Axial meson-exchange currents in nuclear weak interactions

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On symmetry grounds and by a systematic analysis of one-meson exchange diagrams, we obtain the two-body axial meson-exchange current operator which is needed for computing meson-exchange corrections in nuclear systems. Special attention is given to the time-component of this operator and the experimental evidence of its importance.

[RADIOACTIVITY Structure of axial meson-exchange currents; meson-exchange corrections to $0^+ - 0^-$ transitions in β decay and μ capture of $A=12$ (^{12}B, ^{12}C, ^{12}N) and $A=16$ (^{16}O, ^{16}N) systems.]

Recently there has been great interest in meson-exchange effects in the realm of nuclear electroweak interactions. In any realistic description of the electromagnetic and weak behavior of nuclei, meson-exchange effects arising from exchange of virtual mesons among the constituent nucleons must inevitably be present. It is therefore imperative to obtain convincing evidence of their existence and importance. The purpose of this paper is to consider questions concerning meson-exchange corrections (MEC) due to the two-body axial meson-exchange (ME) current $A_\lambda^{(2)}$ in nuclear beta decay and muon capture processes. MEC due to the space components $\vec{A}^{(2)}$ are found to be $\approx 10\%$ in nuclear weak processes such as $^3\text{H} \to {}^3\text{He} + e^- + \bar{\nu}_e$ and $\mu^- + {}^3\text{He} \to {}^3\text{H} + \nu_\mu$, etc.[1] Of late they have been subject to many investigations. Interest for the MEC due to the time component $A_0^{(2)}$ is, however, more recent. β-decay and μ^--capture experiments in the $A=12$ (^{12}B, ^{12}C, ^{12}N) triad[2] and in the $A=16$ (^{16}O, ^{16}N*) system[3] provide experimental results which are believed to be the best sources available from which to draw conclusion on MEC (due to $A_0^{(2)}$). However, the theoretical interpretation as regards the importance of MEC is still quite open to doubt. This is partly because the above mentioned nuclei (A = 12 and 16) do not give ground to a description accurate enough to disentangle MEC from nuclear structure effects, at least for the time being. Furthermore, the expression for the time component $A_0^{(2)}$, already investigated by Kubodera, Delorme, and Rho (KDR)[4] is still subject to clarifications; such improvements in the knowledge of their structure are the aim of this paper.

Thus our discussion is centered around the structural aspects of $A_0^{(2)}$, which is obtained from a set of ME diagrams (shown in Fig. 1) representing the axial part of the two-nucleon weak processes ($N_i \to N_i' + e + \nu_e + M$; $M + N_j \to N_j'$) and ($\mu + N_i \to N_i' + \nu_\mu + M$; $M + N_j \to N_j'$). Basic to our theoretical consideration is the conventional strangeness-preserving V-A theory of weak interaction, together with the validity of the conserved vector current (CVC), partially conserved axial vector current (PCAC) hypotheses, and the complete absence of the second class currents. These latter three fundamental principles are now on much firmer experimental ground than before, thanks to the recent elegant second class current experiments on the $A=12$ triad by Brandle *et al.*[2] and Masuda *et al.*[2] The measured $\beta^\pm$-decay alignment coefficients $\alpha^\pm$ in these experiments may give a potentially reliable quantitative information regarding MEC due to $A_0^{(2)}$.

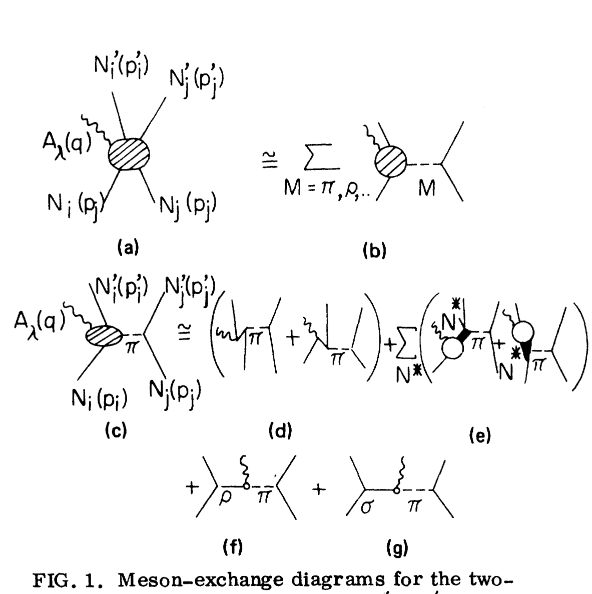

FIG. 1. Meson-exchange diagrams for the two-nucleon weak processes ($N_i + N_j \to N_i' + N_j' + e^+ + \bar{\nu}_e$) and ($\mu + N_i + N_j \to N_i' + N_j' + \nu_\mu$).

1. General approach. We first discuss the deduction of $A_0^{(2)}$ from ME diagrams displayed in Fig. 1, following the general method previously used by one of us[1] in the deduction of $\vec{A}^{(2)}$. The analysis is carried out within the context of CVC, PCAC, and the complete absence of the second class currents. On physical grounds, since the exchange of a lower mass meson is characteristic of longer-range interactions between the two nucleons, ME effects due to one pion exchange are expected to dominate over those due to heavy meson exchange, at least at low energies appropriate for beta decays and muon captures. The general one-exchange diagram is shown in Fig. 1(c). In Figs. 1(d)–1(g) we have displayed those one pion exchange diagrams which we have analyzed; those not shown are either relatively smaller or forbidden by conservation laws: G parity, parity, spin, and isospin. We remark that all exchange diagrams are understood to be accompanied by their respective counterparts with the interchange $N_i(N_i') \to N_j(N_j')$. Also, one meson (M)-exchange diagrams with $M = \rho, \sigma, \ldots$ can be analyzed in a manner similar to that shown in Figs. 1(c)–1(g).

2. Symmetry consideration of the general structure of $A_\lambda^{(2)}$. The general structure of $A_\lambda^{(2)}$ can be obtained on symmetry grounds.[5] As is evident, the two-body axial current $A_\lambda^{(2)}$ must have the same space-time and isospace transformation properties as that of $A_\lambda\ [=A_\lambda^{(1)} + A_\lambda^{(2)}]$. Thus in the $q = 0$ and nonrelativistic limit, $\vec{A}^{(2)}$ and $A_0^{(2)}$ must be suitable combinations of the relative coordinate $\vec{r} = \vec{r}_i - \vec{r}_j$ ($r \equiv |\vec{r}|$), spin ($\vec{\sigma}_i, \vec{\sigma}_j$), and isospin ($\vec{\tau}_i, \vec{\tau}_j$) such that under P (parity) and T (time reversal), $P\vec{A}^{(2)}P^{-1} = +\vec{A}^{(2)}$, $TA^{(2)}T^{-1} = -\vec{A}^{(2)}$, $PA_0^{(2)}P^{-1} = -A_0^{(2)}$, and $TA_0^{(2)}T^{-1} = +A_0^{(2)}$. This symmetry consideration leads to the following results: For $\vec{A}^{(2)}$ there are *six* independent structure terms and for $A_0^{(2)}$ there are *two* independent structure terms. They are listed below.

$\vec{A}^{(2)}$	$A_0^{(2)}$
$g_\pm(r)(\vec{\sigma}_i \pm \vec{\sigma}_j)(\vec{\tau}_i \pm \vec{\tau}_j)^{(+)}$	$d_1(r)(\vec{\sigma}_i \times \vec{\sigma}_j) \cdot \vec{r}(\vec{\tau}_i + \vec{\tau}_j)^{(+)}$
$h_\pm(r)\vec{r}\,\vec{r} \cdot (\vec{\sigma}_i + \vec{\sigma}_j)(\vec{\tau}_i \pm \vec{\tau}_j)^{(+)}$	$d_2(r)(\vec{\sigma}_i + \vec{\sigma}_j) \cdot \vec{r}(\vec{\tau}_i \times \vec{\tau}_j)^{(+)}$
$J_1(r)(\vec{\sigma}_i \times \vec{\sigma}_j)(\vec{\tau}_i \times \vec{\tau}_j)^{(+)}$	
$J_2(r)\vec{r}\,\vec{r} \cdot (\vec{\sigma}_i \times \vec{\sigma}_j)(\vec{\tau}_i \times \vec{\tau}_j)^{(+)}$	

Note that these structure terms are given in configuration space. The structure terms in momentum space for those ME diagrams shown in Fig. 1 are of the type $\vec{k}(\vec{\sigma}_i \pm \vec{\sigma}_j) \cdot \vec{k}(\vec{\tau}_i \pm \vec{\tau}_j)$ and $\vec{k} \times \vec{k} \times (\vec{\sigma}_i \times \vec{\sigma}_j)(\vec{\tau}_i \times \vec{\tau}_j)$ for $\vec{A}(2)$, and $\vec{k} \cdot (\vec{\sigma}_i + \vec{\sigma}_j)(\vec{\tau}_i \times \vec{\tau}_j)$ for $A_0^{(2)}$, $\vec{k}$ being the momentum transfer between N_i and N_j.

However, it is important to point out that in the analysis of the ME diagrams shown in Fig. 1, all six structure terms for $\vec{A}^{(2)}$ are realized, but for $A_0^{(2)}$ only one structure term $d_2(r)(\vec{\sigma}_i + \vec{\sigma}_j) \cdot \vec{r}(\vec{\tau}_i \times \vec{\tau}_j)^{(+)}$ is realized. Immediate consequences of these results are the following. (1) In the case of $\vec{A}^{(2)}$, the relative importance of these six structure terms among themselves and also of the ME diagrams among themselves cannot *a priori* be determined without first calculating their contributions to the transition matrix of $\vec{A}$, using nuclear wave functions. This implies that the assessment of relative importance is dependent on nuclear models.

(2) In contrast, in the case of $A_0^{(2)}$, since there exists only one structure term, the relative importance of ME diagrams among themselves can be determined by comparing the $d_2(r)$ functions which result from the ME diagrams; the conclusion is evidently independent of nuclear models. Note that because of the spin and isospin of the σ meson ($J^PI = 0^+0$), the σ-π exchange diagram[6] cannot give rise to either one of the two structure-terms associated with $A_0^{(2)}$, thus implying that MEC due to the σ-π exchange are absent in β-decay processes. This conclusion has been reached by KDR, but for entirely different reasons.

3. Deduction of $A_0^{(2)}$ from meson-exchange diagrams. We have analyzed the ME diagrams shown in Figs. 1(d)–1(g). The results for Figs. 1(d) and 1(e) are found to be of the order $O(\vec{\sigma}_i\, \vec{p}\,\vec{\sigma}_j \cdot \vec{k})$ and so will not be discussed in the present paper. In what follows we shall consider only the ρ-π and σ-π exchange diagrams [Figs. 1(f) and 1(g)], stressing the structural similarity between them. In so doing, we are well aware that (1) $\rho\pi\pi$ and ρNN couplings are different from $\sigma\pi\pi$ and σNN couplings, and (2) the ρ meson is a well established $\pi\pi$ resonance, while the status of the σ meson remains unclear. Phenomenologically, the σ meson is considered as parametrizing the $J^PI^G = 0^+0^+$ channel, just as the ρ meson does in parametrizing the (1^-1^+) channel in the process $(N\bar{N} \rightleftharpoons \pi\pi)$ in the low energy regime.

The momentum-space expression for $A_\lambda^{(2)}$ is obtained from the transition amplitudes $M_{a\pi}$ ($a = \rho, \sigma$) via the identity $M_{a\pi} = \chi_i^\dagger \chi_j^\dagger A_\lambda^{(2)}(a\pi)\chi_i\chi_j$. The results are,[7,8] with $A_\lambda^{(2)}(a\pi) = \sum_{i<j} A_{\lambda(ij)}^{(2)}(a\pi)$

$$\vec{A}_{(ij)}^{(2)}(\rho\pi) = \frac{\sqrt{2} f_V(1+\kappa) g_{\pi NN} f_A(\rho\pi) m_\pi}{4m^2(k^2+m_\rho^2)(k^2+m_\pi^2)} \times \vec{k} \times [\vec{k} \times (\vec{\sigma}_i \times \vec{\sigma}_j)](\vec{\tau}_i \times \vec{\tau}_j)^{(+)}, \qquad (1)$$

$$A_{0(ij)}^{(2)}(\rho\pi) = \frac{\sqrt{2} f_V g_{\pi NN} f_A(\rho\pi) m_\pi}{2m(k^2+m_\rho^2)(k^2+m_\pi^2)} \times \vec{k} \cdot (\vec{\sigma}_i + \vec{\sigma}_j)(\vec{\tau}_i \times \vec{\tau}_j)^{(+)}, \qquad (2)$$

$$\vec{A}^{(2)}_{(ij)}(\sigma\pi) = \frac{\sqrt{2}\, g_{\sigma NN} g_{\pi NN} f_P(\sigma\pi)}{2m(k^2+m_\sigma^2)(k^2+m_\pi^2)}$$
$$\times [\vec{k}(\vec{\sigma}_i+\vec{\sigma}_j)\cdot\vec{k}(\vec{\tau}_i+\vec{\tau}_j)^{(+)} \qquad (3)$$
$$+\vec{k}(\sigma_i-\vec{\sigma}_j)\cdot\vec{k}(\vec{\tau}_i-\vec{\tau}_j)^{(+)}]\,,$$
$$A_0^{(2)}(\sigma\pi) \cong 0(\vec{\sigma}_i\cdot\vec{p}\,\vec{\sigma}_i\vec{k}) \cong 0\,. \qquad (4)$$

We now turn to discuss $A_0^{(2)}(\sigma\pi)$. As was already noted, the contribution arising from the σ-π exchange is negligible in β-decay processes, a conclusion which is *independent of the details of dynamics and of the σ-meson mass*. An alternative way to see this result is as follows: In the nonrelativistic and $q=0$ limit, the vertex function $A_0(\sigma\pi) = \langle\pi|A_0|\sigma\rangle$ in $M_{\sigma\pi}$ reduces to $P_0 f_P(\sigma\pi)$, f_P being a weak $(\sigma-\pi)$ form factor. But since $P_0 = (p_0^\sigma+p^\pi)_0 - (p_i'-p_j^0)_0+(p_j'-p_j)_0 \cong 0(\vec{p}^2/2m) \cong$ recoil energy of the nucleons, which is negligible, thus $M_{\sigma\pi}\to 0$, which gives Eq. (4). The above argument on $A_0^{(2)}(\sigma,\pi)\cong 0$ remains valid for the muon capture process. This is because for the $0^+\to 0^-$ transition itself, and more generally for the low-lying final states contributing to the bulk of the muon capture strength, the kinetic is such that $q_0\approx$ a few MeV and $P_0\approx$ (nucleons recoil energy).

Moreover, the result $A_0^{(2)}(\sigma\pi)=0$ can also be obtained from a quark model consideration. We assume that the σ meson is a color-singlet two-quark-two-antiquark $(q^2\bar{q}^2)$ state as suggested by Jaffe and Low recently,[9] in contradistinction from ρ and π mesons being $(q\bar{q})$ states. From this standpoint, it follows that the quark-model counterparts of the σ-π exchange diagram are of the type shown in Fig. 2. The two-body meson exchange operator $A_\lambda^{(2)}(\sigma\pi)$ is now proportional to the weak transition amplitude $M_\lambda = \langle u(p') \times |A_\lambda^{(+)}|d(p)\rangle = \bar{u}(p')\gamma_\lambda\gamma_5 d(p)$, where u and d stand for the u and d quarks, respectively. In the nonrelativistic limit, the time-component $M_0 \cong \chi^+\vec{\sigma}_q\cdot(\vec{p}'+\vec{p})/2m_q\chi \cong 0$ and the space components $\vec{M}\cong x^+\vec{\sigma}_q\chi$, where $\frac{1}{2}\sigma_q$ is the quark spin and m_q is the quark mass. (We have assumed that $m_q=m_u=m_d\neq 0$.). Therefore, for beta-decay processes, $A_0^{(2)}(\sigma\pi)\cong 0$ and $\vec{A}^{(2)}(\sigma\pi)\neq 0$, in agreement with the results obtained in the above. On the other hand, if the σ meson is described as a $(q\bar{q})_{s=0,I=0}$ state, then the matrix element $\langle\pi^+|A_0^{(+)}|\sigma\rangle$ is proportional to $\langle(u\bar{d})_{s=0,I=1}|\vec{\sigma}_q|(d\bar{d})_{s=0,I=0}\rangle$, which is zero identically. Thus, in this case, $A_0^{(2)}(\sigma\pi)=0$, a stronger result than that of the other two methods mentioned above.

We would like to now comment on the experimental situation regarding MEC. As was shown in a recent paper[10] there is an intricate interplay between MEC and nuclear structure effects originating in the sensitivity of Γ_μ and Γ_β to the 2p-2h configurations in the $^{16}O(0^+)$ wave function. Thus, it would be premature to attempt to draw a definite conclusion on MEC in the $A=16$ system. Similarly, for $A=12$ nuclei, the difficulty due to the intricate interplay between MEC and nuclear structure effect would persist and, therefore, the present "good" agreement of the nucleon-only-impulse-approximation theory with experiment (see the second paper of Ref. 2) must not be considered as being final.

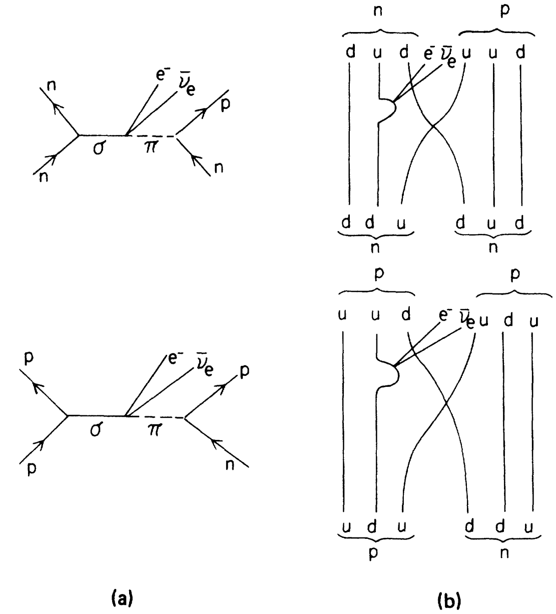

FIG. 2. The σ-π exchange diagrams in the two-nucleon beta decay as viewed from (a) the traditional standpoint and (b) the quark standpoint.

To conclude, we may summarize our results as follows.

(1) On symmetry grounds and by a systematic analysis of ME diagrams, we have gained a deeper insight into the structure aspects of $A_0^{(2)}$. From three independent considerations we have shown that MEC due to $A_0^{(2)}(\sigma\pi)$ is negligible in nuclear β decays and μ captures. This result is rather independent of the details of dynamics. Also, from the same quark model consideration in which the σ meson is assumed to be a $(q^2\bar{q}^2)$ system, it follows that the σ-meson exchange in nuclear force must necessarily be of a short-range character, since it inevitably involves the exchange of four quarks.

(2) We stress again the point that the question of MEC in nuclear weak processes is of fundamental importance and therefore deserves further detailed and careful investigations, in spite of our less optimistic comment that experimental evidence regarding the contribution of MEC is rather difficult to obtain at the present, even in nuclei

such as the $A=12$ and $A=16$ systems. We suggest that nuclear structure studies of these systems which include short-range correlation and/or core polarization would be extremely useful; e.g., separate treatments of Γ_μ and Γ_β and study of photopion production on ^{16}O might provide more experimental data for comparison with theoretical results.

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[6]The exchange diagrams of Figs. 1(f) and 1(g) are, respectively, referred to as the $\rho-\pi$ and $\sigma-\pi$ exchange diagrams; the corresponding two-body axial current operators are referred to by $A^{(2)}(\rho\pi)$ and $A^{(2)}(\sigma\pi)$.

[7]The expression for $A_0^{(2)}(\rho\pi)$ was first derived by one of the present authors (W. K. C.) in 1966 (see Ref. 1, first paper).

[8]We may use Goldberger-Treiman type relations to relate the weak form factor $f_A(\rho\pi)[f_P(\sigma\pi)]$ to the strong coupling constants $f_{\pi NN}$ and $f_{\rho\pi\pi}$ $(g_{\sigma\pi\pi})$ and the neutron β decay axial constant $f_A(np)\cong 1.25$.

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