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Approximate Coulomb correction to elastic *N-d* scattering

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An approximate Coulomb correction that includes Coulomb distortion of the strong amplitudes is used to relate observables for the charge-symmetric branches of *N-N* and *N-d* scattering. The approximate correction is nearly identical with an exact calculation for *N-N* scattering, and tends to account better for the differences between *p-d* measurements and three-body *n-d* calculations than Coulomb corrections currently used.

[NUCLEAR REACTIONS Coulomb correction for mirror reactions; applied to *N-N* and *N-d* scattering; predicted polarizations for *n-d* scattering from *p-d* phase shifts.]

I. INTRODUCTION

The vast majority of three-nucleon scattering data[1] are *p-d* data. They are available at various energies, but are more numerous in the region below 20 MeV deuteron lab energy. There, first and second rank analyzing powers have been measured over a wide range of angles by the Zürich group. Unfortunately, the theoretical treatment, even of elastic *p-d* scattering, lags the experiments, mainly because it is a difficult task to include the Coulomb force in a realistic three-nucleon calculation. Faddeev type calculations, as well as other methods, usually neglect the Coulomb force and are therefore applicable in a strict sense to *n-d* data only. Still, *n-d* calculations have always been used to analyze *p-d* data, assuming that the interplay of the Coulomb force with the strong interaction does not contribute significantly. If one seeks only a qualitative theoretical guideline through experimental points this might very well be a reasonable assumption. But a more quantitative description requires at least an estimate of the Coulomb interference (CI) effect on the strong amplitudes. Configuration-space Faddeev calculations including the Coulomb force have been performed for the three-nucleon bound state.[2,3] There have been also Faddeev type calculations of *p-d* elastic scattering[4] which included the Coulomb force to first order exactly, but the underlying *N-N* interaction was too simple to yield a *p-d* *T* matrix which would have given nonzero polarization. A more refined calculation which would include the Coulomb force exactly and simultaneously provide a fit to *p-d* scattering data seems to be out of reach at the present time.

Given this outlook for near future *p-d* calculations and the possibility of more *n-d* experiments which would allow a comparison of charge symmetric reactions, it seems appropriate to search for methods to describe the Coulomb corrections in *p-d* scattering in an approximate way. In a series of papers, Grüebler *et al.*[5,6] have presented a first attempt to analyze *p-d* data with an approximate *p-d* calculation. In fact, they employ a *n-d* Faddeev calculation, but do take into account the effect of the asymptotic Coulomb phases. This is the minimum Coulomb correction that can be made to obtain *p-d* amplitudes from *n-d*, but it may in some cases be the most important correction. The final determination awaits an exact *p-d* three-body calculation.

In this paper we move a step further seeking an approximation to *p-d* Coulomb effects that arise from the interference of the strong interaction and the Coulomb interaction within the range of the strong interaction. This short-ranged Coulomb distortion effect has been approximated for the two-body scattering system through an on-shell approximation that was quite successfully applied to *p-p* phase shifts[7–9] and later was used for an analysis of *N-N* phase shifts [10] and in the calculation of *N-N* observables.[11]

In Sec. II we briefly recapitulate the essence of this *N-N* approximation method and, having in mind its applicability to the three-nucleon scatter-

ing system, propose how to use the method in a slightly different way. Both versions of the approximation are then used to calculate the CI effect on the nucleon polarization.

Haftel and Zankel[12] have shown that a similar approximation can be derived from three-body equations. An application to p-d scattering phase shifts yielded for most partial waves a good qualitative agreement with the Faddeev calculation of Ref. 4. Utilizing the results of Ref. 12 we present in Sec. III an approximate formula to describe the CI effect on the strong three-nucleon scattering amplitude. The method essentially describes the CI as a two-body effect, but we do account for the finite charge distribution of the deuteron.

The main ingredients for calculating the CI consist of the strong on-shell three-nucleon collision matrix elements and their on-shell momentum derivatives. If there were enough n-d data to allow a reliable phase shift analysis we could construct a realistic collision matrix to calculate the CI. Since the lack of n-d data also makes it difficult to evaluate the success of n-d potential model calculations, we rather prefer to use p-d collision matrix elements, as given by the phase shift analysis of Schmelzbach *et al.*[13] as an input and to iterate our equations to predict pure nuclear p-d observables. These observables, which would be n-d observables if nuclear charge symmetry holds, are shown and discussed in Sec. IV. Also, we give examples which demonstrate the influence of certain coupled states and of higher order effects on the observables.

II. APPROXIMATE COULOMB CORRECTIONS AND APPLICATION TO N-N SCATTERING

The CI correction to N-N hadronic phase phase shifts δ_l^s can be expressed through an extra residual phase shift δ_l^r using the definition

$$\delta_l^{sC}=\delta_l^s+\delta_l^r\,, \tag{1}$$

with δ_l^{sC} being the Coulomb modified nuclear phase shift. In Ref. 7 the following first-order approximation for δ_l^r was found:

$$\delta_l^r(p)=\frac{\mu e^2}{c_l}\left[\delta_l^{s\prime}(p)+\frac{\sin 2\delta_l^s(p)}{2p}\right]. \tag{2}$$

Here, μ denotes the reduced mass, p is the center-of-mass momentum, and c_l is an angular momentum dependent constant

$$\frac{1}{c_l}=\frac{2}{\pi}P\int_0^\infty \frac{dx\,x}{1-x^2}Q_l\left[\frac{1+x^2}{2x}\right]<0\,, \tag{3}$$

with Q_l being the Legendre functions of the second kind. This approximation has been modified to accommodate the finite charge distribution of the proton[8] and has been extended to apply to coupled states.[9] The corrections to the nuclear bar phases and coupling parameters read

$$\bar{\delta}_{j-1}^r(p)=\frac{\mu e^2}{c_{j-1}}\left[\bar{\delta}_{j-1}^{s\prime}(p)+\frac{\sin 2\bar{\delta}_{j-1}^s(p)}{2p\cos 2\bar{\epsilon}_j^s(p)}\right], \tag{4}$$

similarly for $\bar{\delta}_{j+1}^r$, and

$$\bar{\epsilon}_j^r(p)=\frac{\mu e^2}{2}\left[\frac{1}{c_{j-1}}+\frac{1}{c_{j+1}}\right]\left[\bar{\epsilon}_j^{s\prime}(p)+\frac{1}{2p\cos 2\bar{\epsilon}_j(p)}\{\sin 2\bar{\epsilon}_j^s(p)+\sin[\bar{\delta}_{j-1}^s(p)+\bar{\delta}_{j+1}^s(p)]\}\right], \tag{5}$$

with j being the total angular momentum.

If we now take the phase parameters of, e.g., the pure hadronic Paris potential[14] we are able to construct with the help of Eqs. (4) and (5) the Coulomb modified nuclear collision matrix and, once knowing them, we can calculate various p-p observables. For reasons already mentioned in Sec. I we rather wish to proceed in the other direction, namely, to start from the charged scattering system and to predict the pure hadronic p-p observables which are n-n observables if nuclear charge symmetry is assumed. For that purpose we use the phase parameters of the Paris potential which include the Coulomb potential as input for Eqs. (4) and (5) and perform an iteration to obtain the pure hadronic phase parameters. Then we construct the collision matrix elements and calculate the neutron polarization.

Expressions similar to Eqs. (2), (4), and (5) can be derived from three-body theory. However, the possibility of having three partial-wave states coupled for sufficiently large total angular momentum

J creates a problem with the phase correction. To obtain Eqs. (4) and (5), we made use of the additive splitting of the total phase shift

$$\delta_l=\delta_l^{sC}+\sigma_l\ , \tag{6}$$

where σ_l is the asymptotic Coulomb phase shift. For coupled states, this decomposition is possible only for Stapp's "nuclear bar" parametrization[15] of the Coulomb-distorted "nuclear" collision matrix. Unfortunately, no completely general extension of this parametrization has been worked out for 3×3 matrices. Thus, we seek CI corrections to a different quantity that retains the additive and unitary properties of the Stapp phases, but has no restrictions on dimension.

Corrections for such a quantity have already been given in Refs. 7 and 12, which we shall apply here to the case of two- and three-nucleon scattering. Nevertheless, we will continue to show, where possible, comparisons of the new method with the phase-correction method in order to maintain the connection with earlier work, and to show the effects of truncating the T matrix expansion at different orders, which is essentially the difference between the two methods. The nucleon-nucleon calculations are a primary test for any CI approximation, since in that case the Coulomb effects can be treated exactly in terms of the two-body Coulomb and nuclear (e.g., the Paris potential) forces.

It is first necessary to mention some conventions about nomenclature. Unfortunately, the symbols for the various matrices of asymptotic amplitudes for scattering processes are not standardized, and we must switch notation for some of the quantities used in earlier articles[7,12] in order to be consistent with the set most commonly used in descriptions of three-body scattering. This is the set used by Seyler,[16] who adopted the notation of Lane and Thomas,[17] as we do here. In this notation, the collision matrix U (the same as Wheeler's S matrix), is related to the reactance matrix Q (the same as Heitler's K matrix), by

$$U=(1+iQ)(1-iQ)^{-1}\ . \tag{7}$$

If Q is real and symmetric, then U is symmetric and unitary.

A slightly different reactance matrix,

$$R=-\frac{2}{\pi p}Q\ , \tag{8}$$

was introduced in Ref. 12, in which it was shown that the decomposition

$$R=R^{C}+R^{sC} \tag{9}$$

leads to integral equations for the "pure Coulomb" part R^{C} and for the residual R^{sC} that are completely analogous to the two-potential forms for the T matrix. To first order in e^2 and the "pure strong" part R^{s}, the equation for elements of the Coulomb-distorted strong R matrix is

$$R_{mn}^{sC}=R_{mn}^{s}+V_{mm}^{C}G_pR_{mn}^{s}+R_{mn}^{s}G_pV_{nn}^{C}\ , \tag{10}$$

where G_p is the principal-value free Green's function.

The integrals of Eq. (10) can be simplified by making use of an on-shell approximation as described in Ref. 7:

$$\frac{2\mu e^2}{\pi p}P\int_0^\infty dp'R_{mn}^{s}(p,p';E)\frac{p'}{p^2-p'^2}Q_l\left[\frac{p^2+p'^2}{2pp}\right]$$
$$\simeq\frac{\mu e^2}{p}\frac{1}{c_n}\left[R_{mn}^{s}+\frac{p}{2}R_{mn}^{s'}\right]\ . \tag{11}$$

Hereafter, we use the similar equations for Q instead of those for R in order to avoid confusion with Wigner's R matrix, which is the subject of the Lane and Thomas[17] review. The final result for the Coulomb-modified Q matrix is

$$Q_{mn}^{sC}(p)=Q_{mn}^{s}(p)$$
$$+\frac{\mu e^2}{2p}[Q_{mn}^{s}(p)+pQ_{mn}^{s'}]\left[\frac{1}{c_m}+\frac{1}{c_n}\right]. \tag{12}$$

The second term on the right side of Eq. (12) represents the approximate CI correction to the nuclear Q matrix. The simplicity of the correction is striking; it involves essentially only the on-shell Q-matrix elements and their momentum derivatives. Thus, the reactance matrix provides the unitary CI correction we seek which can be extended easily to dimensions greater than two for treating the N-d problem. The Q^{sC} of Eq. (12) (hereafter called method 1) differ from the Q^{sC} that can be constructed with the Coulomb corrected phase parameters of Eqs. (4) and (5) (hereafter called method 2) by higher orders in e^2 and Q^{s}.

To study the quality of the two methods we compare them with the exact CI correction using the Paris potential. In Fig. 1 we show the proton polarization as obtained by taking the phase parameters of the Paris potential, which includes the Coulomb potential (solid curve). These phase parameters have also been used as an input for methods 1 and 2 which then, after performing an iteration of Eqs. (4), (5), and (12), yield pure nu-

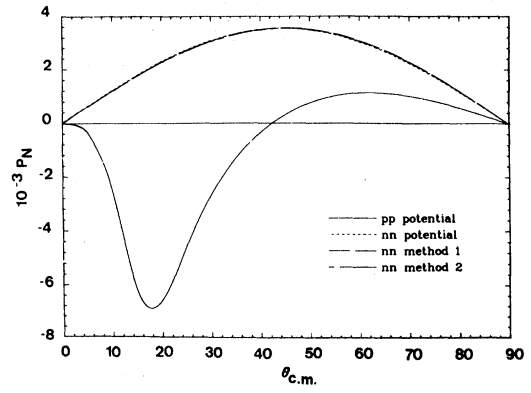

FIG. 1. Nucleon polarizations P at 20 MeV nucleon lab energy.

clear Q matrices. The resulting neutron (or pure hadronic proton) polarization agrees extremely well with the neutron polarization obtained by using the phase parameters of the Paris potential where the Coulomb potential has been switched off (dashed curve). Figure 1 demonstrates that both approximations are almost identical to the exact calculation, implying that there is almost no influence of higher orders. This picture might change slightly as one proceeds to lower energies, but then, of course, the nucleon polarization becomes very small. Similar results are valid for other N-N observables as has been shown in Ref. 11 for the case of method 2. The curves in Fig. 1 give no evidence of the superiority of either method, but a closer inspection of the Q-matrix elements reveals that the off-diagonal elements of method 1 are closer to the exact values than those of method 2. The very small corrections to these small off-diagonal elements clearly do not affect the polarization in the N-N case, but for stronger coupling, as, e.g., in the three-nucleon system, the differences might enlarge and finally influence the observables. Another discrepancy between the two methods arises from the diagonal elements of Q^s whenever the Q^{sC} elements become large, which in the case of N-N scattering happens at low energies where the polarization is small, or at energies above 100 MeV where the Coulomb effect is less important.

III. APPLICATION TO THE THREE-NUCLEON SYSTEM

Expressions analogous to Eqs. (1) and (2) have been derived from three-body theory in Ref. 12. The elastic Coulomb modified scattering amplitude was obtained in a manner similar to Eq. (12) by invoking the channel distortion approximation as proposed by Bencze[18] and by making use of an on-shell approximation previously introduced in Ref. 7. The method treats the CI effect in the three-body system—when two of the particles are charged—as a two-body effect but it takes into account the charge distribution of the deuteron. Although this approach neglects mainly Coulomb polarizing forces, its application to elastic p-d scattering phase shifts has proven to be a surprisingly good approximation (at least in a qualitative sense) when compared to an exact Faddeev calculation and other approximations available. In view of the success of this method it is feasible to think of applying it to N-d observables, which would provide a more stringent test for each approximation. Unfortunately, experimental information on these charge symmetric reactions is very sparse, mainly due to the lack of n-d data. Still, even in the absence of both an exact CI correction and reliable information about experimental differences between n-d and p-d, an approximate CI correction to p-d observables seems to be desirable for studying the order of magnitude of the two-body feature of the CI effect. This information could be relevant for future n-d experiments and might be helpful in explaining some of the shortcomings "realistic" n-d calculations encounter when compared with p-d data.

Trying to utilize the phase correction method we realize its limitation in the three-nucleon case. As mentioned before, we can apply it to 2×2 matrices only, which is not sufficient for treating N-d scattering. If we reduce the 3×3 coupling to a 2×2 by simply dropping the two smallest mixing parameters in each J^π state we can find the Stapp parametrization of the collision matrix and apply the phase corrections. A preliminary calculation of vector analyzing powers has already been reported[19] in which p-d vector analyzing powers were predicted based on n-d phase parameters as given by Stolk and Tjon.[20] The formulas used for each J^π are

$$\bar{\delta}^r_m(q)=\frac{\mu e^2}{\pi\tilde{c}_m(q)}\left[\bar{\delta}^{s'}_m(q)+\frac{\sin2\bar{\delta}^s_m(q)}{2q\cos2\bar{\epsilon}^s(q)}\right]$$

$$-\frac{\pi}{2}q\tilde{V}_m(q)\,, \tag{13}$$

and similarly for the coupled state n, and

$$\bar{\epsilon}^{s'}(q)=\frac{\mu e^2}{\pi}\left[\frac{1}{\tilde{c}_m(q)}+\frac{1}{\tilde{c}_n(q)}\right]\left[\bar{\epsilon}^{s'}(q)+\frac{1}{2q\cos 2\bar{\epsilon}^s(q)}\{\sin 2\bar{\epsilon}^s(q)+\sin[\bar{\delta}^s_m(q)+\bar{\delta}^s_n(q)]\}\right], \tag{14}$$

where q is the on-shell center-of-mass momentum. The labels m and n range over the three possible states $(S=\frac{3}{2}, l=J\pm\frac{3}{2})$, $(S=\frac{1}{2}, l=J\pm\frac{1}{2})$, and $(S=\frac{3}{2}, l=J\pm\frac{1}{2})$, with S being the channel spin and l being the nucleon orbital angular momentum. The quantity $\bar{\epsilon}$ is used in a generic sense to represent the mixing parameter for the strongest coupling, i.e., Eq. (14) holds also for the other mixing parameters ζ and η. $\tilde{V}(q)$ represents the difference of the full electromagnetic potential of the deuteron and the pointlike Coulomb potential. The angular momentum dependent quantity $\tilde{c}$, now also momentum dependent, is

$$\tilde{c}_l(q)=P\int_0^\infty \frac{x^2dx}{1-x^2}\int_{-1}^{1}\frac{dy\,P_l(y)f[q^2(1+x^2-2xy)]}{1+x^2-2xy}<0, \tag{15}$$

where f denotes the form factor for the (spherical) charge distribution of the deuteron. We have employed the same S-wave Yamaguchi form factor as given in Ref. 12. Although the phase corrections are limited to treating matrices no larger than 2×2, we retain them for calculating CI effects in the three-body system in order to see the effects of neglecting certain couplings and higher-order terms in the collision matrix, and to maintain continuity with previous papers.

The Coulomb-modified nuclear three-body Q matrix reads, for each J^π,

$$Q^{sC}_{mn}(q)=Q^s_{mn}(q)+\frac{\mu e^2}{\pi q}\left[\frac{1}{\tilde{c}_m(q)}+\frac{1}{\tilde{c}_n(q)}\right]\left[Q^s_{mn}(q)+q\frac{d}{dq}Q^s_{mn}(q)\right]-\delta_{mn}\frac{\pi}{2}\tilde{V}_{mn}(q). \tag{16}$$

The δ_{mn} reflects the fact that in this nonrelativistic treatment the Coulomb force preserves angular momentum and spin.

Nucleon-deuteron observables were calculated from the p-d phase shifts of Schmelzbach *et al.*,[13] which give good representations of their measurements at deuteron energies between 6 and 11.5 MeV ($E_p=3-5.75$ MeV) for partial waves up through $l=4$ with unsplitted (but spin-dependent) F and G waves. Two conventions need to be specified for the use of these phase shifts: the order in which the nucleon (s_N) and deuteron (s_d) spins are coupled to form channel spins (S), and the order in which the channel spin is coupled to orbital angular momentum (l) to form total angular momentum (J). The reversal of either convention has the effect of changing the signs of all quartet-doublet transition elements.

The Schmelzbach phase shifts can be considered given according to the conventions $S=s_d\otimes s_N$ and $J=S\otimes l$, in agreement with those of Lane and Thomas.[17] They use the spin-one–spin-$\frac{1}{2}$ generalization of Blatt-Biedenharn phase parameters given by Seyler[16] in which the values of the parameters depend on the ordering of the coupled states for a given J^π. [We call attention to a misprint in Seyler's 3×3 rotation matrix. The last element in Eq. (98) should read $u^{j\pi}_{33}=\cos\epsilon^{j\pi}\cos\zeta^{j\pi}$.] An important difference in the ordering of states assumed by Schmelzbach *et al.*[13] and that given by Seyler[16] for $J^\pi=\frac{1}{2}^+$ results in a change of sign for Schmelzbach's largest mixing parameter, $\eta^{\frac{1}{2}}$, if Seyler's ordering is used. This difference from the stated conventions in Ref. 13 prevented us for some time from reproducing their results.

The p-d curves shown in Figs. 2–10 are calculated from spin-space matrix elements of the transition operator

$$\tau=\sum_{JMS'l'Sl}|JMS'l'\rangle T^J_{S'l'Sl}\langle JMSl|, \tag{17}$$

where the reduced matrix elements $T^J_{S'l'Sl}$, taken between spin-angle eigenfunctions of total angular

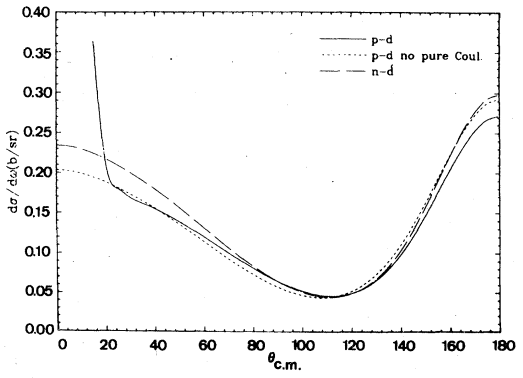

FIG. 2. N-d differential cross sections at 5 MeV nucleon lab energy.

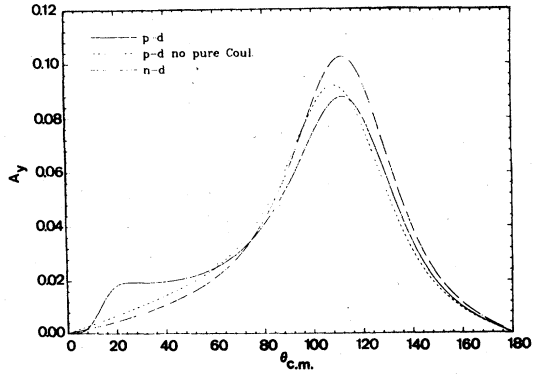

FIG. 3. Nucleon vector analyzing powers at 5 MeV nucleon lab energy.

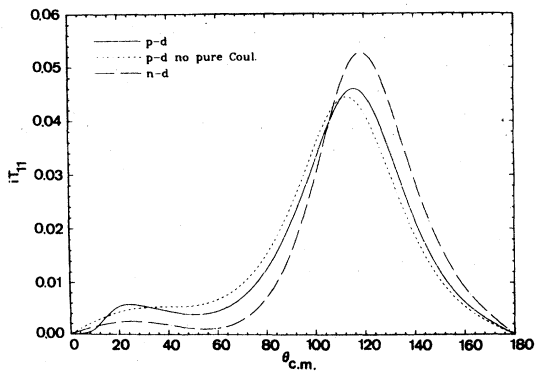

FIG. 5. Deuteron vector analyzing powers at 10 MeV deuteron lab energy.

momentum and parity $|JMSl\rangle$, are related to the phase-shift parameters through the collision matrix, and to the reactance matrix, by

$$T=\frac{1}{2i}(U-1)=Q(1-iQ)^{-1}\ . \tag{18}$$

The relations for the cross section and analyzing powers are given in terms of Wolfenstein's M matrix,

$$M_{S'\mu',S\mu}=(S'\mu'\,|\,\tau\,|\,S\mu)\ , \tag{19}$$

by

$$\sigma=\mathrm{Tr}(MM^{\dagger})/\mathrm{Tr}(1)\ ,$$
$$A_{ij}=\mathrm{Tr}(M\mathcal{O}_{ij}M^{\dagger})/\mathrm{Tr}(MM^{\dagger})\ , \tag{20}$$

where 1 is the unit matrix, and $\mathcal{O}_{ij}$ is the appropriate operator, in the direct-product spin space of the deuteron and nucleon. Since the phases of Ref. 13 parametrize only the Coulomb-distorted nuclear part of the T matrix, the p-d calculations using Eqs. (17)–(20) require the addition of the asymptotic Coulomb phase shift

$$T^{J}_{S'l'Sl}\rightarrow e^{i(\sigma_{l'}+\sigma_l)}T^{J}_{S'l'Sl} \tag{21}$$

and the addition of the Coulomb amplitude to the diagonal elements of M.

The curves labeled "p-d no pure Coulomb" in Figs. 2–10 are calculated as described above, but omitting the asymptotic Coulomb phases and amplitude. This is the type of Coulomb correction ordinarily used[6] to relate p-d measurements and n-d calculations, although it is applied in the opposite direction (n-$d\rightarrow p$-d).

Correcting, in addition, for the Coulomb distortion of the nuclear T matrix parametrized by Schmelzbach's phases gives the curves labeled n-d. The Q-matrix correction of Eq. (16) (method 1) is

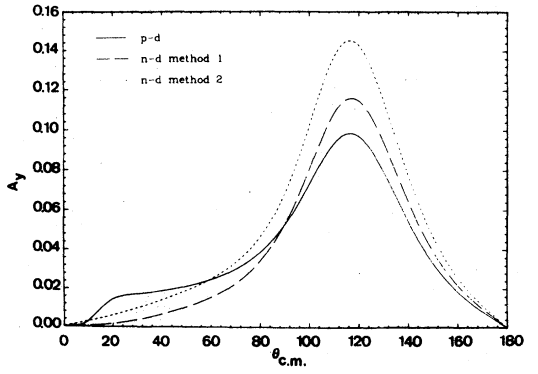

FIG. 4. Nucleon vector analyzing powers at 5 MeV nucleon lab energy when the collision matrix is truncated to two coupled states in each J^{π}.

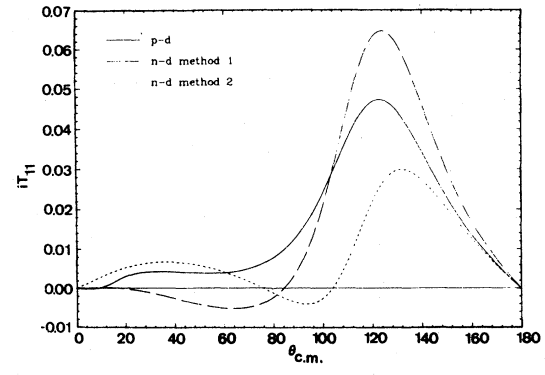

FIG. 6. Deuteron vector analyzing powers at 10 MeV deuteron lab energy when the collision matrix is truncated to two coupled states in each J^{π}.

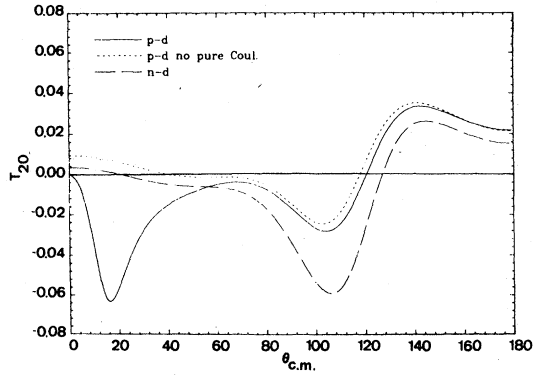

FIG. 7. Deuteron tensor analyzing powers T_{20} at 10 MeV deuteron lab energy.

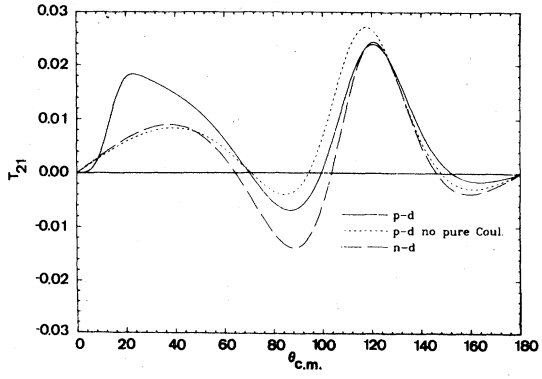

FIG. 9. Deuteron tensor analyzing powers T_{21} at 10 MeV deuteron lab energy.

used for the full-coupling case (up to 3×3 matrices), and for comparison, the phase-parameter correction of Eqs. (13) and (14) (method 2) is used in a limited case where the coupling is restricted to at most 2×2. In both cases, the correction (p-$d\rightarrow n$-d) is applied in the opposite sense to that implied by the equations (n-$d\rightarrow p$-d) in an obvious iterative way. For instance, Eq. (16) can be rewritten as $Q^s=Q^{sC}+Q^r(Q^s)$, where Q^{sC} is inserted as a first approximation to Q^s in Q^r, which contains the higher-order terms in the deviation of Q^s from its exact value. When successive approximations to Q^s are then inserted in Q^r, the procedure typically converges in less than four iterations. The steps used to calculate n-d observables from p-d phase parameters are summarized schematically by

U^{sC} (phase parameters of Ref. 13)

$\rightarrow T^{sC}\rightarrow Q^{sC}$ (Eq. 18),

$Q^{sC}\rightarrow Q^s$ [iterative inversion of Eq. (16)],

$Q^s\rightarrow T^s\rightarrow n$-$d$ observables[Eqs. (17)–(20)].

IV. RESULTS AND DISCUSSION

As was noted in the Introduction, we prefer in this work to obtain the n-d amplitudes from p-d, since the latter are more firmly established from measurements. The price we pay for this approach is that the inevitable fluctuations in the energy dependence of single-energy phase shifts can introduce anomalous values for the momentum derivatives involved in our correction. For the Schmelzbach[13] phase shifts, this effect seemed to be minimal for energies in the neighborhood of $E_N=5$ MeV, and this was chosen as the central energy for our calculations.

The differential cross sections (Fig. 2) reveal the same trend as already shown in Ref. 12, but then in a less realistic calculation. The difference

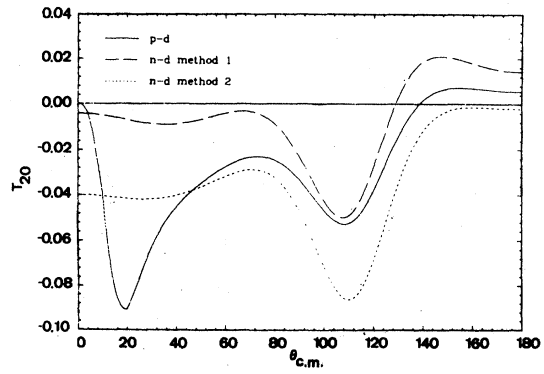

FIG. 8. Deuteron tensor analyzing powers T_{20} at 10 MeV deuteron lab energy when the collision matrix is truncated to two coupled states in each J^π.

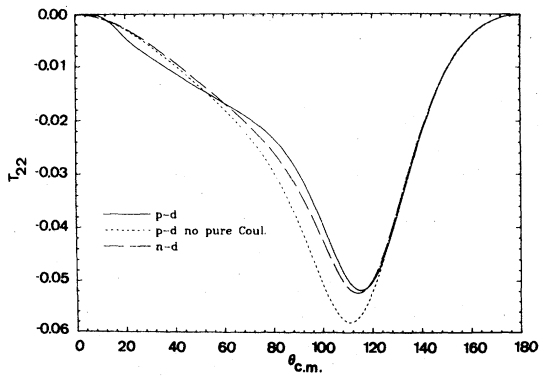

FIG. 10. Deuteron tensor analyzing powers T_{22} at 10 MeV deuteron lab energy.

between the curve "*p-d*" and "*p-d* no pure Coul." is due to neglecting the asymptotic Coulomb phase shifts (and therefore also the Rutherford amplitude), whereas "*n-d*" in addition includes the approximate short-ranged Coulomb corrections as given in Eq. (16). The short ranged corrections are quite significant at forward angles and are less important at backward angles. There, the asymptotic Coulomb phases have a rather big influence making *p-d* about 10% smaller than *n-d*. The CI effect only slightly enhances this difference. The same observation has been made in Ref. 12 at 10 MeV, where experiments[21] indicate only very small differences between *p-d* and *n-d* at backward angles. To reconcile that with our present picture would require that the missing three-body features of the CI cancel the differences, thus contributing a somewhat surprisingly large effect at backward angles. Since *n-d* and even *p-d* experiments at backward angles are rather sparse and since discrepancies have been noted[22] recently in precise measurements of back-angle *p-d* cross sections, it is not certain that our theoretical predictions conflict with the experimental results.

The nucleon vector analyzing power A_y is the only polarization observable where measurements are available for both protons and neutrons incident at the same energy. At present, the lowest energy for neutrons is 7.8 MeV (Ref. 23) but experiments down to about 5 MeV seem to be feasible in the near future.[24] An accurate measurement might test our prediction (Fig. 3) that the peak analyzing power for *n-d* exceeds that for *p-d* by about 0.014. Assuming that charge symmetry holds, such a comparison could reveal to what extent the differences between *p-d* and *n-d* are due to the two-body features of the CI effect. At 8 MeV no significant difference has been observed and it remains to be seen how much the situation changes as one proceeds to lower energies. Experiments at 8 and 12 MeV (Ref. 25) seem to disagree with an experiment at 14.1 MeV,[26] where a measurable difference between *p-d* and *n-d* was reported. In our preliminary calculation[19] using the truncated collision matrix we found good agreement with this experiment but, as pointed out then, the agreement was not to be taken quantitatively. The more complete calculation employing Eq. (16) yields a reduced CI, yet the effect maintains the same direction (Fig. 4). Method 1 is based on Eq. (16) and method 2 on Eqs. (13) and (14). To make a consistent comparison we have, of course, truncated the collision matrix in the same way for both the *n-d* and the *p-d* curves and also have neglected the imaginary part of the phase parameters. The coupled states we have retained are ${}^2P_{3/2} - {}^4P_{3/2}$, ${}^2P_{1/2} - {}^4P_{1/2}$, ${}^4D_{1/2} - {}^2S_{1/2}$, and ${}^2D_{5/2} - {}^4D_{5/2}$. The calculation with method 2 yields a difference already close to the fully coupled result because the analyzing power is dominated by the *p*-wave spin transition amplitudes in the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states, which are both included in by the truncated version of the collision matrix. Figure 4 also displays the influence of higher orders in e^2 and Q^s which is, by far, more important here than it was in the two-nucleon case.

We should also note that the direction of our CI correction does not help to explain why the 5.5 MeV *n-d* Faddeev calculation of Stolk and Tjon[20] predicts an A_y which is too low at the peak value when compared to a *p-d* experiment at 5 MeV (which is already lower than at 5.5 MeV, where the comparison should be made). On the other hand, Doleschall's *n-d* Faddeev calculation with a separable potential yielded a neutron analyzing power at 3.5 and 6.5 MeV which was slightly bigger than the latest *p-d* data points by the Zürich group,[27] thus leaving room for our CI to explain the discrepancy.

CI effects are present in the deuteron vector analyzing power iT_{11} at almost all angles (Fig. 5), although the effect itself is smaller than in A_y. It seems to be questionable whether this difference is measurable, but apparently the direction of the effect is the same as observed in A_y. From Figs. 5 and 6 we conclude that iT_{11} is sensitive to the 3×3 couplings. Higher orders, furthermore, seem to be quite important and explain why the CI effect on the peak value, as reported in the preliminary calculation, had a different sign.

More sensitivity to the CI effect emerges from the tensor analyzing powers, in particular from T_{20}. Figure 7 demonstrates a strong CI around 110°, where *n-d* turns out to be twice as negative as *p-d*. The forward direction is, as usual, strongly governed by the influence of the asymptotic Coulomb phases. T_{20} and T_{21} have been measured lately by the Zürich group[6] at almost all angles for deuteron lab energies between 7 and 13 MeV. These *p-d* data have been analyzed with Doleschall's *n-d* Faddeev calculation where the asymptotic Coulomb phases were included. Not unexpectedly, good agreement was found, particularly in the forward direction, whereas between 90° and 120° a discrepancy persisted. If we add our CI correction to their *n-d* Faddeev calculation we would obtain a very close agreement between data

and theory. Figure 8 demonstrates the significant influence of the neglected couplings, as well as that of higher orders. It is probably fortuitous that method 2 is more in qualitative agreement with the full calculation at backward angles.

The results for T_{21} (Fig. 9) are similar in trend to those for T_{20} with the magnitude of the CI, however, being smaller. The strongest CI takes place around 90°. Again, if we would add our correction to the Faddeev calculation of Ref. 6, we would get good agreement with the data.

Finally, for T_{22} (Fig. 10) we find a rather small CI working in a direction such that *p-d* almost coincides with *n-d* except at forward angles, where we see again the influence of the asymptotic Coulomb phases. As mentioned before, the phase parameters obtained from a data analysis are not always as smooth as those provided by a potential-model calculation. Employing the *p-d* parameters of Schmelzbach *et al.*, therefore, does not enable us to make a reliable prediction of the energy dependence of the CI. For that purpose we would be better off to sacrifice the benefits of having an input derived so closely from experiment, and use an *n-d* Faddeev calculation such as the one of Stolk and Tjon. Such a calculation will be reported in a separate publication.

V. SUMMARY AND CONCLUSIONS

We have adapted a simple method, which originally was devised from three-body theory[12] for calculating the Coulomb interference effect on three-body scattering phase shifts, to apply directly to the three-nucleon Q matrix. This has enabled us to describe the "internal" Coulomb corrections to the scattering amplitude in a quasi-two-body manner, but with the charge distribution of the deuteron accounted for. Before employing it for three-nucleon observables, we tested the two-body version in *N-N* scattering. Comparing with an exact calculation, as is available with the Paris potential, we found that our approximate CI effect on nucleon polarization almost coincides with the exact result. This gave us some confidence that at least the two-body features of the CI correction to three-nucleon observables would be described in a sufficiently reliable manner to allow qualitative predictions of the differences between *N-d* observables.

Employing the *p-d* collision matrix elements as given by the phase shift analysis of Schmelzbach *et al.*, we have calculated the CI corrections to the differential cross section and the vector and tensor analyzing powers at 5 MeV nucleon lab energy. Assuming that charge symmetry is preserved, the Coulomb-corrected *p-d* observables represent predictions for *n-d* observables. The strongest sensitivity was found in $d\sigma/d\Omega$ and T_{20}, where it might be big enough to be measurable. Also, A_y and T_{21} display some sensitivity, whereas iT_{11} and T_{22} are only slightly affected. The direction of the effect in A_y agrees with the experimental results at 14.1 MeV,[26] but quantitatively we obtain a smaller effect, although it is expected to become larger with decreasing energy. We therefore feel that our result supports the 12 MeV result[25] showing only minimal differences between *n-d* and *p-d*. It is also interesting to note that our approximate CI would help reconcile Doleschall's *n-d* calculation of A_y, T_{20}, and T_{21} (between 3.5 and 6.5 MeV nucleon lab energy) with *p-d* measurements. On the other hand, it would not explain why *n-d* Faddeev calculations with local potentials yield too low A_y when compared to *p-d* data.

The sensitivity of $d\sigma/d\Omega$ and T_{20} to other than asymptotic Coulomb effects clearly demonstrates that analyzing *p-d* data with current *n-d* Faddeev calculations, including those which take into account the asymptotic Coulomb phase shifts, is not really adequate. We suggest, in the absence of an exact calculation, to include the CI corrections as represented by our approximation. Of course, measurements of *n-d* analyzing powers for polarized deuterons would provide the most convincing experimental check of the Coulomb-induced differences between *p-d* and *n-d* scattering, as well as the microscopic *n-d* calculations themselves, but these difficult experiments probably will not be done in the near future.

Our approach is not limited to *p-d* scattering, having already roughly reproduced the differences between *p-α* and *n-α* phase shifts[19] in all cases except for the strong resonance ($J^\pi = \frac{3}{2}^-$). If resonances can be included in our approximation scheme,[28] it will be a versatile tool for making Coulomb corrections to elastic scattering in light nuclear systems.

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