Pauli blocking in pion-^{4}He scattering including partial wave mixing

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Detailed calculations are presented of the Pauli blocking correction in π-^{4}He scattering for the energy range $T_\pi = 24-220$ MeV. Pion-nucleon partial wave mixing which is due to the Pauli projection operator is taken into account for the first time. The first-order optical potential of Celenza, Liu, and Shakin is used. The nucleon-core interaction is simulated by means of an effective π-N subenergy. Satisfactory results are obtained for the cross sections at low energies. However, at intermediate energies the elastic cross sections are overestimated by 30–40%. The partial wave mixing which is introduced by the Pauli principle generates a small correction at low energies but in the resonance region it leads to a clear increase in the cross sections by 10–20%.

[NUCLEAR REACTIONS π-^{4}He scattering, $T_\pi = 24-220$ MeV, optical potential, Pauli-principle corrections.]

An essential ingredient in a theory for pion-nucleus scattering is the single-scattering process in which the pion scatters from one nucleon at a time.[1] Usually one takes as the optical potential for this process the ground-state expectation value of the free π-N T operator, summed over all nucleons:

$$U_{\text{opt}} = A\langle\psi_0 | t(\omega) | \psi_0\rangle , \quad (1)$$

where the π-N subenergy ω depends on the way in which the binding and Fermi motion of the target nucleons are treated.[2] In a three-body picture, the π-N subenergy is shifted downwards with respect to the π-A c.m. energy by the kinetic energy of the nuclear core in π-A c.m. and the binding energy E_b of the struck nucleon in the initial state. This is the prescription used by Liu and Shakin.[3] However, in this way the nucleon-core interaction in the intermediate scattering state is ignored. To include this "binding effect" several authors use a reduced downward shift, e.g., Landau and Thomas[4] take an unrealistically small value for E_b and Schmit *et al.*[5] use a prescription in which E_b is cancelled completely.

For an optical potential of the form (1), a mixing of the π-N partial waves into each π-A partial wave in the partial wave expansion of the optical potential only arises from the three-body kinematics of the single-scattering process. However, in Eq. (1) the Pauli principle for the intermediate scattering states is not taken into account, and this can have important effects in π-A scattering. As shown in Ref. 6, its neglect may lead, e.g., to an incorrect description of the threshold behavior of the single-scattering process. In a complete theory, Pauli blocking also leads to a π-N partial wave mixing. Because of the Pauli principle, one should use a G operator for the π-N interaction rather than a T operator. In a G operator the intermediate propagation of the struck nucleon in the occupied states is excluded. And, as is well known from nuclear-structure studies where the G operator was originally introduced, the Pauli-projection operator which appears in the G operator leads to a mixing of partial waves.

In this work I report on calculations of the Pauli-blocking effect for π-^{4}He scattering. In these calculations the π-N partial wave mixing which is due to the Pauli principle is fully taken into account. In this way the importance of this mixing for pion scattering is established for the first time. Furthermore, this approach allows for a reliable evaluation of the Pauli-blocking effect at intermediate energies for which this mixing is especially relevant, as we will see.

Frequently Pauli-principle corrections are calculated by means of nuclear matter procedures in which a local density approximation is applied to the nucleus.[2] Then the phase space of the struck nucleon is restricted by the Fermi sea of occupied states. In these procedures it is standard to use the angle-averaged Pauli operator,[7] which amounts to neglecting the partial wave mixing. For small nuclei one cannot rely on a local-density approxima-

tion and in a calculation of the Pauli blocking effect, rather than a Fermi sea, one should use a shell-model description of the occupied states. This has been done by Hirata *et al.* in the framework of an isobar-hole description of the π-A interaction for the P_{33} π-N partial wave.[8] Recently, we have shown how this can be generalized to all relevant π-N partial waves for an optical-potential description which was applied to π-^{4}He scattering.[9] However, in that work the π-N partial wave mixing was ignored. This restricted the applicability to the low energy regime, where one expects this mixing to be the least important.

The relation between the G operator for π-N scattering τ and the T operator t is given by[8,9]

$$\tau = t - t\frac{1}{e}R\tau \,. \tag{2}$$

Here the π-A propagator is denoted by $1/e$ and R is a projection operator which projects on the space of occupied states. In this way the nuclear ground state is also excluded, as is required for an optical potential. Numerically, however, the ground-state exclusion represents a much smaller correction than the Pauli-blocking effect.[9] Following the approach of Ref. 9, we expand τ and t into partial waves. This gives

$$\tau = \sum_{Tjl} \tau_{Tjl} P^l P^j_l P^T \tag{3}$$

and

$$t = \sum_{Tjl} t_{Tjl} P^l P^j_l P^T \,, \tag{4}$$

where P^l projects on a π-N partial wave with orbital angular momentum l, and where P^j_l and P^T are projection operators which project on the π-N spin and isospin space, respectively. Substituting Eqs. (3) and (4) into Eq. (2) gives

$$\tau_{Tjl} P^l P^j_l P^T = \left[t_{Tjl} P^l - t_{Tjl} P^l \frac{1}{e} R \tau_{Tjl} P^l - \sum_{l',l' \neq l} \sum_{j'} t_{T\,j'l'} P^{l'} P^{j'}_{l'} \frac{1}{e} R \tau_{Tjl} P^l \right] P^j_l P^T \,. \tag{5}$$

In Eq. (5) the last term represents the partial wave mixing contribution. We write the optical potential $U_{\rm opt}$ as the sum of its π-N partial wave contributions

$$U_{\rm opt} = \sum_{Tjl} U^{Tjl}, \quad \text{where } U^{Tjl} = 4 C_T C_{jl} \langle ls \mid t_{Tjl} P^l \mid 1s \rangle \,. \tag{6}$$

We will restrict ourselves to π-N S and P waves. The coefficient C_{jl} is 1 for S waves. For P waves C_{jl} is $\frac{1}{3}$ and $\frac{2}{3}$ for $j = \frac{1}{2}$, and $\frac{3}{2}$, respectively. The coefficient C_T is $\frac{1}{3}$ for $T = \frac{1}{2}$ and $\frac{2}{3}$ for $T = \frac{3}{2}$. The factor 4 comes from the number of nucleons, and $\mid 1s \rangle$ denotes the orbital part of the ^{4}He ground state. We can also write the optical potential $\tilde{U}_{\rm opt}$, which is defined in terms of τ instead of t, as

$$\tilde{U}_{\rm opt} = \sum_{Tjl} \tilde{U}^{Tjl}, \quad \text{and } \tilde{U}^{Tjl} = 4 C_T C_{jl} \langle 1s \mid \tau_{Tjl} P^l \mid 1s \rangle \,. \tag{7}$$

In a shell model in which the four nucleons occupy the $1s$ orbit and in which Coulomb effects are ignored, the projection operator R simply equals $\mid 1s \rangle \langle 1s \mid$. We can then derive from Eq. (5) the following set of relations between $\tilde{U}^{Tjl}$ and U^{Tjl}:

$$\tilde{U}(S_{11}) = U(S_{11}) - \tfrac{3}{4} U(S_{11}) \frac{1}{e} \tilde{U}(S_{11}) - \tfrac{3}{4} \left[U(P_{11}) \frac{1}{e} \tilde{U}(S_{11}) + U(P_{13}) \frac{1}{e} \tilde{U}(S_{11}) \right] , \tag{8a}$$

$$\tilde{U}(S_{31}) = U(S_{31}) - \tfrac{3}{8} U(S_{31}) \frac{1}{e} \tilde{U}(S_{31}) - \tfrac{3}{8} \left[U(P_{31}) \frac{1}{e} \tilde{U}(S_{31}) + U(P_{33}) \frac{1}{e} \tilde{U}(S_{31}) \right] , \tag{8b}$$

$$\tilde{U}(P_{11}) = U(P_{11}) - \tfrac{9}{4} U(P_{11}) \frac{1}{e} \tilde{U}(P_{11}) - \tfrac{3}{4} U(S_{11}) \frac{1}{e} \tilde{U}(P_{11}) \,, \tag{8c}$$

$$\tilde{U}(P_{31}) = U(P_{31}) - \tfrac{9}{8} U(P_{31}) \frac{1}{e} \tilde{U}(P_{31}) - \tfrac{3}{8} U(S_{31}) \frac{1}{e} \tilde{U}(P_{31}) \,, \tag{8d}$$

$$\tilde{U}(P_{13}) = U(P_{13}) - \tfrac{9}{8} U(P_{13}) \frac{1}{e} \tilde{U}(P_{13}) - \tfrac{3}{4} U(S_{11}) \frac{1}{e} \tilde{U}(P_{13}) \,, \tag{8e}$$

$$\tilde{U}(P_{33}) = U(P_{33}) - \tfrac{9}{16} U(P_{33}) \frac{1}{e} \tilde{U}(P_{33}) - \tfrac{3}{8} U(S_{31}) \frac{1}{e} \tilde{U}(P_{33}) \,. \tag{8f}$$

Equations (8a)—(8f) are Fredholm-type integral equations which can be solved numerically by standard matrix-inversion techniques. Here the standard notation $L_{2T+1,2j+1}$ is used. One observes from Eqs. (8a)—(8f) that the Pauli principle leads to a partial wave mixing. In addition to spin-orbit effects mixing comes from charge-exchange reactions with an intermediate Pauli-blocked state. These can take place both for πN S and P waves. It is easily verified that similar equations can also be written for the π-^{4}He partial wave contributions to $U_{\rm opt}$ and $\tilde{U}_{\rm opt}$ separately. Because of this, one can conclude that the effect of the "mixing terms" depends crucially on the π-N partial wave admixture in the π-^{4}He partial wave contributions to $U_{\rm opt}$. If this admixture is small, e.g., at low energies, one expects little effect from the π-N partial wave mixing, introduced by the Pauli principle.

In a calculation of the Pauli-blocking effects by means of Eqs. (8a)—(8f) we need to introduce a model for $U_{\rm opt}$ in which the nucleon-core interaction is taken into account. The proper way to construct $U_{\rm opt}$ is by means of a three-body calculation.[1,10] Only very recently has this problem begun to attract serious attention,[11,12] and the common practice is to use a much simpler method, in which one aims at approximating the results of such a three-body treatment as well as possible by means of a clever choice for the π-N subenergy.[4,5] While a three-body treatment is currently under investigation,[13] it will be interesting to see to what results we are led for this simpler procedure in the present discussion of the Pauli-blocking effect.

As in Ref. 9, I use the Celenza, Liu, and Shakin (CLS) optical-potential model[14] for $U_{\rm opt}$. In the applications of this covariant model by Liu and Shakin,[3] the π-N collision energy $\omega = \sqrt{s}$ is obtained from the four-vector relation

$$s = (P + p - Q)^2 , \tag{9}$$

where P, p, and Q are the four vectors of the nucleus, pion, and spectator, respectively, which is illustrated in Fig. 1, and

$$Q^2 = M_C{}^2 , \tag{10}$$

where M_C is the mass of the spectator nucleus. This, however, is precisely the choice for ω in which intermediate binding effects are completely ignored, as can be seen more clearly in the nonrelativistic limit, for which Eqs. (9) and (10) lead to $\omega = T_\pi - K_C - E_{\rm sep}$, where T_π, K_C, and $E_{\rm sep}$ are the asymptotic kinetic energy of the pion in the π-A c.m., the kinetic energy operator for the spectator, and the single-nucleon separation energy, respectively. To incorporate binding corrections, I adopt a method which is based on an observation made by Schmit *et al.*[5] They have shown that in a first approximation the binding correction cancels the downward energy shift due to the separation energy. This suggests that binding effects can be included by taking instead of Eq. (10)

$$Q^2 = (M_A - M_N)^2 , \tag{11}$$

in which M_A and M_N are the masses for the nucleus and the nucleon, respectively. Now we have in the nonrelativistic limit $\omega = T_\pi - K_C$. One may notice that taking binding effects into account in this way results in a description for $U_{\rm opt}$ which is very similar to the scheme proposed by Schmit *et al.*[5,15] Like CLS they carry out the complete Fermi averaging. However, they ignore Pauli-principle effects. The present results will therefore be relevant also to their optical potential and closely related optical-potential models such as the Landau and Thomas model.[4] Furthermore, the present scheme is essentially equivalent to the isobar-hole description of Hirata *et al.*,[8] if their "spreading potential" is turned off and if the Pauli corrections in the S and the small P waves plus the mixing term for the P_{33} partial wave in $\tilde{U}_{\rm opt}$ are ignored, which also allows for interesting comparisons to their results.

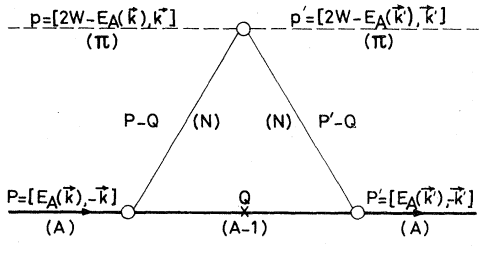
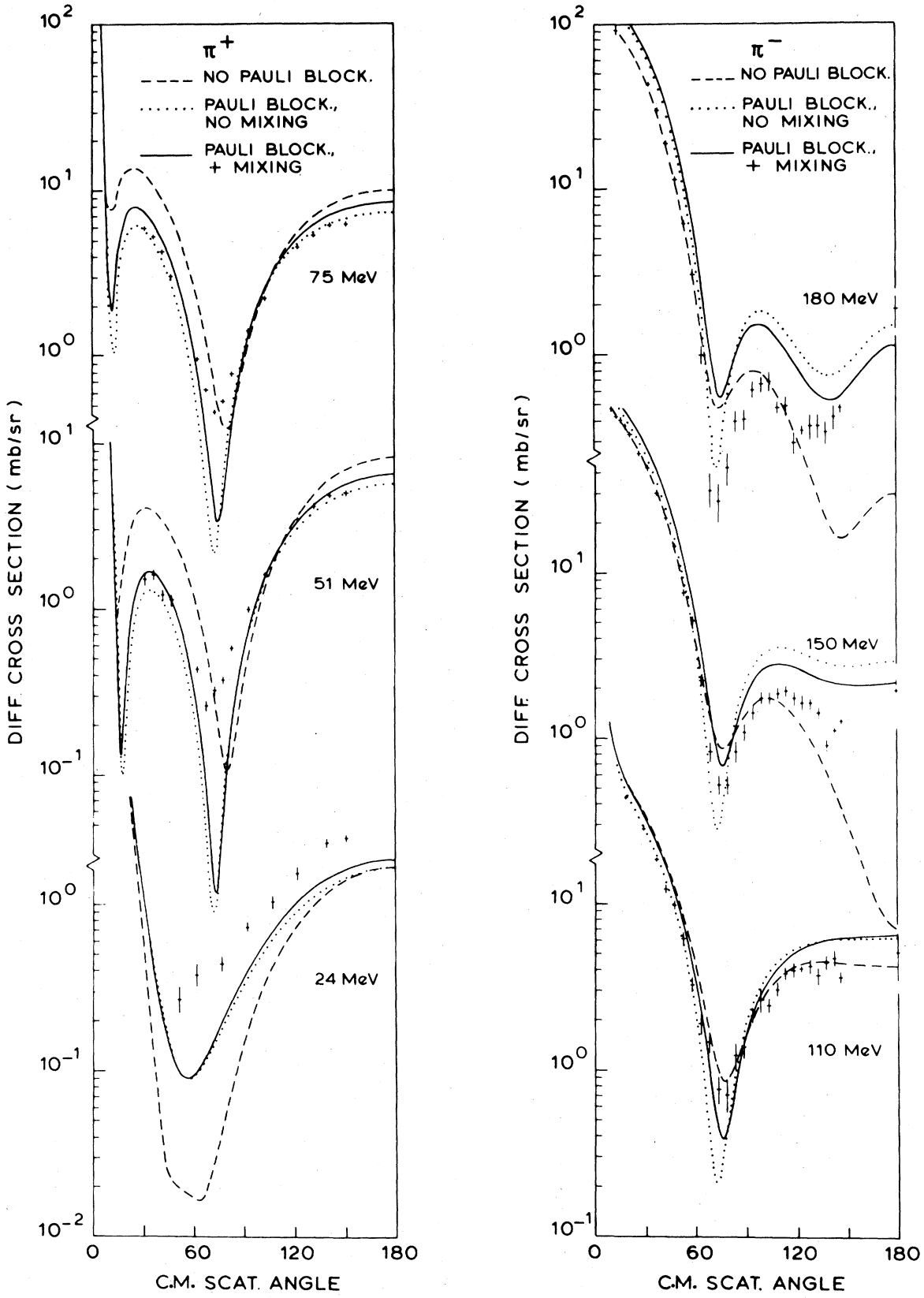


FIG. 1. A graphical representation of the CLS optical potential. The dashed, light, and heavy lines represent the pion (π), nucleon (N), nucleus (A) and residual spectator nucleus ($A-1$), respectively. p, P, and Q are the 4-momenta for the pion, nucleus, and spectator, respectively. The nucleus and the spectator are placed on their mass shell.

Calculations for the differential cross sections have been performed for three cases. First I ignored the Pauli blocking completely, taking $U_{\rm opt}$ for


π+
NO PAULI BLOCK.
PAULI BLOCK., NO MIXING
PAULI BLOCK., + MIXING
75 MeV
51 MeV
24 MeV
DIFF. CROSS SECTION (mb/sr)
C.M. SCAT. ANGLE
π−
180 MeV
150 MeV
110 MeV

the optical potential. Then I included Pauli-principle effects, ignoring the mixing terms. Finally a full calculation was carried out, using $\tilde{U}_{\text{opt}}$. The results are displayed in Figs. 2 and 3. In Fig. 2 π^+ scattering is examined at 24, 51, and 75 MeV. Figure 3 shows the results for π^- scattering at 110, 150, and 180 MeV. For comparison I also plot the available experimental data.[16] Considering first the low energy results for π^+ scattering (Fig. 2), we find a poor agreement with the experimental data if the Pauli-blocking is ignored completely. The curves exhibit a strikingly different behavior for 24 MeV compared to 51 and 75 MeV. While for 24 MeV the cross sections are much too low, they are strongly overestimated at 51 and 75 MeV, particularly at forward angles. This difference reflects the dominance of the repulsive π-N S waves at 24 MeV, while at the higher energies the attractive P_{33} takes over. However, both for 24 and for 51 MeV and 75 MeV, we obtain an important improvement by taking into account Pauli-blocking corrections. One can also judge from these results that the mixing of π-N partial waves due to the exclusion principle indeed represents a minor effect for low energy scattering. It mainly leads to a slight increase of the cross sections at forward angles and a small decrease at backward angles. Examining now the π^- results (Fig. 3) for the higher energies, we notice that the effect of the partial wave mixing is larger. Again we find an increase of the cross sections at forward angles and a reduction at backward angles. For 180 MeV the increase at forward angles even leads to larger cross sections at the first minimum as compared to the results obtained with U_{opt}. On the other hand, if the mixing terms are ignored, a deepening of the minimum results for all three energies considered. Furthermore, we also find that at intermediate energies the Pauli-blocking correction does not improve the agreement with the experimental data (except, perhaps, at extremely backward angles; however, a single scattering theory is not very reliable at these large momentum transfers). At forward angles, the Pauli effect even worsens the agreement, particularly if the mixing terms are included. This contrasts markedly with the low-energy regime. Furthermore, we find that the results for U_{opt} indeed compare very well with those obtained by Maillet *et al.*[17] and that the dotted curves agree with the results of Hirata *et al.*,[18] if they do not include a spreading potential.

The tendencies that we found in the differential cross sections can be seen very clearly also in the integrated elastic cross sections (Fig. 4). At low energies ($T_\pi < 100$ MeV) we see that using U_{opt} strongly overestimates the data, e.g., at $T_\pi = 75$ MeV by almost 50%, and the Pauli-blocking corrections lead to a considerable improvement (Now at 75 MeV the calculated value falls within the experimental error.) Furthermore, it is interesting to see that the results in which the Pauli blocking is taken into account are quite close to the results for U_{opt} in which the Liu and Shakin[3] choice is made for the π-N subenergy [Eqs. (9) and (10)], i.e., in which the intermediate nucleon-core interaction is ignored. This suggests an important cancellation between binding and Pauli-principle corrections, which is very reasonable: The Pauli principle blocks the prohibited bound states, which arise in the nucleon-core interaction.

For the intermediate energies, the results are quite different. Here we have a reasonable agreement with the data using U_{opt}, and much too large values if the Pauli blocking is included. At resonance the elastic cross section is overestimated by 20–30% if partial wave mixing is ignored and by 30–40% in a complete calculation. Although here the magnitude of the cross sections shows no evidence of a cancellation between binding and Pauli effects, we do find an upward energy shift of the resonance peak by about 20 MeV due to the Pauli correction, bringing it very close to the result for U_{opt} in which the binding effect is ignored.

FIG. 2. The differential cross sections for π^+ scattering at T_{lab} equal to 24, 51, and 75 MeV. The dashed, dotted, and solid curves correspond, respectively, to the results in which the exclusion requirements are ignored; the exclusion requirements are taken into account, but in which the mixing terms are ignored; and the full calculation of the exclusion requirements, including the mixing terms. The data for 24 MeV are from Nordberg and Kinsey and for 51 and 75 MeV are from Crowe *et al.* (Ref. 16).

FIG. 3. The differential cross section for π^- scattering at T_{lab} equals 110, 150, and 180 MeV. The dashed, dotted, and solid curves correspond, respectively, to the results in which the exclusion requirements are ignored; the exclusion requirements are taken into account, but in which the mixing terms are ignored; and the full calculation of the exclusion requirements, including the mixing terms. The data are from Binon *et al.* (Ref. 16).

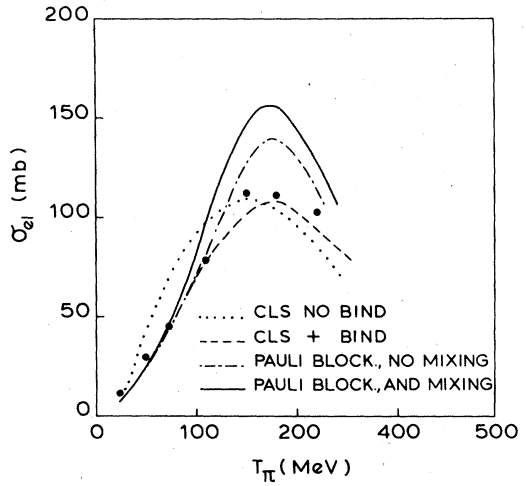


FIG. 4. The integrated elastic cross sections. The data are obtained from Binon *et al.* (Ref. 16). The dotted, dashed, dashed-dotted, and solid curves are, respectively, the results for the CLS optical potential in which the intermediate binding is taken into account; the intermediate binding is ignored; the exclusion requirements are taken into account ignoring the mixing term; and the full calculation of the exclusion requirements including partial wave mixing.

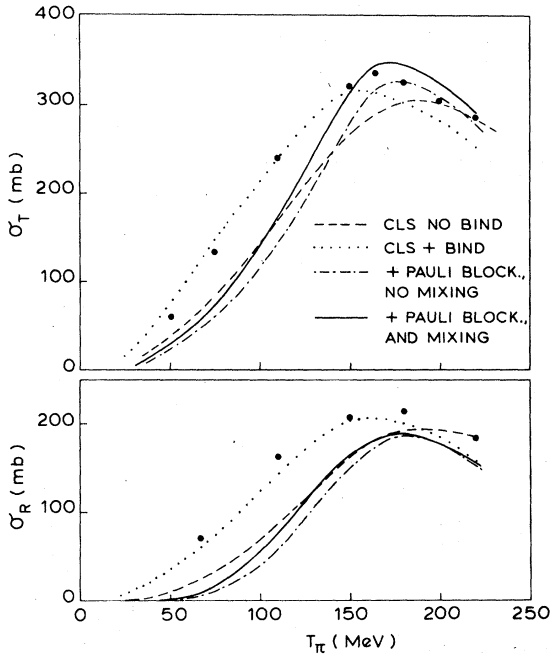


FIG. 5. The total cross section σ_T and the reactive cross section σ_R. The legend of curves is the same as in Fig. 4.

The total cross section σ_T and the reactive cross section σ_R are displayed in Fig. 5. For U_{opt} we find a reactive cross section which is remarkably close to the data. This, however, is not desirable in view of the absorption channel which we have ignored completely. Including the Pauli-blocking corrections leads to a reduction which is considerable at low energies. Compared to the data, the complete calculations fall short by typically 30 mb at 51 MeV. The experimental data[19] on the exclusive absorption cross section suggest that such discrepancies can be explained from our neglect of the absorption channel. The results for σ_{el} and σ_r imply a total cross section which is not too far off compared to the data at resonance, but which underestimates the data at low energies.

Summarizing the results, we have seen that the present model for the single-scattering process, in which the Pauli blocking is treated very carefully, but in which binding effects are taken into account by means of an effective π-N subenergy, leads to satisfactory results at low energies. The discrepancies with the data at these energies are reasonable in view of the neglect of the absorption channel, which is known to be important, especially in the low-energy regime. Furthermore, if the Pauli principle is ignored but if the binding correction is retained, we found cross sections which are very similar to those of Maillet *et al.*[17] This indicates that much of the failure of their model at low energies can be attributed to their neglect of the Pauli principle.

For the resonance region, the total cross sections are in reasonable agreement with the data; however, the present model severely overestimates the elastic cross sections, in particular if the π-N partial wave mixing due to the Pauli principle is taken into account. To a slightly lesser extent, this has also been found by Hirata *et al.* in their isobar-hole calculation. Notice that in an isobar-hole formalism the mixing terms do not arise naturally. Hirata *et al.*[8] could achieve improvement by means of the phenomenological spreading potential. In an attempt to interpret the spreading potential, they emphasized the role of the absorption process. Besides pion absorption, a treatment of the binding correction by means of a genuine three-body calculation, rather than a constant energy shift as is used here, may also lead to more realistic values for $\sigma_r/\sigma_{\text{el}}$. In particular the π-d calculations by Woloshyn *et al.*[20] suggest that the binding correction should lead to an important increase in the absorptive part of the optical potential, giving larger values for the ratio

σ_r/σ_{el}. The energy shift by means of which the nucleon-core interaction is simulated in our discussion indeed leads to an increase of this ratio at lower energies [e.g., at 51 MeV the ratio $\sigma_r/\sigma_{el} = 0.43$ for the Liu and Shakin choice for the π-N subenergy Eqs. (9) and (10), while using Eqs. (9) and (11) leads to $\sigma_r/\sigma_{el} = 0.90$]. In the intermediate-energy regime, however, we do not find such an increase. This indicates that in order to account for the binding correction at resonance energies, a simple energy shift may not be adequate.

Lastly, we have studied in some detail the effects of the π-N partial wave mixing due to the Pauli principle. We found that this mixing leads to small modifications at low energies; however, at resonance energies it gives an increase in the cross sections of typically 15%.

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