

Nuclear levels in  $^{176}\text{Lu}$ 

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The nuclear structure of  $^{176}\text{Lu}$  has been studied with the  $(t, \alpha)$  reaction on an isotopically enriched target of  $^{177}\text{Hf}$  using a 17 MeV triton beam and a quadrupole-triple-dipole-type spectrometer. These measurements were combined with published  $(n, \gamma)$ ,  $(n, e^-)$ , and  $(d, p)$  data to construct a level scheme for  $^{176}\text{Lu}$ . The Nilsson model including Coriolis coupling calculations has been used to interpret the level structure. The previously known Nilsson configurations  $7/2^+[404] + 7/2^- [514]$ ,  $7/2^+[404] - 7/2^- [514]$ ,  $7/2^+[404] - 9/2^+[624]$ , and  $9/2^- [514] - 7/2^- [514]$  with bandhead energies 0, 240.56, 198.01, and 342.48 keV, respectively, have been populated in the  $(t, \alpha)$  reaction. Proposed new Nilsson configurations and bandhead energies are:  $5/2^+[402] - 7/2^- [514]$  at 390.21 keV,  $9/2^- [514] + 7/2^- [514]$  at 485.7 keV,  $5/2^+[402] + 7/2^- [514]$  at 563.89 keV,  $1/2^+[411] + 7/2^- [514]$  at 726.51 keV,  $1/2^+[411] - 7/2^- [514]$  at 838.36 keV,  $7/2^- [523] + 7/2^- [514]$  at 1273 keV, and  $3/2^+[411] + 7/2^- [514]$  at 1390.67 keV. The five new values of the singlet-triplet splitting energies and one new  $K=0$  odd-even shift observed in this study are compared with theory.

[NUCLEAR REACTIONS  $^{20}\text{Ne} + ^{238}\text{U}$ ,  $E_{\text{lab}} = 400$  MeV/nucleon fluid dynamics, viscosity, heat conduction, cross sections.]

## I. INTRODUCTION

The low-lying states in deformed odd-odd nuclei involve the coupling of the unpaired particles to a deformed even-even core. In  $^{176}\text{Lu}$  the admixture of vibrational states is expected to be small,<sup>1</sup> so the low-lying energy spectrum should be made up of nearly pure two-quasiparticle states.

Several excited neutron states in  $^{176}\text{Lu}$  were proposed by Struble and Sheline<sup>2</sup> from  $(d, p)$  reaction studies. On the basis of  $(d, p)$  and  $(d, d')$  reaction spectra, high- and low-energy  $(n, \gamma)$  data, and  $\gamma$ - $\gamma$  coincidences, Minor *et al.*<sup>3</sup> have proposed several now well-established bands. This study also employed crystal-diffraction  $(n, \gamma)$  data of Maier.<sup>4</sup> Balodis *et al.*<sup>1</sup> have used these data, improved  $(n, \gamma)$  data collected at Risø, and previously published  $(n, e^-)$  data<sup>5</sup> to construct a much more complete level scheme for  $^{176}\text{Lu}$ . Eight rotational bands were identified including the ground state configuration and four excited neutron bands. Excited proton bands of the  $9/2^- [514] - 7/2^- [514]$  and  $5/2^+[402] - 7/2^- [514]$  configurations were proposed, although the latter could not be unambiguously assigned. Additionally, bandheads for the  $1/2^+[411] + 7/2^- [514]$  and  $9/2^- [514] + 1/2^- [510]$  bands were proposed.

There has been little success from  $\gamma$ -ray data in assigning excited proton configurations. It would seem useful to perform single-particle proton transfer reactions to directly populate these states. Identification of such states from a proton transfer reaction and remaining unassigned  $\gamma$  transitions should lead to the determination of addi-

tional Gallagher-Moszkowski<sup>6</sup> splitting energies and, in particular, the determination of an experimental value for the odd-even shift<sup>7</sup> in the  $7/2^- [523] - 7/2^- [514]$   $K=0$  band.

We have performed the  $^{177}\text{Hf}(t, \alpha)^{176}\text{Lu}$  reaction to observe excited proton hole states in  $^{176}\text{Lu}$ . The experimental methods are discussed in Sec. II. Six new excited proton bands are identified in Sec. III and compared with theory including Coriolis and residual interaction calculations. In Sec. IV the results of the research are summarized, and the magnitude of the residual interaction and the degree of Coriolis attenuation for the odd-odd  $^{176}\text{Lu}$  nucleus are considered. Some experiments are suggested to test the anomalous residual interaction observed.

## II. EXPERIMENTAL PROCEDURE

The  $(t, \alpha)$  experiments were performed with a 17-MeV triton beam from the Los Alamos Scientific Laboratory FN Tandem Van de Graaff accelerator. The  $^{177}\text{Hf}$  targets were prepared by vacuum evaporation of Hf enriched to 91.6%  $^{177}\text{Hf}$  onto a 100- $\mu\text{g}/\text{cm}^2$  carbon backing. A thickness of  $\sim 45$   $\mu\text{g}/\text{cm}^2$  of  $^{177}\text{Hf}$  was obtained. Outgoing alpha particles from the  $(t, \alpha)$  reaction were momentum analyzed by a quadrupole-triple-dipole (Q3D) magnetic spectrograph, which has a solid angle of 14.3 msr. The particles were detected with a 1-m long, helical proportional counter mounted along the focal plane.<sup>8</sup> Data were collected in 1024-channel bites at angles of 15°, 20°, 25°, 30°, 35°, 40°, and 50°. The observed energy resolution was

typically 13 keV full width at half maximum (FWHM). A representative spectrum is shown in Fig. 1.

The excitation energies of peaks in the alpha particle spectra were determined using a calibration based on the energies of known peaks from the  $^{193}\text{Ir}(t,\alpha)^{192}\text{Os}$  spectrum.<sup>9</sup> A linear function of energy versus channel number was determined, and from that an experimental error curve was drawn to correct for small deviations from linearity as a function of channel number. Then several previously known states associated with the ground state configuration of  $^{176}\text{Lu}$  were used to determine the energy calibration at each angle.

The reaction yield data were corrected for dead time losses, and the data recorded at different scattering angles were normalized to the number of triton elastic-scattering events counted in a solid-state detector placed at  $30^\circ$  with respect to the incident beam. The absolute differential cross sections were derived by normalizing the monitor elastic yields to the distorted-wave Born approximation (DWBA) calculations and by using the known ratio of monitor and spectrometer efficiencies.

### III. RESULTS AND DISCUSSION

Fifty-seven peaks in the range 0 to 1800 keV were identified in the reaction. Table I contains a list of these peaks along with their cross sections at  $40^\circ$  and assignments wherever possible.

The predicted cross sections, excluding Coriolis coupling, appear in column 6. The errors listed were determined from the standard deviations of the energies of each peak averaged over all seven angles.

The DWBA calculations used in the present analysis were performed on the Lawrence Livermore Laboratory CDC 7600 computer system using the code DWUK72.<sup>10</sup> The optical model parameters used in the calculations are those of Lu and Alford<sup>11</sup> (Table II) with a spin-orbit term added to the triton channel. These parameters were determined to be useful from a comparison of the calculated  $l=4$  angular distribution to the experimental angular distribution of the transfer populating the ground state in  $^{176}\text{Lu}$ , which was known to be a very nearly pure  $l=4$  transfer. A nonlocal range parameter,  $\beta=0.25$ , was used in both the incoming and outgoing channels.<sup>10</sup> The lower cutoff for the radial integration was 0.0 fm, and the integration was performed out to 16 fm in intervals of 0.1 fm. Finally, we used a  $(t,\alpha)$  normalization factor of 15.1. This value appears to give the best overall agreement between the measured and calculated absolute differential cross sections.

It was found that the angular distributions, both experimental and those calculated by DWBA, had little structure. Frequently, distinguishing even an  $l=2$  experimental angular distribution from an  $l=5$  was difficult. Measurements at angles more forward than  $15^\circ$  would have been valuable, since

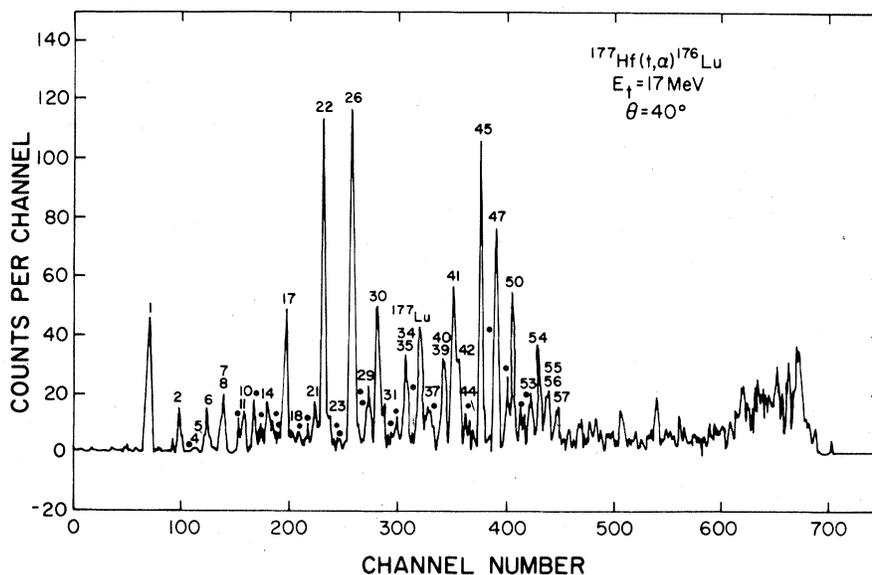


FIG. 1. Alpha spectrum of the  $^{177}\text{Hf}(t,\alpha)^{176}\text{Lu}$  reaction with 17.0 MeV tritons observed at  $40^\circ$ .

TABLE I. Nuclear levels in  $^{176}\text{Lu}$  populated by the  $(t, \alpha)$  reaction on  $^{177}\text{Hf}$ .

	Previous value (keV)	Energy $(t, \alpha)$ (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}^{40^\circ}$	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{calc}}^{40^\circ}$	Assignment		Configuration
				(mb/sr)	(mb/sr)	$I^\pi$	$K$	
1	0	0	1	0.030 6	0.052 4	7-	7	$\frac{7}{2}^+[404] + \frac{7}{2}^-[514]$
2	126.50	123	1	0.009 9	0.015 3	1-	0	$\frac{7}{2}^+[404] - \frac{7}{2}^-[514]$
3	185.4	184	3	0.000 4	0.000 5	8-	7	$\frac{7}{2}^+[404] + \frac{7}{2}^-[514]$
4	198.01	198	3	<0.000 1		1+	1	$\frac{7}{2}^+[404] - \frac{9}{2}^+[624]$
5	236.75	233	4	0.000 3		2+	1	$\frac{7}{2}^+[404] - \frac{9}{2}^+[624]$
6	{ 239.41 240.56					3-	0	
		240	2	0.009 6	0.016 4	0-	0	$\frac{7}{2}^+[404] - \frac{7}{2}^-[514]$
7	302.98	301	1	<0.006 <sup>a</sup>		3+	1	$\frac{7}{2}^+[404] - \frac{9}{2}^+[624]$
8	308.91	308	1	~0.013 <sup>a</sup>	0.015 4	2-	0	$\frac{7}{2}^+[404] - \frac{7}{2}^-[514]$
9	376.12	376				4+	1	$\frac{7}{2}^+[404] - \frac{9}{2}^+[624]$
10	384.98	377	4	0.004 1	0.002 1	2+	1	$\frac{9}{2}^-[514] - \frac{7}{2}^-[514]$
11	390.21	391	3	0.010 6	0.010 6	1-	1	$\frac{5}{2}^+[402] - \frac{7}{2}^-[514]$
12	436.66	433	1	0.011 8	0.010 1	2-	1	$\frac{5}{2}^+[402] - \frac{7}{2}^-[514]$
13	453.73	459	7	0.004 9	0.003 9	3+	1	$\frac{9}{2}^-[514] - \frac{7}{2}^-[514]$
14		486	3	0.008 5	0.005 4	8+	8	$\frac{9}{2}^-[514] + \frac{7}{2}^-[514]$
15	508.49	505	2	0.006 9	0.005 9	3-	1	$\frac{5}{2}^+[402] - \frac{7}{2}^-[514]$
16	536.70	538	3	0.002 3	0.003 9	4+	1	$\frac{9}{2}^-[514] - \frac{7}{2}^-[514]$
17		565	3	0.032 4	0.028 7	6-	6	$\frac{5}{2}^+[402] + \frac{7}{2}^-[514]$
18	599.35	594	3	0.002 8	0.002 3	4-	1	$\frac{5}{2}^+[402] - \frac{7}{2}^-[514]$
19		607	10	0.002 4				
20		653	6	0.005 9				
21		683	3	0.013 2	0.008 4	9+	8	$\frac{9}{2}^-[514] + \frac{7}{2}^-[514]$
22	726.51	723	2	0.073 0	0.086 9	4-	4	$\frac{1}{2}^+[411] + \frac{7}{2}^-[514]$
23		757	4	0.006 7	0.000 8	7-	6	$\frac{5}{2}^+[402] + \frac{7}{2}^-[514]$
24		772	8	<0.002				
25		789	4	0.002 0				
26		840	2	0.071 6	0.071 6	3-	3	$\frac{1}{2}^+[411] - \frac{7}{2}^-[514]$
27		864	6	0.036 0	0.048 5	5-	4	$\frac{1}{2}^+[411] + \frac{7}{2}^-[514]$
28		889	10	<0.001 9				
29		909	2	0.016 4		(2-	2	$\gamma$ vibration)
30		945	2	0.041 1	0.051 5	4-	3	$\frac{1}{2}^+[411] - \frac{7}{2}^-[514]$
31		966	3	0.009 4		(3-	2	$\gamma$ vibration)
32		1006	3	0.004 1				
33		1032	4	0.006	0.010 9	6-	4	$\frac{1}{2}^+[411] + \frac{7}{2}^-[514]$
34		1057	8	~0.013 <sup>a</sup>	0.002 0	(0+	0)	$\frac{7}{2}^-[523] - \frac{7}{2}^-[514]$
35		1074	5	~0.013 4 <sup>a</sup>	0.019 7	5-	3	$\frac{1}{2}^+[411] - \frac{7}{2}^-[514]$
36		1106	12	0.001 7				
37		1162	4	0.007 0				
38		1182	5	0.009 9				

TABLE I. (Continued).

Previous value (keV)	Energy ( $t, \alpha$ ) (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)^{40^\circ}$		Assignment		Configuration
			$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{calc}}$ (mb/sr)	$I^\pi$	$K$	
39	1221	5	$\sim 0.011^a$				
40	1237	4	$\sim 0.019^a$				
41	1273	2	0.041 4	0.041 0	7+	7	$\frac{7}{2}^- [523] + \frac{7}{2}^- [514]$
42	1294	2	0.024 4	0.056 3	(4+	0)	$\frac{7}{2}^- [523] - \frac{7}{2}^- [514]$
43	1326	3	0.005 9				
44	1349	5	0.005 2				
45	1395	1	0.062 5	0.093 0	5-	5	$\frac{3}{2}^+ [411] + \frac{7}{2}^- [514]$
46	1426	9	0.001 5				
47	1462	1	0.055 6	0.082 8	8+	7	$\frac{7}{2}^- [523] + \frac{7}{2}^- [514]$
48	1490	6	$< 0.001$				
49	1510	2	0.031 4	0.036 5	(3+	0)	$\frac{7}{2}^- [523] - \frac{7}{2}^- [514]$
50	1533	2	0.036 4	0.065 8	6-	5	$\frac{3}{2}^- [411] + \frac{7}{2}^- [514]$
51	1569	5	0.008 24				
52	1593	9	0.005 8				
53	1617	5	0.012				
54	1655	2	0.025 8	0.060 6	9+	7	$\frac{7}{2}^- [523] + \frac{7}{2}^- [514]$
55	1679	1	0.009 1				
56	1689	7	0.010 9	0.003 8	7-	5	$\frac{3}{2}^+ [411] + \frac{7}{2}^- [514]$
57	1730	7	0.010 9	0.052 4	(5+	0)	$\frac{7}{2}^- [523] - \frac{7}{2}^- [514]$

<sup>a</sup>Unresolved.

this tends to be an important region in the angular distributions. However, at small forward angles the background problem begins to become significant. Good fits to the  $l=2, 4,$  and  $5$  DWBA distributions are shown in Fig. 2. Additional angular distribution data relevant to the structural features of  $^{176}\text{Lu}$  are presented in Fig. 3. A level scheme including previously known excited two-quasiparticle proton and neutron states and those states pertinent to the present discussion of  $^{176}\text{Lu}$

is shown in Fig. 4. This figure has been compiled from several previous studies<sup>1-5</sup> and includes the  $\gamma$ -ray transitions proposed in those studies. Figures 5 and 6 present assignments of the rotational bands proposed in the present study. In Fig. 6 the unassigned peaks from the transfer reaction of this work are included.

In the sections which follow, the experimental data have been interpreted in terms of specific spins and parities and Nilsson configurations.

TABLE II. Optical model parameters used in the DWBA calculations.

	$V_R$ (MeV)	$r_R$ (fm)	$a_R$ (fm)	$W_I$ (MeV)	$r_I$ (fm)	$a_I$ (fm)	$\lambda$	$r_C$ (fm)	$\beta^b$	$V_{LS}$ (MeV)	$r_{LS}$ (fm)	$a_{LS}$ (fm)
$\alpha$	200	1.4	0.6	20	1.4	0.6	0	1.3	0.25			
$t$	200	1.4	0.6	50	1.4	0.6	0	1.3	0.25	6.0	1.15	0.74
bound state	a	1.25	0.65	0.0	0.0	0.0	25	1.25	0.85			

<sup>a</sup>Adjusted to produce bound state energy.<sup>b</sup>Nonlocal range parameter.

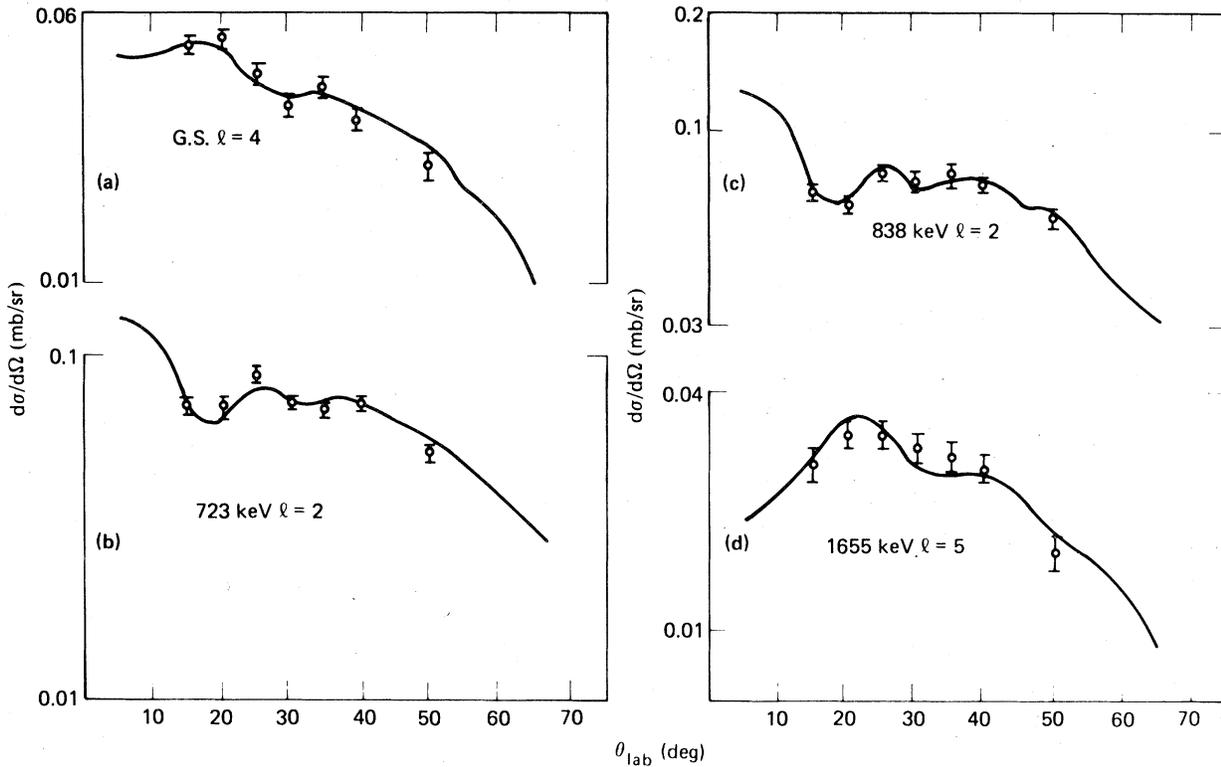


FIG. 2. Comparison of angular distributions calculated by DWBA using Lu and Alford optical model parameters of Table II to the experimental angular distributions of  $l=2$ ,  $l=4$ , and  $l=5$  transfers in the  $^{177}\text{Hf}(t, \alpha)^{176}\text{Lu}$  reaction. The solid curves are the results of DWBA calculations.

Thus it is important to recognize that the interpretation utilizes a number of theoretical concepts of nuclear reaction spectroscopy common to all odd-odd deformed nuclei. Therefore many of the assignments are based at least in part on model considerations. A brief outline of these theoretical considerations follows.

For an odd-odd deformed nucleus we have assumed the Hamiltonian of Motz *et al.*,<sup>12</sup> which is given by

$$\mathcal{H} = H_p + H_n + H_{\text{rot}} + H_{\text{RPC}} + H_{pp} + H_{\text{int}}. \quad (1)$$

Each of the terms in Eq. (1) is explained in Ref. 12, and explicit expressions for the terms  $H_R$ ,  $H_{\text{RPC}}$ , and  $H_{pp}$  are also given there. Solutions to the single-particle terms  $H_p$  and  $H_n$  are presented by Nilsson.<sup>13</sup> The calculation of  $H_{\text{int}}$  has been given by Jones *et al.*<sup>14</sup> The low-lying states are therefore represented by the eigenstates  $\Phi_{MK}^I q_p q_n$ , where the quantum numbers  $I$ ,  $M$ , and  $K$  are explained in Ref. 12, and  $q_p$  and  $q_n$  represent the quantum numbers uniquely identifying each Nilsson intrinsic proton and neutron state, respectively.

The differential cross section for populating a state in the final nucleus by a  $(t, \alpha)$  reaction is given by

en by

$$\left(\frac{d\sigma}{d\Omega}\right) = N \sum_j \left[ \left( \sum_i (S_{ji})_i b_i \right)^2 \frac{\phi_{ji}(\theta)}{2j+1} \right], \quad (2)$$

where  $S_{ji}$  is the spectroscopic amplitude derived by MacFarlane and French,<sup>15</sup> the  $b_i$  are the mixing amplitudes of states with the same total spin  $I$ ,  $\phi_{ji}$  is the intrinsic single-particle cross section determined by DWBA calculations, and  $N$  is the DWBA normalization factor.

For the  $^{176}\text{Lu}$  experiments, the expansion coefficients for the Nilsson orbitals were calculated with a deformation of  $\epsilon_2 = 0.27$  and with  $\mu = 0.625$  and  $\kappa = 0.05$ . Pairing factors were derived from a calculation using single-particle energies and Fermi surface parameters estimated by Immele.<sup>16</sup>

Calculations of  $H_{\text{int}}$  matrix elements were performed by the computer code of Onishi.<sup>17</sup> Separate matrix elements of the spin-, space-, and charge-exchange central force operators and of the spin dependent tensor force were calculated with a Gaussian shaped finite range interaction.<sup>14</sup> Coefficients of each of the matrix elements were varied

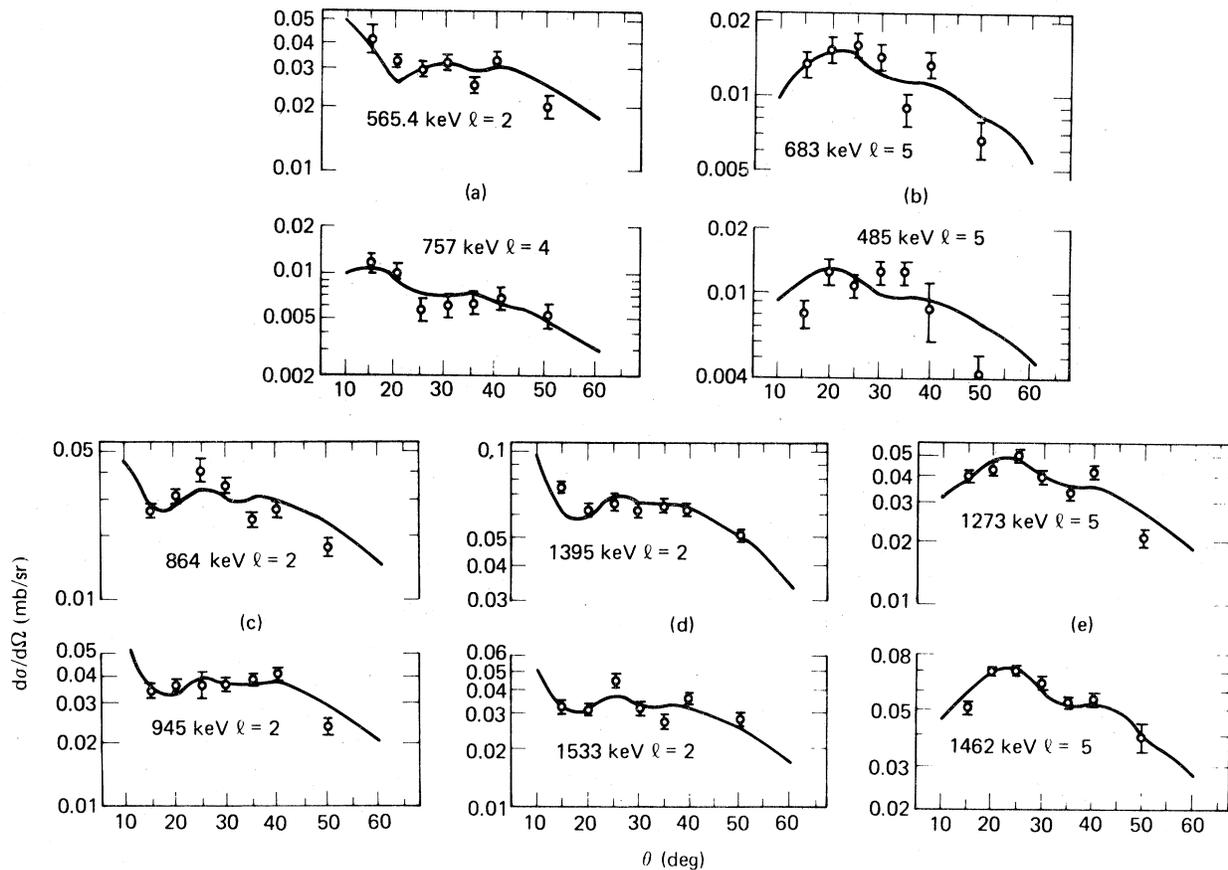


FIG. 3. Angular distributions of the  $^{177}\text{Hf}(t, \alpha)^{176}\text{Lu}$  reaction with 17.0 MeV tritons. The solid curves are the results of DWBA calculations.

by the program to obtain a best fit to the experimental splitting energies and odd-even shifts. Values of previously known splitting energies and odd-even shifts consistent with experiment and with those calculated by Boisson *et al.*,<sup>18</sup> where a more rigorous analysis including core polarization effects was performed, were obtained. Thus it appears that relatively accurate predictions of splitting energies of previously unobserved configurations can be made with the use of this code.

#### A. The $7/2^+[404] \pm 7/2^-[514]$ configuration

The  $7^-$  and  $0^-$  bands expected from the  $7/2^+[404] \pm 7/2^-[514]$  configuration were previously known from gamma-ray studies<sup>2,3</sup> and  $\beta$ -decay studies.<sup>19,20</sup> The levels up to  $I=8$  and  $I=7$  in the respective  $K=7$  and  $K=0$  bands are shown in Fig. 4. In the transfer reaction, the  $7^-$  state is predicted to contain virtually all of the observable cross section in the  $7^-$  band, with population of no

state higher than the  $8^-$  being allowed. The peaks at 0 and 184 keV agree well with the energies of the  $7^-$  and  $8^-$  levels in the ground state band. Their intensities very closely resemble the intensity pattern calculated for this configuration. Since this configuration should be populated by a nearly pure  $l=4$  transfer, the angular distribution of the relatively intense ( $\sim 30 \mu\text{b/sr}$  at  $40^\circ$ ) ground state peak was used to test the applicability of the Lu and Alford optical model parameters. Good agreement between experiment and theory was obtained, as seen in Fig. 2(a).

Levels populated in the  $(t, \alpha)$  spectrum at 123, 240, and 308, keV agree well with the previously assigned  $I^\pi=1^-, 0^-$ , and  $3^-$  (approximately degenerate), and  $2^-$  levels of the  $K^\pi=0^-$  band formed by this configuration. The absolute and particularly the relative intensities of these states support the assignments. Note in Fig. 4 that the  $0^-$  and  $3^-$  levels lie approximately one keV apart and are not resolved from one another in the transfer reaction. The levels at 467.40 and 440.95 keV, which are

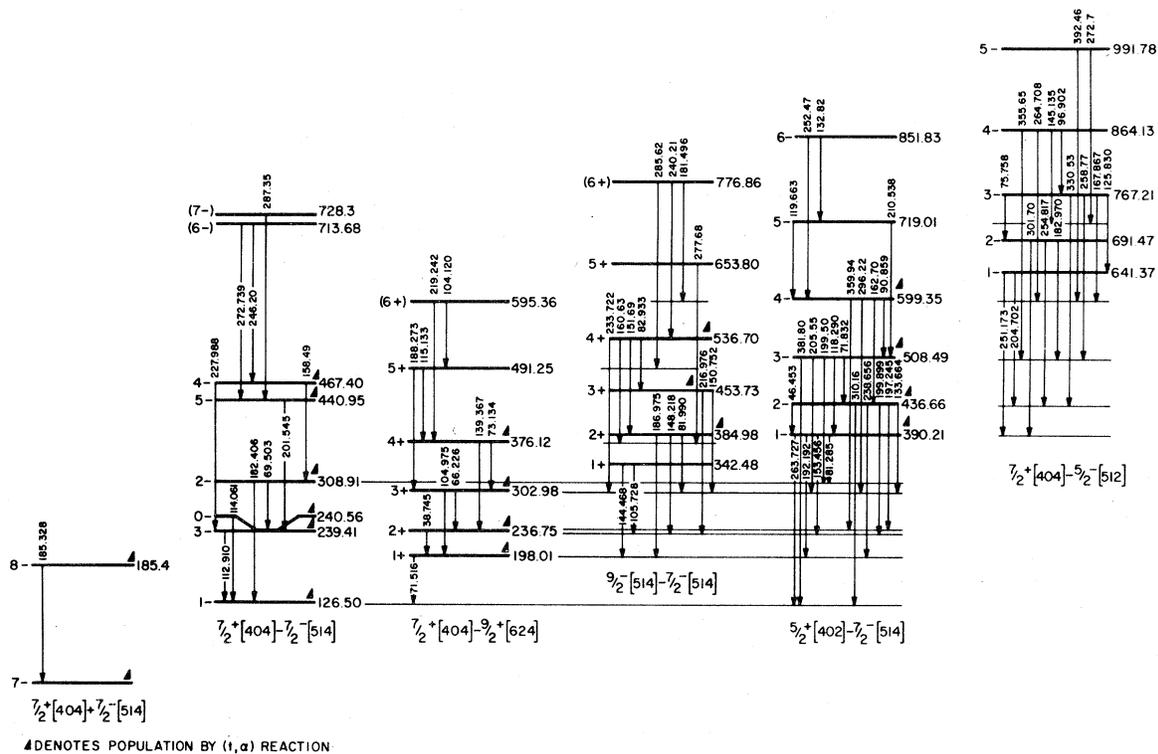


FIG. 4. Previously known levels and  $\gamma$  transitions in  $^{176}\text{Lu}$ . All energies are in keV. Spin and parity assignments lie to the left of each level, those in parenthesis are considered tentative. The  $\frac{5}{2}^+[402] - \frac{7}{2}^- [514]$  and  $\frac{7}{2}^+[404] - \frac{5}{2}^- [512]$   $K^\pi = 1^-$  bands are interchanged from their previous assignments on the basis of this experiment. Population by the  $(t, \alpha)$  reaction is also indicated.

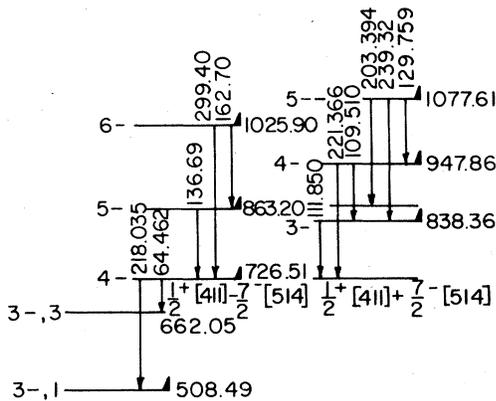
the  $I^\pi$ ,  $K = 4^-, 0$  and  $5^-, 0$  states of this band, are only weakly populated, as predicted by the calculations, and not clearly resolved from the respective 453.73- and 436.66-keV levels of Fig. 4.

#### B. The $5/2^+[402] \pm 7/2^- [514]$ configuration

Previous  $(n, \gamma)$  and  $(d, p)$  studies<sup>2,3</sup> have resulted in the identification of a  $K^\pi = 1^-$  band originating at 390.21 keV with successive spin members at 436.66, 508.49, 599.35, 719.01, and 851.83 keV up to the  $I^\pi = 6^-$  level. This band, appearing in Fig. 4, was believed to be the excited neutron configuration  $\frac{7}{2}^+[404] - \frac{5}{2}^- [512]$ . We have observed population of this band up to the spin  $4^-$  member at all angles, while the  $K^\pi = 1^-$  band beginning at 641.37, previously believed to be the  $\frac{5}{2}^+[402] - \frac{7}{2}^- [514]$  band, was not populated. Spectroscopic calculations assuming the band at 390.21 to be the  $\frac{5}{2}^+[402] - \frac{7}{2}^- [514]$   $K^\pi = 1^-$  excited proton two-quasi-particle band using a fullness factor of 0.4 agree well with experiment. The spin 2, 3, and 4 members have cross sections slightly larger than theoretically expected, but this can be explained by

Coriolis coupling and will be discussed later. We have thus reversed the initial assignments of the two  $K^\pi = 1^-$  bands, and what we believe to be the correct assignments are shown in Fig. 4.

In accordance with the Gallagher-Moszkowski coupling rule,<sup>6</sup> the singlet state coupling of the  $\frac{5}{2}^+[402] \pm \frac{7}{2}^- [514]$  configuration is expected to lie higher in energy than the triplet state. Central force residual interaction calculations indicate that the  $K^\pi = 6^-$  bandhead of the parallel coupling should be observed at approximately 560 keV. Spectroscopic calculations predict that the  $6^-$  level in this band should have an intensity approximately 30 times that of the  $7^-$  level, and should be mostly an  $l=2$  transfer, while the  $7^-$  state should be a pure  $l=4$  transfer. Previously unobserved levels at 565 and 757 keV were seen in the alpha particle spectrum with cross sections that agree with the calculations shown in Table I. The peak at 565 keV has a good  $l=2$  angular distribution, and the 757-keV peak is an excellent  $l=4$  transfer. Angular distributions of the two peaks appear in Fig. 3(a). Assigning the 565-keV level as the  $6^-, 6$  bandhead, it is noted that the only transition which



#### $\blacktriangle$ DENOTES POPULATION BY $(t, \alpha)$ REACTION

FIG. 5. Partial level scheme of  $^{176}\text{Lu}$   $\gamma$  transitions depopulating levels of the  $\frac{1}{2}^+ [411] \pm \frac{7}{2}^- [514]$  configuration in  $^{176}\text{Lu}$ . The  $I^\pi = 4^+$  level at 726.51 keV and  $\gamma$  transitions depopulating it were previously known. Population by the  $(t, \alpha)$  reaction is indicated.

can be expected to depopulate this level would go to the  $7^-, 7$  ground state. Balodis *et al.*<sup>1</sup> have observed an intense  $\gamma$ -ray transition of 563.89(8) keV, which is probably that transition. We have assigned the 565- and 757-keV levels as the  $6^-$  and  $7^-$  members, respectively, of the  $\frac{5}{2}^+ [402] + \frac{7}{2}^- [514]$  band. This is shown in Fig. 6, where previously unassigned  $\gamma$  transitions of 563.89(8), 194.17(12), 527.47(8), and 632.9(7) keV have been used to fix these levels at 563.89 and 758.06 keV. This results in  $(\hbar^2/2\mathcal{I}) = 13.8$  keV compared to 12.6 for the antiparallel coupling of this configuration. The singlet-triplet splitting observed is then  $-104.9$  keV.

#### C. The $9/2^- [514] \pm 7/2^- [514]$ configuration

The  $K^\pi = 1^+$  band resulting from the antiparallel coupling of the  $\frac{9}{2}^- [514] \pm \frac{7}{2}^- [514]$  configuration has been previously assigned by  $\gamma$ -decay studies<sup>2,3</sup> up to the  $6^+$  spin member. Levels in order of increasing spin are at 342.48, 384.98, 453.73, 536.70, 653.80, and 776.86 keV and are shown in Fig. 4. We have populated the  $I^\pi = 2^+, 3^+, 4^+$  members of this band with cross sections quite well reproduced by calculations. The  $1^+$  level is predicted to have a very small spectroscopic factor, and in accordance with this the 342.48-keV level was not observed in the alpha particle spectrum. The inertial parameter of this band is calculated to be  $(\hbar^2/2\mathcal{I}) = 11.1$  keV.

The singlet state coupling of this configuration results in a rotational band with  $K^\pi = 8^+$ . We have

assigned the levels at 486 and 683 keV as the  $8^+$  and  $9^+$  members of this band as shown in Fig. 6. The relative intensities of these two peaks agree very well with spectroscopic factor calculations, though the absolute cross sections are about a factor of 2 larger than predicted. It must be noted that the  $\frac{9}{2}^- [514]$  Nilsson orbital is strongly coupled to the  $\frac{7}{2}^- [523]$  orbital through Coriolis coupling, and this readily accounts for the increased intensity of these two states. The transfers populating these states should be pure  $l=5$  transitions, and the angular distributions of each of these peaks, shown in Fig. 3(b), are in good agreement with the assignments. The experimental inertial parameter of this  $K=8$  band is  $\hbar^2/2\mathcal{I} = 11.0$  keV. Based on these two assignments, a new singlet-triplet splitting of  $-83.1$  keV can be added to the nuclear data. This value is in excellent agreement with tensor force calculations, which predict the splitting to be  $-88.4$  keV.

#### D. The $7/2^+ [404] \pm 9/2^+ [624]$ configuration

The  $\frac{7}{2}^+ [404] - \frac{9}{2}^+ [624]$  band is an excited neutron configuration and to zeroth order should not be populated in a  $(t, \alpha)$  reaction. Levels up to the  $6^+$  state have been previously assigned from the  $(d, p)$  reaction<sup>2</sup> and  $\gamma$ -ray spectroscopy<sup>2,3</sup> as shown in Fig. 4. The first three members of this band have almost certainly been populated in our experiment, and the 376.12 keV  $I^\pi = 4^+$  state has probably been populated at several angles. These are all very weak states, indicative of neutron excited states which gain strength through mixing with proton excited states populated in zeroth order. Coriolis coupling cannot be used to explain the population of any of these  $1^+$  band members. The most reasonable explanation comes from the mixing of this band with the  $K^\pi = 1^+$ ,  $\frac{9}{2}^- [514] - \frac{7}{2}^- [514]$  band discussed earlier. The mixing can best be explained by a combination of the rotational two-particle interaction and the residual interaction perturbations to the Hamiltonian,  $H_{pp}$  and  $H_{int}$ . The  $H_{pp}$  perturbation supports matrix elements of the form  $\delta_{K', K} \delta_{\Omega', \Omega} \delta_{p, p \pm 1} \delta_{\Omega', \Omega} \delta_{n, n \mp 1}$ . For the element  $\langle IM1 \frac{7}{2}^+ [404] - \frac{9}{2}^+ [624] | H_{pp} | IM1 \frac{9}{2}^- [514] - \frac{7}{2}^- [514] \rangle$ , only the  $j_p = \frac{9}{2}$ ,  $j_n = \frac{9}{2}$  and  $j_p = \frac{9}{2}$ ,  $j_n = \frac{11}{2}$  terms are nonzero. First order perturbation calculations indicate that this off-diagonal matrix element is  $-0.1282$  keV, and the wave functions for the two  $K=1^+$  bands remain virtually pure two-quasiparticle states.

The off-diagonal element of the residual interaction between the  $\frac{7}{2}^+ [404] - \frac{9}{2}^+ [624]$  and  $\frac{9}{2}^- [514] - \frac{7}{2}^- [514]$   $K^\pi = 1^+$  bands has been calculated by finite range calculations to be  $-12.6$  keV. Such a perturbation mixes the two bands to the extent



periment and theory is very good for the states in these two bands, and nine  $\gamma$  transitions have been fit into the level scheme. We therefore consider the assignments in this configuration to be firmly based.

#### F. The $3/2^+[411] \pm 7/2^-[514]$ configuration

The  $3/2^+[411]$  proton orbital is a hole state made up almost entirely of  $l=2$  character. There are strong  $l=2$  transitions to levels at 1395 and 1533 keV [see Fig. 3(d)] in the alpha particle spectrum. The relative intensities and energy spacings strongly indicate that these states are the  $I^\pi=5^-$  and  $6^-$  spin members of the  $K=5^-, 3/2^+[411] + 7/2^-[514]$  band. As shown in Table I, the predicted cross sections at  $\theta=40^\circ$  for these two states are  $(d\sigma/d\Omega)=0.0930$  and  $0.0658$  mb/sr, respectively, and the experimental values are  $0.0625$  and  $0.0364$  mb/sr. Including Coriolis coupling slightly improves the agreement between experiment and theory. Spectroscopic calculations predict an  $l=4$  transition weakly populating the  $7^-$  member of this band. The inertial parameter calculated from the energies of the  $6^-$  and  $5^-$  members,  $(\hbar^2/2g)=11.5$  keV, allows the prediction of the  $7^-$  level at an energy of approximately 1695 keV. We have observed a weakly populated level at  $1689 \pm 7$  keV, which we have assigned as this  $7^-$  spin member. Intra-band  $\gamma$ -ray transitions of 159.49(7), 140.481(14), and 299.91(3) keV can be assigned among these three levels. Also, previously unassigned transitions of 194.17(12), 527.47(8), and 637.9(7) keV discussed earlier probably depopulate this band to the  $I^\pi, K=7^-, 6$  state at 758.06 keV and to the  $I^\pi, K=5^-, 4$  state at 863.20 keV, fixing the  $I=5, 6,$  and  $7$  states of this configuration at 1390.67, 1531.15, and 1690.58 keV, respectively, as shown in Fig. 6.

The  $K^\pi=2^-$  band resulting from the antiparallel coupling of the  $3/2^+[411] \pm 7/2^-[514]$  configuration has not been identified. Spectroscopic calculations predict fairly intense peaks representing the  $I^\pi=2^-, 3^-, 4^-,$  and  $5^-$  members of this band should be observed. Tensor force singlet-triplet splitting calculations indicate that the  $K^\pi=2^-$  bandhead should lie near  $E_{ex}=1270$  keV. There is an intense peak at 1293 keV in the alpha spectrum which could be that state; however, no other spin members can be reasonably proposed. Further, the angular distribution of this peak more closely resembles an  $l=5$  transfer than an  $l=2$ , therefore the 1293 keV peak is more likely a member of the  $7/2^-[523] \pm 7/2^-[514]$  configuration.

Since the  $3/2^+[411]$  and the  $5/2^+[402]$  Nilsson orbitals both originate from the  $2d_{5/2}$  shell model state, they can be expected to have a large Coriolis in-

teraction. This interaction is observed in the coupling of the  $5/2^+[402] + 7/2^-[514]$   $K^\pi=6^-$  band to the  $3/2^+[411] + 7/2^-[514]$   $K^\pi=5^-$  band. The  $I^\pi, K=7^-, 6$  and  $6^-, 6$  states have cross section notably enhanced by the mixing of the  $K=6$  band with the  $K=5$  band, which has been quantitatively accounted for quite well by the Coriolis interaction. This interaction may help to explain the absence of a recognizable intensity pattern in the  $K^\pi=2^-$  band. It is, in fact, observed that the intensity pattern of the  $K^\pi=1^-, 5/2^+[402] - 7/2^-[514]$  band is somewhat perturbed; in particular, the spin members higher than the bandhead have cross sections slightly larger than predicted, which is precisely what would be expected from a large Coriolis interaction.

Several examples of the  $3/2^+[411]$  orbital mixing with vibrational states built on the ground states or on the  $1/2^+[411]$  orbital have been observed in this region in odd- $A$  nuclei.<sup>21</sup> In this instance the  $K^\pi=2^-, 3/2^+[411] \pm 7/2^-[514]$  band should be able to mix with a  $K^\pi=2^-$  vibrational band built on the  $K^\pi=0^-$  ground state band or on the  $K^\pi=4, 1/2^+[411] + 7/2^-[514]$  band. Any such mixing would further take intensity from the  $3/2^+[411] - 7/2^-[514]$  band and increase the difficulty of observing it in a transfer reaction. We have proposed a  $K^\pi=2^-$   $\gamma$ -vibrational band, with spin members 2 and 3 at 909 and 966 keV, respectively, built on the ground state  $K^\pi=0^-$  band. The inertial parameter of this band, seen in Fig. 6, is then 9.4 keV. These states can be populated only to the extent that they are composed of the  $3/2^+[411] - 7/2^-[514]$  two-quasiparticle configuration. The angular distributions of these peaks indicate they are  $l=2$  transfers, and their cross sections are approximately one-fifth that which would be expected for the pure two-quasiparticle configuration. The states at 909 and 966 keV thus have the wave function  $\phi \sim 0.9\{3/2^+[404] - 7/2^-[514]; 2^-\} \pm 0.4\{3/2^+[411] - 7/2^-[514]\}$ .

#### G. The $7/2^-[523] \pm 7/2^-[514]$ configuration

Peaks representing levels in the  $7/2^-[523] \pm 7/2^-[514]$  configuration should be very nearly pure  $l=5$  transitions. The peaks at 1273, 1462, and 1655 keV are strongly populated, very good  $l=5$  transitions, as seen in Figs. 2(d) and 3(e). The rotational energy spacings are consistent with the assignment of these levels as the  $7^+, 8^+,$  and  $9^+$  members of a  $K^\pi=7^+$  rotational band, and the relative intensities are well reproduced by theoretical calculations assuming the band to be from the  $7/2^-[523] + 7/2^-[514]$  configuration. The resulting inertial parameter is then  $(\hbar^2/2g)=11.2$  keV. Possible  $\gamma$ -ray transitions between these levels are the previously unassigned transition of 166.33(7),

196.27(12), and 362.70(7) keV. However, no unambiguous assignments can be made of transitions to lower levels. One should observe transitions to the  $K^\pi = 8^+$  band at 486 keV. It is probably more reasonable that this  $7^+$  band is not populated in a thermal neutron capture experiment. The assignments of the  $\frac{7}{2}^- [523] + \frac{7}{2}^- [514]$   $K^\pi = 7^+$  band appear in Fig. 6.

The  $0^+$  band expected from this configuration has not been definitely recognized. This is quite unfortunate as the eigenvalues of a  $K=0$  band are sensitive to the residual interaction of an odd-odd nucleus. Residual interaction calculations indicate that the odd-even shift in this configuration is  $-80$  keV, so the even and odd spin bands should be well separated with the even bandhead lying lower. Spectroscopic calculations predict that each of the spin states  $0^+$  through  $7^+$  should be observable in the alpha particle spectrum. There are many moderately strong peaks in the region 1000 to 1700 keV, some of which very likely belong to the  $K^\pi = 0^+$  band. The most likely candidates are the levels at 1162, 1293, 1510, and 1730 keV, all of which have reasonably good  $l=5$  angular distributions.

Tentative assignments of the  $I^\pi = 0^+$  through  $5^+$  states in the  $K^\pi = 0^+$ ,  $\frac{7}{2}^- [523] - \frac{7}{2}^- [514]$  band appear in Fig. 6. These states are successively at 1057, 1388, 1128, 1510, 1293, and 1730 keV. The inertial parameter of the even member band is 11.8 keV, and that of the odd member band is 12.2 keV. The singlet-triplet splitting is  $-7$  keV, which is not in good agreement with the central plus tensor force calculation of  $-99$  keV although of the same sign, and the odd-even shift is  $-156$  keV, which is in reasonably good agreement with the calculated value of  $-80$  keV. The cross sections fit well into the predicted intensity pattern, as is observable in Table I. The  $I^\pi = 2^+$  level at 1128 keV is about 0.020 mb/sr too large, but we believe much of this peak to be the  $\frac{3}{2}^+, \frac{1}{2}^+ [411]$  impurity state of  $^{177}\text{Lu}$ .

#### H. Coriolis coupling calculations

In the spectroscopy of  $^{176}\text{Lu}$ , the important positive parity proton Nilsson orbitals are the  $\frac{7}{2}^+ [404]$ ,  $\frac{5}{2}^+ [402]$ ,  $\frac{3}{2}^+ [411]$ , and  $\frac{1}{2}^+ [411]$ . Coupling each of these to the  $\frac{7}{2}^- [514]$  neutron orbital yields negative parity bands of  $K=0$  up to 7. Each of these bands with  $K$  quantum number  $K_0$  is connected to all other bands with  $K=K_0+1$  by Coriolis coupling. A rigorous analysis would require the diagonalization of up to an 8 by 8 matrix. After explicit calculation of all off-diagonal  $H_{\text{RPC}}$  matrix elements it was found that certain of the perturbations were smaller by a factor of 3 than the

others, and were therefore ignored. The result was two 3 by 3 matrices involving the  $K=0, 1,$  and 2 bands from the  $\frac{7}{2}^+ [404]$ ,  $\frac{5}{2}^+ [402]$ , and  $\frac{3}{2}^+ [411]$  orbitals and involving the  $K=5, 6,$  and 7 bands from those same orbitals, and a single 2 by 2 matrix involving the  $K=3$  and 4 bands from the  $\frac{1}{2}^+ [411] \pm \frac{7}{2}^- [514]$  configuration.

Since the  $K^\pi = 2^-$  band formed from the  $\frac{3}{2}^+ [411] - \frac{7}{2}^- [514]$  configuration was not identified, it was not possible to perform any calculations on the first of these 3 by 3 matrices. It has been noted however, that since the  $H_{\text{RPC}}$  off-diagonal element is negative, and the amplitudes  $S_{j_i}$  of the pure wave functions  $\frac{3}{2}^+ [411] - \frac{7}{2}^- [514]$  and  $\frac{5}{2}^+ [402] - \frac{7}{2}^- [514]$  are the same sign for all values of  $j$ , any mixing of these two wave functions would tend to enhance the cross sections of the  $I=2, 3,$  and 4 states in the  $K^\pi = 1^-$  band. Experimentally, this enhancement is observed as can be seen by noting the appropriate experimental and calculated cross sections in Table I.

A small computer program was written to determine the effect of Coriolis coupling between the  $\frac{7}{2}^+ [404] - \frac{7}{2}^- [514]$   $K=0$  and  $\frac{5}{2}^+ [402] - \frac{7}{2}^- [514]$   $K=1$  bands. In this program the two unperturbed bandhead energies were slowly varied as free parameters over pertinent ranges, and the Coriolis matrix element, odd-even shift, and the eigenvalues of the  $I=2, 3, 4,$  and 5 states in both bands were calculated at each combination. It was found that the best agreement in a least squares fit between the calculated and experimental eigenvalues occurred when the Coriolis coupling element between the two bands was zero. Thus the  $K=0$  band formed from the  $\frac{7}{2}^+ [404] - \frac{7}{2}^- [514]$  configuration can be considered a pure, unmixed band.

The results of Coriolis interaction calculations for the  $K=5, 6,$  and 7 negative parity bands are shown in Table III. Explicit off-diagonal Coriolis matrix elements were calculated by the equations of Motz *et al.*<sup>12</sup> Then the magnitude of the perturbation was determined, and mixing coefficients were calculated. It is noted that the calculated cross sections and energies agree better with the experiment when Coriolis coupling is included. The theoretical spectroscopic calculations for the  $K^\pi = 5^-$  band were performed with DWBA results calculated at  $E_{\text{ex}} = 1.5$  MeV, while the cross sections for the  $K^\pi = 6^-$  and  $7^-$  bands were determined using DWBA results calculated at  $E_{\text{ex}} = 0$ .

Coriolis interaction calculations involving the  $K=3$  and  $K=4$  bands from the  $\frac{1}{2}^+ [411] \pm \frac{7}{2}^- [514]$  configuration accurately reproduce the energies at which the states in these bands occur. The energies, wave functions, and cross sections after including Coriolis coupling for these two bands are shown in Table IV. Note that the cross sections

TABLE III. Results of Coriolis band mixing for the  $\frac{3}{2}^+[411]$ ,  $\frac{7}{2}^+[404]$ , and  $\frac{5}{2}^+[402]$  orbitals coupled to the  $\frac{7}{2}^-[514]$  neutron orbital. The unperturbed inertial parameters were determined to be 11.19, 14.15, and 11.66 keV for the respective  $K=5$ , 6, and 7 bands.

$I$	$K$	Proton orbital	Energies (keV)			Wave function			Cross section at $40^\circ$ (mb/sr) <sup>a</sup>		
			$\epsilon^0$	$\epsilon^\pm$	$\epsilon^{\text{ex}}$	$b_{K=5}$	$b_{K=6}$	$b_{K=7}$	$\left(\frac{d\sigma}{d\Omega}\right)^0$	$\left(\frac{d\sigma}{d\Omega}\right)^\pm$	$\left(\frac{d\sigma}{d\Omega}\right)^{\text{ex}}$
5	5	$\frac{3}{2}^+[411]$	1391	1391	1391	1.00	0.0	0.0	0.0930	0.0930	0.0625
6	5		1525	1534	1531	0.998	-0.0632	0.0	0.0658	0.0605	0.0365
7	5		1682	1688	1691	0.99	-0.14	0.017	0.0038	0.0035	0.0109
8	5		1861	1877		0.991	-0.133	0.0152			
6	6	$\frac{5}{2}^+[402]$	568.10	563.84	563.89	0.0632	0.998	0.0	0.0287	0.0343	0.0324
7	6		766.2	757.7	758.06	0.14	0.99	-0.017	0.0008	0.0013	0.0067
8	6		993	979		0.133	0.990	-0.0522			
7	7	$\frac{7}{2}^+[404]$	1.1	0	0	0.0022	0.039	1.00	0.0524	0.0414	0.0306
8	7		187.1	185.6	185.4	0.0036	0.050	1.0	0.0005	0.0004	0.0004

<sup>a</sup>The superscript 0 refers to unperturbed quantities, the superscript  $\pm$  refers to mixed quantities, and the superscript ex refers to experimental quantities.

of the  $K=3$  band members are reduced by the Coriolis interaction, and the experimental data agree with this prediction. This is equally true at all other angles. Unfortunately we do not observe the difference in intensity appearing in the  $K=4$  members. The cross sections are all significantly lower than predicted by the calculations. This suggests that the  $K^\pi=4^-$  band mixes with some other unobserved band, possibly a  $\gamma$ -vibrational band.

The only positive parity states populated in the  $(t, \alpha)$  reaction are those arising from the configurations  $\frac{7}{2}^-[523] \pm \frac{7}{2}^-[514]$  and  $\frac{9}{2}^-[514] \pm \frac{7}{2}^-[514]$ . Since the  $\frac{7}{2}^-[523]$  and  $\frac{9}{2}^-[514]$  orbitals both originate from the  $1h_{11/2}$  unique parity shell model state, Coriolis coupling between them should be impor-

tant. The  $K^\pi=0^+$  band of  $\frac{7}{2}^-[523] - \frac{7}{2}^-[514]$  was not positively identified so coupling between the  $K^\pi=0^+$  and  $1^+$  bands is not considered. However, coupling between the  $7^+$  and  $8^+$  bands at 1273 and 486 keV, respectively, should be observed, and it was noted that the 486 and 683 keV peaks both were populated with anomalously large cross sections.

By constructing the  $I=8$  and  $I=9$  Hamiltonian matrices, including Coriolis off-diagonal matrix elements, moments of inertia of the two bands, and the unperturbed bandhead energy of the  $K=8$  band (hereafter referred to as  $E_8$ ) as unknowns, one is able to calculate each of the above quantities plus the eigenvalues of the  $I=9$  matrix from a single choice of  $E_8$  and the known eigenvalues of the  $I=8$  matrix. Therefore it is possible to write

TABLE IV. Results of Coriolis band mixing calculations for the two bands of the  $\frac{1}{2}^+[411] \pm \frac{7}{2}^-[514]$  configuration. The unperturbed inertial parameters were determined to be 12.83 and 14.36 keV for the  $K=3$  and  $K=4$  bands, respectively. The  $\langle \frac{1}{2}^+ | j \pm | \frac{1}{2}^- \rangle$  decoupling value was taken to be 0.818. The superscripts are explained in Table III.

$I$	$K$	Energies (keV)			Wave functions		Cross section at $40^\circ$ (mb/sr)		
		$\epsilon^0$	$\epsilon^\pm$	$\epsilon^{\text{ex}}$	$b_{K=3}$	$b_{K=4}$	$I^0$	$I^\pm$	$I^{\text{ex}}$
3	3	838.36	838.36	838.36	1.00	0.0	0.0716	0.0716	0.0716
4	3	941.00	945.60	947.86	0.99	0.14	0.0515	0.0344	0.0411
5	3	1069.30	1080.12	1077.61	0.97	0.24	0.0197	0.0074	$\sim 0.0134$
6	3	1223.3	1240.7	(1238)	0.96	0.28	0.0047	0.0014	( $\sim 0.0192^a$ )
4	4	730.71	726.11	726.51	-0.14	0.99	0.0869	0.1040	0.0730
5	4	874.31	863.49	863.20	-0.24	0.97	0.0485	0.0607	0.0363
6	4	1046.63	1029.26	1025.90	-0.28	0.96	0.0109	0.0142	0.0063
7	4	1247.67	1218.2	(1221.2)	-0.37	0.93	0.00179	0.0025	(0.0107)

<sup>a</sup>Not well resolved.

TABLE V. Values of the free parameters and eigenvalues of the  $I=9$  energy matrix for several choices of  $E_8$  near  $[d(\chi^2)/dE_8]=0$ . The values of  $(\hbar^2/2g)(K)$  and  $\alpha^2$  are determined by forcing the eigenvalues  $\lambda_{\text{calc}}^{I=8}$  to be equal exactly to the experimental eigenvalues. Therefore the  $\lambda_{\text{calc}}^{I=8}$  do not contribute to the calculation of  $\chi^2$ .

$E_8$	$\frac{\hbar^2}{2g}(K=8)$	$\frac{\hbar^2}{2g}(K=7)$	$\alpha^2$	$\lambda_{K=8}^{I=9}$ (keV)		$\lambda_{K=7}^{I=9}$ (keV)		$\chi^2$
				calc	exp	calc	exp	
501.4	10.87	10.82	3734	664.6	683.1	1673.3	1655	680.8
500.0	10.92	10.78	3408	664.6	683.1	1673.6	1655	691.9
502.0	10.78	10.90	3881	664.4	683.1	1673.2	1655	684.6

the eigenvalues of the  $I=9$  matrix as a function of  $E_8$  only. It is further possible to write the sum of the squares of the differences between the eigenvalues calculated by this method and the experimental eigenvalues as a function of  $E_8$  only. That is, calling that sum  $\chi^2$ ,

$$\chi^2(E_8) = \sum_{K_i} [\lambda_{K_i}^{I=9}(E_8) - \lambda_{K_i}^{I=9}]^2.$$

The value of  $E_8$  at which  $[d(\chi^2)/dE_8]=0$  will be the proper choice of  $E_8$  to produce the best eigenvalues of the  $I=9$  matrix. The results of this calculation, as shown in Table V, indicate that this minimum in  $\chi^2$  occurs for  $E_8=501.4$  keV. Note that the value of the Coriolis matrix element  $\alpha$ , which has been defined by

$$\alpha = \left[ \frac{\hbar^2}{2g}(K=8) + \frac{\hbar^2}{2g}(K=7) \right] \times \langle \frac{9}{2}^- [514] | j_+ | \frac{7}{2}^- [523] \rangle [U_7 U_8 + V_7 V_8],$$

where the  $U_K$  and  $V_K$  are the emptiness and fullness factors of the appropriate orbitals, is  $(3734)^{1/2} = 61.1$ . The theoretical value of  $\alpha$  as determined by evaluating  $\langle \frac{9}{2}^- [514] | j_+ | \frac{7}{2}^- [523] \rangle$  is 52.34, which is an excellent agreement.

The energies, cross sections and wave functions resulting from this Coriolis interaction are shown

in Table VI. The cross sections of the states in the  $K=8$  band are in much better agreement with the experimental values after Coriolis coupling is included. This is also true of the  $K=7$  band members, though the difference between experiment and theory is still large.

#### IV. CONCLUSION

Levels in  $^{176}\text{Lu}$  have previously been studied with particular emphasis on excited neutron configurations. Using the  $(t, \alpha)$  reaction, we have been able to emphasize proton hole configurations in  $^{176}\text{Lu}$ . Fifty-seven peaks in the  $^{177}\text{Hf}(t, \alpha)^{176}\text{Lu}$  reaction were identified. Angular distributions allowed the assignment of  $l$  values to 20 of these transfers. Assignments were made to 33 states in two-quasiparticle rotational bands. Sixteen new assignments were made in the excited proton orbitals  $\frac{5}{2}^+ [402]$ ,  $\frac{1}{2}^+ [411]$ ,  $\frac{3}{2}^+ [411]$ ,  $\frac{9}{2}^- [514]$ , and  $\frac{7}{2}^- [523]$  coupled to the  $\frac{7}{2}^- [514]$  neutron. The previously known  $K^\pi = 1^-$  band beginning at 390.21 keV was reassigned the  $\frac{5}{2}^+ [402] - \frac{7}{2}^- [514]$  configuration.

Little perturbation of the low energy spectrum by vibrational states was observed. A single  $K^\pi = 2^-$  vibrational band built on the  $K^\pi = 0^-$  ground state configuration, strongly mixed with the  $K^\pi = 2^-, \frac{3}{2}^+ [411] - \frac{7}{2}^- [514]$  band, was proposed at  $\sim 910$

TABLE VI. Results of Coriolis band mixing for the  $\frac{9}{2}^- [514]$  and  $\frac{7}{2}^- [523]$  orbitals coupled to the  $\frac{7}{2}^- [514]$  neutron orbital. The unperturbed inertial parameters for the  $K=8$  and  $K=7$  bands were found to be 10.87 and 10.82 keV, respectively. The superscripts are explained in Table III.

$I$	$K$	Proton orbital	Energies (keV)			Wave functions		$\left(\frac{d\sigma}{d\Omega}\right)^{40^\circ} \left(\frac{\text{mb}}{\text{sr}}\right)$		
			$\epsilon^0$	$\epsilon^\pm$	$\epsilon^{\text{ex}}$	$b_{K=8}$	$b_{K=7}$	$I^0$	$I^\pm$	$I^{\text{ex}}$
7	7	$\frac{7}{2}^- [523]$	1273	1273	1273	0.0	1.00	0.0410	0.0410	0.0414
8	7		1446	1462	1462	-0.127	0.991	0.0828	0.0408	0.0556
9	7		1641	1673.3	1655	-0.179	0.984	0.0606	0.0352	0.0258
8	8	$\frac{9}{2}^- [514]$	501.4	485.8	485.7	0.991	0.127	0.0054	0.0110	0.0085
9	8		697.1	664.6	683.1	0.984	0.179	0.0084	0.0168	0.0132

keV.

A manifestation of the particle-particle interaction term ( $H_{pp}$ ) extracted from the rotational Hamiltonian and of the off-diagonal matrix elements of the residual interaction was observed in the mixing of the  $\frac{9}{2}^- [514] \pm \frac{7}{2}^- [514]$  and  $\frac{7}{2}^+ [404] \pm \frac{9}{2}^+ [624]$  configurations.  $H_{pp}$  and preliminary residual interaction off-diagonal matrix element calculations have failed to account for the magnitude of the experimental mixings. It would be useful to perform extensive residual interaction calculations to account for this anomaly. In addition it would be interesting to observe these two configurations in other odd-odd nuclei to measure their overlap. Useful experiments to make this measurement will be the proton stripping reaction on  $^{177}\text{Hf}$  and the stripping and pickup reaction on  $^{179}\text{Hf}$ , and the neutron transfer reactions on  $^{175}\text{Lu}$ .

With respect to the Coriolis coupling calculations, several observations can be made. A large Coriolis interaction between the  $\frac{9}{2}^- [514]$  and  $\frac{7}{2}^- [523]$  proton orbitals coupled to the  $\frac{7}{2}^- [514]$  neutron was observed experimentally. The experimental matrix element between the  $K=7$  and  $K=8$  bands agrees very well with the theoretical calculation. Thus apparently no attenuation of the Coriolis matrix element is observed for these negative parity proton orbitals. This situation has not been experimentally observed in the neighboring odd- $A$  isotopes  $^{175}\text{Lu}$  and  $^{177}\text{Lu}$ , because the  $\frac{7}{2}^- [523]$  proton hole band has not been identified in these nuclei.

The Coriolis interaction of the  $\frac{5}{2}^+ [402] - \frac{7}{2}^- [514]$  band with the  $\frac{7}{2}^+ [404] - \frac{7}{2}^- [514]$  band is observed in these experiments to be nearly zero, while a theoretical value of  $-6.13$  keV excluding the  $[(I-K)(I+K+1)]^{1/2}$  normalization factor has been calculated. Attenuation of the Coriolis force is thus quite apparent in these positive parity proton orbitals.

The attenuation in the positive parity orbitals is, however, not consistent with the observations in the  $K=3$  and  $4$  bands in the  $\frac{1}{2}^+ [411] \pm \frac{7}{2}^- [514]$  configuration. Here using the theoretically calculated decoupling factor of  $a = -0.818$  as the Coriolis matrix element, the experimental eigenvalues in the  $K=3$  and  $K=4$  bands were very accurately reproduced. This is also true for the eigenvalues of the  $\frac{1}{2}^+ [411]$  band in  $^{175}\text{Lu}$ . The experimental eigenvalues from  $I = \frac{1}{2}$  up to  $I = \frac{11}{2}$  are

626.7, 632.9, 757.5, 773.6, 990.2, and 1019.4 keV,<sup>22</sup> and the calculated eigenvalues are 628.9, 635.8, 756.8, 772.5, 990.9, and 1017.4. This agreement between experiment and theory is particularly significant, because while it is always possible to generate a set of eigenvalues of a pure wave function which will accurately reproduce the experimental eigenvalues when the theoretical Coriolis perturbation is turned on (providing the perturbation is not so large that perturbation theory does not work), it is not necessarily possible to generate an "uncoupled"  $K = \frac{1}{2}$  band that will accurately reproduce the experimental eigenvalues when the decoupling perturbation is turned on. A contributing factor to the experimental accuracy of the theoretical decoupling factor is that there is no error introduced by the pairing reduction factor, as it is rigorously equal to 1. When calculating off-diagonal Coriolis elements the pairing factor can not be determined precisely, so the theoretical matrix elements contain some uncertainty.

It is not possible to compare the experimental Coriolis matrix elements in  $^{176}\text{Lu}$  measured in this study to those of  $^{175}\text{Lu}$  and  $^{177}\text{Lu}$  observed in the proton stripping reactions on  $^{174}\text{Yb}$  and  $^{176}\text{Yb}$ . In these latter two cases, the only proton hole state observed is the  $\frac{1}{2}^+ [411]$ , and coupling between the  $\frac{3}{2}^+ [411]$  and  $\frac{5}{2}^+ [402]$  and between the  $\frac{9}{2}^- [514]$  and  $\frac{7}{2}^- [523]$  orbitals could not be experimentally measured. The attenuation in the experimental proton Coriolis coupling in  $^{176}\text{Lu}$  is not consistent from one set of Nilsson states to another, and it is therefore difficult to draw general conclusions.

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