Two-body effects in deuteron photoabsorption sum rules

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The effect of the two-body charge and current densities is considered in the deuteron photoabsorption sum rules. The modifications induced in the Siegert form of the $E1$ operator give rise to an enhancement factor $k_2 \simeq 0.2$ over the Thomas-Reiche-Kuhn value of the integrated cross section σ_0. The bremsstrahlung-weighted cross section $\sigma_{-1}(E1)$ is found practically unchanged with respect to its value in impulse approximation, while $\sigma_{-1}(M1)$ shows a 30% increase. The calculations are performed with hard-core, soft-core, and super-soft-core potentials. The dependence on the potential model is small for σ_0 and quite negligible for σ_{-1}.

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I. INTRODUCTION

Interest in the photonuclear sum rules has been renewed in consequence of the measurements of the Mainz group[1] of the total photoabsorption cross section on several light nuclei, $A \leq 40$, up to the pion threshold $E_\pi = 140$ MeV. When the first results appeared in 1972, the theoretical estimate of the enhancement k over the Thomas-Reiche-Kuhn (TRK) sum rule was still that of Levinger and Bethe,[2] $k \simeq 0.4$, based on the exchange percentage in the central part of the nuclear potential. Instead, the value deduced from these measurements was $k_{\text{expt}} \simeq 1$ for all the nuclei with $A \geq 4$. More recently, Bergère and collaborators[3] have extended the Mainz data in the region $A > 100$, obtaining $k \simeq 0.75$ independently of A.

The theoretical efforts for raising the theoretical k to 1 were successful[4] thanks to the two-body correlations in the wave functions, especially those induced by the tensor force. The large k values also made it necessary to look more deeply into the relation between k and the deviation δg_l from the free value of the orbital g factor in nuclei.[5] From the theoretical point of view the first problem is to what extent it is meaningful to compare the unretarded $E1$ sum rule with the experimental total photoabsorption cross section integrated up to E_π. The arguments against this possibility are straightforward: Multipoles other than $E1$ contribute to the $E1$ cross section; the $E1$ operator should have its complete expression, i.e., with the retardation factors included; and finally, the theoretical sum is for the infinite energy range while the experimental cross section is necessarily integrated over a finite energy interval. The investigations about these points have thrown new light on some classical results such as the Gerasimov sum rule[6] and the finite energy sum rule of Gell-Mann *et al.* (GGT).[7]

Reconsidered by several authors,[8] Gerasimov's proof of the exact cancellation between the $E1$ retardation effect and other multipole contributions does not hold for nonrelativistic systems, the violations being of the order B/M, where B is the binding energy and M the nucleon mass. Besides these binding corrections, the GGT finite energy sum rule must be corrected for the shadowing effects as pointed out by Weise.[9]

The conclusion which can be drawn from these investigations is that the integral up to E_π of the experimental total photoabsorption cross section constitutes the natural comparison value for the unretarded $E1$ sum rule, as long as only the nucleonic degrees of freedom are considered. When the mesonic degrees of freedom are also included, the integration limit of the experimental cross section should be accordingly increased. But a clear-cut criterion to define this limit does not exist. However, when the effect of the Δ-resonance excitation is taken into account, it seems reasonable to extend the integration limit to cover the region of the Δ resonance.[10]

In this paper we shall be concerned with the exchange effects on the deuteron sum rules. More precisely, our aim is to evaluate the effect of the two-body charge density $\rho_{[2]}$ on the unretarded

$E1$ sum rule σ_0 and on the unretarded $E1$ bremsstrahlung-weighted cross section $\sigma_{-1}(E1)$. For the sake of completeness and because the experimental value is known with high precision, we shall also evaluate $\sigma_{-1}(M1)$, including in the $M1$ operator the contributions of the meson-exchange currents (MEC) and of the Δ-excitation current (IC). While the exchange current operators are well defined through low-energy theorems, some uncertainties affect the determination of the two-body charge density. However, it is clear from all the processes studied [for example, the electromagnetic form factors of the two- and three-body nuclei,[11-15] the forward angle deuteron photodisintegration,[16,17] and the charge form factors of ^{4}He, ^{16}O, and ^{40}Ca (Ref. 18)] that the dominant contribution to $\rho_{[2]}$, up to momentum transfer $q^2 < 20$ fm^{-2}, comes from the pionic pair process $\rho_{N\bar{N}}$.

The theoretical problem concerning the definition of $\rho_{[2]}$ and its solution in a consistent scheme will be briefly discussed in the following section. At this point, we want only to stress that we shall consider in our calculations all the contributions to $\rho_{[2]}$ deriving from the one-pion exchange (OPE) but disregarding the nonlocal and frame-dependent terms.

The integrated cross section σ_0 for the deuteron has already been given by Lucas and Rustgi,[19] by Rustgi *et al.*,[20] by Arenhövel and Fabian[21] (but neglecting the two-body charge density), and by Hadjimichael,[17] who finds a further large enhancement k_2 due to $\rho_{N\bar{N}}$.

Our results agree with those of Refs. 20 and 21 for the usual enhancement k_1 coming from the double commutator of the potential with the one-body dipole operator. As for k_2, we obtain values rather different from those of Hadjimichael.[17] Because Ref. 17 is lacking in details, it is not easy to understand the origin of this discrepancy. Strangely enough, the values for k_1 quoted in Ref. 17 also disagree with ours and with those in Refs. 20 and 21.

As for $\sigma_{-1}(E1)$, we find that the inclusion of $\rho_{[2]}$ causes slight increases with respect to the Levinger and Bethe classical result[2] evaluated with realistic wave functions by Rustgi *et al.*[20] and by Arenhövel and Fabian.[21]

As said above, we have also obtained $\sigma_{-1}(M1)$ in closed form with the exchange current contributions included. We take the opportunity to note that the analytic expression of $\sigma_{-1}(M1)$ for the isovector and isoscalar transitions in impulse approximation, i.e., with only the one-body operators, have been incorrectly reported in Ref. 19.

In Sec. II we discuss the problem of the determination of the two-body charge density and we give the expression of the two-body $E1$ operators we shall use. We report and discuss the results for the enhancement factor k over the TRK sum rule in Sec. III, and those for $\sigma_{-1}(E1)$ and $\sigma_{-1}(M1)$ in Sec. IV. Finally, in Sec. V our conclusions are stated.

II. CHARGE DENSITY AND $E1$ OPERATOR

The problem of the two-body corrections to the charge density has been initially tackled by several authors[11-13] developing to a higher order in $1/M$ the exchange currents coming from the Feynman diagrams which give the dominant contributions to the magnetic moment. Recently, the theory of the OPE contributions to $\rho_{[2]}$ has been put on more certain ground[22-24] by reducing through unitary transformations a relativistic meson-nucleon Hamiltonian to the subspace not containing mesons and the negative energy nucleon states. The major advantage of these approaches is to treat the problems of the NN interaction and of the em currents at the same time. For example, the problem of the wave function renormalization has a more consistent and satisfactory solution. As a result, one obtains exact cancellation to the order $1/M^2$ between the contribution of the recoil process and that of the wave function renormalization process at every momentum transfer. On the other hand, the resulting two-body em operators suffer from some ambiguities coming from the arbitrariness of the unitary transformations and of their order. As usually done, we resolve this arbitrariness by choosing the free parameters so that $\rho_{N\bar{N}}$ reduces to the form first given by Kloet and Tjon[13] in pseudoscalar π-N coupling.

For the sake of completeness, we shall also include in the calculations the retardation effect in the recoil and wave function renormalization processes (in short, retardation process). In fact, this process gives the only other contribution to the local part of the charge density to the order $1/M^3$. Besides the nonlocal terms we shall disregard the ω- and ρ-exchange processes,[12] the $\rho\pi\gamma$ process,[12,15,25] and the two-boson exchange process,[26,27] which give negligible contributions.

In conclusion, the OPE contributions to the charge density which we shall use are those related to the pair process $\rho_{N\bar{N}}$ and to the retardation process $\rho_{\rm ret}$ as given by Hyuga and Gari.[23] Their explicit expressions in momentum space are given in the Appendix. We observe that these charge densities do not contribute to the total charge of the nuclear system. In fact, both of them vanish in momentum space for vanishing photon momentum.

The two-body modifications $\vec{D}_{[2]}$ to the Siegert form $\vec{D}_{[1]}$ of the electric dipole operator

$$\vec{D}_{[1]} = \tfrac{1}{2} \sum_i [1+\tau_z(i)]\vec{r}_i \;, \tag{1}$$

$\vec{r}_i$ being the nucleon coordinates, corresponding to $\rho_{N\bar{N}}$ and $\rho_{\rm ret}$, are easily derived (see the Appendix) with the result

$$\vec{D}_{N\bar{N}} = -\frac{f^2}{2M} Y_1(\mu r)\left\{\left[\mu_s \vec{\tau}_1\cdot\vec{\tau}_2 + \mu_v \frac{(\vec{\tau}_1+\vec{\tau}_2)_z}{2}\right]\hat{\vec{r}}\times(\vec{\sigma}_1\times\vec{\sigma}_2) - \mu_v \frac{(\vec{\tau}_1-\vec{\tau}_2)_z}{2}(\vec{\sigma}_1\vec{\sigma}_2\cdot\hat{\vec{r}} + \vec{\sigma}_2\vec{\sigma}_1\cdot\hat{\vec{r}})\right\},$$
$$\vec{D}_{\rm ret} = -\frac{f^2}{4M}\frac{(\vec{\tau}_1-\vec{\tau}_2)_z}{2}\left[Y_1(\mu r)(\vec{\sigma}_1\vec{\sigma}_2\cdot\hat{\vec{r}} + \vec{\sigma}_2\vec{\sigma}_1\cdot\hat{\vec{r}}) - \tfrac{1}{3}\hat{\vec{r}}(e^{-\mu r}\vec{\sigma}_1\cdot\vec{\sigma}_2 + Y_2(\mu r)S_{12})\right], \tag{2}$$

where S_{12} is the usual tensor operator, $f^2 \simeq 0.081$, $\hat{\vec{r}} = \vec{r}/r$, $\mu_s = \mu_p + \mu_n$, $\mu_v = \mu_p - \mu_n$ (μ_p, μ_n are the proton, neutron magnetic moments), μ is the pion mass, and finally,

$$Y_1(x) = \frac{e^{-x}}{x}\left(1+\frac{1}{x}\right),$$
$$Y_2(x) = e^{-x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right). \tag{3}$$

These expressions are for the hadronic form factor $F_{\pi NN}(\vec{q}^{\,2}) = 1$. As we shall see in the next sections, the sum rules for the deuteron should diverge with these forms of $\vec{D}_{[2]}$, unless one uses only hard-core potentials. This divergence is eliminated in the most natural way by exploiting the momentum dependence of $F_{\pi NN}$. In our calculations we shall use the monopole form[28]

$$F_{\pi NN}(\vec{q}^{\,2}) = \frac{\Lambda^2-\mu^2}{\Lambda^2+\vec{q}^{\,2}}\;, \tag{4}$$

with $\Lambda = (1003 \pm 66)$ MeV as obtained by Dominguez and Clark[29] in their fit to the charged pion photoproduction data. With a straightforward calculation one obtains the new radial functions, which substitute those in (2) following the scheme

$$Y_1^{NN} \to \phi(r,\mu,\Lambda),$$
$$Y_1^{\rm ret} \to \phi_1(r,\mu,\Lambda),$$
$$e^{-\mu r} \to \phi_0(r,\mu,\Lambda),$$
$$Y_2 \to \phi_2(r,\mu,\Lambda). \tag{5}$$

As for the explicit form of these functions we refer to the Appendix.

III. INTEGRATED CROSS SECTION

For the deuteron the unretarded $E1$ sum rule is

$$\sigma_0 = \int dE\,\sigma(E) = \frac{2\pi^2 e^2}{3}\sum_m \langle d,m|[D_z,[H,D_z]]\,|d,m\rangle\;, \tag{6}$$

where $\vec{D}$ is the dipole operator, H is the nuclear Hamiltonian, and $|d,m\rangle$ indicates the deuteron state with polarization m. Factoring out the TRK sum rule, deriving from the double commutator of the kinetic energy with $\vec{D}_{[1]}$, σ_0 reads

$$\sigma_0 = \frac{\pi^2 e^2}{M}(1+k). \tag{7}$$

The enhancement factor k can be divided into the sum of k_1, deriving from $\vec{D}_{[1]}$ and the nuclear potential V, and k_2 coming from $\vec{D}_{[2]}$:

$$k_1 = \frac{2M}{3}\sum_m \langle d,m|[D_{[1]z},[V,D_{[1]z}]]\,|d,m\rangle\;, \tag{8}$$

$$k_2 = \frac{2M}{3}\sum_m \langle d,m|[D_{[1]z},[H,D_{[2]z}]] + [D_{[2]z},[H,D_{[1]z}]] + [D_{[2]z},[H,D_{[2]z}]]\,|d,m\rangle\;. \tag{9}$$

In the previous works on the deuteron sum rules, in which $\rho_{[2]}$ was neglected,[19-21] it was possible to calculate explicitly, with realistic potentials, the double commutator defining k_1 owing to the simple form of $\vec{D}_{[1]}$.

Because of the complex structure in spin and angular variables of $\vec{D}_{[2]}$ it is not convenient to follow this method for k_2. Rather, we shall evaluate directly $1+k$, developing the original double commutator (6) and exploiting the property $H|d\rangle = -B|d\rangle$, where B is the binding energy. The isospin matrix elements are easily evaluated with the result

$$1+k = \frac{4M}{3}\sum_m \left\langle d,m\left|\sum_T D_z^T(H^T+B)D_z^T\right|d,m\right\rangle, \tag{10}$$

where the sum is over the $T = 0$, 1 parts of the Hamiltonian and the dipole operator

$$H^T = K + \langle T|V|T\rangle\;,$$
$$\vec{D}^{T=0} = 3\frac{f^2}{2M}\phi\,\mu_s\,\hat{\vec{r}}\times(\vec{\sigma}_1\times\vec{\sigma}_2), \tag{11}$$
$$\vec{D}^{T=1} = \frac{\vec{r}}{2} + \frac{f^2}{2M}(\mu_v\phi - \tfrac{1}{2}\phi_1)(\vec{\sigma}_1\vec{\sigma}_2\cdot\hat{\vec{r}} + \vec{\sigma}_2\vec{\sigma}_1\cdot\hat{\vec{r}}) + \frac{f^2}{4M}\tfrac{1}{3}\hat{\vec{r}}(\phi_0\vec{\sigma}_1\cdot\vec{\sigma}_2 + \phi_2 S_{12}).$$

Here K is the kinetic energy and the arguments of the ϕ's have been omitted.

By means of the Wigner-Eckart theorem, (10) can be expressed as the reduced matrix element of the zero-order tensor obtained by coupling the first order tensors $\vec{D}^T$ and $(H^T+B)\vec{D}^T$,

$$1+k=-\frac{4M}{3}\left\langle d\left\|\sum_T[\vec{D}^T\otimes(H^T+B)\vec{D}^T]^{(0)}\right\|d\right\rangle. \tag{12}$$

By inserting complete sets of spin-angle states we obtain

$$1+k=-\frac{4M}{9}\sum_{jll'sT}(-)^{j}\langle d\|\vec{D}^T\|jls\rangle\langle jls|(H^T+B)|jl's\rangle \times\langle jl's\|\vec{D}^T\|d\rangle\,, \tag{13}$$

using the Hamiltonian property of not coupling states of different spin. A lengthy but straightforward calculation gives the final result

$$1+k=\frac{4}{9\mu}\sum_{JLL'ST}\int dx\left[\langle JLST|U|JL'ST\rangle g_{JL}^{ST}g_{JL'}^{ST}+\delta_{LL'}\left(\left(\frac{dg_{JL}^{ST}}{dx}\right)^2+(g_{JL}^{ST})^2\left(\frac{L(L+1)}{x^2}+\epsilon\right)\right)\right]. \tag{14}$$

where $x=\mu r$, $\epsilon=MB/\mu^2$, $U=MV/\mu^2$, and the functions $g_{JL}^{ST}(x)$ different from zero are defined by

$$\begin{aligned}
g_{01}^{11}&=\frac{1}{\sqrt{3}}(f_1-3f_2-2f_3)u+(\tfrac{2}{3})^{1/2}(2f_3-f_1)w,\\
g_{11}^{11}&=-(f_1+2f_2+f_3)u+\frac{1}{\sqrt{2}}(f_2-f_3-f_1)w,\\
g_{21}^{11}&=\frac{1}{\sqrt{15}}(5f_1-f_3)u+\frac{1}{\sqrt{30}}(15f_2+11f_3-f_1)w,\\
g_{23}^{11}&=3(\tfrac{2}{5})^{1/2}f_3u+\frac{3}{\sqrt{5}}(f_1-f_3)w,\\
g_{11}^{00}&=\sqrt{2}\,f_4u+f_4w,
\end{aligned} \tag{15}$$

in terms of the usual deuteron wave functions u and w, and of the functions $f_i(x)$ given by

$$\begin{aligned}
f_1&=\tfrac{1}{2}x+\frac{1}{12}\frac{\mu}{M}f^2\phi_0,\\
f_2&=\frac{\mu}{M}f^2(\mu_v\phi-\tfrac{1}{2}\phi_1),\\
f_3&=\frac{1}{6}\frac{\mu}{M}f^2\phi_2,\\
f_4&=3\frac{\mu}{M}f^2\mu_s\phi.
\end{aligned} \tag{16}$$

The enhancement factors k_1 and k_2 follow from (14) annihilating the appropriate functions in (16).

In Table I we report $k_2(N\bar{N})$, which corresponds to the pair-process contributions only, and the total k_2, which includes the retardation-process contributions. For reference, the values of k_1 are also listed. In order to see the influence of the short-range behavior of the phenomenological potentials, we have considered the Hamada-Johnston potential (HJ),[30] the Reid soft-core potential (RSC),[31] and the super-soft-core potential (SSC) of de Tourreil *et al.*[32]

Our values of k_1 are in agreement with those calculated by Arenhövel and Fabian[21] and by Rustgi *et al.*,[20] in the common cases. Instead, our results do not agree with those of Hadjimichael[17] for both k_1 and $k_2(N\bar{N})$, even if $k_1+k_2(N\bar{N})$ nearly coincides for the HJ potential. As can be seen from Table I, the inclusion of the retardation charge density has the effect of reducing the pair-process contribution, but by a small amount. This reduction is more easily understandable looking at expressions (2) of $\vec{D}_{N\bar{N}}$ and $\vec{D}_{ret}$ for $F_{\pi NN}=1$. In fact, $\vec{D}_{ret}$ partially cancels the isovector part of $\vec{D}_{N\bar{N}}$.

Unlike k_1, which is essentially the matrix element of r^2V, k_2 could be expected *a priori* to be more sensitive to the NN interaction model because of the short range behavior of the two-body

TABLE I. Enhancement factors over the TRK sum rule for the deuteron, evaluated with the Hamada-Johnston potential (HJ), the Reid soft-core potential (RSC), and the super-soft-core potential (SSC) of de Tourreil *et al.* k_1 is the enhancement for the one-body charge density, k_2 $(N\bar{N})$ for the OPE pair charge density, k_2 $(N\bar{N}+\text{ret})$ for the total OPE charge density, and k is the total enhancement.

Potential \ Enhancement factors	k_1	k_2 $(N\bar{N})$	k_2 $(N\bar{N}+\text{ret})$	$k=k_1+k_2$
HJ	0.525	0.191	0.190	0.715
RSC	0.502	0.162	0.161	0.663
SSC	0.533	0.192	0.192	0.725

operators, which are even divergent for $F_{\pi NN}=1$. As already said above, in order to avoid divergent matrix elements in the calculation of the last term in (9), we have used $\vec{D}_{[2]}$ with the radial functions regularized thanks to the hadronic form factor. Our results for k_2 (see the third column in Table I) show nearly the same dependence on the potentials as k_1. The ratio k_2/k_1 goes from 30% to 35% for the potentials used. Instead, Hadjimichael[17] obtains values of k_2/k_1 remarkably model dependent, ranging from 20% to 50%, even if with a set of potentials which does not completely coincide with ours.

It is difficult for us to comment on this model dependence and the discrepancy with our results, since it is not explained in Ref. (17) how the short-range behavior of $\vec{D}_{[2]}$ and its corresponding divergent matrix elements have been handled.

The last remark about the values of k_2 concerns the amount of uncertainty introduced by the regularization of $\vec{D}_{[2]}$ by means of form (4) of $F_{\pi NN}$. Taking into consideration the error ± 66 MeV for Λ quoted in Ref. 29, we have obtained variations in k_2 of about ± 10%.

The total enhancement k follows by adding k_2 to k_1 and is reported in the last column of Table I. The values are around $k \simeq 0.7$, with little difference among the potentials.

At this point, there still remains to be considered the contribution to k coming from the excitation of the isobar resonances, and, essentially, of the dominant Δ resonance. First of all, we wish to note that the isobaric degrees of freedom do not fall within the theory developed by Gari and Hyuga[22,23] and by Friar,[24] and must be treated separately. Now, the charge density operator for the Δ excitation has been evaluated by Kloet and Tjon,[13] from the corresponding diagram, and turns out to be nonlocal. Thus, consistently with what has been done for the nonlocal OPE terms, it has been disregarded in our calculations. However, besides the approach by means of effective two-body operators, the virtual presence of nucleon isobars can be treated by adding isobar configurations to the usual nucleonic wave functions.[33] In principle, these two methods are equivalent, but, obviously, in practical calculations their results may be different because of the unavoidable approximations. For the deuteron sum rules, the wave function modification method has been applied by Arenhövel and Fabian[21] and a further enhancement factor $k_2(\Delta) \simeq 0.2$ has been found, with a small model dependence. Since the coupling constants entering in the $N\Delta$ transition potential are far from being univocally determined, we recall that these authors take the $\Delta N\pi$-coupling constant from the Δ-decay width $(f_{\Delta N\pi}^2/4\pi \simeq 0.35)$ and the $\Delta N\rho$-coupling constant from the quark model prediction $(f_{\Delta N\rho}^2/f_{NN\rho}^2 = 72/25)$. This contribution cannot simply be added to that calculated in this paper because of the renormalization of the deuteron wave function induced by the introduction of the isobar configurations, and, moreover, because of the interference terms. However, these effects are small when only the one-body charge density is considered, as follows from Table I of Ref. (21). In the reasonable hypothesis that the inclusion of the two-body charge density does not cause a large interference term, the total enhancement factor assumes a value $k \simeq 0.9$.

As far as the experimental enhancement factor k_{expt} is concerned, the problem of establishing the upper limit of integration of the experimental cross section has no clear solution. In fact, in σ_0 (unlike σ_{-1} and, in general, σ_{-n}) there is not a weighting factor with a negative power of the excitation energy, which enhances the importance of the low-energy part of the cross section. This lack has two consequences: On one hand, the theoretical k can only be taken as an estimate because of the low-energy approximations used in the calculations, and, on the other hand, it is difficult to compare k with evaluations $[k_{\text{expt}}(E_\gamma)]$ obtained integrating the experimental cross section up to a finite energy E_γ. The upper limit of integration, E_γ, should be chosen in such a way that all the degrees of freedom considered in the theory have already given, at that energy, their main contribution to σ_0, and no other degrees of freedom are becoming important. Obviously, this requirement is not easily satisfied. For example, partially inconsistent results occur in the deuteron case if E_γ is taken to be the pion threshold $(E_\pi \simeq 140$ MeV$)$ which is commonly considered the natural upper limit when σ_0 is evaluated with $\vec{D}_{[1]}$. In fact, $k_{\text{expt}}(E_\pi) \simeq 0.35$ (Ref. 10) while, from Table I, $k_1 \simeq 0.5$. On the other hand, this discrepancy is not so large as to make the comparison completely meaningless.

When the effects of $\rho_{[2]}$ and of the Δ-resonance excitation are included in σ_0, a convenient E_γ can be found looking at the behavior of the experimental cross section. In fact, beyond E_π it shows a broad resonance just below 300 MeV, and then, towards 500–600 MeV, falls off by an order of magnitude. This resonance is attributed to the pion-exchange effects and, mainly, to the Δ-resonance excitation. In consequence, it seems quite reasonable to compare our result $k \simeq 0.9$ with k_{expt} (540 MeV) $= 0.80 \pm 0.10$.[21] To the extent that the validity of the comparison is sensible, we can conclude that there is a good agreement between these two values.

IV. BREMSSTRAHLUNG-WEIGHTED CROSS SECTIONS

The electric dipole bremsstrahlung-weighted cross section for the deuteron may be written as

$$\sigma_{-1}(E1)=\frac{4\pi^2e^2}{3}\sum_m \langle d,m\,|D_z^{\,2}\,|d,m\rangle, \tag{17}$$

which becomes

$$\sigma_{-1}(E1)=\frac{4\pi^2e^2}{9\mu^3}\sum_{JLST}\int dx(g_{JL}^{ST})^2 \tag{18}$$

when the dipole operator is modified with respect to its Siegert form by the inclusion of the OPE-two-body charge density. The functions g_{JL}^{ST} are given in (15) and (16). from where it is easily seen that (18) reduces to the Bethe-Levinger form when the exchange contributions are ignored.

The values of $\sigma_{-1}(E1)$ obtained with the potentials considered are reported in Table II for the following cases of charge density: only $\rho_{[1]}$, $\rho_{[1]}+\rho_{N\bar{N}}$, and $\rho_{[1]}+\rho_{N\bar{N}}+\rho_{\rm ret}$. The two-body contributions produce a small increase of σ_{-1} with respect to the classical value. As for σ_0, the inclusion of $\rho_{\rm ret}$ in addition to $\rho_{N\bar{N}}$ lowers the effect of the exchange charge density very slightly.

If we take into account the decrease of σ_{-1} due to the IC effects as calculated in Ref. 21(with the same cares previously underlined concerning the interference terms and the wave function renormalization), we must conclude that MEC and IC contributions nearly cancel each other.

As far as the magnetic dipole bremsstrahlung-weighted cross section $\sigma_{-1}(M1)$ is concerned, Lucas and Rustgi[19] give the result of its evaluation in impluse approximation exploiting the completeness of the states, while Arenhövel and Fabian[21] give the result of the explicit integration of the theoretical cross section up to E_π. The closed form of σ_{-1} $(M1)$ does not have a cumbersome expression when the two-body $M1$ operators corresponding to the meson exchange and Δ-excitation currents are included.

TABLE II. Deuteron bremsstrahlung-weighted $E1$ cross sections in mb for the same potentials as in Table I. The first column corresponds to the one-body charge density, the second column includes the contributions of the OPE pair charge density, and the third one includes the contributions of the retardation charge density.

Potential	$\sigma_{-1}(E1)$ (mb) $\rho_{[1]}$	$\rho_{[1]}+\rho_{N\bar{N}}$	$\rho_{[1]}+\rho_{N\bar{N}}+\rho_{\rm ret}$
HJ	3.668	3.704	3.703
RSC	3.677	3.712	3.712
SSC	3.746	3.781	3.780

We start, for the sake of clarity, from the expression

$$\sigma_{-1}(M1)=\frac{\pi^2e^2}{6M^2}\sum_{\lambda,m,f}|\langle f|\vec{\epsilon}_\lambda\cdot\vec{\mu}\,|d,m\rangle|^2, \tag{19}$$

where the sum over the possible final states $|f\rangle$ has yet to be performed. The other sums in (19) run over the photon and deuteron polarizations; $\vec{\epsilon}$ is the photon polarization vector and $\vec{\mu}$ is the magnetic dipole operator in units $e/2M$, which for the n-p system can be written as

$$\vec{\mu}=\tfrac{1}{2}\vec{L}+\mu_s\vec{S}+\mu_v\frac{(\vec{\tau}_1-\vec{\tau}_2)_z}{2}\frac{(\vec{\sigma}_1-\vec{\sigma}_2)}{2}+\vec{\mu}_{\rm ex}. \tag{20}$$

For the exchange part $\vec{\mu}_{\rm ex}$ we follow the classification of Chemtob and Rho[34]:

$$\vec{\mu}_{\rm ex}=\tfrac{1}{2}\{(\vec{\tau}_1\times\vec{\tau}_2)_z[\vec{\sigma}_1\times\vec{\sigma}_2 g_I+\vec{T}_{12}^{(\times)}g_{II}] + (\vec{\tau}_1-\vec{\tau}_2)_z[(\vec{\sigma}_1-\vec{\sigma}_2)h_I+\vec{T}_{12}^{(-)}h_{II}]\}, \tag{21}$$

with the definition

$$\vec{T}_{12}^{\odot}=\hat{\vec{r}}(\vec{\sigma}_1\odot\vec{\sigma}_2)\cdot\hat{\vec{r}}-\tfrac{1}{3}\vec{\sigma}_1\odot\vec{\sigma}_2, \tag{22}$$

where $\hat{\vec{r}}=\vec{r}/r$. Let us note that operating on the deuteron wave function

$$\vec{\mu}_{\rm ex}|d\rangle=\tfrac{1}{2}(\vec{\tau}_1-\vec{\tau}_2)_z[(\vec{\sigma}_1-\vec{\sigma}_2)(g_I+h_I) + \vec{T}_{12}^{(-)}(g_{II}+h_{II})]\,|d\rangle. \tag{23}$$

The pair current and the pionic current contribute only to the g factors,[34] which added together are

$$\begin{aligned} g_I(N\bar{N})+g_I(\pi)&=\frac{2}{3}\frac{M}{\mu}f^2(2\mu r-1)Y_0(\mu r),\\ g_{II}(N\bar{N})+g_{II}(\pi)&=-2\frac{M}{\mu}f^2(1+\mu r)Y_0(\mu r). \end{aligned} \tag{24}$$

As for the isobar-excitation processes, we limit ourselves to the dominant Δ excitation, which, according to Riska and Brown,[35] gives the contributions

$$\begin{aligned} g_I(\Delta)+h_I(\Delta)&=0,\\ g_{II}(\Delta)+h_{II}(\Delta)&=-\tfrac{48}{25}f^2\frac{\mu}{M_\Delta-M}\mu_vY_2(\mu r), \end{aligned} \tag{25}$$

where M_Δ is the Δ-resonance mass.

After this digression on the $M1$ operators, let us come back to expression (19) of $\sigma_{-1}(M1)$. In order to obtain $\sigma_{-1}(M1)$ in closed form we must sum over all n-p states except the deuteron state.[36] This exclusion makes a difference only for the isoscalar transitions, because the isovector operator in (23) does not connect states with the same T. The results are

$$\sigma_{-1}^{\Delta T=1}(M1)=\frac{\pi^2 e^2}{3M^2}\left[\int dr(u^2+w^2)(G_1{}^2+2G_2{}^2) - \int dr\, w(\sqrt{8}u-w)(2G_1G_2+G_2{}^2)\right],$$

$$\sigma_{-1}^{\Delta T=0}(M1)=\frac{2\pi^2 e^2}{3M^2}(\mu_s-\tfrac{1}{2})^2[1-(1-\tfrac{3}{2}P_D)^2], \tag{26}$$

where

$$G_1=\mu_v+2[g_I(N\bar{N})+g_I(\pi)],$$
$$G_2=\tfrac{2}{5}[g_{II}(N\bar{N})+g_{II}(\pi)+g_{II}(\Delta)+h_{II}(\Delta)], \tag{27}$$

and P_D is the D-wave probability.

As a first remark we can observe that, dropping the exchange contributions, $\sigma_{-1}^1(M1)$ reduces to the form given by Rustgi and Levinger,[37]

$$[\sigma_{-1}^1(M1)]_{\mathrm{IA}}=\frac{\pi^2 e^2}{3M^2}\mu_v{}^2 \tag{28}$$

(where IA is for impulse approximation), which retains its validity when the D-wave component of the deuteron is taken into account. In passing, we note that Lucas and Rustgi[19] report the same $\sigma_{-1}^1(M1)$ multiplied by $[P_S+\frac{1}{10}P_D]$, P_S being the S- wave probability.

Our second remark concerns $\sigma_{-1}^0(M1)$. Its analytic expression previoulsy given in Ref. 19 does not agree with ours: The difference seems to derive mainly from the inclusion in Ref. 19 of the deuteron state in the set of possible final states.

Numerically, from expression (26) if follows $\sigma_{-1}^0(M1)\cong(0.5-0.6)\times10^{-3}$ mb depending on the D-wave percentage, which is three times smaller than that given in Ref. 19. This does not change the total $\sigma_{-1}(M1)$ which is nearly completely determined by the isovector transitions, and which assumes in IA the model independent value of 0.235 mb.

Of course, $\sigma_{-1}^1(M1)$ should be divergent if the contributions to the $M1$ operator coming from the Δ-excitation current are taken as in (25), where the hadronic form factors are assumed to be $F_{\pi NN}=F_{\Delta N\pi}=1$. To regularize the behavior of $[g_{II}(\Delta)+h_{II}(\Delta)]$ at $r=0$ we have introduced in the calculations the momentum dependence of $F_{\pi NN}$ in the form (4), and of $F_{\Delta N\pi}$ in the form

$$F_{\Delta N\pi}(\vec{q}^2)=\frac{\alpha^2}{\alpha^2+\vec{q}^2}. \tag{29}$$

Using these expressions of the form factors, derivation of the radial function substituting $Y_2(\mu r)$ in (25) is straightforward and we do not report it here.

We list in Table III the results for the isovector $\sigma_{-1}^1(M1)$, calculated with the same value as before for Λ and with $\alpha=5$ fm^{-1}, which corresponds to the explicit inclusion of the ρ-exchange effect in the ΔN transition potential $V_{\Delta N}$.[33(b)] This value agrees with the estimate obtained in dispersive theory by Dillig and Brack.[38] Also reported in Table III is the isoscalar $\sigma_{-1}^1(M1)$ together with the total $\sigma_{-1}(M1)$.

TABLE III. Deuteron bremsstrahlung-weighted $M1$ cross sections σ_{-1} in mb for the same potentials as in Table I. The values in the first column are for the isovector transitions, and those in the second one are for the isoscalar transitions. The values of the total σ_{-1} are in the third column.

Potential	$\sigma_{-1}(M1)$ (mb) $\Delta T=1$	$\Delta T=0$	Total
HJ	0.2987	0.61×10^{-3}	0.299
RSC	0.2974	0.56×10^{-3}	0.298
SSC	0.2978	0.52×10^{-3}	0.298

The values of $\sigma_{-1}^1(M1)$ are practically independent of the potential model. As for their sensitivity to the $F_{\pi NN}$ parameter, we can add that they are unchanged for Λ running in the range (1003 ± 66) MeV.[29] Higher uncertainties affect the $\Delta N\pi$ vertex. Indeed, α drops to 2 fm^{-1} if only the π-exchange effect is considered in $V_{\Delta N}$.[33(b)] With this lower α, the values of $\sigma_{-1}^1(M1)$ diminish by ~5%, independently of the potential.

For comparison, we remember that Arenhövel and Fabian[21] find a nearly exact cancellation between MEC and IC effects in σ_{-1} calculated from explicit integration up to E_π of the theoretical cross section, in which all multipoles up to $L=4$ and the retardation factors are included.

V. CONCLUSIONS

We have evaluated, for the deuteron, the unretarded $E1$ sum rule and the bremsstrahlung-weighted $E1$ and $M1$ cross sections including the two-body contributions to the charge and current densities. The exchange magnetic moments used are the standard ones corresponding to the pair current, π current, and the Δ-excitation current. For the exchange charge densities, which are less firmly established, we have taken the expressions associated with OPE as derived by Hyuga and Gari.[23]

We find that $\rho_{[2]}$ produces a further enhancement $k_2\simeq0.2$ over the TRK sum rule; this enhancement, unlike that of Hadjimichael,[17] is not so sensitive to the potential model. The dominant contribution to k_2 derives from the pair charge density and is slightly lowered by the inclusion of the retardation terms.

The total enhancement becomes $k\simeq0.9$, taking into account the IC effects as evaluated by Arenhövel and Fabian.[21] This theoretical value is in good agreement with $k_{\text{expt}} = 0.80 \pm 0.10$, obtained from the integration of the experimental cross section up to a 540 MeV.[21]

As far as the two-body effects in σ_{-1} are concerned, σ_{-1} $(E1)$ remains practically unchanged with respect to its value in impulse approximation, while $\sigma_{-1}(M1)$ increases from its IA model independent value of 0.235 mb to 0.3 mb. This value presents insignificant variations with the potential model.

Finally, we have given the analytic expressions of $\sigma_{-1}^{0}(M1)$ and $\sigma_{-1}^{1}(M1)$ which in IA correct those previously obtained by Lucas and Rustgi.[19]

APPENDIX

The two-body charge densities $\rho_{N\bar{N}}$ and ρ_{ret}, in momentum space and for real photons, are[23]

$$\rho_{N\bar{N}}(\vec{k}, \vec{q}_1, \vec{q}_2) = \frac{e}{2M}\left(\frac{g}{2M}\right)^2 \frac{\vec{\sigma}_1 \cdot \vec{k}\,\vec{\sigma}_2 \cdot \vec{q}_2}{\vec{q}_2^{\,2} + \mu^2} \times (\mu_s \vec{\tau}_1 \cdot \vec{\tau}_2 + \mu_v \tau_{2z}) F_{\pi NN}^{2}(\vec{q}_2^{\,2}) + (1 \rightleftarrows 2),$$

$$\rho_{\text{ret}}(\vec{k}, \vec{q}, \vec{q}_2) = -\frac{e}{4M}\left(\frac{g}{2M}\right)^2 \frac{\vec{k} \cdot \vec{q}_2\,\vec{\sigma}_1 \cdot \vec{q}_2\,\vec{\sigma}_2 \cdot \vec{q}_2}{(\vec{q}_2^{\,2} + \mu^2)^2} \times (\vec{\tau}_1 \cdot \vec{\tau}_2 + \tau_{2z}) F_{\pi NN}^{\;2}(\vec{q}_2^{\,2}) + (1 \rightleftarrows 2), \tag{A1}$$

where $\vec{k}$ is the photon momentum, g is the pseudoscalar πN coupling constant, $\vec{q}^{\,2}$ is the pion momentum, μ the pion mass, $\mu_s \simeq 0.88$ and $\mu_v \simeq 4.71$ are the isoscalar and isovector magnetic moments of the nucleon in units of nuclear magnetons, and $F_{\pi NN}(\vec{q}^{\,2})$ is the hadronic form factor.

By Fourier transforming (A1) with respect to $\vec{q}_1$ and $\vec{q}_2$, one gets the expression of the charge densities in configuration space,

$$\rho(\vec{x}, \vec{r}_1, \vec{r}_2) = \int \frac{d^3 q_1}{(2\pi)^3} \frac{d^3 q_2}{(2\pi)^3} \exp\{i[\vec{q}_1 \cdot (\vec{r}_1 - \vec{x}) + \vec{q}_2 \cdot (\vec{r}_2 - \vec{x})]\} \times \rho(\vec{k}, \vec{q}_1, \vec{q}_2). \tag{A2}$$

The corresponding dipole operators are obtained, in a straightforward way from the definition

$$\vec{D} = \int d^3x\, \vec{x} \rho(\vec{x}). \tag{A3}$$

With $F_{\pi NN}(\vec{q}^{\,2}) = 1$ one has expressions (2) of the text, while, with the monopole form[28]

$$F_{\pi NN}(\vec{q}^{\,2}) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + \vec{q}^{\,2}} \tag{A4}$$

the radial functions in (2) become, with self-explanatory notations,

$$\begin{aligned}
Y_1^{NN} \to \phi &= Y_1(x) - \lambda^2 Y_1(y) - \frac{\lambda^2 - 1}{2} e^{-y}, \\
Y_1^{\text{ret}} \to \phi_1 &= Y_1(x) + \lambda^2 Y_1(y) - \frac{4}{\lambda^2 - 1}\left[\frac{Y_2(x)}{x^2} - \lambda^4 \frac{Y_2(y)}{y^2}\right], \\
e^{-x} \to \phi_0 &= e^{-x} + \lambda^2 e^{-y} - \frac{4}{\lambda^2 - 1}\left[Y_1(x) + 2\frac{Y_2(x)}{x^2} - \lambda^4\left(Y_1(y) + 2\frac{Y_2(y)}{y^2}\right)\right], \\
Y_2 \to \phi_2 &= Y_2(x) + \lambda^2 Y_2(y) - \frac{4}{\lambda^2 - 1}\left[Y_3(x) - \lambda^4 Y_3(y)\right],
\end{aligned} \tag{A5}$$

where $x = \mu r$, $y = \Lambda r$, $\lambda = \Lambda/\mu$, and

$$Y_3(x) = \frac{e^{-x}}{x}\left(1 + \frac{6}{x} + \frac{15}{x^2} + \frac{15}{x^3}\right). \tag{A6}$$

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