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Unified theory of $NN\to\pi d$, $\pi d\to\pi d$, and $NN\to NN$ reactions

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Using a set of equations that couple the N-N to the π-d channel and satisfy two- and three-body unitarity, we have studied in detail the reaction $pp\to\pi^+d$, and $\pi d\to\pi d$ and $NN\to NN$ scattering. We find that the equations give a very good description of the differential cross sections and some of the polarization data for both $\vec{p}p\to\pi^+d$ and $\pi d\to\pi\vec{d}$ over the energy region $47<T_\pi<256$ MeV. We observe that for $pp\to\pi^+d$, higher partial waves ($l_{\pi d}>2$) contribute significantly to both the differential cross section and analyzing power. Including the P_{11} π-N interaction and thus true absorption in π-d elastic scattering, we find a cancellation between the pole and nonpole parts of the P_{11} amplitude. A major part of the effect of true absorption is in $J^\pi=0^+$, which contributes little to the absorption cross section. The sensitivity of the results to the D state of the deuteron and the choice of the πNN form factor is investigated. For N-N scattering the agreement with the experimental phase shifts is better in the singlet than triplet channels. This is due to the absence of vector meson exchange in the calculation. The $^1D_2$ phase shift exhibits resonance behavior in the absence of a pole in the amplitude.

[NUCLEAR REACTIONS $pp\leftrightarrow\pi d$, $\pi d\to\pi d$. Total and differential cross section, polarization. $47<T_\pi<256$ MeV. N-N $I=1$ phase shifts.]

I. INTRODUCTION

Historically, the reactions

$$N+N\to N+N\ , \tag{1a}$$

$$\pi+d\to\pi+d\ , \tag{1b}$$

$$\pi+d\to N+N\ , \tag{1c}$$

have always been considered as three distinct reactions with different models developed for each of them separately. Thus, until very recently, N-N scattering was described by a real potential and most of the data and analyses were restricted to below the threshold for pion production. For that reason it was not considered important to include into the theory the coupling to the π-d or N-Δ channels. On the other hand, the quality of experimental π-d elastic data did not warrant the inclusion of real pion absorption into models that described pion-deuteron scattering. However, with the advent of the new meson facilities we have had an increase in the quality of data for both N-N scattering above the pion production threshold, and pion-nucleus scattering over the full range from $T_\pi=25$ MeV up through the resonance region. This new high quality data required more refined calculations and, in particular, the inclusion of the coupling between the different reactions. Thus, in low energy pion-nucleus scattering, it became apparent that true absorption plays an important role and should be included to achieve a fit to the data.[1,2] On the other hand, N-N scattering above the pion production threshold required a model for the inelasticity, as well as a possible explanation of the resonance behavior observed in $\Delta\sigma_L=\sigma^{\rm tot}(\rightleftarrows)-\sigma^{\rm tot}(\rightrightarrows)$ for p-p scattering.[3] For example, this resonance behavior might be due to coupling between the N-N and N-Δ channels, where the Δ is a π-N resonance.

The first attempt, in the spirit of the present theory, at a coupling between the reactions in Eq. (1) was due to Varma.[4] He calculated N-N scattering in the $I=0$ channels and allowed coupling to the πNN channel. This coupling was achieved by having a bound state in the P_{11} channel as suggested by Lovelace.[5] This model, commonly referred to as the bound state model (BSM), treats one of the nucleons in the NN channel as a π-N bound state. Although the results of Varma for the singlet N-N phase shifts were reasonable for high partial waves, the discrepancy with experiment in the $^1P_1$ channel was attributed to the inadequacy of the one term separable P_{11} interaction. This basic idea was further extended by Afnan and Thomas[6] to describe π-d scattering, pion production (and absorption), as well as N-N scattering, with reasonable success. However, it soon became apparent that treating one of the nucleons as a π-N bound state implies that only one of the nucleons can emit a pion and the resultant model excludes a whole class of diagrams.

The success of the BSM in giving a unified description of the reactions in Eq. (1) led to several formulations[7–15] of a unified theory of the NN-πNN system. These different approaches can be shown to give the same final set of coupled integral equations that satisfy two- and three-body unitarity.[16] These equations, which can be derived from field theory, have the form of Faddeev equations and overcome the difficulty of the BSM. In Sec. II, we present a simplified derivation of the equations, starting from the Faddeev equations and removing the undercounting problem. We then show how the equations reduce when one assumes that the two-body π-N and N-N input interactions are represented by separable potentials. We proceed in Sec. III to show what the origin of the two-body input is, and therefore how critical the final results might be to the parametrization of the input two-body interactions. In particular, we construct the π-N amplitude in the P_{11} channel to fit the experimental phase shifts, and show how we can divide this amplitude into a pole part and nonpole part. The pole part of this amplitude then determines the πNN form factor. Since our final results depend on the choice of P_{11} amplitude, we present various parametrizations corresponding to different values of the P_{11} scattering volume and range of the πNN form factor.

There have been several calculations that have examined (a) the effect of absorption on π-d elastic scattering in the resonance region,[17,18] and (b) the inclusion of production in N-N scattering.[19] However, with the exception of a preliminary report on the present work,[20] and the results of Betz and Lee,[21] who use a relativistic Hamiltonian[22] to calculate π-d elastic scattering and pion production, ours is the first detailed calculation of the three reactions in Eq. (1) where all three amplitudes are simultaneously calculated from a single set of integral equations. We therefore devote Sec. IV to a detailed analysis of each of the above reactions by comparing the theory with most of the available data (both elastic cross section and polarization) for pion laboratory energy $47 \leq T_\pi \leq 256$ MeV. We find that for pion production we can reproduce the experimental total and differential cross sections, except at high energies where the total cross section is slightly too large. This is most likely due to the fact that we have used nonrelativistic kinematics for the nucleons.[23] We also show that one needs to include more than s-, p-, and d-wave pions, and thus, the analysis of the experimental differential cross section in terms of γ_0, γ_2, and γ_4 (the coefficients of 1, $\cos^2\theta$, and $\cos^4\theta$, respectively) can be misleading. This is particularly the case since γ_6 (the coefficient of $\cos^6\theta$) can be large. A similar ambiguity exists in the analysis of the polarization asymmetry data in terms of λ_0, λ_1, and λ_2 only. Here we find λ_3 and λ_4 to be larger than λ_1, which might be the reason why most calculations[24–26] disagree with the experimental value of λ_1. The most serious discrepancy between our results and experiment is in λ_0. However, we find λ_0 to be very sensitive to choices of both the deuteron D-state probability and the range of the πNN form factor.

Turning to π-d elastic scattering we find that true absorption is quite important at all energies. Furthermore, the contribution of the pole and nonpole part of the P_{11} amplitude to the π-d differential cross section almost cancel each other. The extent of this cancellation depends to a large measure on the choice of the πNN form factor. In this way the π-d differential cross section is found to be sensitive to the choice of the πNN form factor. Comparing our results with the latest tensor[27] and vector[28] polarization data we find good agreement at 140 MeV. At 256 MeV, where theory and experiment do not agree very well, the uncertainty in the theory, especially in view of the sensitivity to the P_{11} interaction, is large enough to warrant further investigation before one can point to a failure in the theory.

Finally, we present our results for the $I=1$ N-N phase shifts. Here the agreement with experiment is not as good. This is due to the lack of heavy meson exchange which the theory allows. In general, the agreement in the singlet channels is better than in the triplet, indicating the need for vector meson exchange. We also show that one can extract a resonance width for the $^1D_2$ channel on the basis of speed analysis even though one might not have a resonance pole. In Sec. V we present some concluding remarks.

II. THE THEORY OF THE NN-πNN SYSTEM

In this section we present a derivation of the coupled equations for the NN-πNN system. The method is based on the removal of the major discrepancies in the three-body model of Afnan and Thomas (AT).[6] The advantage of this approach over previous derivations, which are more rigorous, is in the simplicity of the method and the fact that it presents most of the basic physical ideas.

The model of Afnan and Thomas[6] considers the πNN system as a three-body problem with the basic input being the two-body π-N and N-N amplitudes. The coupling to the N-N channel is achieved by taking the π-N interaction in the P_{11} channel to have a bound state with binding energy equal to the pion mass. In other words, the π-N amplitude in the P_{11} channel has a pole at a total energy equal to the nucleon mass. In this model pion absorption is nothing more than the rearrangement reaction $\pi+d \rightarrow N+N'$ in which N' is the π-N bound state in the P_{11} channel. Similarly, nucleon-nucleon scattering is taken as $N+N' \rightarrow N+N'$. Since the model is a pure three-body problem, the corresponding amplitudes satisfy the Faddeev[29] equations or the Alt-Grassberger-Sandhas[30] (AGS) equations

$$U^{(0)}_{\alpha\beta}=G_0{}^{-1}\bar{\delta}_{\alpha\beta}+\sum_{\gamma}\bar{\delta}_{\alpha\gamma}t_\gamma G_0 U^{(0)}_{\gamma\beta} , \tag{2}$$

where $\bar{\delta}_{\alpha\beta}=1-\delta_{\alpha\beta}$ and $G_0=(E-H_0)^{-1}$ is the free Green's function for the πNN system. For the two-body amplitude t_γ we use the labeling scheme where t_i $(i=1,2)$ is the π-N amplitude for the pion scattering off the ith nucleon, while t_3 is the N-N T matrix. The superscript in $U^{(0)}_{\alpha\beta}$ is used to indicate that this is a pure three-body amplitude. These equations, used by AT to calculate s-wave pion production and the effect of absorption on the π-d scattering length, suffer from two major flaws that lead to undercounting: (i) only one of the nucleons N' can emit the pion, and (ii) the intermediate state of two nucleons (NN') violate the Pauli exclusion principle in that N and N' are not identical. This lack of symmetry between N and N' leads to nonconservation of total spin in nucleon-nucleon scattering.[31] Both of these problems have been overcome in the more recent theories of the NN-πNN system.[14,15]

We now show how Eq. (2) can be modified to overcome the above problem of undercounting. Although the procedure we follow does not put in full perspective the problem associated with the dressing of the πNN form factor and NN propagator, it does give the correct equations and illustrates the source of some of the dressing. Using the notation that $i,j,\ldots=1,2$ and $\alpha,\beta,\ldots=1,2,3$ we can rewrite Eq. (2) as

$$U^{(0)}_{\alpha\beta}=G_0{}^{-1}\bar{\delta}_{\alpha\beta}+\sum_{i}\bar{\delta}_{\alpha i}t_i G_0 U^{(0)}_{i\beta} +\bar{\delta}_{\alpha 3}t_3 G_0 U^{(0)}_{3\beta} . \tag{3}$$

The π-N interaction t_i in the P_{11} channel can be written as the sum of a pole term $(t_i{}^{\rm P})$ and a nonpole term $(t_i{}^{\rm NP})$, i.e.,

$$\begin{aligned} t_i &= t_i{}^{\rm P}+t_i{}^{\rm NP} \quad (\text{in } P_{11} \text{ channel}) ,\\ &= t_i{}^{\rm NP} \quad (\text{otherwise}) , \end{aligned} \tag{4}$$

with

$$t_i{}^{\rm P}=f_i g_i f_i{}^{+} . \tag{5}$$

Here, $f_i{}^{+}$ is the dressed $\pi N \rightarrow N$ form factor and g_i is the N-N propagator with only the ith nucleon dressed. This form for the π-N amplitude can be deduced from a two-body equation involving the potential given by the diagrams in Fig. 1.[16] In this case $t_i{}^{\rm NP}$ is the amplitude due to a potential represented by the diagram in Fig. 1(b); it corresponds to the sum of all diagrams with at least two pions in every intermediate state. In Fig. 1(a) we have the diagram in terms of the bare πNN vertex and nucleon propagator.

We now can write the AGS equations as

$$U^{(0)}_{\alpha\beta}=G_0{}^{-1}\bar{\delta}_{\alpha\beta}+\sum_{i}\bar{\delta}_{\alpha i}f_i g_i f_i{}^{+}G_0 U^{(0)}_{i\beta} +\sum_{i}\bar{\delta}_{\alpha i}t_i{}^{\rm NP}G_0 U^{(0)}_{i\beta}+\bar{\delta}_{\alpha 3}t_3 G_0 U^{(0)}_{3\beta} . \tag{6}$$

The second term on the right hand side of Eq. (6) is presented diagrammatically in Fig. 2(a). To overcome the undercounting problem in the AT model we need to include the diagram in Fig. 2(b) on an equal basis with the diagram in Fig. 2(a),

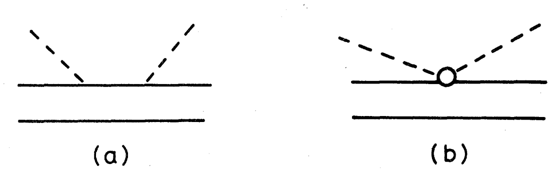

FIG. 1. The effective π-N potential in the P_{11} channel. (a) The pole part. (b) The nonpole part.

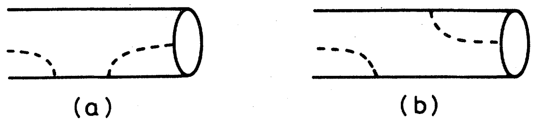

FIG. 2. Possible N-N intermediate states. (a) This diagram is included in the BSM. (b) Diagram to be included to overcome undercounting.

i.e., we need to add a term of the form

$$f_i\bar{\delta}_{ij}gf_j^{+}G_0U_{j\beta}\equiv V_{ij}G_0U_{j\beta}\ . \tag{7}$$

Here we have dropped the (0) superscript in the $U_{j\beta}$ on the grounds that the final amplitude will be modified due to the additional diagram in Fig. 2(b).

The introduction of the mechanism whereby the pion gets absorbed on one nucleon and is emitted by the other nucleon solves both deficiencies associated with the AT model. Firstly, the nucleon that absorbs the pion does not have to be the one that will emit a pion at a later stage. In fact, either nucleon can emit a pion with equal probability. Secondly, by introducing the above mechanism into the AGS equations, we have introduced contributions from diagrams such as those in Fig. 3. Here the diagrams in Figs. 3(a) and 3(b) give rise to the dressing of the other (spectator) nucleon. This puts the two nucleons on equal footing, and the NN intermediate states satisfy the Pauli exclusion principle. The inclusion of the two diagrams in Figs. 3(a) and 3(b) allows us to use the fully dressed N-N propagator g in Eq. (6) rather than g_i. On the other hand, the diagrams in Figs. 3(c) and 3(d) give the form factor dressing to guarantee the fact that f_i and f_j^{+} in Eq. (7) have been dressed to the same extent. Having included the contribution of the diagrams in Fig. 3 in the dressing of the NN propagator and πNN form factor, we need to guarantee that with the dressing completed we do not have any process whereby the pion is emitted by the ith nucleon and then gets absorbed or rescattered by the same nucleon; such diagrams lead to overdressing. This is achieved by introducing a $\bar{\delta}_{\alpha i}$ in front of the expression in Eq. (7). We now can write our modified equation as

$$\begin{aligned}U_{\alpha\beta}&=\bar{\delta}_{\alpha\beta}G_0^{-1}+\sum_i\bar{\delta}_{\alpha i}\hat{t}_iG_0U_{i\beta}+\sum_{ij}\bar{\delta}_{\alpha i}V_{ij}G_0U_{j\beta}\\&\quad+\delta_{\alpha 3}t_3G_0U_{3\beta}\\&=\bar{\delta}_{\alpha\beta}G_0^{-1}+\sum_{\gamma\rho}\delta_{\alpha\gamma}B_{\gamma\rho}G_0U_{\rho\beta}\ ,\end{aligned} \tag{8}$$

where

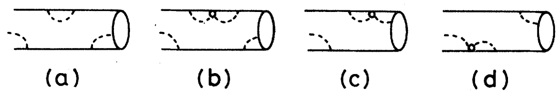

FIG. 3. Dressing diagrams that arise as a consequence of including the diagram in Fig. 2(b).

$$\underline{B}=\begin{bmatrix}\hat{t}_1 & V_{12} & 0\\ V_{21} & \hat{t}_2 & 0\\ 0 & 0 & t_3\end{bmatrix} \tag{9}$$

with

$$\hat{t}_i=f_igf_i^{+}+t_i^{\mathrm{NP}}\ . \tag{10}$$

We note that $\hat{t}_i$ as defined in Eq. (10) is an operator in three-body Hilbert space to the extent that both nucleons in g are equally dressed. This set of equations is identical to those derived by Afnan and Stelbovics[16] provided one used the π-N amplitude in the P_{11} channel either in the form suggested by Mizutoni and Koltun,[9] or as a solution of the Lippmann-Schwinger equation for the potential given by the diagrams in Fig. 4, as was suggested by Stingl and Stelbovics.[12]

It is now straightforward to derive the equations of Avishai and Mizutani (AM) (Ref. 14) and those of Afnan and Blankleider (AB) (Ref. 15) by introducing a new set of amplitudes given by

$$T_{\mu\nu}=U_{\mu\nu}\ , \tag{11a}$$

$$T_{\mu N}=\sum_j U_{\mu j}G_0f_j\ , \tag{11b}$$

$$T_{N\nu}=\sum_i f_i^{+}G_0U_{i\nu}\ , \tag{11c}$$

$$T_{NN}=\sum_{ij} f_i^{+}G_0U_{ij}G_0f_j\ , \tag{11d}$$

and using Eq. (8) to get the corresponding equations for the amplitudes $T_{\mu\nu}$, $T_{\mu N}$, $T_{N\nu}$, and T_{NN}.

For practical calculations it is essential to introduce separable interactions for both the π-N and N-N input amplitudes. In this way, and after partial wave expansion, we reduce our equations to a set of coupled linear integral equations in one

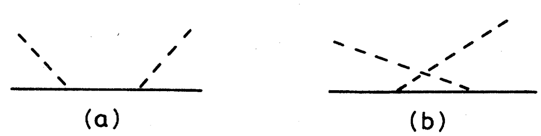

FIG. 4. The lowest order π-N interaction in the P_{11} channel.

dimension. To achieve this we write the nonpole part of the π-N amplitude as

$$t_i^{\mathrm{NP}} = |\phi_{\Delta_i}\rangle \tau_{\Delta_i} \langle \phi_{\Delta_i}| \;, \tag{12}$$

and the nucleon-nucleon amplitude as

$$t_3 = |\phi_d\rangle \tau_d \langle \phi_d| \;. \tag{13}$$

In writing Eqs. (12) and (13) we have introduced the simplified notation of "d" for all N-N interactions and "Δ" for all π-N interactions. In particular, the nonpole part of the P_{11} interaction is included in Δ.

The physical amplitudes for the reactions

$$\begin{aligned} \pi+d &\to \pi+d \\ &\to N+\Delta \\ &\to N+N \end{aligned} \tag{14}$$

are now given by

$$\begin{aligned} X_{dd} &= \langle \phi_d | G_0 U_{33} G_0 | \phi_d \rangle \\ X_{\Delta_i d} &= \langle \phi_{\Delta_i} | G_0 U_{i3} G_0 | \phi_d \rangle \;, \\ X_{Nd} &= \left\langle \chi_N \left| \sum_i f_i^{+} G_0 U_{i3} G_0 \right| \phi_d \right\rangle , \end{aligned} \tag{15}$$

where χ_N is the asymptotic NN wave function (i.e., it is a plane wave). In a similar manner we can define the physical amplitudes for the reactions

$$\begin{aligned} N+N &\to N+N \\ &\to N+\Delta \\ &\to \pi+d \end{aligned} \tag{16}$$

to be

$$\begin{aligned} X_{NN} &= \left\langle \chi_N \left| \sum_{ij} f_i^{+} G_0 U_{ij} G_0 f_j \right| \chi_N \right\rangle , \\ X_{\Delta_i N} &= \left\langle \phi_{\Delta_i} \left| \sum_j G_0 U_{ij} G_0 f_j \right| \chi_N \right\rangle , \\ X_{dN} &= \left\langle \phi_d \left| \sum_j G_0 U_{3j} G_0 f_j \right| \chi_N \right\rangle . \end{aligned} \tag{17}$$

We observe from the above definition of the physical amplitudes that either nucleon can emit the pion. It is now straightforward to derive a set of equations for the physical amplitudes using Eq. (8) and the definitions of the physical amplitudes [Eqs. (15) and (17)]. However, to minimize the number of coupled equations, and at the same time satisfy the Pauli exclusion principle, we will antisymmetrize the amplitudes in both the NN and πNN sector. To include the antisymmetry in the NN channel we will take $|\chi_N\rangle$ to be antisymmetric, while in the πNN channel we introduce the antisymmetric amplitudes for $\pi d \to N\Delta$ and $NN \to N\Delta$ by taking

$$X_{\Delta\alpha} = \frac{1}{\sqrt{2}} \left[X_{\Delta_2 \alpha} - X_{\Delta_1 \alpha} \right] \text{ for } \alpha = N, d \;. \tag{18}$$

We now can write a set of coupled integral equations for the antisymmetrized physical amplitudes, which in matrix form are

$$\underline{X} = \underline{Z} + \underline{Z}\,\underline{\tau}\,\underline{X} \;. \tag{19}$$

Here the matrix elements of $\underline{X}$ and $\underline{Z}$ are $X_{\alpha\beta}$ and $Z_{\alpha\beta}$ with $\alpha,\beta = N,\Delta,d$ and the matrix τ given by

$$[\underline{\tau}]_{\alpha\beta} = \tau_\alpha \delta_{\alpha\beta} \tag{20}$$

with

$$\tau_\Delta = \tau_{\Delta_1} = \tau_{\Delta_2} \tag{21}$$

and

$$\tau_N = \tfrac{1}{2} g \;. \tag{22}$$

The elements of the matrix $\underline{Z}$ are now given by

$$Z_{NN} = \left\langle \chi_N \left| \sum_{i,j} \bar{\delta}_{ij} f_i^{+} G_0 f_j \right| \chi_N \right\rangle , \tag{23a}$$

$$Z_{\Delta N} = \frac{1}{\sqrt{2}} \{ \langle \phi_{\Delta_2} | G_0 f_1 | \chi_N \rangle - \langle \phi_{\Delta_1} | G_0 f_2 | \chi_N \rangle \} \;, \tag{23b}$$

$$Z_{Nd} = \left\langle \chi_N \left| \sum_i f_i^{+} G_0 \right| \phi_d \right\rangle , \tag{23c}$$

$$Z_{\Delta\Delta} = -\langle \phi_{\Delta_2} | G_0 | \phi_{\Delta_1} \rangle \;, \tag{23d}$$

$$Z_{\Delta d} = \frac{1}{\sqrt{2}} \{ \langle \phi_{\Delta_2} | G_0 | \phi_d \rangle - \langle \phi_{\Delta_1} | G_0 | \phi_d \rangle \} \;. \tag{23e}$$

These equations are identical to those of AM (Ref. 14) and AB (Ref. 15) in the absence of heavy boson exchange. Although these equations are similar in form to those of AT,[6] they do not have any undercounting problems. In fact, one can relate the Z's defined in Eq. (23) to those used by AT (Ref. 6) as

$$Z_{\alpha\beta} = C_{\alpha\beta} Z_{\alpha\beta}^{\mathrm{AT}} \;, \tag{24}$$

where

$$C = \begin{bmatrix} 4 & 2 & 2\sqrt{2} \\ 2 & 1 & \sqrt{2} \\ 2\sqrt{2} & \sqrt{2} & 0 \end{bmatrix} . \tag{25}$$

We note that part of this difference is due to the fact that the AT amplitudes $X^{\rm AT}_{\alpha\beta}$ need to be normalized to get the physical amplitudes. In writing Eq. (24) we have assumed that the initial and final states are restricted to those allowed by the Pauli principle.

The fact that our final equations are identical in form to the Faddeev equations with separable potentials allows us to use the same methods used in solving the standard three-body problem for π-d scattering. In particular, we will use relativistic kinematics for the pion, maintaining a nonrelativistic description of the nucleons. This approximation was first used by Thomas[32] for π-d scattering with good success. In the present investigation we shall adopt the same approximation used by Thomas,[32] and we refer the reader to his paper for details. This will allow us to use his π-N amplitudes in the S_{11}, S_{31}, P_{33}, P_{31}, and P_{13}. For the P_{11} we will need an amplitude which can be written as the sum of a pole part plus a nonpole part. This we will present in the next section.

III. THE TWO-BODY INTERACTIONS

In setting up the equations for the NN-πNN system we have found it necessary to specify as input both the π-N and N-N interactions. Furthermore, to reduce the dimensionality of the final equations we need to take these interactions to be separable. Since we will be dealing with π-d scattering it might be obvious that we need, as input, the π-N amplitude to describe the multiple scattering of the pion off the two nucleons, and the N-N interaction to generate the deuteron wave function. However, because of the coupling between the π-d and N-N channels, our equations give both amplitudes simultaneously. We now have a bootstrap situation where we input the N-N amplitude to get an N-N amplitude out. Before we proceed to specify the input two-body amplitudes we should examine how they were introduced into the theory.

The input N-N and π-N amplitudes, in fact, arise from the truncation in the field theory to eliminate any explicit two or more pion intermediate states from the equations. This is most simply illustrated if one considers the Hamiltonian where the basic interaction is the πNN vertex. In this case the input N-N interaction to lowest order is given by one pion exchange with a spectator pion [Fig. 5(a)]. This diagram with a two pion intermediate state was replaced by a static N-N interaction with a spectator pion. This substitution removes the coupling to four particle states and in the process we lose four-body unitarity (i.e., the threshold for two pion production in N-N scattering). Thus the input N-N amplitude which is described by a potential incorporates in a phenomenological manner the contribution from multipion intermediate states. Here we hope our final results will not be very sensitive to the details of this N-N interaction. The other role the input N-N interaction plays is to describe the deuteron in the π-d channel. Here one needs to introduce an N-N interaction that gives a good description of the deuteron wave function. This is especially important as pion production ($NN\to\pi d$) and absorption ($\pi d\to NN$) are particularly sensitive to the short range behavior of the deuteron wave function and its D-state probability.

Having established the origin of our input N-N interaction we are now in a position to specify the N-N interactions used in the present investigation. For the ${}^3S_1$-${}^3D_1$ channel we need an interaction that both gives a good description of the deuteron wave function and is separable. This is best obtained by taking the unitary pole approximation (UPA) (Ref. 33) to one of the more sophisticated N-N potentials. In this way, we have a rank-one separable potential that gives the same deuteron wave function as the sophisticated N-N interaction. For the present investigation we use the UPA to the Reid soft core (RSC) (Ref. 34) and Bryan-Scott (BS) (Ref. 35) potentials. For the other N-N partial waves we have neglected the tensor coupling (i.e., $V_{ll'}=0$ for $l\neq l'$) and used the separable potentials of Mongan[36] which give a reasonable fit to the experimental phase shifts. From Mongan's many parametrizations of the N-N potential we have taken those of the form

$$V_l(k',k)=\sum_{i=1}^{n} g_{li}(k')\lambda_{li}g_{li}(k)\ , \tag{26}$$

where

$$g_{li}(k)=\frac{C_{li}k^l}{(k^2+\beta_{li}{}^2)^{\gamma}}\ , \tag{27}$$

and, for convenience, the parameters in the different channels are given in Table I. The above choice of N-N interaction is consistent with our choice of nonrelativistic kinematics for the nucleons.

The input π-N interaction in our model can be divided into two parts: (i) The nucleon pole part of the amplitude ($t^{\rm P}$) which gives the mechanism for pion absorption and production ($\pi d\rightleftharpoons NN$). This

TABLE I. Parameters for the Mongan N-N potentials for the form factors in Eq. (27).

N-N channel	γ	C_{l1} ($\mathrm{fm}^{1+l-2\gamma}$)	β_{l1} (fm^{-1})	λ_{l1}	C_{l2} ($\mathrm{fm}^{1+l-2\gamma}$)	β_{l2} (fm^{-1})	λ_{l2}
${}^1S_0$	1	21.498 67	6.157	+1	1.945 558	1.786	−1
${}^1P_1$	$\frac{3}{2}$	35.593 82	2.951	+1			
${}^3P_0$	2	94.608 38	5.000	+1	1.922 066	1.462	−1
${}^3P_1$	2	14.258 89	2.661	+1			
${}^3P_2$	2	8.720 49	2.720	−1			
${}^1D_2$	2	1.501 35	1.944	−1			
${}^3D_2$	2	1.466 47	1.468	−1			
${}^3D_3$	2	35.152 46	6.558	+1	0.549 284	1.451	−1

has been written in terms of the dressed πNN vertex and nucleon propagator [see Eqs. (4) and (5)]. (ii) The nonpole part of the amplitude (t^{NP}) which describes the multiple scattering of the pion off the two nucleons. This second part of the amplitude arises from the elimination of explicit two-pion intermediate states in our equations. Thus, for the Hamiltonian, where the only interaction is the πNN vertex, the lowest order contribution to t^{NP} is given by the diagram in Fig. 5(b). To eliminate two pion intermediate states from the equations, we replace diagrams like Fig. 5(b) by a static potential that gives the nonpole part of the π-N amplitude with a spectator nucleon. Because of isospin and angular momentum conservation, the pole part of the π-N amplitude contributes only to the P_{11} channel. Thus t^{NP}, in all channels but the P_{11}, has to be adjusted to fit the experimentsl π-N phase shifts. Such a parametrization of the π-N amplitude in terms of one-term separable potentials is given by Thomas[32] and was used by him to solve the Faddeev equations for π-d scattering. Since our choice of kinematics is identical to that of Thomas, we will use his parametrization of the π-N amplitude.

In the P_{11} channel the full π-N amplitude is the

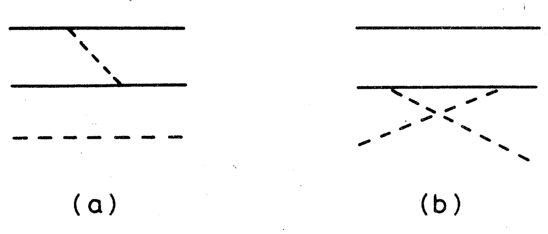

FIG. 5. Two-pion intermediate states that are replaced by a static N-N (a) and π-N (b) potential.

sum of a pole part plus a nonpole part (i.e., $t=t^{\mathrm{P}}+t^{\mathrm{NP}}$). The theory of the NN-πNN system[14,15] demands that the pole part be of the form given in Eq. (5) with the dressing in both the πNN vertex and nucleon propagator written in terms of the nonpole part of the amplitude. Such a parametrization of the P_{11} channel leads to both mass and wave function renormalization, as well as energy dependence for the πNN vertex. To avoid the renormalization problem and the energy dependence of the πNN vertex, we have fit the experimental phase shifts and scattering volume with a two-term separable potential. The requirement of a two-term potential is dictated by the fact that the P_{11} phase shifts change sign at an energy $T_\pi \sim 150$ MeV. To divide the amplitude into a pole and nonpole part we have adjusted the parameters of the potential so that the amplitude has a pole at the nucleon mass. To see how this division is achieved, we write our two-term potential in matrix form as

$$V=|\underline{v}>\underline{\lambda}<\underline{v}| \ , \tag{28}$$

where the form factor $|\underline{v}\rangle=(|v_1\rangle\,|v_2\rangle)$ and the strength matrix is given by

$$[\underline{\lambda}]_{ij}=\delta_{ij}\lambda_i, \quad i,j=1,2 \ . \tag{29}$$

The corresponding amplitude can be written as

$$t(E)=|\underline{v}\rangle\underline{\tau}(E)\langle v| \ , \tag{30}$$

where

$$\underline{\tau}(E)=[\underline{\lambda}^{-1}-\underline{G}(E)]^{-1} = \frac{\underline{N}(E)}{D(E)} \tag{31}$$

with

$$[\underline{G}(E)]_{ij}=G_{ij}(E)$$
$$=\langle v_i \mid G_0(E) \mid v_j \rangle \ , \qquad (32)$$

$$D(E)=\det[\underline{\tau}^{-1}(E)] \ , \qquad (33)$$

and

$$N_{11}(E)=\lambda_2{}^{-1}-G_{22}(E) \ ,$$
$$N_{22}(E)=\lambda_1{}^{-1}-G_{11}(E) \ , \qquad (34)$$
$$N_{21}(E)=N_{12}(E)=G_{12}(E) \ .$$

In the above equations $G_0(E)$ is the free π-N Green's function. For $E \simeq M$ we can write

$$D(E)=D(M)+(E-M)A$$
$$+(E-M)^2 B(E) \ . \qquad (35)$$

The requirement that the π-N amplitude $t(E)$ have a pole at the nucleon mass (i.e., $E=M$) is equivalent to taking $D(M)=0$. Thus our amplitude in the vicinity of the pole is given by

$$t(E) \underset{E\sim M}{\sim} \frac{|\underline{v}\rangle \underline{N}(M) \langle \underline{v}|}{(E-M)A}$$
$$=\frac{|f\rangle\langle f|}{E-M} \equiv t^{\mathrm{P}}(E) \ , \qquad (36)$$

where

$$|f\rangle=\frac{1}{|A|^{1/2}}\{\ |v_1\rangle \, |N_{11}(M)|^{1/2}$$
$$-\ |v_2\rangle \, |N_{22}(M)|^{1/2} s\} \qquad (37)$$

with s being the sigh of $[\lambda_1{}^{-1}-G_{11}(M)]$. Having defined the pole part of the amplitude, we can write the nonpole part as

$$t^{\mathrm{NP}}(E)=t(E)-t^{\mathrm{P}}(E) \ . \qquad (38)$$

In this way we have divided the amplitude and thus determined the πNN vertex $|f\rangle$, which in this case is independent of energy. If one thinks of the pole as a π-N bound state, then the normalized wave function for the bound state is given by

$$|\psi\rangle=G_0(M)\,|f\rangle \ . \qquad (39)$$

To fit the experimental data we take the form factor for the potential to be

$$v_i(k)=\frac{C_i k_i^{\,n_i}}{(k^2+\beta_i^2)^{m_i}}, \quad i=1,2 \ . \qquad (40)$$

The requirement that the wave functions have the correct asymptotic behavior implies that $n_1=1$, $n_2 \geq 1$, and $2m_i > n_i$.

To test the sensitivity of our final results to our choice of P_{11} interaction, we have constructed several parametrizations of the potential (see Table II). These potentials give different values for the scattering volume a_{11} and πNN coupling constant, but give similar fits to the phase shifts for $T_\pi < 250$ MeV as illustrated in Fig. 6. The experimental data is that of Carter, Bugg, and Carter.[37] The failure of all these potentials to fit the higher energy phase shifts is due to the fact that our amplitude does not incorporate the Roper resonance. However, it is possible to build this resonance into the amplitude and improve the high energy phase shifts if found necessary.[38] Two of the potentials, $B\phi8$ and $C\phi8$, have different analytic expressions for $v_i(k)$ but give approximately the same scattering volume and πNN coupling constant. In this way we can test the sensitivity of our results to the "range" of the πNN form factor. In Fig. 7, we give the πNN form factor $k^{-1}f(k)$ as a function of momentum. In categorizing the differences between the πNN form factors shown in Fig. 7, we shall use both the scattering volume a_{11}, which to a large extent corresponds to the maximum values

TABLE II. Parameters for the Yamaguchi form factors of Eq. (40) used to describe two-term separable potentials in the P_{11} channel. The resulting potentials are named according to the values of n_i and m_i, and the values of the scattering volume a_{11} they generate.

Potential label	m_1	n_2	m_2	C_1 (fm^{2-2m_1})	β_1 (fm)	C_2 ($\mathrm{fm}^{n_2-2m_2+1}$)	β_2 (fm)	a_{11} $m_\pi{}^{-3}$	$\frac{f^2}{4\pi}$
$B\phi5$	2	5	3	24.359291	3.10	1.3067105	4.45	−0.0506	0.040
$B\phi8$	2	5	3	20.560579	2.80	1.2546282	4.00	−0.0807	0.068
$B11$	2	5	3	16.092255	2.50	1.1343013	3.05	−0.1099	0.102
$C\phi8$	3	3	2	906.18622	3.65	1.1312287	3.80	−0.0814	0.073

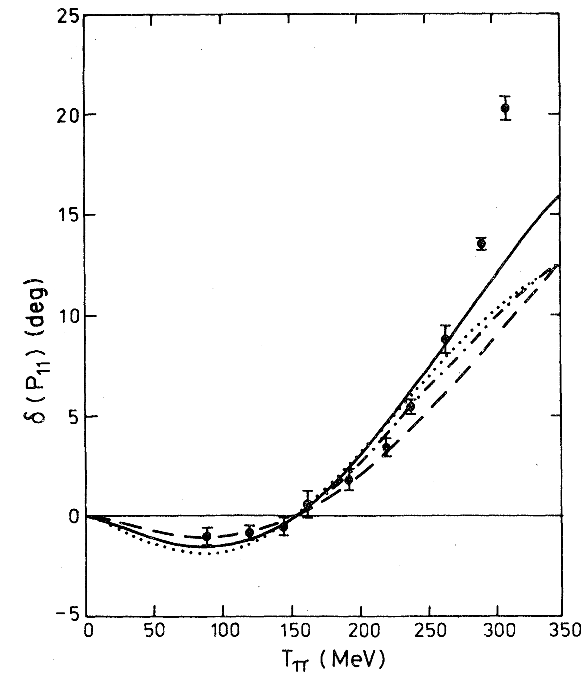

FIG. 6. The P_{11} phase shifts resulting from two-term separable potentials $B\phi5$ (-----), $B\phi8$ (——), $B11$ (·····), and $C\phi8$ (—·—·—) The experimental points are from Carter *et al.* (Ref. 37).

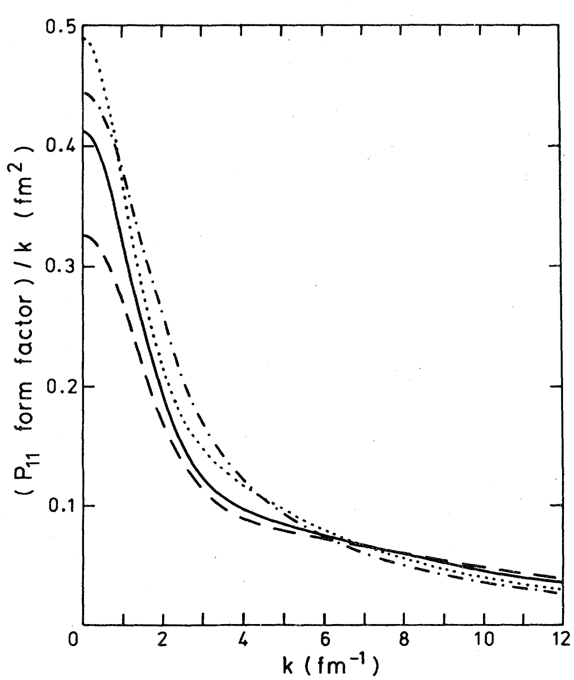

FIG. 7. Plot of $k^{-1}f(k)$, where f is given by $t^{\mathrm{P}} = f^2(E+m_\pi)^{-1}$ and results from a two term separable potential. The curves are labeled as in Fig. 6.

of the form factors at $k=0$, and the term range to specify the behavior of the form factors in the region $0<k<6$ fm^{-1}. In particular we note that the $B\phi5$, $B\phi8$, and $B11$ form factors, which all arise from the same analytic forms, have similar falloff rates and therefore never cross, while $C\phi8$, originating from a different analytic form, has a slower falloff and, in fact, crosses the $B11$ curve at $k\sim1$ fm^{-1}. This, in effect, means that by range we shall imply some particular analytic form for the P_{11} interaction. Beyond 6 fm^{-1} all form factors are similar and small. We will see that our final results for pion production are very sensitive to the choice of this form factor.

IV. NUMERICAL RESULTS

Having established our equations for the NN-πNN system which give the amplitudes for the reactions

$$\begin{aligned} \pi+d &\to \pi+d \\ &\to N+N \end{aligned} \tag{41a}$$

and

$$\begin{aligned} N+N &\to N+N \\ &\to \pi+d\ , \end{aligned} \tag{41b}$$

and having specified the input to these equations, we can proceed to discuss the success of this model in predicting the experimental results for the above reactions. We will restrict our results to the energy region $47<T_\pi<256$ MeV, i.e., from just above the threshold for pion production through the Δ resonance. However, before we proceed to the discussion of our results, we should point out certain limitations that have been imposed on our calculations:

(i) In the general formulation of the NN-πNN equations there is the scope for introducing heavy meson ($\rho,\omega,\ldots$) exchange as an approximation to nonladder multipion exchange diagrams.[15] In this initial calculation we have neglected such heavy meson exchange contributions. This can have its most serious effect in our description of N-N scattering, where the absence of vector meson exchange leads to almost no spin-orbit interaction. The absence of ρ exchange from our description of $pp\leftrightarrow\pi d$ is not as serious, in that we may be able to compensate for it by our choice of the πNN form factor. Furthermore, it is not definite that ρ exchange is important in pion absorption and elastic scattering.[39]

(ii) In the last section we showed how we constructed a P_{11} amplitude that fits the phase shifts and scattering volume, and how we split that amplitude into a pole and nonpole part. In this way we obtained a πNN form factor that was a real function and independent of energy. However, the formal theory prescribes how the P_{11} is to be split.[9,15] In that case the πNN form factor is energy dependent and complex, and the energy dependence of this form factor is required to prove three-body unitarity.[16] In that respect our results do not strictly satisfy three-body unitarity. We will also discover that the π-d elastic cross section may be sensitive to the way the P_{11} amplitude is divided.

(iii) Considering the fact that we will be investigating the reaction $pp\to\pi d$ up to proton laboratory energies of 800 MeV, we should use relativistic kinematics for the nucleons. Unfortunately, there are ambiguities in the choice of a relativistic theory short of the Bethe-Salpeter equations.[40] To avoid this ambiguity, and be consistent with the common description of pion elastic scattering and production in heavier nuclei, we have used relativistic kinematics for the pion and nonrelativistic kinematics for the nucleons. This procedure was first used by Thomas[32] for low energy π-d elastic scattering, and we refer the reader to that paper for the details of the kinematics.

(iv) Finally, we have made two approximations in order to perform the numerical calculations. First we had to restrict the two-body input. For the π-N interaction we limited ourselves to s- and p-wave amplitudes since the d-wave π-N phase shifts are small in the energy region of interest. On the other hand, for the N-N input interaction we have included the S-, P-, and D-wave amplitudes given in the last section. Even with this restriction on two-body input, the number of coupled integral equations is too large for all practical purposes. We therefore have restricted our three-body channels to those found most important for the final cross sections.[41] Second, in converting our integral equations to algebraic equations, we have used the method of contour rotation commonly used in the solution of the Faddeev equations.[43] For the Gauss quadratures we used 20 points for all integrals except the $J^\pi=2^+$ channel, where we needed 32 points. The need for a 32 point mesh in $J^\pi=2^+$, the largest amplitude, was required to get a smooth energy dependence for the total cross sections. With this choice of quadratures and a contour rotation angle of $-10.2°$, we have at most an error of 5% in all amplitudes except the N-N, where in some cases the error was as large as 10%.

In this section we will basically examine the three distinct reactions $\pi d\to\pi d$, $pp\to\pi^+d$, and $pp\to pp$ in some detail. Although in our formulation the corresponding amplitudes come from a solution of a single integral equation, traditionally these reactions have been discussed separately. For convenience we will continue this tradition and discuss our results for each reaction in turn.

A. Pion production in N-N scattering

One of the main features of the present theory is the coupling between the π-d and N-N elastic channels. With it we can calculate the effect of pion absorption (production) on π-d (N-N) elastic scattering. However, before we can study these effects on the elastic channels we need to know how successful the theory is in describing the coupling between the elastic channels, i.e., the reaction $pp\leftrightarrow\pi^+d$. Furthermore, pion absorption on the deuteron has been of considerable interest in recent years because it is the (π,p) reaction on the simplest nucleus, and several models have been developed to describe the mechanism for this reaction. We will compare our results with these models.

As most of the recent experimental data has been devoted to measurements of $pp\to\pi^+d$ and $np\to\pi^0d$ where only the projectile proton is polarized, we will restrict our discussion to the predictions of our model for the observables of the reaction $\vec{p}p\to\pi^+d$. In this way we also will be able to compare our results with those of other models. The differential cross section for $\vec{p}p\to\pi^+d$ is usually written as

$$\frac{d\sigma}{d\Omega}=\frac{1}{32\pi}[(\gamma_0+\gamma_2\cos^2\theta+\gamma_4\cos^4\theta+\cdots)+\vec{P}\cdot\hat{N}\sin\theta(\lambda_0+\lambda_1\cos\theta+\lambda_2\cos^2\theta+\cdots)]\,, \tag{42}$$

where $\vec{P}$ is the polarization of the incident beam, $\hat{N}$ is a unit vector in the direction $\vec{k}_p\times\vec{k}_\pi$ (with the z axis defined to be along the incident proton's momentum $\vec{k}_p$), and θ is the angle between $\vec{k}_p$ and k_π; the coefficients γ_i ($i=0,2,4,\ldots$) and λ_i ($i=0,1,\ldots$) may be expressed in terms of bilinear combinations of the transition amplitudes $a^J_{l's',ls}$ for the reaction $pp \to \pi^+ d$.[42] In the partial wave analysis we have retained amplitudes up to h-wave pions in the final state. The resultant possible amplitudes are enumerated in Table III, and for convenience we have given them a sequential designation a_i ($i=0,\ldots,14$). Note that $a_0,\ldots a_6$ are the same amplitudes (including normalization) as defined by Mandl and Regge[44] and used by Dolnick.[45] They are related to our amplitudes X of Sec. II by

$$a_I = \left[4\pi^3 \frac{k_f}{k_i}\mu_i\mu_f(2l_i+1)\right]^{1/2} X_I \ , \tag{43}$$

where k_i (k_f) is the initial (final) center of mass momentum, l_i specifies the initial orbital angular momentum, and μ_i (μ_f) is the "reduced" mass in the initial (final) channel and is given by

$$\mu_i = \frac{\omega_1(k)\omega_2(k)}{\omega_1(k)+\omega_2(k)} \quad \left[\mu_f = \frac{\omega_3(k)\omega_4(k)}{\omega_3(k)+\omega_4(k)}\right] , \tag{44}$$

where $\omega_j(k)=(k^2+m_j{}^2)^{1/2}$; here, m_1 and m_2 are the masses of the initial particles while m_3 and m_4 are the masses of the final particles.

Because of the sensitivity of the cross section for $\pi^+ d \to pp$ to the off-shell behavior of the π-N interaction in the P_{11} channel (i.e., the πNN vertex), we have chosen the potential in this channel ($B\phi 8$) which fits the π-N data and gives the best fit to the total cross section at one energy. In this way we might have compensated for the lack of ρ exchange in our description of $\pi^+ d \to pp$. In Fig. 8 we compare our total cross section for $\pi^+ d \to pp$ with the corresponding experimental data from Richard-Serre *et al.*[46] The agreement between theory and experiment is, in general, very good except at high energies, where our model gives a slightly larger total cross section. This good agreement between theory and experiment indicates that the inelasticity in both π-d and N-N scattering is of the right magnitude.

To see if the mechanism for pion production (absorption) is correctly represented, we need to examine the differential cross section with unpolarized incident proton, the coefficient of $\sin\theta$ in Eq. (42) is zero. In Fig. 9, we compare the results of our model with experiment for the differential cross section for $pp \to \pi^+ d$ at proton laboratory energies T_p =382.9, 425.0, 451.4, 492.9, 533.0, 567.4, 647.3, 751.3, and 799.3 MeV. We have followed the common practice of plotting these versus $\cos^2\theta$, as then any deviations from a straight line will imply the presence of π-d partial waves with $l_{\pi d} > 1$. The displayed experimental data of Hürster *et al.*[47] (T_p =382.9, 425.0, 451.4, 492.9, 533.0, and 567.4 MeV) have been scaled, as in their original form they were normalized to give $d\sigma/d\Omega = 1$ at $\theta = 0°$. The other experimental data are due to Axen *et al.*[48] (T_p=382.9 MeV), Dolnick[45] (T_p=425 MeV), Aebischer *et al.*[49] (T_p=451.4 MeV), Richard-Serre *et al.*[46] (T_p=567.4, 647.3, 751.3 MeV), and Nann *et al.*[50] (T_p=799.3 MeV). Up to

TABLE III. The partial waves, up to h-wave pions, contributing to the reaction $pp \to \pi^+ d$.

Designation for $a^J_{l'S',lS}$	J^π	Pion wave	l' (πd)	S' (πd)	l (pp)	S (pp)	p-p state
a_0	0^+	p	1	1	0	0	$^1S_0$
a_1	1^-	s	0	1	1	1	$^3P_1$
a_2	2^+	p	1	1	2	0	$^1D_2$
a_3	1^-	d	2	1	1	1	$^3P_1$
a_4	2^-	d	2	1	1	1	$^3P_2$
a_5	2^-	d	2	1	3	1	$^3F_2$
a_6	3^-	d	2	1	3	1	$^3F_3$
a_7	2^+	f	3	1	2	0	$^1D_2$
a_8	4^+	f	3	1	4	0	$^1G_4$
a_9	3^-	g	4	1	3	1	$^3F_2$
a_{10}	4^-	g	4	1	3	1	$^3F_4$
a_{11}	4^-	g	4	1	5	1	$^3H_4$
a_{12}	5^-	g	4	1	5	1	$^3H_5$
a_{13}	4^+	h	5	1	4	0	$^1G_4$
a_{14}	6^+	h	5	1	6	0	$^1I_6$

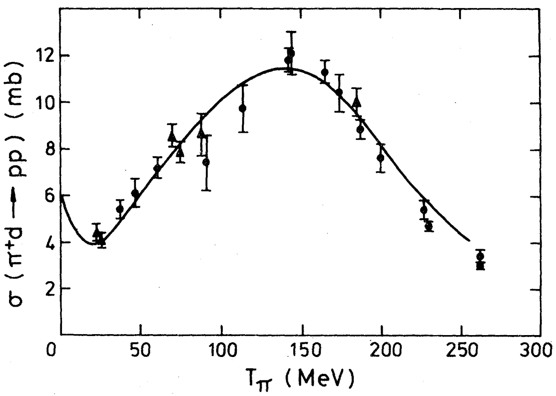

FIG. 8. The total cross section for $\pi^+d\to pp$. Experimental data are of Richard-Serre *et al.* (Ref. 46).

the resonance region (T_p =567.4 MeV), the calculated differential cross section is in very good agreement with experiment and shows very little deviation from a straight line. At 425 MeV, our calculation favors the data of Hüster *et al.*[47] over the ones of Dolnick,[45] which seem to have too large a slope. Above the resonance region, our cross sections become progressively larger than experiment, although we still reproduce the shape of the angular distribution quite well. A similar effect appears in π-d scattering and has been shown by Rinat and Thomas[23] to be due to the use of nonrelativistic kinematics for the nucleons. Thus our choice of kinematics might be the reason for the calculated cross section being too large at high energies.

The experimental data is often analyzed in terms of the parameters γ_i and λ_i [see Eq. (42)]. This analysis is commonly based on including only s- and p-wave pions and more recently d-wave pions, i.e., in Eq. (42) one usually retains only γ_0, γ_2, γ_4, λ_0, λ_1, and λ_2 terms. Furthermore, some of the models for $pp\to\pi^+d$ present their results by comparing the theoretical values of γ_i and λ_i with those extracted from experiment. To check the convergence of the partial wave expansion given in Eq. (42), we have included the parameters γ_6, γ_8, λ_3, and λ_4 in our calculation. They arise from at least f-, g-, d-, and f-wave pions, respectively. In Fig. 10 we present our values for γ_0 to γ_8 as a function of the pion laboratory energy T_π. Perhaps the most interesting feature of our results is the relatively large size of the commonly ignored γ_6. This throws doubt on any analysis in terms of just γ_0, γ_2, and γ_4, especially regarding extracted values of γ_4. In addition, the parameter γ_8 be-

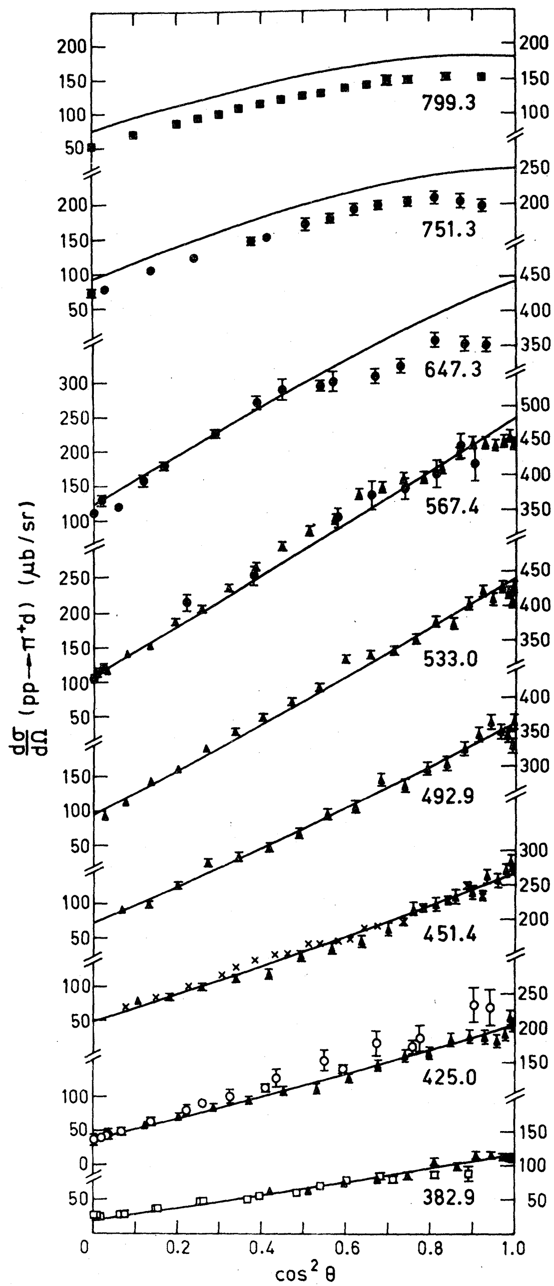

FIG. 9. Differential cross section for $pp\to\pi^+d$ using the UPA to the Reid soft core potential and P_{11} interaction $B\phi8$. The experimental results are those of Axen *et al.* (Ref. 48) (□), Dolnick (Ref. 45) (○), Aebischer *et al.* (Ref. 49) (×), Richard-Serre (Ref. 46) (●), Nann *et al.* (Ref. 50) (■), and Hürster *et al.* (Ref. 47) (▲). The data of Hürster was scaled since it was normalized to 1 at zero degrees.

comes appreciable above the resonance and may not be neglected, especially around T_π =200 MeV, where γ_4 goes to zero.

Having established the importance of the higher

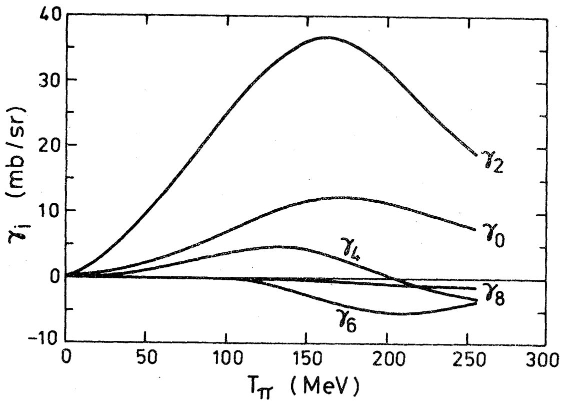

FIG. 10. The γ_i coefficients defined in Eq. (42), as a function of pion laboratory energy.

γ_i terms ($l_{\pi d} > 2$), we see from Table III that one needs to retain partial wave amplitudes at least up to $J^\pi = 4^+$ ($l_{\pi d} = 3$). Although Niskanen[24] first pointed out the importance of the "higher partial wave" $J^\pi = 3^-$ ($l_{\pi d} = 2$), the role of the $J^\pi = 4^+$ wave has so far not been investigated. To examine this point we compare, in Fig. 11, the effects of neglecting the $J^\pi = 3^-$ and 4^+ on the differential cross sections at both the low energy $T_\pi = 383$ MeV and the resonance energy $T_\pi = 567$ MeV. We find that neglecting these higher partial waves

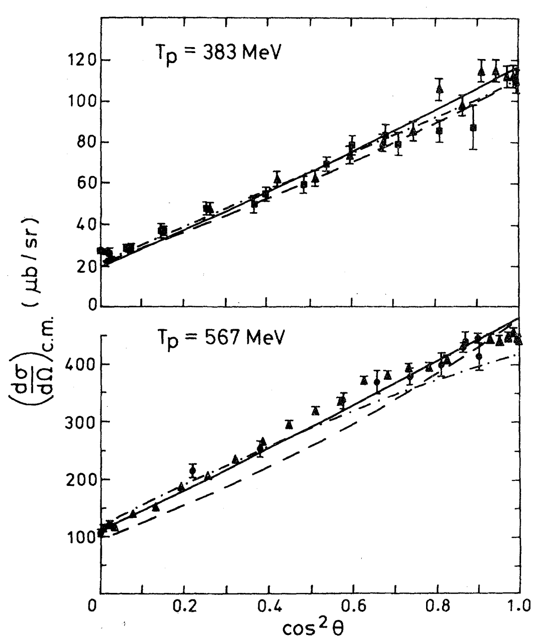

FIG. 11. The effect of excluding the $J^\pi = 3^-$ or 4^+ amplitude on the differential cross section for $pp \rightarrow \pi d$. (——) full calculation, (-----) no 3^- contribution, (—·—·—) no 4^+ contribution. The experimental results as in Fig. 9.

changes the shape of the angular distribution. In fact, with all partial waves included, the differential cross section versus $\cos^2\theta$ is almost a straight line even at the higher energy. This has often been interpreted as absence of any contribution from partial waves with $l_{\pi d} > 1$. Yet removal of the $J^\pi = 3^-$ or 4^+ amplitude from the calculation of the cross section gives an angular distribution that deviates from a straight line. An analysis of the data in terms of γ_0, γ_2, and γ_4 could have given a small value for γ_4 which in turn could be erroneously considered as evidence for small contributions from amplitudes with $l_{\pi d} \geq 2$. This illustrates the extreme care that must be taken when interpreting data in terms of the γ_i parameters. To make matters far worse, the parameters γ_4, γ_6, and γ_8 are highly correlated, so that they cannot be unambiguously determined even from the most accurate available experimental data. Thus, the importance of higher partial waves leads us to suggest that one compare theory directly with the measured cross section.

Comparing our results with those of Chai and Riska[25] we note that they found ρ exchange important in determining γ_4. In particular, γ_4 calculated with just π exchange was always positive and ρ exchange was needed to bring its value down to the usual negative "experimental" value. In view of the uncertainty in the experimental value of γ_4, the large contribution from γ_6, and the fact that we fit the experimental cross section, it is no longer clear that (for pion production) one needs a mechanism-like ρ exchange to produce agreement with the experimental unpolarized differential cross sections.

We now turn to the polarization parameters λ_i ($i = 0, 1, \ldots$). In Fig. 12 we compare the calculated values of λ_0, λ_1, and λ_2 with the corresponding experimental values.[50–52] Although we get a good agreement for the largest parameter λ_2, we fail to reproduce the experimental values of λ_0 and λ_1. As Chai and Riska[25] have shown, including ρ exchange does not improve the situation at low energies (although above the resonance region there might be some effect). A careful examination of the contributions to λ_0 shows that it is very sensitive to the $J^\pi = 3^-$ (i.e., ${}^3F_3$ N-N channel) amplitude. The absence of ρ exchange in our N-N channel can affect the distortion in the triplet N-N channels and in this way affect our calculated value of λ_0. In addition λ_0 is very sensitive to both the range of the πNN form factor and the D-state probability of the deuteron. This is illustrated in Fig. 18, where the analyzing power (which at 90°

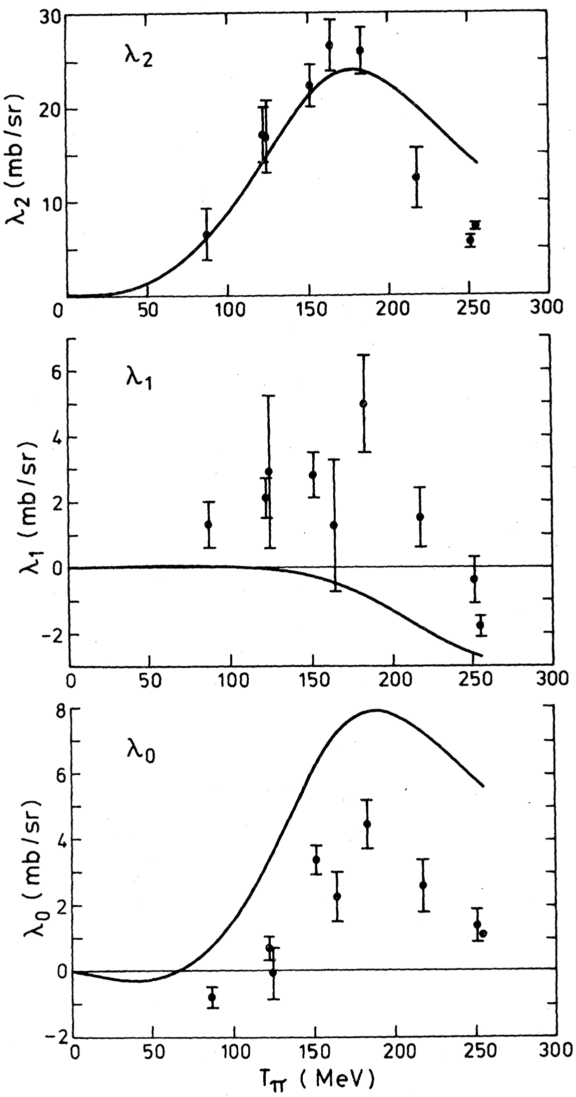

FIG. 12. The asymmetry parameters defined in Eq. (42). The experimental data are from Refs. 50–52.

is just λ_0/γ_0) is given at $T_p=383$ MeV for different deuterons and πNN form factors.

For comparison with experiment of the parameter λ_1 (Fig. 12), it is interesting to note that our calculation together with those of Chai and Riska,[25] Niskanen,[24] and Maxwell *et al.*[26] all give negative values around the resonance energy regions, while the empirical values are positive. As was the situation with γ_4, we attribute this to the fact that most experimental analyses ignore the contribution of higher angular momentum [i.e., they neglect γ_i $(i=3,4\ldots)$]. In Fig. 13 we compare the relative sizes of the asymmetry parameters λ_0 to λ_4. Not only are the usually ignored λ_3 and λ_4 significant, they are comparable in magnitude to λ_0 and larger

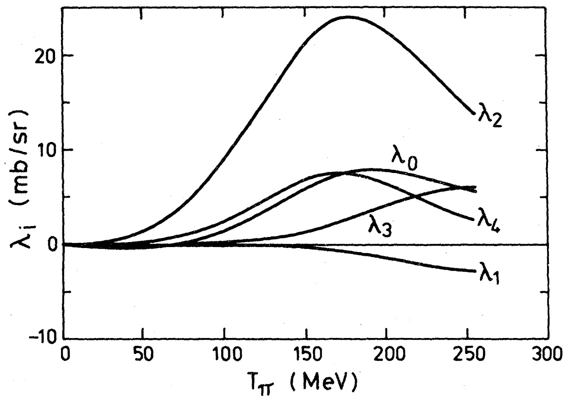

FIG. 13. The asymmetry parameters defined in Eq. (42) as a function of pion laboratory energy.

than λ_1 in the present model. Moreover, both λ_3 and λ_4 are positive and may therefore account for the positive values of λ_1 extracted from experiment.

It is interesting to note that while λ_0, λ_2, and λ_4 display resonance behavior, λ_1 and λ_3 do not, at least in the energy region we have considered. This is easy to understand when one considers that the large P_{33}-dominated amplitude a_2 does not contribute at all to λ_1 and λ_3. Curiously, though, both Chai and Riska[25] and Niskanen[24] get λ_1 to obtain a strong minimum value at about the resonance energy.

It is by now clear that the use of the expansion given by Eq. (42) is not satisfactory for comparing theory with experiment. As we have shown, this is due to the slow convergence of the expansion. However, the decisive problem is the large degree of correlation among the higher γ_i and λ_i parameters making their determination from experiment very inaccurate. To remove the problem of correlations and possibly improve convergence, it has recently been proposed[53] to expand the $pp\rightarrow\pi^+d$ observables in terms of orthogonal functions rather than powers of $\cos\theta$ as done in Eq. (42). In particular, it has been suggested that the differential cross section for $\vec{p}p\rightarrow\pi^+d$ be written as

$$\frac{d\sigma}{d\Omega}=\frac{1}{4\pi}\{[a_0+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)+\cdots] +\vec{\mathrm{P}}\cdot\hat{N}[b_1P_1{}^1(\cos\theta)+b_2P_2{}^1(\cos\theta) + b_3P_3{}^1(\cos\theta)+\cdots]\}\,, \quad (45)$$

where P_l and $P_l{}^1$ are the associated Legendre functions P_l^m with $m=0$ and $m=1$, respectively,[54] and the coefficients a_i (not to be confused with the amplitudes of Table III) and b_i are the counterparts of the parameters γ_i and λ_i, respectively. To test the convergence of this series we have calculated the a_i and b_i, and compare their relative sizes in Figs. 14 and 15. We find rapid convergence for the a_i with a_8 being two orders of magnitude smaller than the already small a_6. Likewise for the b_i we find that b_2, b_4, and b_5 are, in general, a magnitude smaller than b_1 and b_3, with higher terms a magnitude smaller again. This may be contrasted with the convergence of the γ_i, where γ_8 is appreciable, and the especially slow convergence of the λ_i, where we find even λ_6 to be of a comparable size to that of λ_3 and λ_4. We conclude, therefore, that Eq. (45) holds promise of providing a much more practical partial wave decomposition of $d\sigma/d\Omega$ for $\vec{p}p\to\pi^+d$ than does the standard one of Eq. (42), and we look forward to its use in future analyses of experimental data.

Before we proceed to a discussion of the effect of absorption (production) on π-d (N-N) elastic scattering, we should demonstrate the sensitivity of our results to the input two-body interactions. Having established the importance of comparing theory with the directly measured cross section, we have chosen to study the effect of the deuteron and P_{11} interactions on the differential cross section and analyzing power $A_\pi(\theta)$ given by

$$A_\pi(\theta)=\frac{1}{P}\frac{\dfrac{d\sigma}{d\Omega}(\uparrow)-\dfrac{d\sigma}{d\Omega}(\downarrow)}{\dfrac{d\sigma}{d\Omega}(\uparrow)+\dfrac{d\sigma}{d\Omega}(\downarrow)}\,, \tag{46}$$

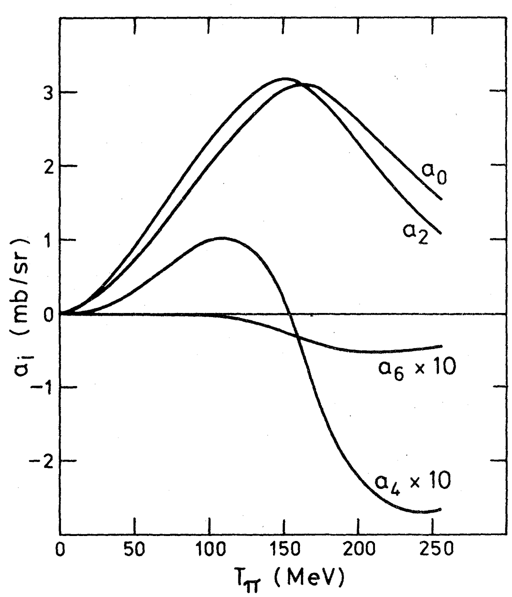

FIG. 14. The a_i coefficient $(d\sigma/d\Omega)(pp\to\pi^+d)$ as defined in Eq. (45), as a function of pion kinetic energy.

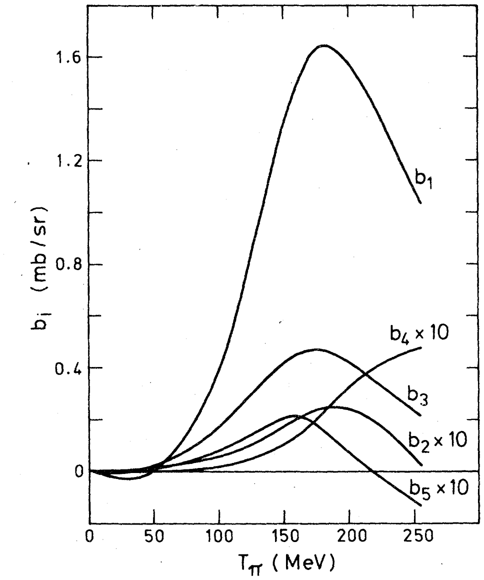

FIG. 15. The asymmetry parameters b_i for $\vec{p}p\to\pi^+d$ as defined in Eq. (45), as a function of pion kinetic energy.

where P is the polarization of the incident beam. In Fig. 16 we present the differential cross section at $T_p=383$ and 567 MeV using the Reid soft core[34] ($P_d=6.56\%$) and Bryan-Scott[35] ($P_d-5.36\%$) potentials. We find that the larger the D-state pro-

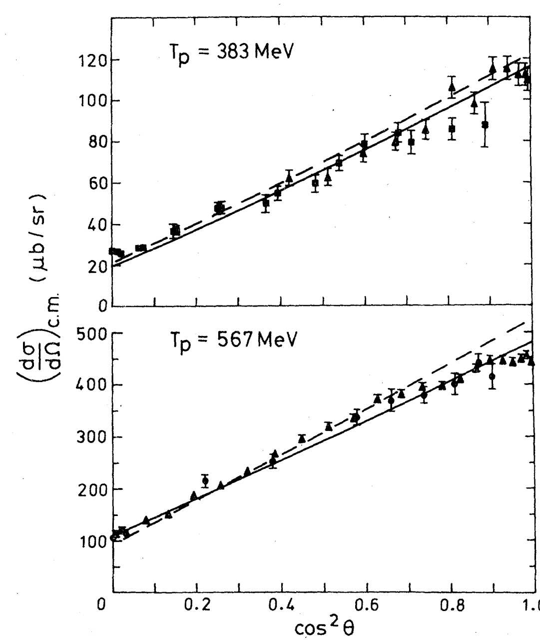

FIG. 16. The effect of the deuteron D-state probability on $(d\sigma/d\Omega)(pp\to\pi d)$. (——) UPA to Reid Soft core ($P_d=6.56\%$), (-----) UPA to Bryan-Scott ($P_d=5.36\%$). Experimental data as in Fig. 9.

bability of the deuteron (P_d) the flatter the angular distribution. Furthermore, the total cross section increases with a decrease in the D-state probability. This is consistent with the results of Niskanen.[24] To show the sensitivity of our result to the choice of P_{11} interaction we present, in Fig. 17, the differential cross section for the different π-N potentials. We find that for a given form factor the cross section increases with increasing P_{11} scattering volume or coupling constant which is naturally expected. However, we get a substantial increase in cross section even when the scattering volume a_{11} is kept constant but the analytic form (or range) of the πNN form factor is changed (compare cross sections for potentials $B\phi 8$ and $C\phi 8$).

Finally, in Fig. 18 the analyzing power is given for the two different deuteron wave functions and the four πNN form factors. We observe that decreasing the D-state probability brings down $A_\pi(\theta)$ and this is mainly due to change in λ_0, which again is consistent with the results of Niskanen.[24] On the other hand, we find that $A_\pi(\theta)$ is not sensitive to the scattering length but very sensitive to the range of the πNN form factor. All this indicates that the major source of uncertainty is the πNN form factor. To overcome this uncertainty, we need to resort to some other experimental constraint on the form factor. Alternatively, we might be able to resort to more fundamental models such as the bag model to determine the πNN vertex.[55,56]

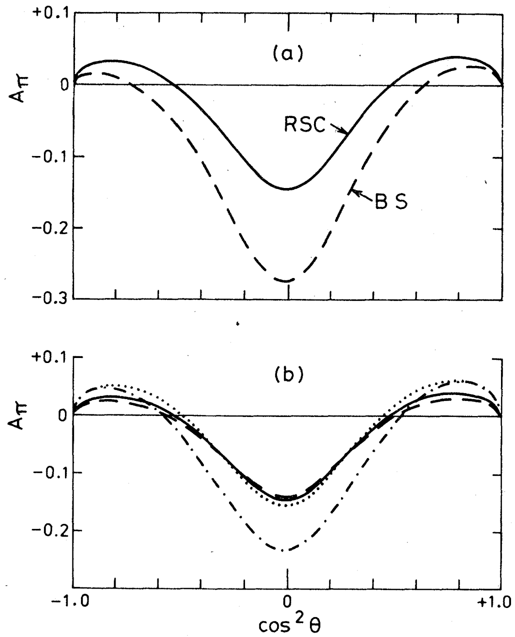

FIG. 18. The sensitivity of the analyzing power $A_\pi(\theta)$ to (a) different deuterons and (b) different P_{11} interactions. The curves in (a) are labeled as in Fig. 16 and those in (b) as in Fig. 6.

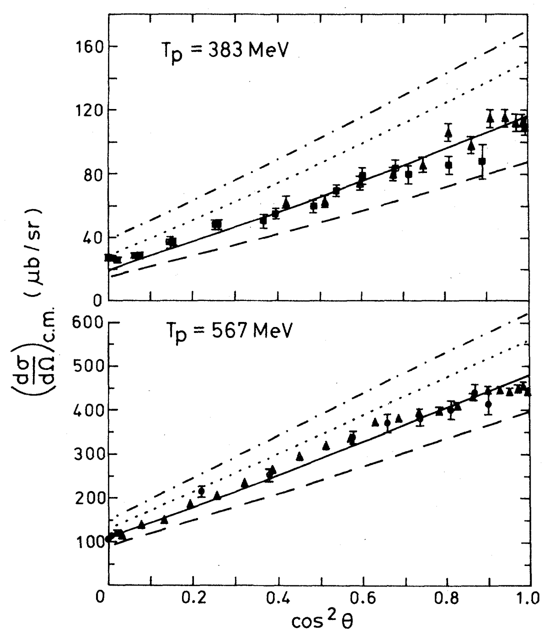

FIG. 17. Sensitivity of $(d\sigma/d\Omega)(pp\to\pi d)$ to P_{11} amplitude. The curves correspond to different P_{11} interaction and are labeled as in Fig. 6.

B. Pion-deuteron elastic scattering

Having established that our model gives a good description of pion absorption (production) we now can proceed to the study of the effect of real pion absorption on π-d elastic scattering. Although we have discussed the effect of real pion absorption on low energy π-d elastic scattering elsewhere,[20] for the sake of completeness we will cover the full energy range from $T_\pi=47$ to 256 MeV. In all calculations we have included as many partial waves as was necessary up to a total angular momentum of $J=7$. In Figs. 19 and 20 we present the differential cross section for $T_\pi=47.7$, 140, 180, 217, 232, and 256 MeV. The calculations correspond to the cross section with absorption (solid line) and with no absorption (dashed line). Also, at $T_\pi=47.7$, 140, and 256 MeV we have included our results for the case of no absorption or P_{11} rescattering (dotted line). The experimental data are due to Axen *et al.*[48] ($T_\pi=47.7$ MeV), Pewitt *et al.*[57] ($T_\pi=140$ MeV), Holt *et al.*[27] ($T_\pi=140$ MeV), Gabathuler *et al.*[58] ($T_\pi=140$, 180, 217, and 256 MeV), and

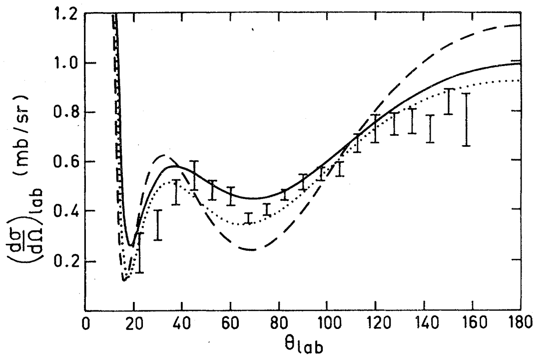

FIG. 19. $(d\sigma/d\Omega)(\pi^+d\to\pi^+d)$ for $T_\pi=47.7$ MeV. The curves correspond to full calculation (——), no absorption (-----), no absorption or P_{11} rescattering (·····). The results are for the Reid Soft core and the P_{11} potential $B\phi8$. The experiment is that of Axen *et al.* (Ref. 48).

Cole *et al.*[59] ($T_\pi=232$ MeV). We observe that at all energies considered the effect of absorption is considerable. In fact, above the resonance ($T_\pi>140$ MeV), the result with absorption gives a better fit to the experimental data.

We may compare our results with those of Fayard *et al.*,[18] who used the same NN-πNN equations in a calculation of π-d elastic scattering. In their work, however, they used relativistic kinematics and divided the P_{11} amplitude in a way different from the one employed by us. In general, we find only limited agreement regarding the effect of true absorption, which might not be surprising in view of the different description of the P_{11} channel. In particular, our results are in better agreement with the experimental differential cross sections at backward angles ($\theta>90°$) for $T_\pi=180$ and 256 MeV where we find, contrary to Fayard *et al.*, that inclusion of absorption lowers the dip at $\theta\sim100°$. On the other hand, our results are similar to those of Rinat *et al*,[17] although too strict a comparison is not possible due to the fact that they use equations that do not self-consistently couple all the physical amplitudes. Also, in their calculation they did not include P_{11} rescattering. Nevertheless, we agree with their results at 217 and 232 MeV in that the curve including absorption has a deeper minimum than the curve without absorption. We also agree with their result in that at 217, 232, and 256 MeV the curve without absorption is flat at backward angles, while the one including absorption has a characteristic rise past the 100° minimum. Finally, we compare our results with those of Betz and Lee.[21] Their theory, which includes three-body unitarity, is based on a relativistic Hamiltonian approach and couples the NN to the πNN channel. They take the mechanism for absorption to be $\pi d\to N\Delta\to NN$, where the $N\Delta\to NN$ is described by a potential. We find that, in general, we agree with their results that the inclusion of absorption lowers the cross section in the backwad angles ($\theta>100°$). However, their cross sections with and without true absorption are flatter than ours past the minimum ($\theta\simeq100°$).

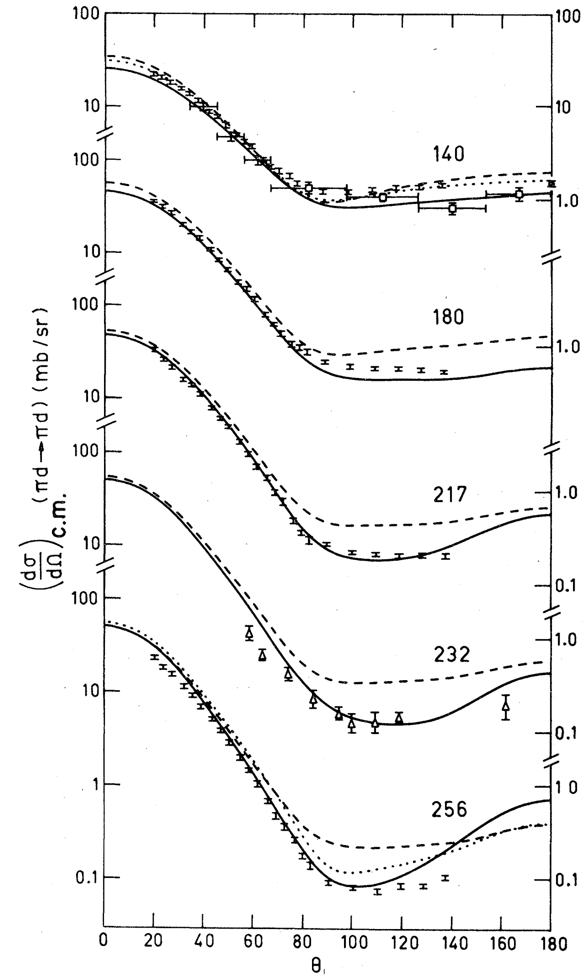

FIG. 20. The differential cross section for πd elastic scattering. Curves labeled as in Fig. 19. The experimental data are those of Cole *et al.* (Ref. 59), Gabathuler *et al.* (Ref. 58) (I), Holt *et al.* (Ref. 27), and Pewitt *et al.* (Ref. 57).

The phase shifts for π-N scattering in the P_{11} channel at low energies ($T_\pi<200$ MeV) are generally small. We therefore might expect the π-d cross section to be insensitive to the inclusion of the P_{11} channel. This, in fact, is the case if we take a P_{11} amplitude that fits the low energy phase shifts with no nucleon pole.[60] To investigate the overall contribution of the P_{11} amplitude to π-d elastic scattering in the present investigation, we compare the solid and dotted curves in Figs. 19 and 20. We find, in general, that the total contribution of the P_{11} is smaller than the effect of absorption alone. To understand this difference, we recall that the π-N amplitude is the sum of a pole part and a nonpole part. The inclusion of the nonpole part of the P_{11} amplitude tends to increase the differential cross section in the background directions. This is the result of π-N rescattering in the P_{11} channel. On the other hand, when we include the pole part of the amplitude as well, in the calculation of the cross section, we observe that the curve moves in the opposite direction to that obtained on including P_{11} rescattering. This is evidence for a cancellation between the pole and nonpole parts of the amplitude. Indeed, the polar part of the amplitude gives negative phase shifts and is repulsive, while the nonpole part is attractive and gives positive phase shifts. The fact that these two parts of the π-N amplitude are weighted differently in the π-N and π-d systems might give us a tool to study the off-shell behavior of the π-N amplitude in P_{11} channel.

To test the sensitivity of the differential cross section to the strength of the πNN vertex (or P_{11} amplitude), we have compared in Fig. 21 the differential cross section at $T_\pi=47.7$ and 140.0 MeV for the three P_{11} scattering volumes $a_{11}=-0.05$ (dashed curve), -0.08 (solid curve), and -0.11 m_π^{-3} (dotted curve). We observe that the sensitivity to the scattering volume is more at low energies, where the cross section increases with increasing scattering length, than is the case at higher energies, where the opposite effect takes place. On the other hand, if we keep the scattering volume approximately the same and change the range of the πNN form factor we see a more dramatic change in the cross section, particularly at higher energies, i.e., compare the cross section for the potentials $B\phi8$ (solid line) and $C\phi8$ (dash dot line). This means one might be able to adjust the range of the πNN form factor and the scattering volume or coupling constant to fit both π-d elastic cross section and pion production. Of course, this procedure is not only impractical but aesthetically not pleasing. Here again it is seen that we need to know the πNN form factor before we can uniquely predict the π-d elastic cross section.

To gain further insight into the mechanism for true pion absorption and its effect on elastic π-d scattering, we have examined the effect of including absorption in certain partial waves. Since the cross section for $\pi^+d\to pp$ is dominated by the $J^\pi=2^+$ channel in the energy region we are investigating, we might expect most of the contribution due to true pion absorption to be in this channel. In fact, in heavier nuclei one often uses the total cross section for $\pi d\to pp$ to determine the contribution of true absorption to the pion nucleus optical potential.[2] In Fig. 22 we demonstrate the effect of absorption on the π-d elastic cross section at $T_\pi=47.7$ and 140 MeV by successively including absorption in the $J^\pi=0^+$, 2^+, 1^-, and 2^-. The first surprising fact is that a considerable contribution due to absorption comes from the 0^+ even though this channel contributes minimally to the cross section for $\pi^+d\to pp$. In particular, for $T_\pi=47.7$ MeV the absorption in the 0^+ gives most of the change in the differential cross section for $40°<\theta<120°$. In addition, for $\theta>140°$, the 0^+ gives approximately half the contribution to the change in cross section due to absorption. Thus, had we neglected the effect on the cross section due to absorption in the 0^+ on the ground that the contribution of this channel to $\pi^+d\to pp$ is negligible, we would have missed a major contribution of absorption to π-d elastic scattering. In fact, if we compare the total and total elastic cross sections

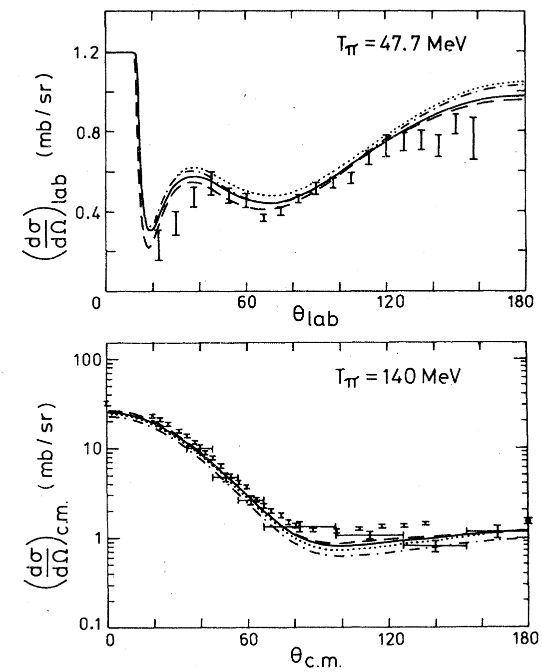

FIG. 21. The sensitivity of the differential cross section for π^+d elastic scattering to the π-N interaction in the P_{11} channel. Curves labeled as in Fig. 6.

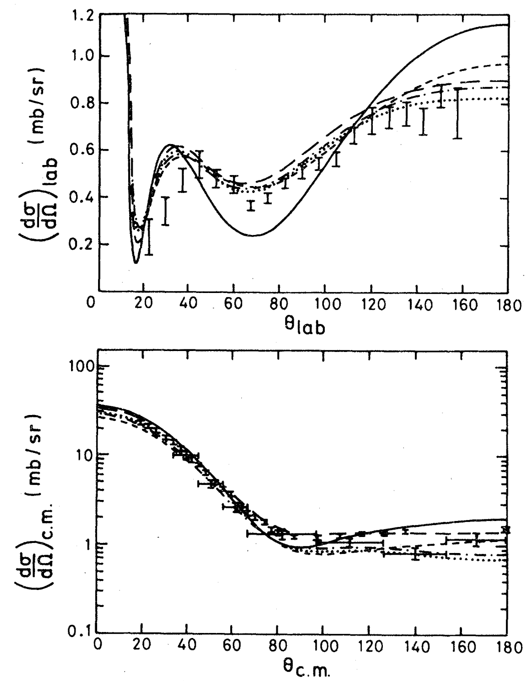

FIG. 22. The effect of including absorption in different channels on $(d\sigma/d\Omega)(\pi^+d\to\pi^+d)$. (——) no absorption; (-----) absorption in 0^+; (·····) absorption in 0^+, 2^+; (—·—·—) absorption in 0^+, 2^+, 1^-; (—··—··—) absorption in 0^+, 2^+, 1^-, 2^-.

for π-d scattering when absorption is included in the 0^+ and 2^+ channels separately (see Table IV) we find:

(i) The change in the elastic cross section due to inclusion of absorption in either 0^+ or 2^+ is of the same magnitude in both cases. This implies that both channels are of equal importance in the contribution of absorption to the elastic cross section.

(ii) Most of the change in the total cross section due to inclusion of absorption in the 0^+ comes from change in the elastic cross section, i.e., the inclusion of absorption in the 0^+ does not change the reaction cross section. On the other hand, almost all the change in the total cross section due to absorption in the 2^+ comes from the change in the reaction cross section.

TABLE IV. The effect of absorption on total and elastic cross sections for π-d scattering at 47.7 MeV.

Channels with absorption (J^π)	None	0^+	2^+	All
σ_{el}(mb)	8.55	9.02	8.05	8.54
σ_{tot}(mb)	17.68	18.13	22.81	23.89
$\Delta\sigma_{el}$		0.47	−0.50	−0.01
$\Delta\sigma_{tot}$		0.45	5.13	6.21

To examine the large contribution of absorption through the 0^+ we examine the first term in the multiple scattering series for π-d elastic scattering and pion absorption. For $\pi^+d\to pp$, the first term, given by the diagram in Fig. 23(a), gives an on-shell amplitude of -0.004 fm^2 at $T_\pi=47.7$ MeV. On the other hand, for $\pi d\to\pi d$, the lowest order diagram given in Fig. 23(b) gives an on-shell amplitude of $-0.048-i\,0.001$ fm^2 which is an order of magnitude larger. This means that the off-shell value of the amplitude given by the diagram in Fig. 23(a) is much larger than its on-shell value. This illustrates the error one can incur in constructing the pion-nucleus optical potential when one relies on the cross section for $\pi^+d\to pp$ to obtain the contribution of true absorption.

We now turn to the polarization experiments in π-d elastic scattering. To date there are two measurements; the first is a measurement of the tensor polarization t_{20} at 140 MeV of Holt *et al.*,[27] the second is a recent measurement of vector analyzing power ($i\,T_{11}$) by Bolger *et al.*[28] at 143 and 256 MeV. In Fig. 24, we compare our results with the above experiments. Here the solid curve is our full calculation, the dashed curve is with no absorption, while the dotted curve corresponds to the calculation with no P_{11} (i.e., no P_{11} rescattering or absorption). The agreement between theory and experiment is very good at $T_\pi\simeq140$ MeV, although the experimental errors are too large to put a constraint on the theory. The results at $T_\pi=256$ MeV, though not in good agreement with experiment, are very interesting. This data has been recently used as evidence for a $^3F_3$ dibargon resonance.[28] If we compare our different theoretical results with experiment, we note that the shape of the polarization curve is very sensitive to the inclusion of absorption. In particular, our results at $T_\pi=256$ MeV with absorption have a small dip at 70° and are negative at backward angles. This suggests that it might not be necessary to assume a

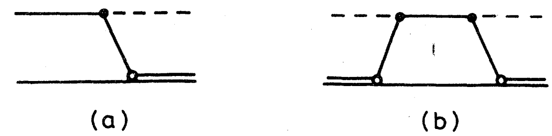

FIG. 23. Lowest order diagrams for (a) $\pi d\to pp$ and (b) $\pi d\to\pi d$.

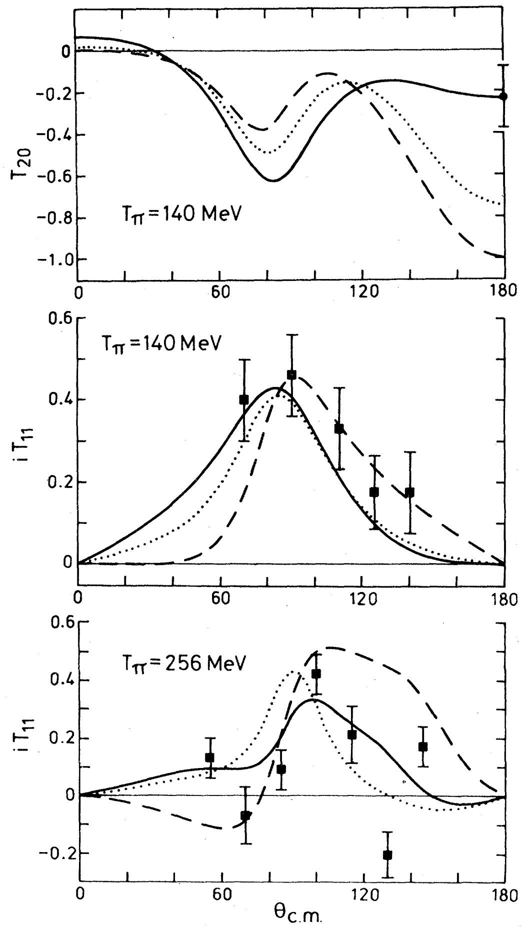

FIG. 24. The tensor (t_{20}) and vector (iT_{11}) polarization of the deuteron in π-d scattering. The curves are labeled as in Fig. 17. The experimental data are those of Holt *et al.* (Ref. 27) (●), and Bolget *et al.* (Ref. 28) (■).

new mechanism to explain the data. We will elaborate on this possibility in the next section. At this stage, we can conclude that both theory and experiment need to be refined before any definite conclusion can be drawn on the failure of the theory to fit experiment.

Finally, in Fig. 25 we present our results for the π-d total cross section σ_{tot}. The very accurate data are due to Pedroni *et al.*[61] Since we reproduce the experimental cross section for $\pi^+ d \to pp$ very well except at high energies, the present discrepancy between theory and experiment is due to our failure to reproduce the π-d elastic cross section. This can be due either to the way we have split the P_{11} amplitude, or the choice of the πNN form factor, as well as relativistic effects. These points are currently under investigation.

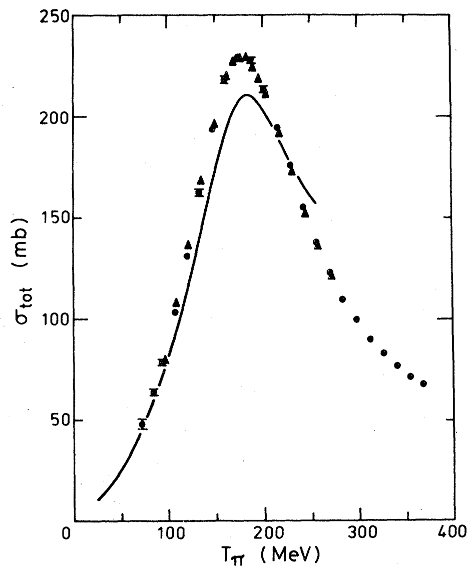

FIG. 25. The total π-d cross section. Experimental data are from Pedroni *et al.* (Ref. 61).

C. Nucleon-nucleon scattering

Although we do not expect the model, as it stands, to reproduce the experimental N-N phase shifts for the lack of heavy meson exchange, there are several features of the model that are not included in most other descriptions of N-N scattering. In particular, we have (i) included inelasticity through pion production in a unitary way, and since we fit the production cross section, our inelasticity is of the right magnitude, and (ii) we have included the coupling of the N-N to the $N\Delta$ channel with our Δ being a genuine π-N resonance. Furthermore, this coupling is included to all orders in our solution of the equations.

In Fig. 26 we present some of the $I=1$ N-N phase shifts obtained from the same calculation that gave our results for pion production and π-d elastic scattering. The experimental data are from the analysis of Bystricky, Lechanoine, and Lehar.[62] Except for the higher partial wave phase shift $^3F_3$, and to some extent $^1G_4$, our model does not reproduce the experimental data very well. We note, however, that for the singlet channels $^1S_0$ and $^1D_2$, our results are much more reasonable than for the triplet channels. If we take into consideration the fact that there are no vector meson (i.e., ρ,ω) exchanges in our calculation, and thus little spin-orbit interactions, then we are able to understand the above observations. In fact, since the spin-orbit interaction is repulsive for $J \le L$, and attractive for $J = L+1$, then the $J = L$, $L-1$ phase shifts would

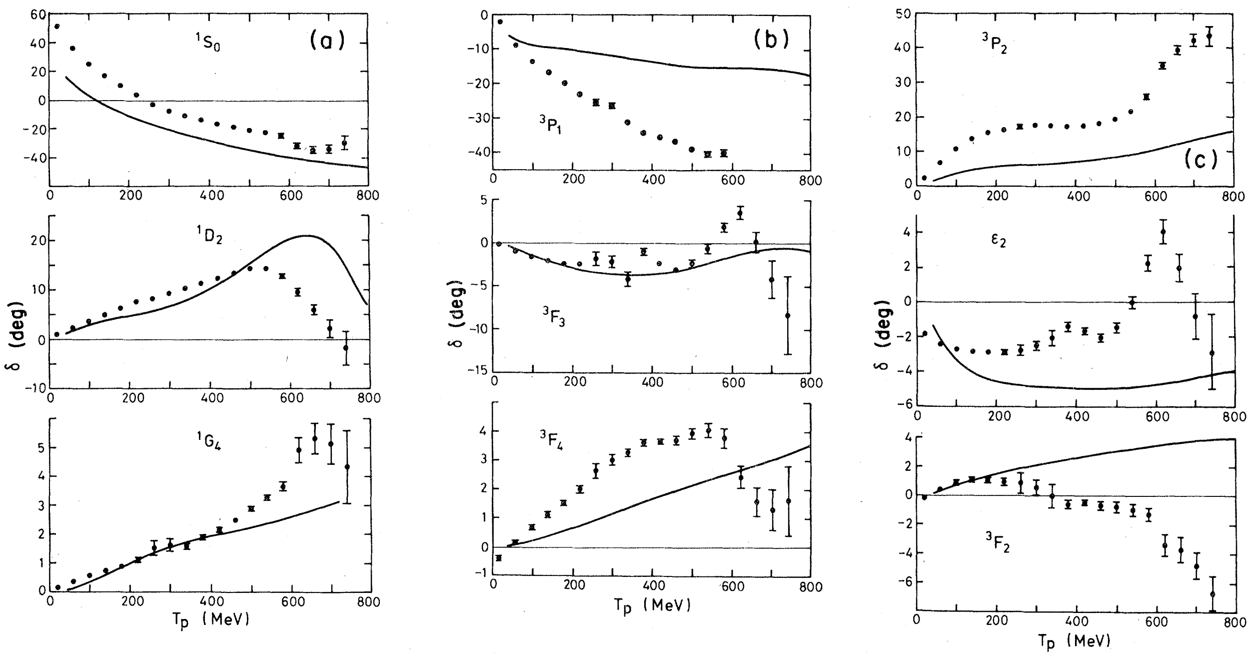

FIG. 26. Some $I=1$ N-N phase shifts (deg) calculated with present model. Experimental data are from Ref. 62.

become more repulsive on including the spin-orbit interaction, while $J=L+1$ phase shifts become more attractive. This is consistent with the present discrepancy between the model and the results of phase shift analyses for the triplet channels. For the singlet channels, we would expect some effect from short-range interactions on inclusion of vector meson exchanges. Also, the inclusion of ρ exchange would cancel part of the tensor force, and would thus reduce the value of the coupling parameter ϵ_2.

If we compare our results with the ones of Kloet and Silbar,[19] who use a covariant version of BSM,[63] in which the undercounting is overcome by adding half a static one pion exchange, we find qualitative agreement in all channels except the $^3P_2$. This is probably due to the fact that they do not have any N-N rescattering in intermediate states, which we find to be very important. Comparing with a different unitary three-body approach, namely that of Brayshaw's boundary condition formalism,[64] we again find agreement with his quoted $^1D_2$ phase shifts. We conclude that all the considered three-body unitary approaches give approximately the same results (except possibly for the $^3P_2$ channel), and all therefore reflect similar needs for higher meson exchanges. These have recently been included by Kloet and Silbar in their model[65] resulting in improved agreement with experiment.

One of the most important recent developments in nucleon-nucleon scattering has been the subject of dibaryon resonances. Perhaps the strongest evidence for these has come from a series of measurements by Auer *et al.*[3] of the total cross section difference $\Delta\sigma_L = \sigma^{\text{tot}}(\rightleftarrows) - \sigma^{\text{tot}}(\rightrightarrows)$ for p-p scattering in initial longitudinal spin states. In particular, the authors found that $\Delta\sigma_L$ displays a remarkable energy dependence at a beam momentum of about 1.5 GeV/c. These results were interpreted by Hidaka *et al.*[66] as evidence for a $^3F_3$ diproton resonance. Later measurements[3] discovered more spin structure in p-p scattering, suggesting the possibility of $^1D_2$ and $^1G_4$ resonances.

If an elastic amplitude T contains a resonance, then one can, in principle, separate out the resonant (T_R) and background (S_∞, T_∞) components.[67] These are related by

$$T = S_\infty T_R + T_\infty \ . \tag{47}$$

T_R is usually parametrized by a Breit-Wigner form

$$T_R = \frac{\Gamma_{\text{el}}/2}{E_R - E - i\Gamma/2} \ , \tag{48}$$

where E is the two-body center of mass energy, E_R is the resonance energy, Γ is the total width of the resonance, and Γ_{el} is the partial width for the elastic channel. The "speed" is then defined as the derivative $|dT/dE|$, and for Eq. (48) this gives

$$|dT_R/dE| = \frac{\Gamma_{\rm el}}{\Gamma}\frac{1}{(\Gamma/2)}\frac{1}{\left|\dfrac{(E_r-E)^2}{(\Gamma/2)^2}+1\right|} . \tag{49}$$

For slowly varying background, speed is well represented by Eq. (49). Then both E_R and Γ can be deduced from the peak energy and the full width at half-maximum, respectively.

Despite the deficiencies of our model for N-N scattering, we suspect that resonance behavior might occur because of the coupling to the N-Δ channel. It would therefore be very interesting to perform an analysis on our N-N amplitudes in terms of argand plots and speed curves, to examine whether our model can simulate the behavior usually attributed to a dibaryon resonance. As we are solely interested, at this stage, in the effect of coupling to the N-Δ channel, we have performed a calculation in the dominant ${}^1D_2$ ($J^\pi=2^+$) partial wave, retaining only the deuteron, nucleon, and P_{33} channels.

In Figs. 27(a) and 27(b) we present our results for the ${}^1D_2$ argand plot and speed curve, respectively. The argand plot displays the typical counter-clockwise motion of a resonant amplitude. Even more typical of a resonance is our result for the speed curve. In fact, we obtain from Eq. (49) that $E_R=2160$ MeV and $\Gamma=200$ MeV. This may be compared with the phase shift analysis of Hoshizaki,[68] who obtained $E_R\approx2170$ MeV and $\Gamma\sim50-100$ MeV. Although the two results look suspiciously similar, we cannot draw any definite conclusions without further investigation. For instance, we have not as yet examined the complex E plane for any poles on the second sheet. (However, a preliminary investigation of eigenvalue trajectories, suggests that no such poles exist in our model.) We also have not investigated the true effect of background, or the effect of short-range forces. Consequently, we cannot say with certainty whether coupling to the N-Δ channel is able to reproduce all the structure seen in N-N scattering. Indications are, however, that this is a distinct possibility.

V. CONCLUSION

The model that we have been investigating couples the N-N to the πNN system and is a linearization of a field theory of pions and nucleons which preserves two- and three-body unitarity. In the last section we have demonstrated that the present

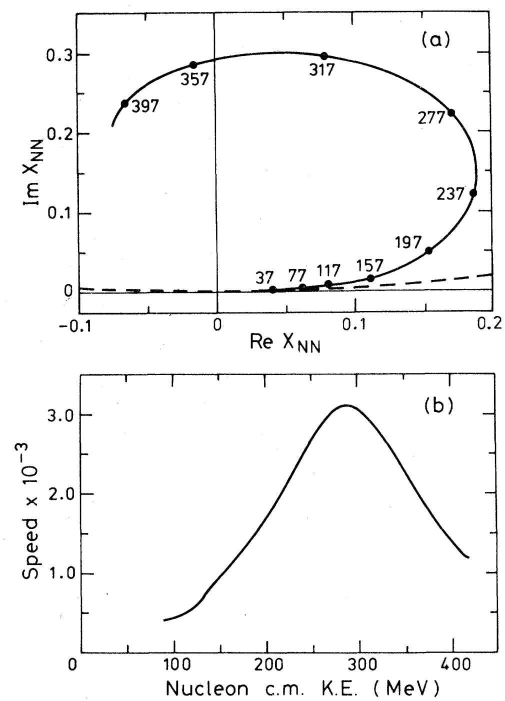

FIG. 27. (a) The ${}^1D_2$ argand plot. Points on the curve are labeled by the nucleon c.m. kinetic energy (in MeV). (b) The speed corresponding to the amplitudes in (a).

theory gives a very good unifying description of all three reactions in Eq. (1) below the threshold for two pion production. The basic input into the equations are the πNN form factor together with the π-N and N-N interactions. These two-body interactions arise from a truncation in the field theory to states of no more than one pion. We have shown that much of the discrepancy between the theory and experiment is due to uncertainty in the input. The biggest source of uncertainty at this stage is the πNN form factor and particularly its range. A knowledge of that form factor not only determines the magnitude of the cross section for $pp\leftrightarrow\pi^+d$ but the division of the P_{11} amplitude into a pole and nonpole part. This in turn governs the net contribution of the P_{11} to π-d elastic scattering, and may remove the discrepancy in the total cross section. Another source of discrepancy is the use of nonrelativistic kinematics which causes the total cross section at high energies to be too large. One could overcome this problem by employing one of the available covariant equations.[23,63] Unfortunately, however, they do not have the correct clustering properties.[40] Alternatively one can use a relativistic Hamiltonian approach[22] in which case pion absorption and production is through the Δ resonance and one does not have a basic πNN vertex.

The largest discrepancy between theory and experiment was in N-N scattering. Here the source of the disagreement is the lack of multipion (or heavy boson) exchange in our calculation. This is straightforward to incorporate and has been included in a similar model by Kloet and Silbar[65] to improve the fit to experiment. Of course, the inclusion of heavy meson exchange ($\rho,\omega,\ldots$) will improve the fit to the N-N channel and also improve the distortion in the initial state for the reaction $pp \to \pi^+ d$. This can possibly explain some of the discrepancy in our value of λ_0.

Having established that one can account for the present discrepancy between theory and experiment, we can conclude that the present theory has enough flexibility to describe the reactions in Eq. (1) in a unified way with a minimum number of parameters. Finally, we remark that it is possible to adjust the input parameters to improve the fit to experiment for one of the reactions. This, however, might destroy the fit for other observables. Thus, within the framework of the present theory one should describe all three basic reactions simultaneously.

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