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High energy potential scattering into back angles

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The sums in the potential scattering series are carried out by permitting only one large angle scattering. An early result of Schiff is thus rederived and in the process the connection between it and the recent successful work of Chen is made clear. The amplitudes are convenient to use. The results are applied to the local Laplacian optical potential used in pion-nucleus scattering. The large angle amplitude is very sensitive to three different treatments of the coordinate systems transformation of the πN amplitude that have been proposed.

[NUCLEAR REACTIONS Large angle potential scattering theory; π-nucleus optical potentials, $^{12}C(\pi, \pi)$ calculated $\sigma(\theta)$.]

I. INTRODUCTION

The eikonal approximation has been used successfully and widely in calculating scattering amplitudes for both atomic and hadronic processes. The approximation is usually not applicable for backward scattering angles. This is unfortunate since a number of factors influence the large angle cross section more sensitively than the forward one. Specifically, we are motivated here by many recent investigations of pion-nucleus optical potentials in which, for example, a variety of off energy shell extensions of the πN amplitude or transformation from the πN system to the c.m. frame are proposed. An eikonal-like formalism is relatively simple to use and one specifically appropriate to back angles should be useful for investigating the above mentioned effects among others. The goal is to assess and test such a formalism.

Many studies have been carried out to examine and account for the success of the eikonal or Glauber approximation and to extend the region of validity.[1-9] There has been much less done at extreme back angles but the recent investigations by Chen provide a stimulus.[2,3] His potential scattering results compare very well with exact ones when calculations are made for a variety of parameters using exponential potentials.[2] Earlier work of Schiff,[4] which is well known for its corrections to the eikonal, also contains a specific expression for very large angle scattering but it differs from that of Chen.

In Sec. II, we examine back angle scattering theory in terms of the infinite Born series. We use a propagator that permits only forward or backward propagation. The nth term then represents n potential interactions with intervening propagation only forward or backward. However, a backward scattering is allowed to occur only once within any of the terms of the series. Scattering into back angles that comes from three, five, or more reversals is discarded as negligible due to multiple integration over rapidly oscillating functions. Within the nth term, we are able to sum over the contributions from the back scattering that occurs only once but anywhere in the multiple interactions of that order. It is then easy to sum the remaining infinite series. The final expression for the amplitude is the old result of Schiff. With these methods we can then see that the potential scattering result of Chen is due to back scatterings that occur either last or next to last within the multiple interactions of a Born term. Earlier back scatterings are not included. The standard eikonal form includes back scattering only during the last collision. The relationship between the two results is thus understood. The results of Chen should be and are excellent when there is weak coupling. They hold even when it is not very weak probably because the back scattering during the penultimate interaction provides the important correction to the eikonal form.

In Sec. II numerical results are discussed and presented. First, a simple Gaussian potential is used to demonstrate the expected relationship between the two forms of the back angle amplitude.

Then calculations are carried out for potentials that have been used to account for data, almost all of it at forward angles. The goal is to determine how sensitive the back angle expression is to some of the alternatives that have been proposed. Specifically, we consider pion-nucleus scattering and carry out calculations for a local Laplacian optical potential. We treat three transformations of the πN parameters.[8,10-12] The back angle cross sections that result for π-^{12}C scattering are very sensitive to the three variations.

The amplitude is convenient to use. It should provide a useful first test when proposed treatments of a variety of effects are considered. For example, off energy shell extensions and proton-neutron mass density differences also affect back angle scattering sensitively.

II. POTENTIAL SCATTERING THEORY FOR BACK ANGLES

The scattering amplitude is given by

$$f(q) = -\frac{1}{4\pi}\langle \vec{k}' | T | \vec{k} \rangle . \tag{1}$$

The momentum transferred $\vec{q} = \vec{k}' - \vec{k}$ and the transition operator T satisfies the Lippman-Schwinger equation

$$T = U + Ug_0 T ,$$

where g_0 is the free propagator. We are interested in the elastic scattering case so $k = |\vec{k}'| = |\vec{k}|$. Units of $\hbar = c = 2m = 1$ are used throughout.

The propagator is an operator which includes a continuum of propagation directions. However, at high energies and for a smooth potential U of range a and strength U_0 such that $ka \gg 1$ and $U_0/k^2 \ll 1$, a good approximation for forward scattering is obtained if the propagator only includes propagation in the forward direction (which is chosen to be the z direction). Under these circumstances, the Green's function, G_+ that pertains to this forward direction propagator, g_+, is given by

$$G_+(\vec{r}, \vec{r}') = \frac{1}{2ik} e^{ik(z-z')} \delta(x - x') \times \delta(y - y') \Theta(z - z') ,$$

where

$$\Theta(z - z') = \begin{cases} 1, & z > z' \\ 0, & z < z' \end{cases}$$

the subscript + is used to denote propagation in the forward direction. This propagator produces the eikonal amplitude,[2,6] which is formally given by

$$T_{eik} = U + Ug_+ T_{eik} .$$

In order to treat backward angles as well as forward ones, Chen includes both directions in the propagator.

$$g_0 \sim g_+ + g_- . \tag{2}$$

This operator is central to what follows. It permits propagation in the forward or backward direction, or very close to them, i.e., any momentum transverse to z must be very much smaller than the z component. The Green's function pertaining to the operator $g_+ + g_-$, is

$$G_+ + G_- = \frac{1}{2ik} e^{ik|z-z'|} \delta(x - x')\delta(y - y') .$$

It is clear that we are motivated by the work of Chen.[2,3] Before proceeding, we will summarize his potential scattering results. His ansatz is

$$T \simeq U + U(g_+ + g_-)T_{eik} . \tag{3}$$

In the case of forward scattering, the term $Ug_- T_{eik}$ contains a rapidly oscillating function. This term is neglected when the integral for the scattering amplitude, Eq. (1), is evaluated. The result is the usual eikonal amplitude. In the case of backward scattering, $\vec{k}' \simeq -\vec{k}$, both terms have a rapidly oscillating function and both are retained. The amplitude for back angles is then obtained by Chen and Hoock[2] as

$$f(q) = \frac{2ik}{4\pi}\int d^3r\, e^{i\vec{q}\cdot\vec{r}} \chi e^{\chi/2ik} , \tag{4}$$

where

$$\chi = \int_{-\infty}^{z} U(x, y, s) ds .$$

We now begin the investigation of the transition operator by using the propagator of Eq. (2).

$$\begin{aligned} T &= U + U(g_+ + g_-)T \\ &= U + U(g_+ + g_-)U + U(g_+ + g_-)U(g_+ + g_-)U + \cdots . \end{aligned} \tag{5}$$

This is an extension or correction of Eq. (3) since, in any of the terms, the propagation between any of the interactions is either in the original *or* in the reversed direction. With the choice of $\vec{k}$ as the z direction and the specific representations of the Green's functions given above, the propagations are either in the forward or backward z directions.

When the series of Eq. (5) is placed in Eq. (1), the amplitude is given in terms of the nth ordering scattering amplitudes,

$$f = \sum_{n=1}^{\infty} f_n .$$

For example, the triple scattering amplitude is

$$f_3 = -\frac{1}{4\pi}\langle \vec{k}' | Ug_+Ug_+U + Ug_+Ug_-U + Ug_-Ug_+U + Ug_-Ug_-U | \vec{k} \rangle .$$

In the case of back scattering, $\vec{k}' \simeq -\vec{k}$, the first term represents two very small forward scatterings followed by a third scattering into back angles. The second term represents a scattering into back angles, then a second scattering back into forward angles, and finally a third scattering into back angles, i.e., three reversals of direction. The third term $\langle -\vec{k} | Ug_-Ug_+U | \vec{k} \rangle$ represents a small forward scattering followed by a reversal, and finally a small deflection but still in the backward direction. The last term represents a scattering from the initial direction into back angles followed by propagation in the backward direction with two small rescatterings. On physical grounds one would expect the second term, i.e., the three reversals, to contribute negligibly compared to the other three terms which involve one reversal and two small deflections.

Incidentally in the case of forward scattering $\vec{k}' \simeq \vec{k}$, any term which involves g_- results in two reversals and should be neglected. In this case, of course, the eikonal amplitude is obtained.

Returning to the back angle case, we conclude that contributions from three, five, or more reversals should be neglected. This means that in any multiple scattering term a g_+ must occur only on the right of a g_-. There can be any number of g_- in sequence as long as they are on the left of any g_+. Such a term means that a back scattering has occurred when the first g_- appears and propagation continues in the backward direction with only very small deflections. In this way, the nth order multiple scattering term, instead of being made up of 2^{n-1} terms, is made up of n terms. The nth order amplitude at back angles now is

$$f_n = -\frac{1}{4\pi} \sum_{m=1}^{m} \langle -\vec{k} | U^{(n)} g_- U^{(n-1)} g_- \cdots g_- U^{(m)} g_+ U^{(m-1)} \cdots g_+ U^{(1)} | \vec{k} \rangle , \tag{6}$$

where the superscripts denote serial order, i.e., back scattering occurs at the mth interaction.

It is clear that the above version of the large angle amplitude is an extension of that provided by Eq. (3). In the current version, scattering into back angles can occur at any time; whereas Eqs. (3) and (4) include backward scattering only during the last or second to last rescatterings. The result is that the nth order amplitude contains n terms rather than two.

In order to sum the series, we need the explicit integrals that represent f_n.

$$f_n = -\frac{1}{4\pi}\left(\frac{1}{2ik}\right)^{m-1} \int_{-\infty}^{\infty} dx_n dy_n dz_n dz_{n-1} \cdots dz_1 e^{-i\vec{k}'\cdot\vec{r}_n} U(\vec{r}_n) e^{ik|z_n - z_{n-1}|} U(\vec{r}_{n-1})$$
$$\times e^{ik|z_{n-1}-z_{n-2}|} U(\vec{r}_{n-2}) \cdots U(\vec{r}_2) e^{ik|z_2 - z_1|} U(\vec{r}_1) e^{i\vec{k}\cdot\vec{r}_1} ,$$

where the δ functions in the Green's functions have been used to put $x_n = x_{n-1} = \ldots = x_1$ and $y_n = y_{n-1} = \ldots y_1$. Note that $\vec{k}\cdot\vec{r}_1 = kz_1$. The back angle condition $\vec{k}' \simeq -\vec{k}$ has not yet been included, nor have the scatterings leading to multiple reversals been neglected.

The next step is to break up the z integrals into two parts so that the various absolute values $|z_i - z_{i-1}|$ can be removed. When the back angle condition is included, the multiple reversals appear as multiple integrals over the rapidly oscillating function e^{i2kzi} and thus there are mathematical grounds for neglecting these terms. After these manipulations, the explicit form for Eq. (6), the nth order term, is obtained,

$$f_n = -\frac{1}{4\pi}\left(\frac{1}{2ik}\right)^{n-1} \int_{-\infty}^{\infty} dx_n dy_n e^{i(q_x x_n + q_y y_n)} \sum_{m=1}^{n} z_{nm} ,$$

where

$$z_{nm} = \int_{-\infty}^{\infty} dz_n U(\vec{r}_n) \int_{z_n}^{\infty} dz_{n-1} U(\vec{r}_{n-1}) \int_{z_{n-1}}^{\infty} dz_{n-2} U(\vec{r}_{n-2}) \cdots$$
$$\times \int_{z_{m+1}}^{\infty} dz_m e^{2ikz_m} U(\vec{r}_m) \int_{-\infty}^{z_m} dz_{m-1} U(\vec{r}_{m-1}) \cdots \int_{-\infty}^{z_2} dz_1 U(\vec{r}_1) .$$

We now let the operator $\int_{z_{m+1}}^{\infty} dz_m e^{2ikz_m} U(\vec{r}_m)$ migrate to the left, changing the limits of integration appropriately. Likewise, the operator $\int_{-\infty}^{\infty} dz_n U(\vec{r}_n)$ migrates to the right so as to interchange the two operators. We then have

$$z_{nm} = \int_{-\infty}^{\infty} dz_m e^{2ikz_m} U(\vec{r}_m) \left[\int_{-\infty}^{z_m} dz_{n-1} U(\vec{r}_{n-1}) \int_{-\infty}^{z_{n-1}} dz_{n-2} U(\vec{r}_{n-2}) \cdots \int_{-\infty}^{z_{m+1}} dz_n U(\vec{r}_n) \right]$$
$$\times \left[\int_{-\infty}^{z_m} dz_{m-1} U(\vec{r}_{m-1}) \cdots \int_{-\infty}^{z_2} dz_1 U(\vec{r}_1) \right] .$$

The quantities in the two sets of brackets are independent of each other, although each is a function of the last variable of integration z_m. To consider them further, we note that

$$\int_{-\infty}^{z} dz' U(x,y,z')\chi^{m-1}(z') = \frac{1}{m}\chi^m(z).$$

In the first quantity in brackets, the rightmost integral is $\chi(z_{m+1})$ and then the entire quantity is

$$\frac{\chi^{n-1-(m-1)}(z_m)}{[(n-1)-(m-1)]!}.$$

With a similar treatment for the second set of brackets, we obtain

$$z_{nm} = \int_{-\infty}^{\infty} dz_m\, e^{2ikz_m} U(\vec{r}_m) \frac{\chi^{n-1}(z_m)}{(m-1)!\,(n-m)!}.$$

Therefore, since

$$\sum_{m=1}^{n} \frac{1}{(m-1)!\,(n-m)!} = \frac{2^{n-1}}{(n-1)!},$$

we can carry out the sum over m to obtain, after putting $q_z \sim 2k$,

$$f_n = -\frac{1}{4\pi}\left(\frac{1}{ik}\right)^{n-1} \int d^3r\, e^{i\vec{q}\cdot\vec{r}} U(\vec{r}) \frac{\chi^{n-1}}{(n-1)!}.$$

The Born series can now also be summed to obtain our final expression for the scattering amplitude

$$f(q) = -\frac{1}{4\pi}\int d^3r\, e^{i\vec{q}\cdot\vec{r}} U(\vec{r}) e^{\chi/ik}. \tag{7}$$

Here $\vec{q} = (q_\perp, q_z)$ is understood to be the momentum transferred in very large angle scattering, i.e., $\theta = \pi - \epsilon, q_\perp = k\epsilon, q_z = 2k + O(\epsilon^2)$.

In fact, the result Eq. (7) amounts to a rederivation of the amplitude obtained by Schiff[4,7] for very large angle scattering.

Comparison of this result with that of Chen is facilitated if Eq. (4) is written in an alternate form. When the exponential in Eq. (4) is represented by its series, and then an integration by parts is carried out [of the same kind as used in deriving Eq. (4)], upon resumming one obtains[13]

$$f(q) = -\frac{1}{4\pi}\int d^3r\, e^{i\vec{q}\cdot\vec{r}} U(\vec{r})\left(1 + \frac{\chi}{2ik}\right) e^{\chi/2ik}. \tag{8}$$

This form strongly suggests that Eq. (8) can be obtained from Eq. (7) by treating the eikonal-like phase factor in the latter as

$$e^{\chi/ik} = e^{\chi/2ik} e^{\chi/2ik} \cong e^{\chi/2ik}(1 + \chi/2ik).$$

Furthermore, if the χ in the parentheses is dropped then the eikonal amplitude results

$$f_{eik} = -\frac{1}{4\pi}\int d^3r\, e^{i\vec{q}\cdot\vec{r}} U(\vec{r}) e^{\chi/2ik}.$$

Our results thus determine which formulation for the scattering amplitude is obtained when the backscattering is allowed to occur during different interactions within the nth order Born term. When the backscattering is allowed to occur only during the last interaction, the eikonal expression is the result. When the backscattering occurs during the last or next to last interaction, Eq. (8) [or equivalently Eq. (4)] is the result. In the most general case of allowing the backscattering to occur during any of the interactions, the Schiff form Eq. (7) is the result. This is the most comprehensive result. The Chen form, and the eikonal at back angles, involve further approximations.

The effect of being more exact, of using the Schiff form, is to include higher order terms of $\chi/2k$ in the factor that multiplies the eikonal integrand. The contributions from earlier backscatterings become more negligible as $\chi/2k$ decreases, i.e., the Chen expression and even the eikonal, approximate the Schiff result in this limit.

A rough measure of $\chi/2k$ is $U_0 a/2k$. As this parameter decreases the calculations of Chen and Hoock[2] appear to verify for the exponential potential the trends mentioned above. The most extreme case presented in Ref. 3 is for $U_0 a/2k \sim 0.8$. Even for this large value the results due to Eq. (8) are not bad. We can go some way towards explaining this somewhat surprising accuracy by noting that the ratio of the first term neglected in the series to the last one included, i.e., the ratio of third to second is $U_0 a/4k \sim 0.4$. Evidently this is small enough to obtain good results with Eq. (8). It is important to include the second term; the first term alone—the eikonal amplitude—does not yield good results.

III. NUMERICAL RESULTS: PION-NUCLEUS OPTICAL POTENTIALS

A. Sample calculations

Some earlier work indicates that the Schiff result works reasonably well for a spherically symmetric parabolic well[14] and for a square well.[15] Berriman and Castillejo[9] test an exponential well in a variety of eikonal like amplitudes, including one due to Schiff which should reduce to Eq. (7) at back angles. However, they do not calculate with the explicit form of Eq. (7) nor do they indicate results at extreme back angles. Their largest angle results are in the vicinity of 125° and are at least as good for the Schiff form as for any of the other variants.

At any rate, we can consider that Eq. (7) has been numerically verified for exponential potentials, at least for $U_0 a/2k$ not too large, since Eq. (4) is more approximate than Eq. (7) and the former has been found to work well by Chen.

We now try

$$U(r)=U_0e^{-r^2/a^2}$$

in order to investigate the Gaussian form[16] and to further compare Eqs. (4) and (7).

We then have

$$\chi=\frac{\sqrt{\pi}a}{2}U_0e^{-b^2/a^2}[1+\mathrm{erf}(z/a)],$$

where $b^2=x^2+y^2$, and in calculations a rational approximation is used for the error function.[17] The scattering amplitude from Eq. (7) then is

$$f=-\frac{U_0}{4\pi}\int_0^\infty db\, b\, J_0(q_\perp b)e^{-b^2/a^2}\times\int_{-\infty}^\infty dz\exp[i(q_z z-\chi/k)-z^2/a^2].$$

An analogous expression can readily be obtained from Eq. (4) or Eq. (8). Although the expressions hold only for $q_\perp \ll q_z \simeq 2k$, in the calculations there is no need to put in this restriction.

In Fig. 1 we show the real and imaginary parts of the amplitude given both by Eq. (7) and Eq. (8). The figure also shows exact results obtained from a partial wave solution. With the parameters used

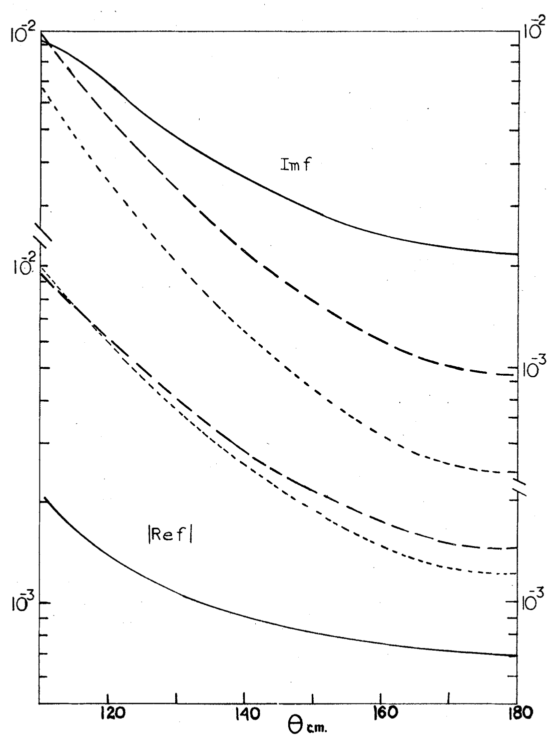

FIG. 1. Real and imaginary parts of the amplitude obtained from exact calculation, solid curves; from Eq. (7), dashed curves; and from Eq. (8), dotted curves. The Gaussian potential is used with $U_0=1.0$ fm^{-2}, $a=1.75$ fm, and $k=2$ fm^{-1}.

the partial wave solution has converged very well after 31 terms. In the case shown in Fig. 1, i.e., $U_0a/2k=0.44$, the imaginary part of the amplitude obtained from the Schiff result is much more accurate than that from Chen. The real part of the amplitude is ambiguous but it is dominated in the cross section by the imaginary amplitude. Our other calculations show that as the combination $U_0a/2k$ decreases the results from Eq. (7) and Eq. (8) approach each other for both the real and imaginary amplitudes. As $U_0a/2k$ increases, the real amplitudes obtained from the Schiff and Chen forms cross at larger angles. The effect is that they do not seem to differ significantly at back angles. However, these other calculations do show that as $U_0a/2k$ increases, the imaginary part of the amplitude, and the cross sections, obtained from the Schiff and Chen forms become more discrepant for the Gaussian potential.

B. Application to π-nucleus optical potentials

We now apply the back angle amplitudes to potentials that have been used in the literature to account for experimental results. It is known that back angles are sensitive to a number of factors. The back angle amplitudes, Eqs. (7), (4), or (8) are easier to use than ones obtained from more fundamental microscopic theories. The goal then is to determine to what extent the back angle amplitude is sensitive to various cases that have been proposed.

There has been much work recently in developing pion-nucleus optical potentials. A variety of effects and treatments have been considered. Our intent is to carry out some trial calculations of cases that are relatively easy to apply. Accordingly, we focus on the local Laplacian form of the optical potential.[8,10-12,18]

$$U(r)=-(b_0+b_1)k^2\rho(r)-\frac{b_1}{2}\nabla^2\rho(r), \qquad (9)$$

where b_0 and b_1 are the complex parameters that come from the s and p wave parts of the πN amplitude and $\rho(r)$ is the matter density of the target nucleus. We choose for the density the modified Gaussian appropriate to sp shell nuclei,

$$\rho(r)=\frac{4A}{\pi^{3/2}a^3}\left[1+\frac{\Delta-4}{6}\left(\frac{r}{a}\right)^2\right]e^{-r^2/a^2},$$

which we shall apply to carbon, $A=12$.

More particularly we will investigate some of the ways of treating the b_0 and b_1 parameters when they appear in the optical potential. Dedonder[10] and Faldt[8] were the first to note that, when these parameters are used to describe π-nucleus scattering, the transformation from the πN to the π-

nucleus system must not neglect the mixing of partial waves since the p wave plays such a dominant role. Accordingly, they suggest that a more exact procedure is to multiply the coefficient of the Laplacian by $(k_l/k_c)^2$ where k_l is the pion-nucleon laboratory momentum and k_c the corresponding pion-nucleon c.m. momentum. (Both parameters b_0 and b_1 are changed under this procedure but only the coefficient of the Laplacian changes since the sum b_0+b_1 is not altered.) We shall be doing calculations for pion laboratory energies ranging from 180 to 280 MeV. In this region the $(k_l/k_c)^2$ factor varies from 1.7 to 1.9.

Subsequently Miller[12] has suggested another modification which includes the so called p wave threshold kinematics in the angle transformation. His result is that the coefficient of the Laplacian should be multipled by the factor $(1+\omega_l/m)$ where ω_l is the total pion laboratory energy and m is the nucleon mass. In the energy region under consideration this produces a factor ranging from 1.34 to 1.45.

To sum up, we have applied the back scattering amplitude to elastic scattering of pions from ^{12}C at energies from 180 to 280 MeV. This has been done for the local Laplacian optical potential with untransformed or original parameters, with the Laplacian modified by the Faldt or Dedonder factor and then with the Miller factor. The upshot is that three different potential strengths—complex ones—are used in each case.

It is worth reiterating that we do not intend to resolve here the question of how one must treat the transformation of the πN amplitude or what coefficient must be used with the Laplacian part of the potential. These are rather trial calculations of the back angle amplitude in order to see how it behaves with specific potentials that are considered to be reasonable.

The Fermi averaged parameters of Sternheim and Auerbach[19] are used as the starting values of b_0 and b_1. When they are used directly in the optical potential Eq. (9) the solid curves shown in

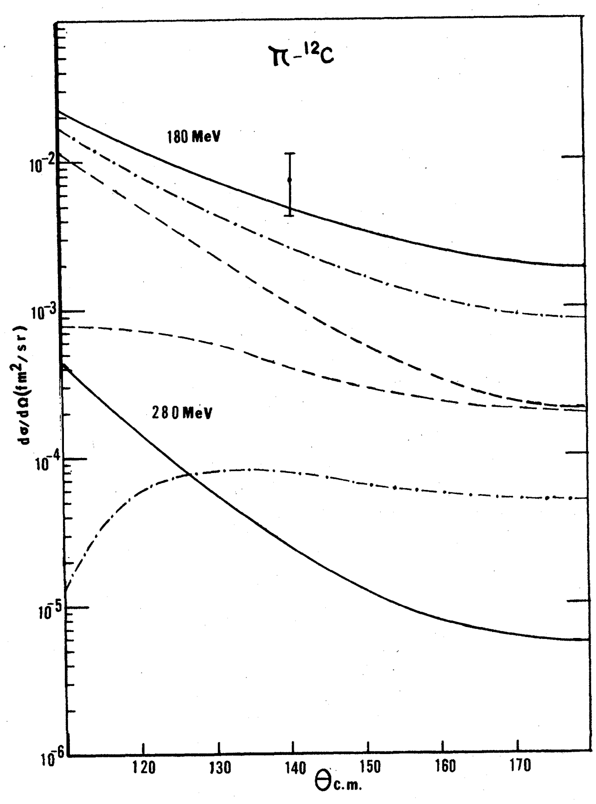

FIG. 2. Angular distributions for back angle elastic scattering of π from ^{12}C at 180 MeV (upper curves) and at 280 MeV (lower curves). The solid curves result when no mixing of partial waves is included in transforming the πN parameters. The dashed curve results when the modification of Faldt (Ref. 8) or Dedonder (Ref. 16) is used. The dot-dashed curve results when the modification of Miller (Ref. 18) is used (see text). The data point is from Binon *et al.* (Ref. 20).

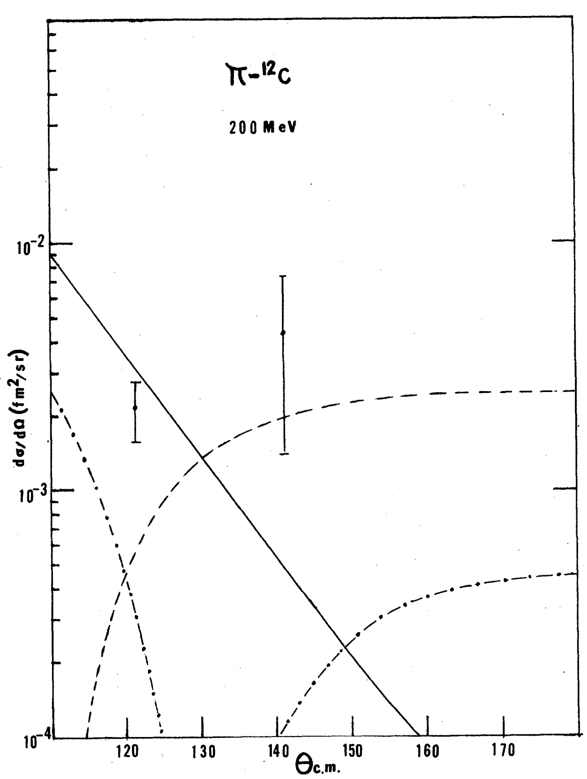

FIG. 3. The same as Fig. 2 but at $T=200$ MeV.

Figs. 2 and 3 are obtained. The range parameter a is taken to be 1.67 fm for ^{12}C. Some typical cross sections are shown in the figures. The data points of Binon *et al.*[20] are included for reference.

The 180 and 280 MeV cross sections are shown in Fig. 2 and the 200 MeV cross section is shown, in greater detail, in Fig. 3. The solid curves are the ones obtained using the original, untransformed parameters of Ref. 19. The dashed curve is obtained when b_1, as it appears in the Laplacian term only, is increased by the factor $(k_l/k_c)^2$, i.e., this is the Faldt or Dedonder suggestion. The dot-dashed curve is obtained using the $(1+\omega_l/m)$ factor in the coefficient of the Laplacian, i.e., the Miller suggestion. Results at other energies up to 280 MeV show the same sort of variation with the three cases. It is clear that back angles are very sensitive to the b_1 parameter. We have also used a range parameter of $a=1.5$ fm. The cross sections again are very sensitive. Overall the sensitivity is such that it is likely that, as the range parameter varies between 1.5 and 1.67 fm and the Laplacian coefficient varies between the untransformed case and the Faldt case, one could completely fill in the bottom half of Fig. 3, for example.

IV. CONCLUDING REMARKS

In this work we have elucidated the relationship between the back angle amplitudes of Chen and Schiff. The Schiff result is obtained from summing the Born series in which the one backward scattering occurs anywhere within the rescatterings. It is evident that including a back scattering during the next to last interaction, as Chen does, is a significant improvement over the back angle eikonal amplitude. In the latter, back scattering occurs during the last interaction only.

These back angle amplitudes are sensitive to πN amplitude transformations that have been proposed. Since the expression is convenient to use, it should provide a useful first test when proposed treatments of a variety of effects are considered. This will become increasingly the case when the difficulties of obtaining large angle data are overcome.

One caveat is that contributions to scattering into back angles that come about from many small deflections or a few intermediate ones have not been included in this theory. They may be significant.

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[16] In their discussion, Sugar and Blankenbeckler, Ref. 5, indicate that the Gaussian potential may not be as well suited to the multiple scattering considerations carried on here as potentials that are less smooth. However, since the potential is so frequently used, it is thought worthwhile to do some trial calculations.

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