Derivation of breakup-fusion cross sections from the optical theorem

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It is shown that the formula for the breakup-fusion cross section, used successfully to explain observed massive transfer cross sections, having been derived originally by Kerman and McVoy, can be rederived, based on the optical theorem involving a complex potential.

[NUCLEAR REACTIONS Breakup-fusion cross section.]

In our recent publications,[1,2] we showed that massive transfer reactions[3] can be very well explained in terms of the two-step, breakup-fusion mechanism; i.e., the process in which breakup of the projectile takes place first, and is followed by an absorption of one member of the broken-up pair by the target nucleus. Such a process may symbolically be written as

$$a+A\to b+x+A\to b+B^{*}\to b+y+C\,. \tag{1}$$

The last step in the above equation indicates the eventual decay of the compound system, B^*, formed by the fusion of x and A. A characteristic feature of the process is that, after the first breakup, b behaves as a spectator, and thus the remaining process can be viewed essentially as a compound process; $x+A\to B^*\to y+C$. [We shall, henceforth, call the channel $x+A(y+C)$ simply the $x(y)$ channel.]

In calculating the massive transfer cross sections (i.e., singles cross sections of the light α particles) in Refs. 1 and 2, use was made of a cross section formula derived originally by Kerman and McVoy (KM).[4] KM obtained this formula by manipulating the so-called channel correlation matrix using certain statistical assumptions. In the present article, we show that the same formula can be derived in a straightforward manner, based on the optical theorem (or equivalently a unitarity condition). We introduce the optical model Hamiltonian explicitly, which helps us to avoid making any statistical argument explicitly. (Both the formalism of KM and ours can be used to calculate singles cross sections in any reaction that creates a three-body system at the intermediate step preceding the fusion or absorption. For simplicity of presentation we keep using the terminology of the breakup fusion.)

Our starting basis is the relation

$$\sigma_a^A=(2m_a/\hbar^2k_a)\langle\chi_a^{(+)}|W_a|\chi_a^{(+)}\rangle\,, \tag{2}$$

where σ_a^A is the total reaction or (absorption) cross section in the incident channel a, while $\chi_a^{(+)}$ and W_a are, respectively, the optical model wave function in the incident channel, and the imaginary part of the optical potential used in generating $\chi_a^{(+)}$. Equation (2) is obtained directly from the usual optical theorem that the total cross section can be given in terms of the forward scattering amplitude. Thus, we may still call Eq. (2) the optical theorem.

In order to calculate the breakup and the breakup fusion cross sections, we first single out σ_d^A and W_d, i.e., the respective contributions from the breakup channel $d(=b+x+A)$. Using the formalism of Feshbach,[5,6] they can be given explicitly as

$$W_d=\mathrm{Im}\langle\phi_A|VG_d^{(+)}V|\phi_A\rangle$$

and (3a)

$$\sigma_d^A=(2m_a/\hbar^2k_a)\langle\chi_a^{(+)}|W_d|\chi_a^{(+)}\rangle\,.$$

Here ϕ_A is the target ground state wave function, V is the coupling Hamiltonian that causes the breakup reaction, and $G_d^{(+)}$ is the Green's function for the propagation in the d channel, for which we may assume an optical model Green's function

$$G_d^{(+)}=1/(E-H_0-T_x-T_b-U_x-U_b+i\epsilon)\,. \tag{3b}$$

In (3b) H_0 denotes the intrinsic Hamiltonian of the three ions b, x, and A, while T_i and U_i ($i=x$ and b) are the kinetic energy and optical potential for the relative motion between x and A ($i=x$) and $b+B$ ($i=b$), respectively. The absorption of x by A is thus described by the imaginary part of U_x introduced in the above Green's function.

In order to separate out the absorption due to this imaginary potential, let us use the following identity for $G_d^{(+)}$:

$$G_d^{(+)}=\Omega_b^{(-)}[\omega_x^{(-)}g^{(+)}\omega_x^{(-)\dagger}-G_x^{(+)\dagger}U_x^{\dagger}G_x^{(+)}]\Omega_b^{(-)\dagger}$$
$$-\Omega_x^{(-)}G_b^{(+)\dagger}U_b^{\dagger}G_b^{(+)}\Omega_x^{(-)\dagger}\,, \tag{4a}$$

where $\omega_i^{(-)}$, $\Omega_i^{(-)}$, $G_i^{(+)}$, and $g^{(+)}$ are defined as

$$\begin{aligned} \omega_i^{(-)} &= 1 + G_i^{(+)\dagger} U_i^\dagger , \\ G_i^{(+)} &= 1/(E - H_0 - T_x - T_b - U_i + i\epsilon) , \\ \Omega_i^{(-)} &= 1 + G_d^{(+)\dagger} U_i^\dagger , \\ g^{(+)} &= 1/(E - H_0 - T_x - T_b + i\epsilon) . \end{aligned} \tag{4b}$$

Note that $\omega_b^{(-)}$ introduced above is nothing but the wave operator that generates a distorted wave from the free wave. Also $g^{(+)}$ is the free Green's function, while $G_i^{(+)}$ is the optical model Green's function for the motion in channel i. We now simplify (4), based on our fundamental assumption mentioned above that after the breakup b can be treated as a spectator. This means that we neglect the last term in (4a), and approximate $\Omega_b^{(-)}$ by $\omega_b^{(-)}$

$$\Omega_x^{(-)} G_b^{(+)\dagger} U_b^\dagger G_b^{(+)} \Omega_x^{(-)} \simeq 0 \text{ and } \Omega_b^{(-)} \simeq \omega_b^{(-)} . \tag{5}$$

Using (4a) and (5) in (3a), it is straightforward to obtain[7]:

$$\sigma_d^A = \int dE_b\, d\Omega_b \frac{m_a m_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} \left(\left| \langle \chi_b^{(-)} \chi_x^{(-)} \phi_b\, \phi_x | V | \chi_a^{(+)} \phi_a \rangle \right|^2 + 4\pi \langle x | W_x | x \rangle \right) . \tag{6a}$$

Here ϕ_i ($i = a$, b, and x) denotes the intrinsic wave function of the ion i, while $|x\rangle$ is the wave function in the channel x and is given by

$$|x\rangle = \frac{1}{2\pi} (\chi_b^{(-)} \phi_b\, G_x^{(+)} | V | \chi_a^{(+)} \phi_a\, \phi_A \rangle , \tag{6b}$$

$\chi_b^{(-)}$ and $\chi_x^{(-)}$ being the distorted waves for the relative motion in channels b and x.

The physical significance of the two terms in (6a) is clear. The first term is the total one-step DWBA cross section of the breakup process, while the second term describes the contribution from the breakup-fusion process. This second term is given, in full, as

$$\frac{d^2\sigma}{dE_b d\Omega_b} = \frac{m_a m_b}{(2\pi\hbar^2)^2} \frac{k_b}{k_a} 4\pi \langle x | W_x | x \rangle , \tag{7}$$

which is exactly identical to Eq. (33) of KM.

From (7), together with (6b), one can rederive Eq. (1) of Ref. 1 that was used there for the analysis of the massive transfer reaction. Let us for this purpose introduce the on-the-energy-shell approximation for the Green's function $G_x^{(+)}$ in $|x\rangle$. Representing the resultant Green's function by a biorthogonal set of the optical model wave functions, it is easy to show that

$$4\pi \langle x | W_x | x \rangle \simeq \sum_{l_x} (2 l_x + 1) P_{l_x} / (4 |s_{l_x}|^2) \left| \langle \chi_b^{(-)} \chi_x^{(-)} \phi_b\, \phi_x | V | \chi_a^{(+)} \phi_a \rangle \right|^2 . \tag{8}$$

This is the same as Eq. (1) of Ref. 1, except for a factor of 4 in the denominator. As we discussed in Ref. 1, the cross section of (8) is to be multiplied by a factor of 4. This factor originates from the contribution from the off-energy shell, as was explained in some detail in Ref. 2.

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[7]Note that here $\chi_x^{(-)}$ is normalized such that $\langle \chi_x^{(-)}(E) | \chi_x^{(-)}(E) \rangle = \delta(E - E')$, which is different from the usual normalization of the distorted waves, $\chi_b^{(-)}$ and $\chi_a^{(+)}$.