are also important. The width of the distribution of nearest neighbors around the mean value is perhaps an important parameter, but we have not established this point.

The influence of the distribution of the γ_μ^a on W can be summarized as follows: As one increases the higher moments μ_K $(k \geq 4)$ by extending the tail of the distribution, W increases. This increase is mainly due to the increase of μ_4, and depends very little on the μ_k with $k \geq 6$. There is some slight evidence that keeping the μ_k fixed and increasing the number N_0 of equivalent channels reduces W.

We have presented some arguments to understand the influence of the pole distribution on W. We are led to expect that a pole distribution given by random-matrix theory and a Gaussian distribution for the residues yields $W=2.0$. Our arguments also offer a late justification for the use of a picket-fence model in Ref. 12.

If the influence of the higher moments μ_k, $k \geq 4$ on W indeed were to decrease with increasing N_0, it appears that a reasonably stiff spectrum of eigenvalues would lead to $W=2.0$. In this case, the behavior of the compound nucleus for strongly overlapping resonances would be determined entirely by $\overline{(\gamma_\mu^a)^2}$, and by the mean level spacing. It would be independent of other dynamical features. This would be in keeping with thermodynamic ideas, and with the maximum entropy approach of Mello.[14] A precise experimental determination of W, with an error significantly smaller than the present one,[15] would obviously be very interesting.

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