Strong j-dependent effects in ($\vec{^7}$Li, ^{6}Li) reactions

F. D. Santos*
University of Wisconsin, Madison, Wisconsin 53706

A. M. Gonçalves
Laboratório de Física, Universidade de Lisboa, Lisbon, Portugal

(Received 27 February 1981)

The analyzing powers of heavy ion induced transfer reactions are discussed within the framework of the distorted-wave Born approximation using a semiclassical model to describe the conservation of kinetic energy in the transfer process. It is found that the analyzing powers T_{kq} with k odd have a strong dependence on Q value. Those T_{kq} with k even have a weaker Q dependence and are specially sensitive to the deformed shape of the projectile. Strong j-dependent effects are found in the analyzing powers of ($\vec{^7}$Li, ^{6}Li) reactions, particularly in T_{2q} and iT_{3q}. Calculations for the ^{58}Ni($\vec{^7}$Li, ^{6}Li)^{59}Ni reaction at $E_{\mathrm{Li}} = 20.3$ MeV are in good agreement with cross section and iT_{11} angular distribution data.

[NUCLEAR REACTIONS: DWBA analysis of heavy ion induced transfer. ^{58}Ni($\vec{^7}$Li, ^{6}Li)^{59}Ni, $E = 20.3$ MeV; calculated $\sigma(\theta)$, $iT_{11}(\theta)$, $T_{2q}(\theta)$, $iT_{3q}(\theta)$, j-dependent effects.]

I. INTRODUCTION

In the past few years there has been an increased interest in polarization measurements for nuclear heavy ion induced transfer reactions.[1-3] This work has been mostly directed to the determination of the type of reaction mechanism involved in the transfer process. More recently it became possible to polarize heavy ion beams, in particular ^{7}Li and ^{6}Li, and measurements of the analyzing powers of transfer reactions were reported.[4]

It is well known that the analyzing powers of light-ion induced transfer reactions are a powerful spectroscopic tool since they can be used to identify the spin and parity of states in the residual nucleus. For instance, this is the case of the vector and tensor analyzing powers of $(\vec{d}, p)$ and $(\vec{d}, \alpha)$ reactions.[5-7] The analyzing powers in these reactions are usually treated with the distorted wave Born approximation (DWBA). This theory generally provides angular distributions which are in agreement with the data. The applicability of the DWBA in heavy ion induced transfer reactions is considerably more doubtful and the theory has not been always successful in describing the angular distributions of polarization observables.[3]

The semiclassical nature of quasielastic transfer processes between heavy ions makes it possible to use semiclassical models[8] to interpret most of the qualitative features of polarizations produced in these reactions. However, reliable quantitative predictions are necessary if these reactions are to be used in high precision nuclear spectroscopy. Here we present a reaction model which is founded in the DWBA formalism but where the balance of kinetic energy in the transfer process is treated semiclassically following an approximation proposed by Brink.[9,10] This model takes into account the recoil effects due to the finite mass of the transferred particle and is specially adapted to describe the Q-value dependence of the transfer amplitude.

We consider an unpolarized target and take the case of no spin-orbit interactions in the entrance and exit channels. Particular emphasis is given to the discussion of the dependence of the analyzing powers on the total angular momentum j of the transferred particle in the projectile and residual nucleus. This discussion is applied to (^{7}Li, ^{6}Li) and (^{7}Li, ^{6}He) reactions. In particular we consider the ^{58}Ni($\vec{^7}$Li, ^{6}Li)^{59}Ni reaction at 20.3 MeV, where Tungate *et al.*[4] have recently observed strong Q-value effects in the vector analyzing power.

II. SEMICLASSICAL MODEL IN THE DWBA

We consider a transfer reaction $A(a, b)B$, where $a = b + x$ and x is the transferred particle or cluster. Following the notation of Ref. 11 we write for the DWBA transition amplitude in the "post" form and in the absence of spin-orbit interactions

$$T = \int d^3R \int d^3r\, \chi_b^{(-)*}(\vec{r}_b) \langle Bb \,|\, V_{bx} \,|\, Aa \rangle \chi_a^{(+)}(\vec{r}_a) . \tag{1}$$

Here χ_a and χ_b are the distorted waves, the vectors $\vec{R}$, $\vec{r}$, $\vec{r}_a$, and $\vec{r}_b$ are illustrated in Fig. 1, and V_{bx} is the interaction between b and x. Performing an expansion of the transition amplitude into terms with definite angular momentum we can write

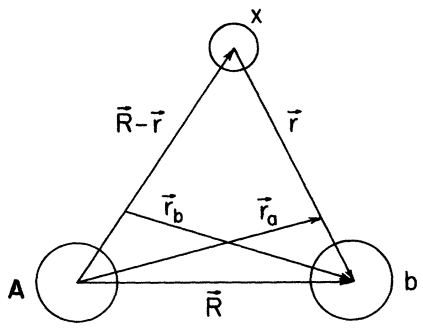

FIG. 1. Coordinate vectors for a transfer reaction.

$$T=\sum_{j_1 j_2 L}(J_A M_A j_2 M_2 | J_B M_B)(LM j_1 M_1 | j_2 M_2)(-1)^{s_b-m_b} \times (s_a m_a s_b - m_b | j_1 M_1) B^{LM}_{j_1 j_2}, \quad (2)$$

where J_A, J_B, s_a, s_b are the spins of A, B, a, b and L is the orbital angular momentum transfer in the reaction. The dynamics of the reaction are contained in the $B^{LM}_{j_1 j_2}$ which, in the notation of Satchler[11] are given by

$$B^{LM}_{j_1 j_2}=A_{L j_1 j_2}\beta^{LM}_{j_1 j_2}. \quad (3)$$

Here

$$\beta^{LM}_{j_1 j_2}=\int d^3R \int d^3r\, \chi_b^{(-)*}(\vec{r}_b) F^{LM}_{j_1 j_2}(\vec{R}-\vec{r},\vec{r})\chi_a^{(+)}(\vec{r}_a) \quad (4)$$

are the reduced transition amplitudes, where the transfer form factor is given by

$$F^{LM}_{j_1 j_2}(\vec{r}',\vec{r})=i^{l_1-l_2}\sum_{m_1 m_2}(l_2 m_2 l_1 m_1 | LM) R_{l_2 j_2}(r') \times Y^{m_2*}_{l_2}(\hat{r}') V_{bx}(\vec{r}) R_{l_1 j_1}(r) Y^{m_1*}_{l_1}(\hat{r}). \quad (5)$$

l_1, j_1 are the orbital and total angular momentum of x in the projectile, l_2, j_2 are the orbital and total angular momentum of x in the residual nucleus, and $R_{l_1 j_1}$, $R_{l_2 j_2}$ are the corresponding normalized radial bound state wave functions. The coefficient $A_{L j_1 j_2}$ in Eq. (3) is given by

$$A_{L j_1 j_2}=S_{l_1 j_1} S_{l_2 j_2}\hat{s}_a \hat{L}(-1)^{j_2-j_1-L-l_1} W(l_2 s_x L j_1; j_2 l_1). \quad (6)$$

Here $S_{l_1 j_1}$, $S_{l_2 j_2}$ are spectroscopic amplitudes, s_x is the spin of the transferred particle, and $(2s+1)^{1/2}$ is abbreviated as $\hat{s}$. Unless stated otherwise we use the Madison convention[12] coordinate system S with z axis along $\vec{k}_a$ and y axis along $\vec{k}_a\times\vec{k}_b$, with $\vec{k}_a$ and $\vec{k}_b$ being the asymptotic momenta in the entrance and exit channels. This system is represented in Fig. 2.

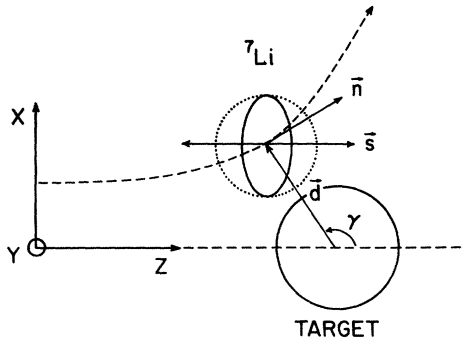

FIG. 2. Polarization effects on the ($^7\vec{\text{Li}}$, ^{6}Li) reaction from the deformed shape of the ^{7}Li-^{6}Li overlap. $\vec{n}$ is the tangent to the Rutherford orbit at the point of closest approach defined by the vector $\vec{d}$. The deformed full curve represents ^{7}Li with a spin alignment axis parallel to the incident beam direction (z axis of the S coordinate system). The dotted circumference represents ^{7}Li with the spin alignment axis perpendicular to the plane of the reaction.

In heavy ion transfer reactions the integrand on the right hand side of Eq. (4) is strongly localized in the displacement $\vec{R}$ between the heavy ion cores b and A. This localization results from the combined action of the Coulomb barrier and of strong absorption for small R with the lack of overlap between the bound states at large R. It is therefore appropriate to make approximations when dealing with the deviations from $\vec{R}$ in the argument of the distorted waves. Performing a Taylor expansion in $\vec{r}$, we can write

$$\chi_b^{(-)*}(\vec{r}_b)\chi_a^{(+)}(\vec{r}_a)=e^{i\vec{r}\cdot\vec{P}}\chi_b^{(-)*}\left(\frac{m_A}{m_B}\vec{R}\right)\chi_a^{(+)}(\vec{R}), \quad (7)$$

where

$$\vec{P}=-i\left(\frac{m_x}{m_A}\nabla_b-\frac{m_x}{m_a}\nabla_a\right). \quad (8)$$

Here m_i is the mass of particle i and $\nabla_a(\nabla_b)$ is the gradient operator with respect to $\vec{R}$ and acts only on the function $\chi_a(\chi_b)$. The substitution of Eq. (7) into Eq. (4) shows that $\hbar\vec{P}$ plays the role of the momentum conjugate with the variable $\vec{r}$ in the transition matrix element.

We wish to replace the operator $\vec{P}$ by its eigenvectors. This type of approximation is well known.[13,14] The usual procedure, used, for instance, by Braun-Munzinger *et al.*,[13] is to replace $\hbar\vec{P}$ by a recoil momentum $\vec{q}$ determined in a local momentum approximation for the distorted waves. Here we follow a different approach in which the eignevectors of $\vec{P}$ are generated using a semiclassical model of Brink.[9,10] Energy conservation in the transfer process gives the reaction Q value as approximately

$$Q=(\vec{P}_1^{\,2}-\vec{P}_2^{\,2})/2m_x, \tag{9}$$

where $\vec{P}_1$ is the momentum of x relative to the heavy ion core b in the projectile a and $\vec{P}_2$ is the momentum of x relative to the heavy ion core A in the residual nucleus B. Hence $\vec{P}_1$ is the momentum conjugate with $\vec{r}$ and $\vec{P}_2$ is the momentum conjugate with $\vec{R}-\vec{r}$ (see Fig. 1). We now write $\vec{P}_i$ as a sum of two orthogonal vectors

$$\vec{P}_i=\vec{p}_i+\vec{p}_i', \quad i=1,2, \tag{10a}$$

where

$$\vec{p}_i=p_i\vec{n}, \quad i=1,2 \tag{10b}$$

is the projection along the unit vector $\vec{n}$. We choose $\vec{n}$ to be in the reaction plane and tangent to the projectile trajectory at the point where the transfer is more probable. Hence in reactions where the projectile can be approximately localized in a Rutherford orbit $\vec{n}$ makes an angle of $\theta/2$ (θ is the scattering angle) with the incident beam direction, as shown in Fig. 2. The peripheral nature of the reaction means that the larger contribution to the kinetic energy balance in Eq. (9) comes from the tangential components $\vec{p}_i$. With this assumption $p_1'\cong p_2'$ in the transfer process and using Eqs. (9) and (10) we can write

$$p_1=-\frac{Q}{v}-\tfrac{1}{2}m_x v, \tag{11a}$$

$$p_2=-\frac{Q}{v}+\tfrac{1}{2}m_x v. \tag{11b}$$

Here $v=(p_2-p_1)/m_x$ is the relative velocity between the heavy ions in the region of space where the transfer is more probable.

When Eq. (7) is substituted into Eq. (4) the eigenvectors of the operator $\vec{P}$ must be necessarily in the reaction plane in order to conserve parity in the transition matrix. In fact, it is easily proved that only with this condition do the amplitudes $B^{LM}_{j_1j_2}$ (referred to the coordinate system S) satisfy the relation[11]

$$B^{LM}_{j_1j_2}=\pi_{ab}(-1)^{L-M}B^{L-M}_{j_1j_2}. \tag{12}$$

Here π_{ab} is the sign of parity change in the reaction. Since $\vec{p}_1$ is the component of $\vec{P}_1$ in the reaction plane we obtain from Eqs. (4) and (7)

$$\beta^{LM}_{j_1j_2}=\int d^3R\,\chi_b^{(-)*}\left(\frac{m_A}{m_B}\vec{R}\right)\times\int d^3r\,F^{LM}_{j_1j_2}(\vec{R}-\vec{r},\vec{r})\,e^{i\vec{r}\cdot\vec{p}_1/\hbar}\,\chi_a^{(+)}(\vec{R}). \tag{13}$$

We notice that $p_1=0$ occurs when the Q value is symmetric in the energy $m_x v^2/2$ of the transferred particle. In this case of optimum energy balance the transfer is more likely for x orbits relative to b that are perpendicular to the reaction plane. The phase factor in the integrand of Eq. (13) is then equal to one and we get the well known no-recoil approximation.[15] If, however, $|p_1|\gg 0$, part of the momentum of x relative to b must be used to conserve energy in the transfer process. The rapid oscillations of the phase factor $\exp(i\vec{r}\cdot\vec{p}_1/\hbar)$ considerably reduces the transfer amplitude and the no-recoil approximation is not expected to be reliable.

It has been shown by various authors[15-18] that the peripheral nature of heavy ion transfer reactions makes it convenient to represent the radial bound state wave functions as sums of Hankel functions. This representation provides an accurate description in the important asymptotic region of large r, and also it has the advantage of allowing the d^3r integration in Eq. (13) to be performed analytically. It then becomes simpler to study the dependence of the reduced transition amplitudes on the reaction parameters and in particular on the momentum $\vec{p}_1$. In order to simplify the argument we represent the radial bound state wave functions $R_{l_1j_1}$ and $R_{l_2j_2}$ by only one Hankel function that describes the asymptotic behavior. Thus we write

$$R_{l_nj_n}\cong N_{l_nj_n}\,i^{l_n}h_{l_n}(i\beta_n r), \quad n=1,2, \tag{14}$$

where $N_{l_nj_n}$ are the asymptotic normalization constants and β_1, β_2 the wave numbers corresponding to the separation energies of x from the projectile and residual nucleus. Using the methods developed in Ref. 17 we can perform the d^3r integration in Eq. (13) for the wave functions (14) without requiring further approximations. This integration gives

$$\int d^3r\,F^{LM}_{j_1j_2}(\vec{R}-\vec{r},\vec{r})\,e^{i\vec{r}\cdot\vec{p}_1/\hbar}=\frac{2\pi\hbar^2}{\mu_1}\sum_{lL_c}i^{l_1-l_2+l}\,\Gamma X_{L_c}(R)\left(\frac{|p_1|}{\hbar}\right)^l\sum_{mM_c}(lmL_cM_c|LM)Y^{M_c^*}_{L_c}(\hat{R})Y^{m*}_{l}(\hat{p}_1). \tag{15}$$

Here μ_1 is the reduced mass of x relative to b,

$$\Gamma=(-1)^{L_1}\hat{l}_1\hat{L}_1\hat{L}_c\hat{l}W(l_1l_2lL_c;LL_1)(l_10l0|L_10)(L_10L_c0|l_20) \tag{16}$$

is an angular momentum coupling coefficient, where

$$L_1=l_1-l, \tag{17}$$

and

$$X_{L_c}(R)=\frac{(2l_1+1)!!}{(2l+1)!!\,(2L_1+1)!!}N_{l_1j_1}N_{l_2j_2}\frac{\beta_2^{L_1}}{\beta_1^{l_1+1}}i^{L_c}h_{L_c}(i\beta_2R) \tag{18}$$

is the radial form factor $|(2l+1)!!=(2l+1)(2l-1)\ldots]$. The angular momenta in Eq. (15) have a simple physical interpretation. In fact, the form of the right hand side of Eq. (15) shows that L_c is the orbital angular momentum associated with the displacement $\vec{R}$ between the heavy ion cores. Therefore L_c is the orbital angular momentum transferred between the heavy ion cores in the reaction. The l provides for the total balance of orbital angular momentum and is usually called the recoil angular momentum. The selection rule (17) implies $l \leq l_1$ and thus strongly restricts the range of values that l can take. Finally we note that the term with $l=0$ on the right hand side of Eq. (15) corresponds to the no-recoil approximation and has no dependence on the momentum $\vec{p}_1$.

III. J-DEPENDENT EFFECTS IN THE ANALYZING POWERS

To be definite we consider the analyzing powers of the transfer reaction $A(a,b)B$. In a spherical tensor basis they are defined by

$$T_{kq}=\mathrm{Tr}[T\tau_{kq}(s_a)T^\dagger]/\mathrm{Tr}(TT^\dagger), \tag{19}$$

where τ_{kq} are the usual spherical tensor operators.[12] Using Eq. (2) we can show in a straightforward manner that

$$\sigma T_{kq}=\sum_{j_1 j_1' j_2}\hat{s}_a\hat{j}_1\hat{j}_1'(-1)^{k+j_1-j_2}W(ks_a j_1 s_b; s_a j_1')\sum_{LML'M'}W(j_1 j_1' LL';\ kj_2)(-1)^M(L-ML'M'\,|kq)B^{LM}_{j_1 j_2}B^{L'M'*}_{j_1' j_2}, \tag{20}$$

where

$$\sigma=\sum_{j_1 j_2 LM}|B^{LM}_{j_1 j_2}|^2/(2L+1). \tag{21}$$

The particular dependence of the T_{kq} on the quantum numbers l_2 and j_2 can be a useful spectroscopic tool since it can lead to the identification of the spin and parity of the residual nucleus. Usually this dependence is an intricate effect and highly sensitive to the reaction dynamics. However, in certain cases the j_2 dependence of the T_{kq} has systematic features which can be useful. In order to study them it is most straightforward to consider the case where $J_A=0$, so that $J_B=j_2$.

The substitution of Eqs. (3) and (6) into Eq. (20) gives

$$\sigma T_{kq}(J_B)=\sum_{j_1 j_1' LL'}C(J_B)\hat{s}_a^3 W(ks_a j_1 s_b; s_a j_1')\sum_{MM'}(-1)^{L-M}(L-ML'M'\,|kq)\hat{L}\hat{L}'\beta^{LM}_{j_1 J_B}\beta^{L'M'}_{j_1' J_B}, \tag{22}$$

where

$$C(J_B)=\hat{j}_1\hat{j}_1' W(LJ_B kj_1';j_1 L')W(s_x J_B l_1' L';l_2 j_1')W(s_x J_B l_1 L;l_2 j_1). \tag{23}$$

To simplify the discussion it was assumed that all spectroscopic amplitudes are equal to one. The right hand side of Eq. (22) depends on J_B through two different factors: (a) the angular momentum coupling factor $C(J_B)$; (b) the product of two reduced transition amplitudes. However, the J_B dependence of $\beta^{LM}_{j_1 J_B}$ is much weaker than that of the $C(J_B)$ because it only originates in Q-value effects and through differences in the radial wave functions resulting from the spin dependence of the binding potential. The range of L values allowed by the selection rules, in the last two Racah coefficients on the right hand side of Eq. (23) is a function of J_B. This mechanism is known to produce a J_B dependence in the cross section. Recently this has been observed by Kim *et al.*[19] in the ^{50}Cr(^{7}Li, ^{6}He)^{51}Mn reaction at 28 MeV and for $\theta \leq 10°$.

The particular dependence of C on J_B is made clear if we consider the relation

$$\sum_{J_B}(2J_B+1)C(J_B)=(-1)^{L'-l_2+j_1-s_x}\hat{j}_1\hat{j}_1' W(L'Ll_1' l_1;kl_2)W(j_1' j_1 l_1' l_1;ks_x) \tag{24}$$

which is derived from Eq. (23) using a well known sum rule for Racah coefficients.[20] We notice that the right hand side of Eq. (24) contains a triangular relation between l_1, l_1', k which is not present in $C(J_B)$. Thus we conclude that

$$\sum_{J_B}(2J_B+1)C(J_B)=0 \text{ for } k>l_1+l_1'. \tag{25}$$

Using Eqs. (22) and (25) and neglecting the weaker dependence of the reduced transition amplitudes and of the cross section σ on J_B, we obtain

$$\sum_{J_B}(2J_B+1)T_{kq}(J_B)\cong 0 \text{ for } k>2l_1. \tag{26}$$

No l_1 mixing was assumed in the projectile; therefore $l_1=l_1'$.

Equation (26) describes the well known sign rule type j dependence in which the angular distributions with different J_B tend to be out of phase by π and tend to have unequal amplitudes. This j dependence was predicted by Newns[21] in (d,p) reactions. Since then it has been extensively observed in the $k=1$ and $k=2$ analyzing powers of light ion reactions where the transferred particle is initially

bound in an s state. It is the case, for instance, of (d, α) reactions.[7] In the transfer from a p state, as for example in (^{7}Li, ^{6}Li) and (^{7}Li, ^{6}He) reactions, the same type of j dependence is only present in the T_{3q}.

IV. CALCULATIONS FOR THE ^{58}Ni($^7\vec{\text{Li}}$,^{6}Li)^{59}Ni REACTION

The reduced transition amplitudes $\beta_{j_1 j_2}^{LM}$ were computed using Eq. (13) and the d^3r integration was performed analytically through the representation of the radial bound state wave functions as sums of Hankel functions. A program was developed to this effect for wave functions generated in Woods-Saxon potentials. Using a variational method we find that by choosing carefully the initial estimates of the parameters in the arguments of the Hankel functions, the wave functions can be satisfactorily reproduced down to distances of the order of 2 fm. The relative error in the Hankel function representation of neutron bound states was always smaller than 1% in the asymptotic region and of the order of 2% down to distances of about 1 fm. For proton states a maximum relative error of the order of 4% is obtained down to distances of 2 fm.

A computer code[22,23] has been programmed in Fortran to generate the distorted waves χ_a, χ_b and to calculate the reduced transition amplitudes given by Eq. (13), with $\vec{p}_1$ determined by Eqs. (10) and (11). Good agreement was obtained in comparisons of differential cross sections of one nucleon transfer reactions calculated with this code and with full finite range DWBA codes.[24,25] Reactions where recoil effects are known to be large were specially considered. This is the case, for instance, of the ^{12}C(^{14}N, ^{13}N)^{13}C ground state reaction.[24] The spectroscopic factors for this transition determined in calculations using the present model and the full finite range code LOLA differ by less than 10% at ^{14}N incident energies of 78 and 100 MeV.[24,26]

Calculations of analyzing power angular distributions were performed for the ^{58}Ni($^7\vec{\text{Li}}$, ^{6}Li)^{59}Ni reaction at $E_{\text{Li}} = 20.3$ MeV.[4] Generally we find that the analyzing powers T_{kq} with k odd are very sensitive to p_1 and therefore have a strong Q dependence. Those with k even have a weaker dependence on $\vec{p}_1$ and Q but are more sensitive to shape effects in the projectile. Figure 3 shows the result of calculations of the cross section and iT_{11} angular distributions to the ground state and two excited states of ^{59}Ni using Li optical model potentials of Ref. 27. The calculations were performed assuming a pure $p\frac{3}{2}$ neutron configuration in ^{7}Li and $p\frac{1}{2}$ and $f\frac{5}{2}$ states were assumed for the unresolved first excited state in ^{59}Ni. A $g\frac{9}{2}$ configuration was assumed for the 3.06 MeV level in ^{59}Ni. We obtain a reasonable description of the iT_{11} angular distributions, in particular of its strong Q dependence (the ground state Q value is 1.8 MeV) for $\theta > 60°$. The change of sign of iT_{11} occurs for $Q \cong -0.5$ MeV, which corresponds to the condition that $p_1 = 0$ in the present reaction. This Q dependence could not be reproduced when using the approximation of Ref. 13, where $\vec{p}_1$ in Eq. (13) is replaced by a local recoil momentum $\vec{q}$.

For bell shaped angular distributions, such as those of Fig. 3, Bond[8] has shown that the vector analyzing power is expected to be independent of θ over a wide range of angles near the peak in the cross section. This type of behavior is clearly seen in Fig. 3, but for angles smaller than the peak, iT_{11} is strongly reduced in a way that depends on the values of l_2 and j_2. Thus, although iT_{11} depends weakly on l_2, j_2 in the angular region where it is almost constant, l_2 and j_2 dependent effects are present at forward angles.

The calculations of Fig. 4 show that the T_{2q} have a much stronger angular dependence than iT_{11}. It is straightforward to show, using the model of Knutson *et al.*,[28] that the particular form of these angular distributions can be understood as a consequence of the oblate shape of ^{7}Li. We take, for instance, T_{20}, which can be measured using a beam with the alignment axis parallel to the incident beam direction, as shown in Fig. 2. With this spin orientation the cross section obtained with the aligned beam σ_a is related to the unpolarized cross section σ_u by

$$\sigma_a = \sigma_u(1 + T_{20}/\sqrt{2}) . \tag{27}$$

Since the projectile follows approximately a Coulomb trajectory, the transfer for large θ ($\theta > 90°$) is less favored in the aligned beam than in the unpolarized beam because of the smaller overlap between the ^{7}Li and the target. This is illustrated in Fig. 2. Hence Eq. (27) implies that T_{20} is negative for $\theta > 90°$ (see Fig. 4). The same argument also shows that T_{20} must change sign as θ decreases.

The results of Fig. 4 suggest that the T_{2q} have a strong dependence on l_2 and j_2 at forward angles. To study these effects it is most convenient to consider the observable

$$A_{yy} = -(\sqrt{3}\, T_{22} + T_{20}/\sqrt{2}) , \tag{28}$$

in which the angular dependence from the deformed shape of ^{7}Li is reduced to a minimum. In fact, the measurement of A_{yy} involves a ^{7}Li beam aligned along the y axis and with this spin orientation the ^{7}Li wave function has radial symmetry in planes parallel to the reaction plane, as shown in Fig. 2. The result of calculations assuming different values of l_2 and j_2 in the $Q = -1.3$ MeV transition are

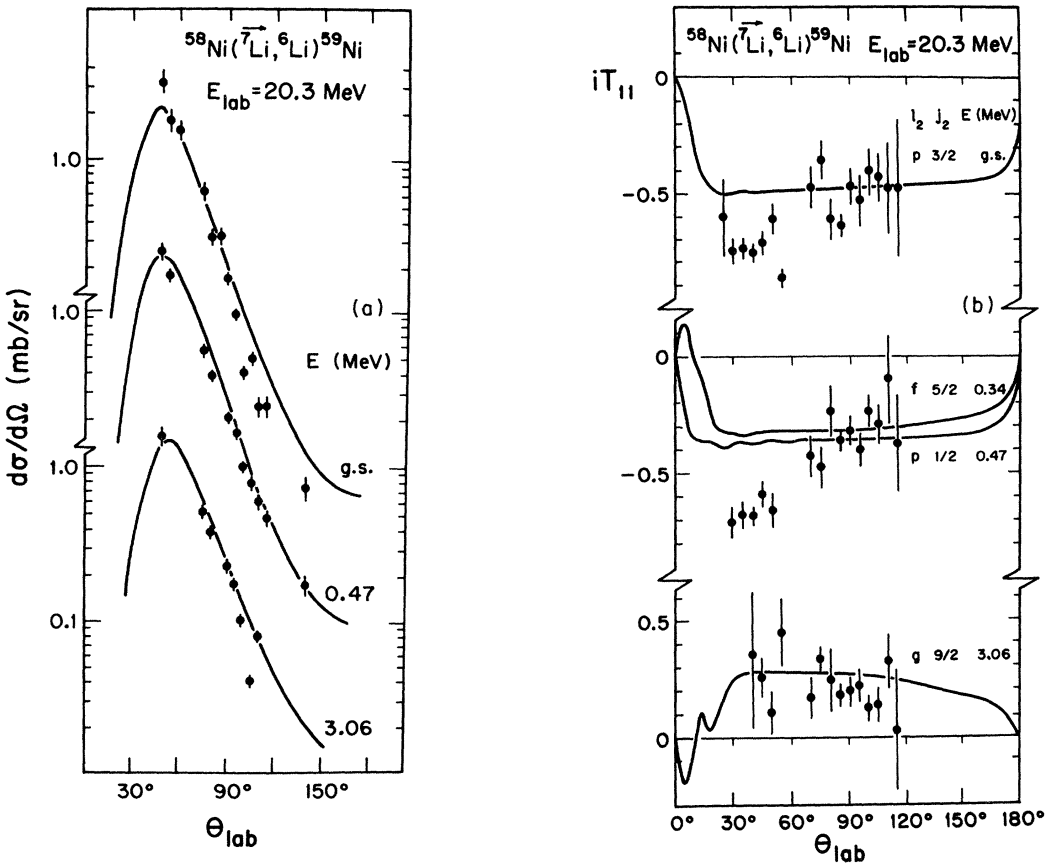

FIG. 3. Comparison between distorted wave calculations using Eq. (13) and data from Ref. 4 for (a) cross section and (b) iT_{11} angular distributions to the ground state and two excited states of ^{59}Ni. The neutron final state configuration was assumed to be $p\frac{3}{2}$, $p\frac{1}{2}$, and $g\frac{9}{2}$, respectively. For the unresolved first excited state (Ref. 4) the calculated iT_{11} is shown for $p\frac{1}{2}$ and $f\frac{5}{2}$ states. E are the excitation energies.

shown in Fig. 5. For angles larger than the position of the cross section peak, A_{yy} is approximately independent of θ, but its value is strongly dependent on l_2 and j_2. This result indicates that A_{yy} measurements may be useful to identify the l_2 and j_2 values in a given transition. In this context we note that A_{yy} has a much weaker Q-value dependence than iT_{11}, as is illustrated in Fig. 6.

The transitions to s states are particularly interesting because we predict A_{yy} to be independent of θ (see Fig. 5) and to have the value

$$A_{yy} = (5\sqrt{2})^{-1}. \tag{29}$$

This relation, proved in the Appendix, is exact on the condition that we neglect spin dependent distortion effects and the $p\frac{1}{2}$ admixture in ^{7}Li.

Finally we consider the analyzing powers iT_{3q} of ($^7\vec{\text{Li}}$,^{6}Li) reactions. In view of Eq. (26) they are predicted to satisfy the relation

$$iT_{3q}(J_B = l_2 + \tfrac{1}{2}) = -\frac{l_2}{l_2+1}\, iT_{3q}(J_B = l_2 - \tfrac{1}{2}). \tag{30}$$

The calculations of Fig. 7 for $p\frac{3}{2}$ and $p\frac{1}{2}$ final states show that the j dependence described by Eq. (30) is quite distinct at forward angles although the iT_{3q} are relatively small.

Up to now it was assumed that the neutron is initially in a pure $p\frac{3}{2}$ state. This is the case of (^{7}Li, ^{6}He) reactions. In (^{7}Li, ^{6}Li) reactions the effect of a $p\frac{3}{2}$ and $p\frac{1}{2}$ admixture in the ^{7}Li-^{6}Li overlap on the analyzing powers is determined by the difference between the reduced transition amplitudes corresponding to these two states. Since the transfer process is preferentially localized at large R, particularly at Coulomb energies, the difference in $\beta^{LM}_{1/2\, j_2}$ and $\beta^{LM}_{3/2\, j_2}$ have their origin mainly in the tail of the bound state wave functions. Hence it is a good approximation to assume

$$\beta^{LM}_{1/2\, j_2} = (N_{1\,1/2}/N_{1\,3/2})\beta^{LM}_{3/2\, j_2}. \tag{31}$$

This relation is exact when the neutron radial wave functions in ^{7}Li are chosen as in Eq. (14).

Using Eqs. (22) and (31) one can relate in a straightforward manner the analyzing powers of a

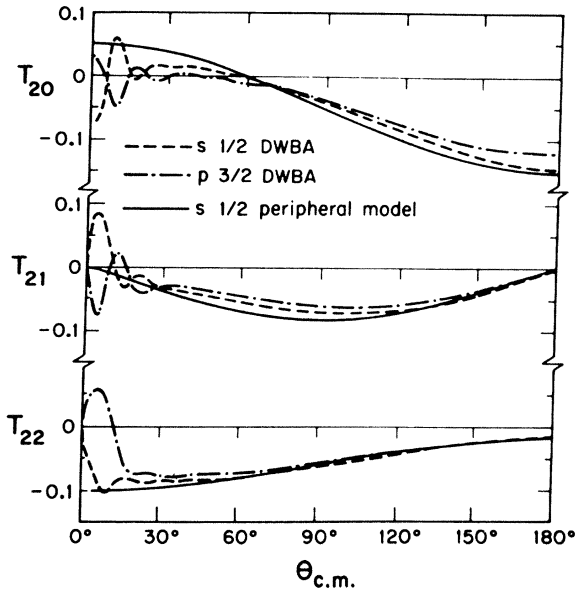

FIG. 4. Calculated T_{2q} for the same reaction of Fig. 3 to the g.s. of ^{59}Ni. The full curve is obtained with the peripheral model applied to the transfer from a $p\frac{3}{2}$ to an s state. The dash and dot-dashed curves are distorted wave calculations using Eq. (13) corresponding to $s\frac{1}{2}$ and $p\frac{3}{2}$ final states.

mixed initial state with those of the pure initial states. In the particular case of transitions to s states this gives

$$iT_{11} = C_1 iT_{11}(3/2), \tag{32a}$$

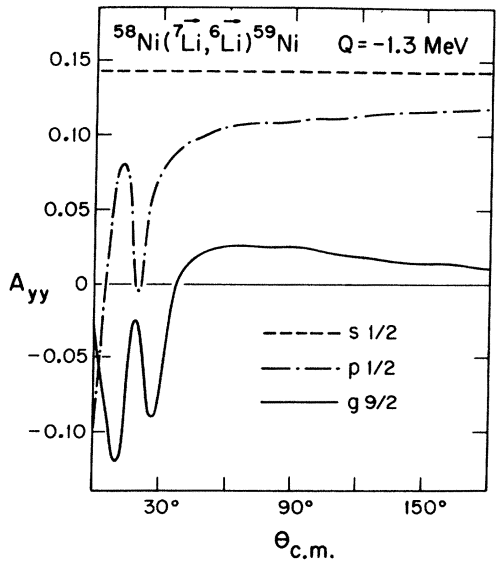

FIG. 5. Distorted wave calculations using Eq. (13) for transitions from a $p\frac{3}{2}$ to an s state (dashed curve), $p\frac{1}{2}$ state (dot-dash curve), and $g\frac{9}{2}$ state (full curve). The effect of a $p\frac{1}{2}$ admixture in the initial state is to multiply A_{yy} by C_2.

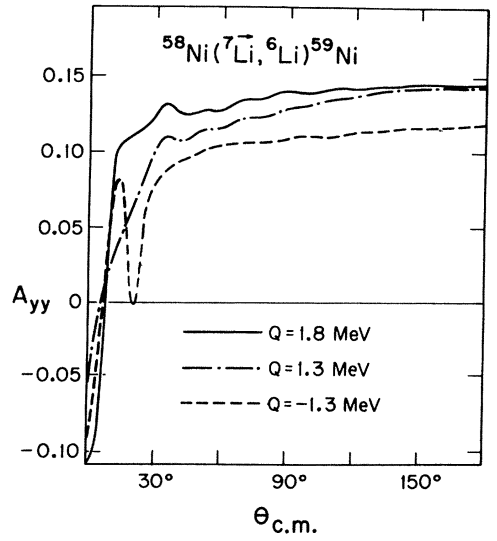

FIG. 6. Distorted wave calculations as in Fig. 5 for the Q-value dependence of A_{yy} in a transition from a $p\frac{3}{2}$ state to a $p\frac{1}{2}$ final state.

$$T_{2q} = C_2 T_{2q}(3/2). \tag{32b}$$

Here $T_{kq}(3/2)$ are the analyzing powers for a pure $p\frac{3}{2}$ configuration. The proportionality factors in Eq. (32) are given by

$$C_1 = (1 + 4\sqrt{5}\,R/11 + 10R^2/11)/(1+R^2), \tag{33a}$$

$$C_2 = (1 + 4\sqrt{5}\,R)/(1+R^2). \tag{33b}$$

In Eq. (33)

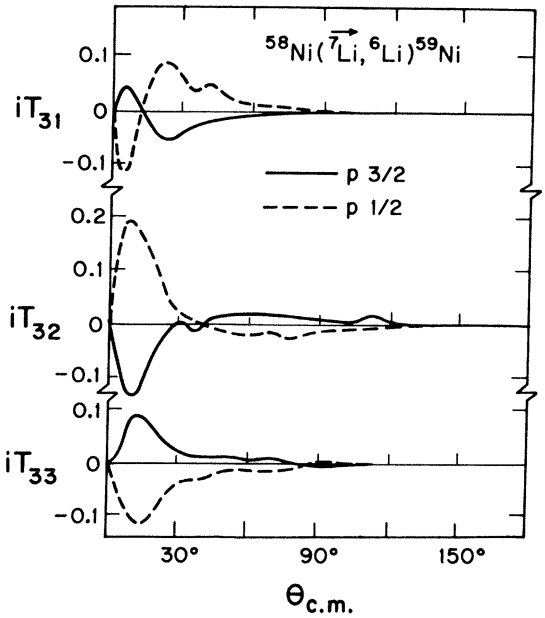

FIG. 7. Distorted wave calculations as in Fig. 5 for the g.s. of ^{59}Ni assuming it to be a $p\frac{3}{2}$ state (full curve) and a $p\frac{1}{2}$ state (broken curve).

$$R = S_{1\,1/2} N_{1\,1/2} (S_{1\,3/2} N_{1\,3/2})^{-1} \tag{34}$$

is the ratio of vertex constants. Using values of asymptotic normalization constants generated in Woods-Saxon potentials and spectroscopic amplitudes from Ref. 29, we obtain $R = 0.93$. Hence $C_1 = 1.36$ and $C_2 = 5.00$. Equation (32) shows that the effect of the $p\frac{1}{2}$ state is considerably larger in T_{2q} than in iT_{11}. The observable A_{yy} is particularly suitable to extract information on R since it is predicted to be independent of angle and to have the value

$$A_{yy} = C_2 (5\sqrt{2})^{-1} . \tag{35}$$

The anisotropy in A_{yy} provides information on spin dependent distortion effects.

V. PERIPHERAL MODEL

The essential features of the analyzing powers in the ^{58}Ni($^7\vec{\text{Li}}$, ^{6}Li)^{59}Ni reaction can also be interpreted using a considerably simplified peripheral model for the transfer process. In view of the relatively good l matching and because only a relatively narrow band of partial waves contributes to the reaction, it is now assumed that the transfer is completely localized at the point of closest approach in the Rutherford orbit of the projectile. This type of model was first suggested by Knutson *et al.*[28] for deuteron stripping reactions at sub-Coulomb energies. Since the distorted waves tend to peak at the distance of closest approach $\vec{d}$ (represented in Fig. 2), the product of distorted waves in Eq. (13) is replaced by $\delta(\vec{R} - \vec{d})$. In this approximation the reduced transition amplitudes are given by

$$\beta^{LM}_{j_1 j_2} = \int d^3r\, F^{LM}_{j_1 j_2}(\vec{d} - \vec{r}, \vec{r})\, e^{i\vec{R}\cdot\vec{p}_1/\hbar} , \tag{36}$$

and using Eq. (15) we obtain

$$\beta^{LM}_{j_1 j_2} = \frac{2\pi\hbar^2}{\mu_1} \sum_{lL_c} i^{l_1 - l_2 + l}\, \Gamma X_{L_c}(d) \left(\frac{|p_1|}{\hbar}\right)^l \times \sum_{mM_c} (lmL_cM_c|LM) Y^{M_c *}_{L_c}(\hat{d}) Y^{m*}_l(\hat{p}_1) . \tag{37}$$

Equation (37) provides some insight on how the orientation of the transferred orbital angular momentum $\vec{L}$ depends on the scattering angle and on p_1. In particular we notice that high values of $|M_c|$ are favored for $\theta < \pi/2$, since $\vec{d}$ makes an angle of $(\theta + \pi)/2$ with the z axis. This result means that for small $|p_1|$, $\vec{L}$ preferentially points in the reaction plane. However, for $|Q| \gg m_x v^2/2$, $|p_1|$ is large and the contributions from high $|m|$ can align $\vec{L}$ perpendicular to the reaction plane. In the particular case where $p_1 = 0$, Eq. (37) reduces to the form

$$\beta^{LM}_{j_1 j_2} = \frac{\sqrt{\pi}\hbar^2}{\mu_1} (-1)^{l_1} \hat{l}_1 (l_1 0 L 0 | l_1 0)\, i^{l_1 - l_2} X_L(d) Y^{M*}_L(\hat{d}) . \tag{38}$$

Using Eq. (22) one can easily show that because of the selection rule $l_1 + l_2 + L = $ even, the analyzing powers T_{kq} for k odd are identically zero. In particular iT_{11} vanishes for $p_1 = 0$, as already noted in Sec. III. However, the $k = $ even analyzing powers do not necessarily vanish for $p_1 = 0$.

The substitution of Eq. (37) into Eq. (22) gives closed analytical expressions for the T_{kq} as a function of $\vec{d}$ and $\vec{p}_1$. We consider here the explicit form of these expressions for a case of particular interest in the (^{7}Li, ^{6}Li) reaction. It is shown in the Appendix that in a transfer from a mixed $p\frac{3}{2}$, $p\frac{1}{2}$ state to an s state

$$iT_{11} = C_1 \frac{11}{6\sqrt{10}} \tanh(2u) , \tag{39a}$$

$$T_{2q} = -C_2 \frac{1}{5} \left(\frac{4\pi}{5}\right)^{1/2} [\sin^2\omega Y^q_2(\delta, 0) + \cos^2\omega Y^q_2(\gamma, 0)] . \tag{39b}$$

Here C_1, C_2 are given in Eq. (33),

$$u = \tanh^{-1} x , \tag{40a}$$

$$\omega = \tan^{-1} x , \tag{40b}$$

$$x = p_1 d[\hbar(1 + \beta_2 d)]^{-1} , \tag{40c}$$

and γ and δ are the angles that $\vec{d}$ and $\vec{p}_1$ make with the incident beam direction. Equation (39a) shows clearly that iT_{11} is independent of θ (except for the weak dependence of d on θ) and has a strong Q dependence since it is approximately linear in p_1. In particular iT_{11} changes sign with p_1. It is interesting to note that the formula of Tungate *et al.*[4] for iT_{11} can be derived from Eq. (39a). In fact, $u \cong p_1/(\hbar\beta_1)$ for $p_1 < \beta_1$ since in the prior form of the transition amplitude, used in Ref. 4, $x = p_1/(\hbar\beta_1)$ in the limit $d \to \infty$. This result is understandable because the projectile trajectories are assumed to be straight lines in Ref. 4, which is a condition obtained in the limit $d \to \infty$. The strong Q dependence of iT_{11}, as given by Eq. (39a), cannot be obtained when $\vec{p}_1$ is replaced by the recoil momentum $\vec{q}$ obtained in a local momentum approximation. This is a consequence of the fact that q is a slowly varying function of Q value.

The T_{2q} have a quadratic dependence on p_1 according with Eq. (39b). For small p_1 the $\sin^2\omega$ term is negligible and the angular dependence of T_{2q} is approximately given by $Y^q_2[(\pi + \theta/2), 0]$. Figure 4 shows that there is good agreement between the predictions of the peripheral model and calculations using Eq. (13), except at forward angles where the complete localization of the transfer process is not a good approximation.

VI. CONCLUSIONS

We have discussed the analyzing powers of heavy ion induced transfer reactions within the framework of the DWBA and using a semiclassical approximation to describe the conservation of kinetic energy in the transfer process. Calculations with this model reproduce satisfactorily the cross section and iT_{11} angular distributions in the ^{58}Ni($^{7}\vec{\text{Li}}$, ^{6}Li)^{59}Ni reaction at an energy of 20.3 MeV. In particular, good agreement is obtained with the Q-value dependence observed in iT_{11}.

In general we find that the analyzing powers T_{kq} with $k=1$ and 3 have a strong dependence on the reaction Q value and tend to change sign at $Q=-m_x v^2/2$. The T_{kq} with $k=2$ do not have this property and show a weaker dependence on Q. Furthermore, their angular distributions are largely determined by the deformed shape of the overlap between the projectile and ejectile.

We showed that strong j-dependent effects are present in the analyzing powers of ($^{7}\vec{\text{Li}}$, ^{6}Li) reactions. Analogous effects are also expected in ($^{7}\vec{\text{Li}}$, ^{6}He) reactions. A_{yy} is particularly interesting in this context. Calculations for the ^{58}Ni($^{7}\vec{\text{Li}}$, ^{6}Li)^{59}Ni reaction show that A_{yy} is approximately isotropic for angles larger than the cross section peak. The value of A_{yy} in this angular region shows a strong dependence on the quantum numbers of the neutron final state. In the particular case of transitions to s states $A_{yy}\cong C_2(5\sqrt{2})^{-1}$, where C_2 is a function of the asymptotic mixing ratio between the $p\frac{3}{2}$ and $p\frac{1}{2}$ components of the ^{7}Li-^{6}Li overlap. Calculations of the iT_{3q} analyzing powers show that they have a strong sign rule type j dependence. This j dependence is similar to that observed in the vector analyzing powers of light-ion induced transfer reactions.

Closed analytic expressions for the analyzing powers are obtained using a simplified reaction model, where it is assumed that the transfer process is completely localized in space. This peripheral model provides a qualitative description of the essential feature of the j and Q-value effects in the ^{58}Ni($^{7}\vec{\text{Li}}$, ^{6}Li)^{59}Ni reaction.

We assumed that there are no spin dependent interactions in the Li optical potentials. The determination of the type and strength of these forces is presently being actively investigated in the elastic scattering of polarized Li beams.[30-32] Spin dependent interactions are likely to affect the calculated polarizations in transfer reactions. We are presently studying the analysis of these effects in the ($^{7}\vec{\text{Li}}$, ^{6}Li) reaction.

ACKNOWLEDGMENTS

The authors are grateful to D. Fick, G. Tungate, and R. C. Johnson for many illuminating discussions during the course of this work. We also wish to thank H. T. Richards for his critical reading of the manuscript. One of us (F.D.S.) would like to thank the University of Wisconsin for its hospitality and the NATO Science Programme (Grant No. 5-2-03B154) for its financial assistance. This work was supported in part by the U. S. Department of Energy.

APPENDIX: TRANSFER TO AN s STATE

A. Isotropy of A_{yy}

In a nucleon transfer from a $p\frac{3}{2}$ state into a $s\frac{1}{2}$ state, $l_2=0$, $L=l_1=1$, and $j_2=s_x=\frac{1}{2}$ in Eq. (20). With these conditions the inversion of Eq. (20) gives

$$B^{1M}_{3/2\,1/2}B^{1M'*}_{3/2\,1/2}=\sigma\sum_k(-1)^M(1-M1M'|kq)T_{kq}/A_k\,, \tag{A1}$$

where

$$A_k=8(-1)^{k+1}W(k\tfrac{3}{2}\tfrac{3}{2}1;\tfrac{3}{2}\tfrac{3}{2})W(\tfrac{3}{2}\tfrac{3}{2}11;k\tfrac{1}{2})\,. \tag{A2}$$

In a coordinate system S' with z axis along $\vec{k}_a\times\vec{k}_b$ the amplitudes $B^{1M}_{3/2\,1/2}$ vanish unless $(-1)^M=\pi_{ab}=-1$. Thus for $M=M'=0$, Eq. (A1) gives

$$\sum_k(1010|k0)T_{k0}(S')/A_k=0\,. \tag{A3}$$

Noting that $A_{yy}(S)=\sqrt{2}\,T_{20}(S')$, Eqs. (A2) and (A3) give

$$A_{yy}(S)=(5\sqrt{2})^{-1}\,. \tag{A4}$$

With $p\frac{3}{2}$ and $p\frac{1}{2}$ mixing in the projectile, Eq. (20) gives

$$A_{yy}=\frac{1}{5\sqrt{2}}(1-4\sqrt{5}\,r/b_{3/2})/[1+(b_{1/2}/b_{3/2})^2]\,, \tag{A5}$$

with

$$r=\mathrm{Re}\Big(\sum_M B^{1M}_{1/2\,1/2}B^{1M*}_{3/2\,1/2}\Big),\quad b_j=\sum_M|B^{1M}_{j\,1/2}|^2\,. \tag{A6}$$

Here the amplitudes $B^{1M}_{j_1 1/2}$ are referred to the coordinate system S'. Notice that Eq. (A5) reduces to (35) with the approximation (31). The relation (A4) is not valid when spin dependent interactions are included in the optical potentials. In this case, L in Eq. (2) is the overall orbital angular momentum transfer and therefore it is not restricted to the value 1 in transfers to s states.

B. Peripheral model

In a transition to an s state the angular momentum coupling in Eq. (16) implies that $L_1=L_c$ and l

can be either 0 or 1. For $l=0$, $L_c=1$ and $\Gamma=1$, while for $l=1$, $L_c=0$ and $\Gamma=-1$. Using these values in Eqs. (3) and (37) gives

$$B^{1M}_{3/2\,1/2} = is[aY_1^{M*}(\hat{d}) + ip_1 bY_1^{M*}(\hat{p}_1)] , \qquad \text{(A7)}$$

where

$$s = -S_{1\,3/2}N_{1\,3/2}S_{1\,1/2}N_{1\,1/2}\left(\frac{2\pi}{3}\right)^{1/2}\frac{\hbar^2}{\mu_1\beta_1^2} ,$$

$$a = i\beta_2 h_1(i\beta_2 d) ,$$

$$b = h_0(i\beta_2 d) .$$

The substitution of Eq. (A7) into Eq. (20) gives

$$iT_{11} = C_1\frac{11}{6\sqrt{10}}\frac{2|x|}{1+x^2}\sin(\gamma-\delta) ,$$

$$T_{2q} = -C_2\frac{1}{5}\left(\frac{4\pi}{5}\right)^{1/2} \times\left[\frac{x^2}{1+x^2}Y_2^q(\delta,0)+\frac{1}{1+x^2}Y_2^q(\gamma,0)\right] .$$

Here γ and δ are the angles that $\vec{d}$ and $\vec{p}_1$ make with with the z axis and

$$x = p_1 d[\hbar(1+\beta_2 d)]^{-1} .$$

---

*On leave from Laboratório de Física, Universidade de Lisboa, 1294 Lisboa Codex, Portugal.

[1]K. Sugimoto, N. Takahashi, A. Mizobuchi, Y. Nojiri, T. Minamisono, M. Ishihara, K. Tanaka, and H. Kamitsubo, Phys. Rev. Lett. 39, 323 (1977).
[2]C. Lauterbach, W. Dünnweber, G. Graw, W. Hering, H. Puchta, and W. Trautmann, Phys. Rev. Lett. 41, 1774 (1978).
[3]F. Pougheon, P. Roussel, M. Bernas, F. Diaf, B. Fabbro, F. Naulin, E. Plagnol, and G. Rotbard, Nucl. Phys. A325, 481 (1979).
[4]G. Tungate, R. Böttger, P. Egelhof, K.-H. Möbius, Z. Moroz, E. Steffens, W. Dreves, D. Fick, D. M. Brink, and T. H. Hill, Phys. Lett. 95B, 35 (1980).
[5]W. Haeberli, in *Proceedings of the International Conference on Nuclear Structure, Tokyo, 1977*, J. Phys. Soc. Jpn. 44 Suppl., 435 (1978).
[6]F. D. Santos, Phys. Rev. C 13, 1145 (1976).
[7]S. A. Tonsfeldt, T. B. Clegg, E. J. Ludwig, Y. Tagishi, and J. F. Wilkerson, Phys. Rev. Lett. 45, 2008 (1980).
[8]P. D. Bond, Phys. Rev. C 22, 1539 (1980).
[9]D. M. Brink, *Les Houches, Session XXX* (North-Holland, Amsterdam, 1978), p. 1.
[10]H. Hasan and D. M. Brink, J. Phys. 65, 771 (1979).
[11]G. R. Satchler, Nucl. Phys. 55, 1 (1964).
[12]*Proceedings of the 3rd International Symposium on Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (University of Wisconsin Press, Madison, 1971), p. 25.
[13]P. Braun-Munzinger, H. L. Harvey, and S. Wenneis, Nucl. Phys. A235, 190 (1974).
[14]P. Hauge, Nucl. Phys. A236, 61 (1974).
[15]P. J. A. Buttle and L. J. B. Goldfarb, Nucl. Phys. 78, 409 (1966).
[16]G. L. Payne and P. L. Von Beheren, Phys. Rev. 56, 1955 (1972).
[17]F. D. Santos, Nucl. Phys. A212, 341 (1973).
[18]Q. K. K. Liu and P. J. A. Buttle, Z. Phys. A 275, 283 (1975).
[19]J. E. Kim and W. W. Daehnick, Phys. Rev. C 23, 742 (1981).
[20]M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1963).
[21]H. C. Newns, Proc. Phys. Soc. London, Sect A 66, 477 (1953).
[22]A. M. Gonçalves, Ph.D. thesis, Lisbon 1979 (unpublished).
[23]A. M. Gonçalves,and F. D. Santos, Port. Phys. 10, 117 (1979).
[24]R. M. DeVries and K. I. Kubo, Phys. Rev. Lett. 30, 325 (1973).
[25]P. J. A. Buttle, Comp. Phys. Commun. 14, 133 (1978).
[26]R. M. DeVries, M. S. Zisman, J. G. Cramer, K. L. Liu, F. D. Becchetti, B. G. Harvey, H. Homeyer, D. G. Kovar, J. Mahoney, and W. Von Oertzen, Phys. Rev. Lett. 32, 683 (1974).
[27]W. Dreves, P. Zupranski, P. Egelhof, D. Kassen, E. Steffens, W. Weiss, and D. Fick, Phys. Lett. 78B, 36 (1978).
[28]L. D. Knutson, E. J. Stephenson, and W. Haeberli, Phys. Rev. Lett. 32, 690 (1974).
[29]S. Cohen and D. Kurath, Nucl. Phys. A101, 1 (1967).
[30]H. Amakawa and K.-I. Kubo, Nucl. Phys. A266, 521 (1976).
[31]F. Petrovich, D. Stanley, L. A. Parks, and P. Nagel, Phys. Rev. C 17, 1642 (1978).
[32]H. Nishioka, R. C. Johnson, and K.-I. Kubo (private communication).