Treiman-Yang criterion as a test of the pole approximation in the $^9Be(^3He,\alpha\alpha)^4He$ reaction

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The $^9Be(^3He,\alpha\alpha)^4He$ reaction has been studied at low incident energy to test the predictions of the pole approximation. Treiman-Yang distributions have been deduced from the measured differential cross sections for a wide range of the Treiman-Yang angle and of the spectator momentum. The data are consistent with the isotropic distribution predicted by the pole mechanism.

[NUCLEAR REACTIONS $^9Be(^3He,\alpha\alpha)^4He$, $E_{^3He} = 2.8$ MeV; measured $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$; deduced Treiman-Yang distributions.]

I. INTRODUCTION

In recent years there has been a considerable interest in the study of quasifree scattering (QFS) and of quasifree reactions (QFR). The experiments have been carried on at energies of the order of 100 MeV, and the plane wave impulse approximation (PWIA) as well as the distorted wave impulse approximation (DWIA) have been used to interpret the data. Evidence for QFR has also been found even at low incident energies,[1–15] where the conditions for the validity of the impulse approximation are generally considered to be no more satisfied.

The reaction $N(0,12)S$ is represented by the polar graph of Fig. 1, under the condition that only the first term in the Feynman series is retained. Here N is the target nucleus, 0 is the projectile, 1 and 2 are the two detected outgoing particles, and S is the spectator of the process. The amplitude of the reaction can then be factorized into two parts corresponding to the two vertices of the graph. Factorization tests, such as the Treiman-Yang (TY) criterion,[16,17] can then be performed to verify the polar nature of the reaction.

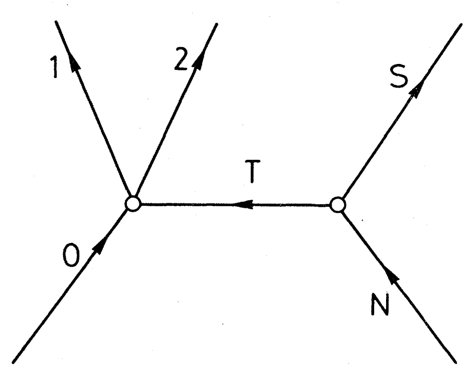

FIG. 1. Pole diagram for the quasifree reaction $N(0,12)S$.

The TY test has been applied to $(p,2p)$, $(\alpha,2\alpha)$, and $(\pi,\pi p)$ QFS.[18–22] Experiments have also been performed to apply this criterion to QFS at low incident energies,[23–26] but the results do not allow any definitive conclusion to be drawn. For instance, the need for further graphs is indicated by the TY distributions in the cases of the $^1H(d,2p)n$ and $^2H(d,dp)n$ reactions at 20 MeV,[23,24] whereas the TY distributions were consistent with the theoretical predictions for the $^6Li(p,pd)^4He$ reaction at 19 MeV (Ref. 25) and the $^1H(d,pn)^1H$ reaction at 12.2 MeV.[26]

The aim of the present work was to apply the TY criterion to the $^9Be(^3He,\alpha\alpha)^4He$ reaction at 2.8 MeV incident energy. This reaction has been already studied extensively at low energies.[7,8,14,15] Information on the QF mechanism and on the momentum distribution of the ^5He-^4He relative motion in ^9Be has been obtained. This test seemed then to be useful in order to see to what extent the measured cross section could be accounted for by a single pole graph. The application of the TY criterion to this reaction at low incident energies is, however, complicated by the presence of the sequential contributions that make the study of the TY distribution difficult in a sufficiently wide angular range. This has required a careful choice of the detection configurations to have little disturbance from other mechanisms.

II. THE TREIMAN-YANG TEST

Nuclear reactions with three bodies in their final state may proceed through different reaction mechanisms. The Feynman graph technique has been widely used[27] to describe such reactions. However, it is very difficult in general to select the graphs that dominate a given process. In particular the role of the pole mechanism has been investigated in detail and attempts have been made to find sensitive criteria in order to establish its relative importance.

When the amplitude of the reaction $N(0,12)S$ is written for the pole graph in the Feynman series, and the spin of the intermediate particle is zero, it can be expressed as[17]

$$M = \text{const}\frac{\Gamma(t)F(s',t',t)}{t-2m_T\epsilon_T} , \qquad (1)$$

where

$$s' = -(\vec{p}_1+\vec{p}_2)^2 + 2(m_1+m_2)(E_1+E_2) ,$$
$$t' = -(\vec{p}_1+\vec{p}_0)^2 + 2(m_1-m_0)(E_1-E_0) , \qquad (2)$$
$$t = -(\vec{p}_S-\vec{p}_N)^2 + 2(m_S-m_N)(E_S-E_N) ,$$

are three invariants quantities, $\vec{p}_j$, m_j, and E_j being the momentum, rest mass, and total energy of the particle j, respectively. In Eq. (1) the term $\Gamma(t)$ is the amplitude of the process $N \rightarrow S+T$ and the term $F(s',t',t)$ is the amplitude of the reaction $0+T \rightarrow 1+2$. ϵ_T is the binding energy of the system $T+S$ in the nucleus N. The units used are $\hbar = c = 1$ throughout.

According to the TY criterion, the amplitude of the reaction should be invariant under rotation of the $(\vec{p}_1,\vec{p}_2)$ plane about the sum of these momenta, in a reference frame in which the projectile or the

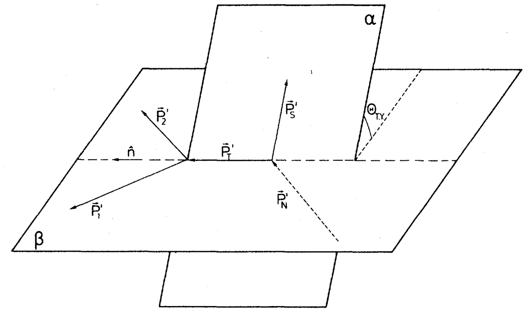

FIG. 2. The $N(0,12)S$ reaction in the antilaboratory system.

target cluster T is at rest. Therefore, in the present case since the ^{4}He cluster in ^{9}Be is a spectator of the virtual reaction $^3\text{He}+{}^5\text{He}\rightarrow \alpha+\alpha$, the TY criterion is most conveniently considered in the antilaboratory system; that is, the system in which $\vec{p}_0=0$. In this case, the direction $\hat{n}$ of $(\vec{p}_1+\vec{p}_2)$ is the same of that of the momentum of the intermediate particle T. In Fig. 2 the pole diagram for the $N(0,12)S$ reaction in the antilaboratory system is shown. The primed quantities refer to the antilaboratory system. The TY angle θ_{TY} is the angle between the $(\vec{p}_N',\vec{p}_S')$ plane (α) and the $(\vec{p}_1',\vec{p}_2')$ plane (β). In this framework the rotation of the $(\vec{p}_1', \vec{p}_2')$ plane around the sum of these momenta is seen as a rotation of α around β by an angle θ_{TY}. The measured reaction amplitude as a function of the TY angle θ_{TY} for a fixed value of the intermediate particle momentum $\vec{p}_T'$ is called the TY distribution.

The TY criterion has been shown to be valid for $L=0$ at nonrelativistic energies.[17] Thus the criterion must hold for the case in question since the relative motion of the ^{5}He and ^{4}He in ^{9}Be is known to be well described by a $3S$ state.[22]

The analysis made by Shapiro *et al.*[17] requires the comparison of different measurements of the quantity

$$\langle |M|^2\rangle = \frac{I(2\pi)^3}{4m_Sm_T}\frac{(1+m_1/m_2)p_2 - p_0\cos\theta_{01}+p_1\cos\theta_{12}}{p_2{}^2}\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} , \qquad (3)$$

where $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$ is the experimental differential cross section, $\langle |M|^2\rangle$ is the square of the matrix element modulus averaged over spin states of the initial particles and summed over those of the final particles. I is the density of the relative flux of particles T and 0

$$I = \left[\frac{2}{m_{OT}}\left[\frac{s'}{2(m_1+m_2)} - Q'\right]\right]^{1/2} ,$$

m_{OT} is the reduced mass of the 0-T system and $Q' = m_0 + m_T - m_1 - m_2$.

The region where the pole diagram is expected to be predomiant in the reaction mechanism was suggested to be[27]

$$p_T'^2 = q^2 \leq 2\mu \mid Q \mid \ , \tag{4}$$

where μ is the reduced mass in the target nucleus and Q is the Q value for the virtual decay. In the case of the ^{9}Be(^{3}He, $\alpha\alpha$)^{4}He reaction, this condition gives a value of about 100 MeV/c for the maximum spectator momentum. In this experiment the presence of the sequential contributions in the reaction further limits the maximum spectator momentum to about 60 MeV/c; this value is comparable with the half-width at half maximum (HWHM) of the ^{5}He-^{4}He $3S$ momentum distribution, which is known[14] to be around 60 MeV/c.

According to Shapiro,[27] the shape of the pole distribution should be given in general by a polynomial in $\cos\theta_{\rm TY}$, whose degree is related to the different angular momenta and spins involved in the reaction. In the case of the ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He reaction the above considerations lead to an isotropic TY distribution.

III. THE EXPERIMENT

The TY distributions were obtained by measuring the absolute differential cross section in kinematically complete experiments at nine sets of angles. The angles were chosen to cover a wide range of $\theta_{\rm TY}$ and spectator momenta, at the same time keeping constant the values of the quantities s', t', and t.

The experiment was performed at the Centro Siciliano di Fisica Nucleare/ Struttura della Materia (CSFN/SM) Van de Graaff laboratory in Catania. A 50 μg/cm^2 ^{9}Be target, evaporated on a 40 μg/cm^2 carbon backing was used. The ^{3}He$^+$ beam energy was 2.8 MeV, with a current of about 200 nA. The outgoing particles were detected by two solid-state detectors. One of them could be moved only in the horizontal plane ($\phi_1 = 0$), while the other could be set out of the horizontal plane. A monitor was set at 140° and was used to normalize the data with respect to the ^{3}He ions elastically scattered from ^{9}Be.

The energy pulses were sent to a 4096-channel analyzer operating in a bidimensional mode. Timing signals were sent to a time-to-amplitude converter (TAC) through constant fraction discriminators. The multichannel analyzer was gated by setting a time window on the output of the TAC. The spurious counting rate on the kinematical locus in the $E_1 - E_2$ plane was estimated to be less than 1%.

The bidimensional $E_1 - E_2$ spectra were then projected on the E_1 axis. A few examples of the triple differential cross section thus obtained are shown in Fig. 3.

IV. DATA ANALYSIS AND RESULT

Sequential contributions were subtracted in each spectrum by assuming a Breit-Wigner spreading of the involved 2.9, 11.4, and 19.9 MeV levels of ^{8}Be, with FWHM's of 1.5, 5, and 0.9 MeV, respectively. The amplitudes of these contributions were summed coherently and the resulting cross section[28] added incoherently to a QF cross section calculated according to Refs. 8 and 14. The weights of all sequential and QF contributions were adjusted as free parameters for each spectrum.

The data were analyzed by assigning each event to a TY angle and a spectator momentum q. TY distributions for different ranges of q were obtained by adding events in equal bins of $\theta_{\rm TY}$, and normalizing the differential cross section by the kinematical factor according to Eq. (3).

The resulting TY distribution is shown in Fig. 4. Some of these data have been published elsewhere.[29] Horizontal bars represent the uncertainty on $\theta_{\rm TY}$ due to the finite angular and energetic resolution in the experiment. Vertical bars represent the statistical errors only. The dashed line is the weighed average of the experimental reaction amplitudes.

The analysis of the data was performed according to two different approaches. Each method has in fact inherent limitations so that the use of both of them was intended to give us more confidence in the conclusions.

(i) Only those events falling within a narrow range of q were taken into account in the first analysis, according to the procedure reported in Ref. 24. This allowed for negligible variations in s', t, and t'. The value of the q range, namely $q = (30 \pm 2)$ MeV/c was chosen so as to satisfy Eq. (4).

The data are in good agreement with the prediction of the pole approximation, where for $L = 0$ the expected TY distribution is isotropic. A contamination of $L = 2$, which is allowed by the cluster model, would not contribute appreciably to the small q values we are dealing with since the corresponding momentum distribution is zero at $q = 0$. The main drawback of this kind of analysis is the introduction of large statistical errors, due to the narrowness of the q range.

(ii) A second data analysis was performed by considering all events obtained from all the measured

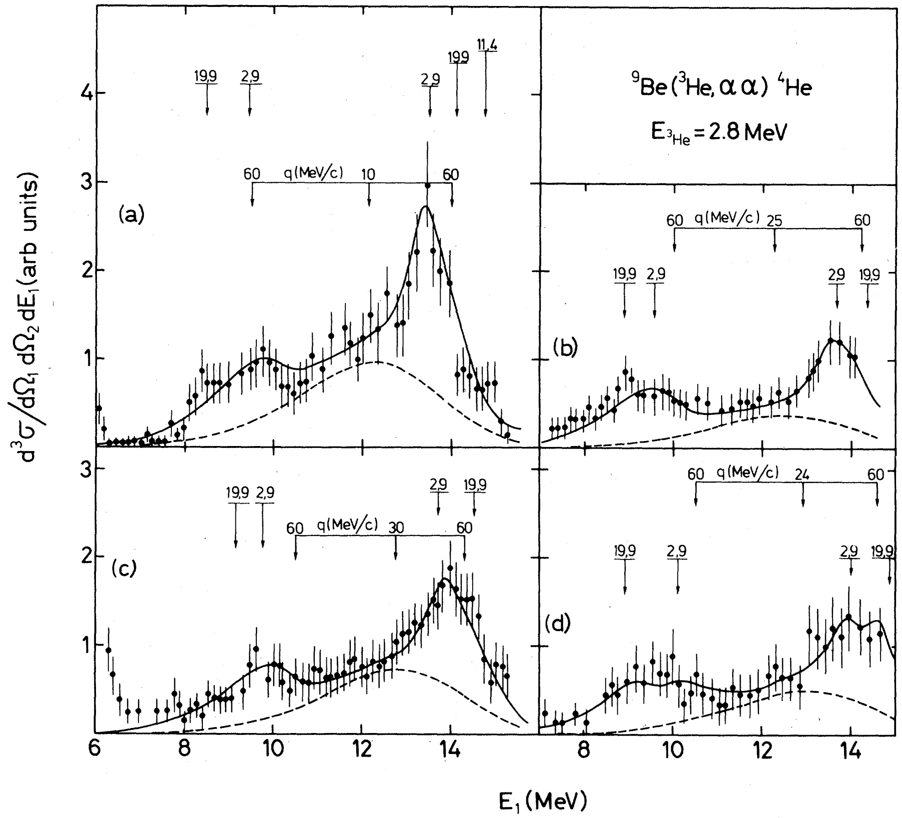

FIG. 3. Projection of the $E_1 - E_2$ spectrum on the E_1 axis. The TY angles are 20° (a), 60° (b) 100° (c), and 140° (d) for $q = 30$ MeV/c. A few values of q are indicated for each spectrum. The arrows mark the position of the sequential peaks. The continuous curve is a fit obtained by adding a QF contribution (dashed curve) to the sequential contribution.

spectra after subtraction of the sequential contributions.The values of q and θ_{TY} were calculated for each event, which was then classified within finite bins both of q and θ_{TY}. Only events with $q < 60$ MeV/c were retained, thus satisfying condition (4).

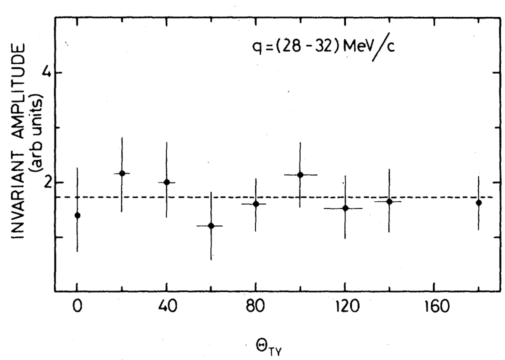

FIG. 4. Treiman-Yang distribution for the 28—32 MeV/c q range after subtraction of the estimated contributions from sequential mechanisms.

In Fig. 5 the resulting TY distributions are reported for the different intervals (0—20, 20—40, and 40—60 MeV/c) of the spectator momentum q, while Fig. 6 reports the TY distribution corresponding to the whole interval 0—60 MeV/c. The missing bins in the distributions correspond to values of θ_{TY} not kinematically allowed in our experiment. The distributions of Figs. 5 and 6 are consistent with the isotropic behavior predicted by the theory and reported as a straight line.

The advantage of this analysis with respect to the previous one is the possibility to look on larger ranges of q, at the same time reducing statistical errors. However, the finite extension of the q intervals introduces variations in the quantities (2) so that the condition of the constancy of the amplitude (1) may not be well satisfied.

V. CONCLUSIONS

The analyses reported in the previous section give a clear indication in favor of the QF nature of the

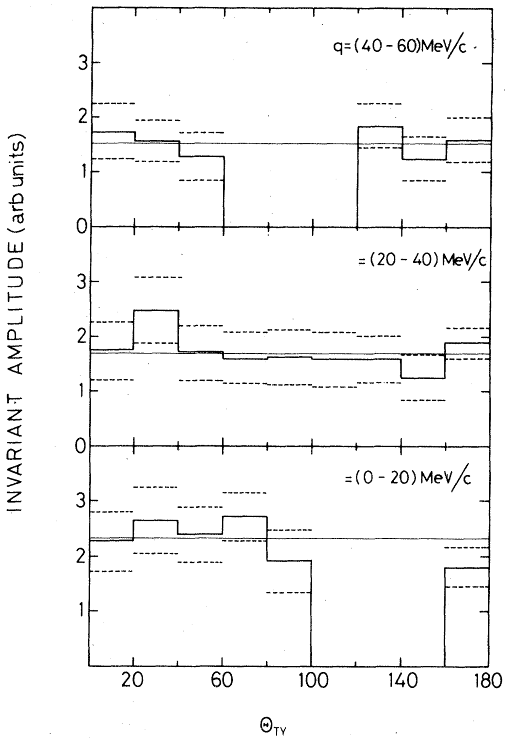

FIG. 5. Treiman-Yang distribution corresponding to the ranges 0–20, 20–40, and 40–60 for the spectator momentum q.

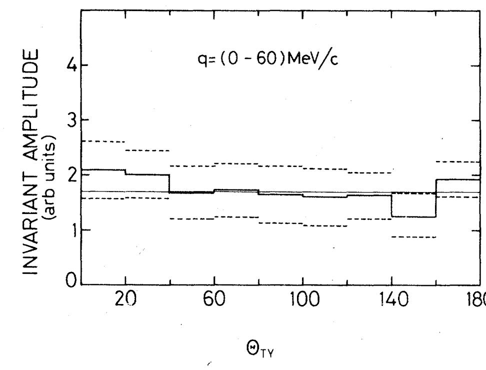

FIG. 6. Treiman-Yang distribution for the entire 0–60 MeV/c region of the spectator momentum q.

^{5}He(^{3}He,α)^{4}He virtual process which takes places in ^{9}Be while the residual ^{4}He nucleus acts as a spectator.

It may be argued that, while a nonisotropic TY distribution would have indicated the presence of nonpolar mechanisms, an isotropic distribution could be the accidental result of many reaction graphs contributing to the process. However, the isotropic distribution has been obtained within two different complementary types of analysis, and for different values of q (Fig. 5).

The QF contribution to the ^{9}Be(^{3}He,$\alpha\alpha$)^{4}He reaction at a low incident energy is now a well established fact for the following reasons.

(i) The QF peak has been found only at QF angles for various incident energies.[7,14]

(ii) For most of the measurements there is no possible contribution or contamination from sequential processes, at least in the region where $q < 30$ MeV/c. A series of measurements has been performed at 2.8 MeV and for different angle settings to confirm this point.[15]

(iii) The momentum distribution which can be extracted from the low energy data, in the PWIA, is consistent with the findings obtained from other QF reactions at higher energies.[8,15]

(iv) The TY distribution agrees with the theoretical predictions (this work).

No satisfactory theoretical explanation has been given so far, to our knowledge, to justify the existence of QF processes at such low energies. However, the high Q of the reaction (19.09 MeV) is responsible for high momenta ($\approx$ 300 MeV/c) transferred to the outgoing particles. This is the case also for (π^+,$2n$) and (π^-,$2p$) reactions with low energy pions which have been reported for many years as showing evidence for QF processes.

Apart from the well known difficulties (Sec. I) of describing the QF process at low energies in the PWIA or even in the DWIA, one may argue which kind of information can be obtained by a more detailed study of the TY distribution. A higher resolution in θ_{TY} and spectator momentum, in addition to a better statistics would be highly desirable in order to measure significant deviation from the prediction of the pole approximation.

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[1]R. E. Warner, G. C. Ball, W. G. Davies, A. J. Ferguson, and J. S. Forster, Phys. Lett. 27, 961 (1971).

[2]M. Furic, R. K. Cole, H. N. Forster, C. C. Kim, D. Y. Park, J. Rucker, H. Spitzer, and C. N. Waddel, Phys. Lett. 39B, 629 (1972).

[3]L. L. Gadeken and E. Norbeck, Phys. Rev. C 6, 1172 (1972).

[4]D. Miljanic, T. Zabel, R. B. Liebert, and G. C. Phillips, Nucl. Phys. A215, 221 (1973).

[5]J. Y. Grossiord, C. Coste, A. Guichard, M. Gusakow, A. K. Jain, J. R. Pizzi, G. Bagiev, and R. de Swiniarski, Phys. Rev. Lett. 32, 173 (1974).

[6]D. Miljanic, J. Hudomalj, G. S. Mutchler, E. Andrade, and G. C. Phillips, Phys. Lett. 50B, 330 (1974).

[7]J. Kasagi, T. Nakagawa, N. Sekine, T. Tohei, and H. Ueno, Nucl. Phys. A239, 233 (1975).

[8]N. Arena, D. Vinciguerra, F. Riggi, and C. Spitaleri, Lett. Nuovo Cimento 17, 231 (1976).

[9]M. Lattuada, F. Riggi, S. Barbarino, C. Spitaleri, and D. Vinciguerra, Lett. Nuovo Cimento 20, 193 (1977).

[10]I. Slaus, R. G. Allas, L. A. Beach, R. O. Bondelid, E. L. Petersen, J. M. Lambert, P. A. Traedo, and R. A. Moyle, Nucl. Phys. A286, 67 (1977).

[11]J. Roth, W. Bretfeld, W. Burgmer, H. Eichner, D. Gola, CHR. Heinrich, H. J. Helten, H. Kretzer, K. Drescher, and W. Schnorrenberg, Z. Phys. A 283, 345 (1977).

[12]D. Miljanic, E. Andrade, J. Hudomalj-Gabitzsch, G. S. Mutchler, and G. C. Phillips, Nucl. Phys. A290, 27 (1977).

[13]C. Spitaleri, M. Lattuada, F. Riggi, N. Arena, and D. Vinciguerra, Lett. Nuovo Cimento 21, 345 (1978).

[14]N. Arena, M. Lattuada, F. Riggi, C. Spitaleri, and D. Vinciguerra, Nuovo Cimento A45, 405 (1978).

[15]S. Barbarino, M. Lattuada, F. Riggi, C. Spitaleri, and D. Vinciguerra, Lett. Nuovo Cimento 25, 249 (1979).

[16]S. B. Treiman and C. N. Yang, Phys. Lett. 8, 140 (1962).

[17]I. S. Shapiro, V. M. Kolybasov, and G. R. Augst, Nucl. Phys. 61, 353 (1965).

[18]V. M. Kolybasov and N. Y. Smorodinskava; Yad Fiz. 14, 590(1971) [Sov. J. Nucl. Phys. 14, 330 (1972)].

[19]G. V. Avakov, E. I. Dolinsky, and V. V. Turoutev, Nucl. Phys. A196, 529 (1972).

[20]Yu. D. Dayukov, V. B. Fedorov, V. D. Khovansky, V. M. Kolybasov, G. A. Leksin, V. L. Stolin, and L. S. Vorobyev, Phys. Lett. 33B, 416 (1970).

[21]A. O. Aganyants, Yu. D. Bayucov, V. N. Deza, S. V. Donskov, V. B. Fedorov, N. A. Ivanova, V. D. Kohvansky, V. M. Kolybasov, G. A. Leksin, V. L. Stolin, and L. S. Vorobyev, Phys. Lett. 27B, 590 (1968).

[22]L. J. Gutay, J. E. Lannutti, P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Phys. Lett. 16, 343 (1963).

[23]R. Corfù, J. P. Egger, C. Lunke, C. Nussbaum. J. Rossel, E. Schwarz, J. L. Durand, and C. Perrin, Phys. Rev. Lett. 27, 1661 (1971).

[24]W. Von Witssch, W. Viefers, H. Mommsen, P. David, and F. Hinterberger, Nucl. Phys. A195, 617 (1972).

[25]R. B. Liebert, K. H. Purser, and R. L. Burman, Nucl. Phys. A216, 335 (1973).

[26]N. T. Okumusoglu, C. O. Bloth, P. B. Dunscombe, N. Berovic, and J. S. C. McKee, Nucl. Phys. A231, 391 (1974).

[27]I. S. Shapiro, *Interaction of High-Energy Particles with Nuclei, International School of Physics "Enrico Fermi," Course 38,* edited by T. E. O. Ericson (Academic, New York, 1967); Usp. Fiz. Nauk 92, 549 (1967); [Sov. Phys. Usp. 10, 515 (1968)], and references therein.

[28]P. Gil, L. Marquez, J. L. Quebert, and H. Sztark, J. Phys. 33, 315 (1972).

[29]P. G. Fallica, M. Lattuada, F. Riggi, C. Spitaleri, C. M. Sutera, and D. Vinciguerra, Lett. Nuovo Cimento 30, 241 (1981).