Interacting boson approximation model analysis of $E2$ transition probabilities for transitions in $^{166-170}$Er

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The experimental results from Coulomb excitation of vibrational-like 2^+ states in $^{166-170}$Er are compared with the predictions of the interacting boson approximation model. The breaking of the SU(3) symmetry by the pairing term plays a crucial role in giving a much better account of the $B(E2)$ values for decay of the $K,J^\pi=0,2^+$ state. In general, the interacting boson approximation model provides a detailed quantitative description of the level energies and of the $E2$ transition probabilities.

[NUCLEAR STRUCTURE IBA model predictions of the $B(E2)$ values for the different decay modes of the 2^+ states in $^{166-170}$Er.]

The occurrence of a rather low-lying $K=2$ γ-vibrational-like band is a systematic feature of the even-A deformed nuclei. For example,[1] the observed nuclear structure properties of the $2'^+$ member in $^{156-160}$Gd and $^{160-164}$Dy are remarkably similar, viz., the $B(E2,0\rightarrow2')$, the large value of $\delta=[T(E2)/T(M1)]^{1/2}$ for the $2'^+\rightarrow2^+$ transition, and the branching data for decay of the $2'^+$ state. On the other hand, the $K=0$ vibrational-like states are highly variable and usually the $B(E2,0\rightarrow2)$ values are at least an order of magnitude less than the $B(E2,0\rightarrow2')$ values. The interacting boson approximation (IBA) model of Arima and Iachello[2] has provided a reasonable account of collective low-lying states in deformed nuclei.[3] The pairing interaction of the IBA Hamiltonian removes the degeneracy which exists in an energy level spectrum with SU(3) symmetry, viz., the degeneracy of the states with a given J in the γ- and β-vibrational-like bands. This breaking of the SU(3) symmetry by the pairing term played a crucial role in giving a much better account of the $B(E2)$ values for decay of the $K,J^\pi=0,2^+$ state in $^{156-160}$Gd and $^{160-164}$Dy. The displacement of the $K,J^\pi=0,2^+$ state above the $K,J^\pi=2,2^+$ increases with increasing pairing interaction and the $B(E2,0\rightarrow2)$ for excitation of the $K,J^\pi=0,2^+$ state decreases. This behavior seems to be a characteristic feature of the even-A deformed nuclei in the rare-earth region. This is true of the $^{156-160}$Gd and $^{160-164}$Dy nuclei.

In this paper, the $B(E2)$ values[4] for the different decay modes of the 2^+ states in $^{166-170}$Er are compared with theoretical predictions from the IBA model. These calculations were done using the IBA computer code[5] PHINT. The interaction parameters in the IBA Hamiltonian were adjusted to fit typically 14 state energies in each nucleus. In the calculations of the transition probabilities, the parameters $E2SD$ and $E2DD$ in the $E2$ transition operator were adjusted to reproduce the experimental $B(E2,0\rightarrow2)$ values for excitation of the 2^+ members of the ground-state[6] and γ-vibrational-like bands. Values of the parameters used in the IBA calculations are given in Table I. The average deviation between experimental and theoretical energies is 8.9 keV for a fit to 13 state energies in ^{170}Er. The values of the parameters in Table I correspond to a truncation of the total number of bosons to 14. Actually, the total number of bosons are 15, 16, and 17 for $^{166-170}$Er, respectively. To test the influence of the truncation on the predictions, a calculation for ^{170}Er was also done with the total number of bosons truncated to $N=11$. The results are indistinguishable from those for $N=14$.

The results of the calculations are given in Table II. It is evident from Table II that the $B(E2)$ values for the $2\rightarrow2$ and $2\rightarrow4$ transitions from the decay of the $K,J^\pi=2,2^+$ state are reproduced reasonably well by the IBA predictions. The breaking of the SU(3) symmetry by the pairing interaction in the IBA Hamiltonian plays a crucial role in giving a quantitative account of the $B(E2)$

TABLE I. Values of the parameters used in the IBA calculations. These parameters correspond to the variable names in program PHINT for the IBA Hamiltonian expressed in terms of the "multipole expansion."

Nucleus	Pair	*ELL* (keV)	*QQ*	*E2SD* (*e* b)	*E2DD* (*e* b)
^{166}Er[a]	16.51	18.33	−17.92	0.1478	−0.1156
^{168}Er[a]	21.50	19.20	−18.80	0.1468	−0.1300
^{170}Er[a]	1.06	17.08	−21.23	0.1482	−0.1367
^{170}Er[b]	1.37	14.83	−27.28	0.1812	−0.1988

[a]Total number of bosons truncated to 14.
[b]Total number of bosons truncated to 11.

values for decay of the $K,J^\pi=0,2^+$ state to the ground-state band. The experimental $B(E2)$ values for excitation of this 2^+ state are at least an order of magnitude smaller than the $B(E2)$ values for excitation of the $K,J^\pi=2,2^+$ state and thus provide a comprehensive test of the IBA predictions.

The IBA prediction of the $B(E2)$ for excitation of the $K,J^\pi=0,2^+$ state in ^{168}Er is consistent with the upper limit of the $B(E2,0\rightarrow2)$ for excitation of a state at 1261 keV [1276 keV from the (n,γ) reaction]. An extensive test of the IBA predictions for a complete set of states with $J<6$ below 2 MeV excitation in ^{168}Er from the (n,γ) reaction and for the $B(E2)$ branching ratios has already been reported by Warner *et al.*[7] The IBA model provides a detailed quantitative description of the level energies and of the $E2$ transition branching ratios. One feature pointed out by Warner *et al.* in the predictions and in the data is the dominant strength of $E2$ transitions from the β-vibrational-like band to the γ-vibrational-like band compared to those to the ground-state band. This feature appears to be present in the decay of the 2^+ state at 1159 keV in ^{166}Er. However, the $B(E2)$ for the 373-keV transition in ^{166}Er is six times smaller than the IBA prediction. Discrepancies of this magnitude were also observed in the comparison of the experimental and IBA predicted $B(E2)$ values for corresponding decay modes of the $K,J^\pi=0,2^+$ state at 1517 keV in ^{158}Gd.

In general, the IBA model provides a detailed

TABLE II. Experimental and IBA predicted $B(E2)$ values.

Nucleus	Initial state K_i,J_i^π	Initial state E (keV)	Final state K_f,J_f^π	Final state E (keV)	$B(E2,J_i\rightarrow J_f)$ (10^{-50} e^2 cm^4) Exp	IBA
^{166}Er	2,2$^+$	785.9	0,0$^+$	0	2.80 ±0.16	2.80
			0,2$^+$	80.6	5.39 ±0.29	4.27
			0,4$^+$	265.0	0.42 ±0.04	0.27
	0,2$^+$	1159	0,0$^+$	0	0.084 ±0.12	0.084
			0,2$^+$	80.6	0.096 ±0.020	0.132
			0,4$^+$	265.0		0.29
			2,2$^+$	785.9	0.45 ±0.09	2.75
^{168}Er	2,2$^+$	821.1	0,0$^+$	0	2.62 ±0.16	2.62
			0,2$^+$	79.8	4.77 ±0.29	3.97
			0,4$^+$	264.1	0.28 ±0.03	0.24
	0,2$^+$	1261	0,0$^+$	0	<0.13	0.047
^{170}Er	2,2$^+$	934	0,0$^+$	0	2.04 ±0.12	2.04
			0,2$^+$	78.6	3.68 ±0.26	3.25
			0,4$^+$	260	0.22 ±0.03	0.20
	0,2$^+$	960	0,0$^+$	0	0.16 ±0.02	0.27
			0,2$^+$	78.6	0.28 ±0.06	0.47
			0,4$^+$	260	0.95 ±0.15	1.01

quantitative description of the level energies and of the $E2$ transition probabilities for decay of 2^+ states. The displacement of the $K,J^\pi=0,2^+$ state above the $K,J^\pi=2,2^+$ increases and the $B(E2,0\rightarrow2)$ for excitation of the $K,J^\pi=0,2^+$ state decreases with increasing pairing interaction in the IBA Hamiltonian. This characteristic feature occurs in several even-A nuclei in the rare-earth region ($^{156-160}Gd$, $^{160-164}Dy$, and $^{166-170}Er$). A knowledge of $B(E2)$ values provides a more comprehensive test of the IBA model predictions.

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