Gamma-neutrino angular correlations in muon capture by ^{28}Si. II

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We report here the results of the numerical calculations of the gamma-neutrino angular correlation coefficients in the capture of polarized muons by the ^{28}Si nucleus. Closed expressions for these coefficients are derived taking into account the relativistic terms in the muon capture Hamiltonian, in the impulse approximation. Relations among these coefficients, the average and the longitudinal polarization of the recoil nucleus in muon capture, are derived and are shown to be independent of nuclear models and muon-capture coupling constants. By comparing with experimental data, we obtain a value for the induced pseudoscalar coupling constant g_P, as $(13.5 + 3.5, - 5.5)g_A$, indicating a possible absence of quenching of g_P in the ^{28}Si nucleus. The effect of meson exchange currents on these correlation coefficients are discussed.

[NUCLEAR REACTIONS ^{28}Si$(\mu^-, \gamma\nu_\mu)^{28}$Al, γ-neutrino angular correlation, polarized muon capture, particle-hole model, induced pseudoscalar, second class current, induced tensor coupling constants, meson exchange corrections.]

I. INTRODUCTION

In an earlier paper[1] (hereafter referred to as I) we considered a simple formalism based on the density matrix methods to study the γ – neutrino angular correlations in the capture of unpolarized muons by the ^{28}Si nucleus, namely,

$$\mu^- + {}^{28}\mathrm{Si}(0^+) \to {}^{28}\mathrm{Al}^*(1^+; 2202\ \mathrm{keV}) + \nu_\mu\,,$$
$$^{28}\mathrm{Al}^*(1^+; 2202\ \mathrm{keV}) \to {}^{28}\mathrm{Al}(0^+; 973\ \mathrm{keV}) + \gamma\,. \quad (1)$$

It is the purpose of this paper to extend the same formalism as in I, to the capture of polarized muons, in the above process. Now the angular distribution of γ rays with respect to the neutrino direction, can be given in the following form so as to make an easy comparison with the measurement of the William and Mary group[2]:

$$I(\theta_{\gamma\nu}) = I(0)[1 + \alpha P_2(\cos\theta_{\gamma\nu}) + \beta_1(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})P_2(\cos\theta_{\gamma\nu}) + \beta_2(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})]\,, \quad (2)$$

where $\vec{P}$ is the polarization of the muon in the atomic K orbit at the instant of capture[3] ($|\vec{P}| \sim 16\%$ in ^{28}Si), $\hat{\gamma}$ and $\hat{\nu}$ are the unit vectors along photon and neutrino momenta, respectively. It is clear from Eq. (2) that β_1 and β_2 arise due to the muon polarization and they involve different angular dependence from that of α in I. These are the coefficients of our present interest, for the following reasons: (i) The correlation coefficient α, studied in I, provided a wide range for g_P when compared with the experiment,[2] due to the large experimental uncertainties. As β_2 has been measured by the same group rather accurately, we hope to obtain a closer range for g_P which will be useful in discussing the possible existence of the second class axial currents within the impulse approximation. (ii) We wish to examine possible relations among α, β_1, β_2, and other observables in nuclear muon capture, similar to that of α and P_L in I. (iii) The disagreement between the earlier calculations of Ciechanowicz[4] and the experiment[2] for β_1 and β_2 necessitates further analysis. (iv) In view of the importance of meson exchange corrections to the impulse approximation in muon capture,[5] we wish to study the effect of these corrections on α, β_1, and β_2.

The theory of γ-neutrino angular correlation in polarized muon capture has been considered in a series of papers by Popov *et al.*[6] and Oziewicz and Pikulskii,[7] using the multipole expansion of the weak hadronic currents in close analogy with the orbital electron capture and by Devanathan and Subramanian[8] using the density matrix methods. We[1] have considered a simple approach to the γ-neutrino angular correlation in an allowed Gamow-Teller muon capture transition followed by $M1$ gamma decay. The choice of a simple Hamiltonian for the $M1$ gamma decay part,[9] greatly simplifies the treatment and is particularly useful for our present purpose. Recently Kobayashi, Ohtsuka, Ohtsubo, and Morita[10] applied the formalism based on the spherical representation of the effective muon capture Hamiltonian developed by Morita[11] and Ohtsubo, Kume, and Ohtsuka[12] to the calculation ^{12}B$(1^+; g\cdot s)$ average recoil polarization, but so far they have not considered the γ-neutrino angular correlations. In this paper, we extend our formalism in I. It consists in using the effective muon capture Hamiltonian of Fujii and Primakoff[13] to construct the density matrix of the final nucleus in its spin space preserving the angular identity of the neutrino, using the operator for γ emission as $\vec{j}_N\cdot\vec{A}_p$ following Rose,[9] where $\vec{j}_N$ is the nucleon current and $\vec{A}_p$ is the vector potential of the γ ray of circular polarization $p(=\pm 1)$, to construct the density matrix of the final nucleus after gamma decay, using the impulse approximation and summing over $p(=\pm 1)$. The details of these steps are given in Sec. II and the relevant expressions in Appendix A. As discussed in I, we use the particle-hole wave functions of Donnelly and Walker[14] to describe $^{28}Al^{*}(1^{+}; 2202$ keV) as this can be considered to be the isobaric analog of $^{28}Si(1^{+}; 13.67$ MeV). These particle-hole wave functions are obtained by diagonalizing the nuclear Hamiltonian in the 2 $\hbar\omega$ shell model space and the residual interaction is described by the Serber-Yukawa force. These wave functions satisfactorily reproduce the inelastic electron scattering data.[14] The closed expressions for β_1 and β_2 are given in the Appendix B. In Sec. III we give the relations among α, β_1, β_2, P_N, and P_L and obtain bounds on the numerical values of β_1 and β_2. Numerical results for β_1, β_2, and α (for comparison) for the process (1) are presented in Sec. IV along with the comparison with the experiment.[2] The effect of the meson exchange currents on α, β_1, and β_2 are discussed in the same section.

We briefly summarize our main results as follows: (i) β_2 is found to be nuclear model insensitive to a large extent while β_1 is sensitive to the nuclear wave functions. (ii) Using the experimental[2] value of β_2, we obtain $g_P=(13.5+3.5, -5.5)$ g_A, a value to a large extent free from nuclear wave function uncertainties. (iii) With a partially conserved axial-vector current (PCAC) estimate of g_P, our results indicate $g_T=(6+3.5, -5.5)g_A$. (iv) With canonical values of muon capture coupling constants, β_1 and β_2 are found to be in satisfactory agreement with the measurement of the William and Mary group,[2] indicating the absence of quenching of g_p in the ^{28}Si nucleus. (v) The effect of the meson exchange corrections on α, β_1, and β_2 are found to be small.

Throughout this paper we use the $\hbar=m_\mu=c=1$ system of units and the notation of Rose[15] for angular momentum coefficients.

II. γ-NEUTRINO ANGULAR CORRELATION COEFFICIENTS IN POLARIZED MUON CAPTURE

Consider the nuclear transition

$$|J_i M_i\rangle \xrightarrow{\mu^-} |J_f M_f\rangle \xrightarrow{\gamma} |J_F M_F\rangle \qquad (3)$$

with the initial nuclear state unpolarized and with the muon of polarization $\vec{P}$. Let us confine ourselves to the Gamow-Teller muon capture transition followed by $M1$ γ decay, relevant to our purpose of discussing the process (1). The construction of the density matrix ρ_f for the intermediate nucleus $|J_f M_f\rangle$ in its $(2J_f+1)$-dimensional spin space, in polarized muon capture has been discussed in general by Devanathan, Parthasarathy, and Ramachandran[16] [their Eq. (15)] and in particular for a $0^+ \to 1^+$ transition by Devanathan, Parthasarathy, and Subramanian[17] [their Eq. (11)]. As we are now interested in the γ-neutrino angular distribution, the integration over neutrino directions should not be carried out, thus the expression for ρ_f will be different from that given in Ref. 17. The expression for the matrix element of ρ_f in the case of unpolarized muon capture has already been given by Eq. (2) of I and we give here only the additional terms due to the muon polarization, in the Appendix A. It is to be noted that these additional terms involve the spherical harmonics $y_J^{M_J}(\hat{\nu})(\vec{P}\cdot\hat{\nu})$ and $[y_{\mathcal{L}}(\hat{\nu})\times y_1(\hat{P})]_J^{M_J}$ which are different from the simple $y_J^{M_J}(\hat{\nu})$ of Eq. (2) of I. Thus Eq. (A1) and Eq. (2) of I define the matrix element of ρ_f in the polarized muon capture, appropriate to our purpose of discussing the process (1).

Denoting this density matrix element by $(\rho_f)_{M_f M_f'}$, that of $|J_F M_F\rangle$ in Eq. (3) after the γ decay can be obtained by using Eqs. (5) and (6) of I. The details of this part are contained in I. However, owing to the additional spherical harmonics in Eq. (A1), it seems to be convenient to use the following kinematics: γ direction is chosen to be the z axis and an integration over the unphysical azimuth angle ϕ_ν of the neutrino is carried out using Eqs. (A2) and (A3). The resulting angular distribution of γ rays with respect to neutrino direction is given in Eq. (A4), relevant for our present purpose. From Eq. (A4), it can be seen that when the summation over J is carried out, the terms with $J=0$ and which do not contain $\vec{P}$, become independent of the angle $\theta_{\gamma\nu}$. These terms with appropriate muon capture coupling constants define $I(0)$ and are factored out. The terms with $J=1$ do not contribute due to the property of the parity Clebsch-Gordan coefficients. The terms with $J=2$ and without the muon polarization yield the $P_2(\cos\theta_{\gamma\nu})$ angular dependence discussed in I. The terms with $J=2$ and with muon polarization yield $(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})P_2(\cos\theta_{\gamma\nu})$ and $(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})$, with the coefficients defining β_1 and β_2, respectively, in Eq. (2).[18] The explicit expressions for β_1 and β_2 are given in Eqs. (B1) and (B2), respectively. These constitute one of our main results. It is to be noted that these are derived including the relativistic terms in the Fujii-Primakoff Hamiltonian and taking into account higher order partial waves of the neutrino.

In order to obtain an idea of the dependence of β_1 and β_2 on the muon capture coupling constants without going through the details of the numerical computation (which are carried out in Sec. IV), we shall examine them in the Fujii-Primakoff approximation by neglecting the relativistic terms and confining our study to S-wave neutrinos only. In this scheme the nuclear reduced matrix elements are cancelled and we obtain

$$\beta_1 = G_P^2/(3G_A^2 + G_P^2 - 2G_P G_A)\,, \tag{4}$$

$$\beta_2 = (3G_A^2 - G_P^2)/(3G_A^2 + G_P^2 - 2G_P G_A)\,. \tag{5}$$

From these equations it is clear that β_1 will be more sensitive to g_P than β_2 and $\beta_1 = 0$; $\beta_2 = 1$ for a pure Gamow-Teller transition, neglecting the induced interactions.

III. RELATIONS AMONG $\alpha, \beta_1, \beta_2, P_N$, AND P_L

In I, we derived a simple relation between α and P_L and discussed its usefulness. In this section, we shall attempt similar relations for the additional correlation coefficients β_1 and β_2. We start first by examining the expression for β_1, β_2, P_N, and P_L under the Fujii-Primakoff approximation. Using Eqs. (10) and (11) of I for α and P_L, respectively, Eqs. (4) and (5) of the previous section for β_1 and β_2, and Eq. (37) of Ref. 17 for P_N, we obtain

$$\begin{aligned} &\beta_1 = 1 - \tfrac{3}{2}P_N/P\,, \\ &\beta_2 = -1 + \tfrac{3}{2}P_N/P - \tfrac{3}{2}P_L\,, \\ &\beta_1 + \beta_2 = 1 + \alpha\,. \end{aligned} \tag{6}$$

We have examined the validity of these relations taking into account the relativistic terms in the Fujii-Primakoff Hamiltonian and higher order partial waves of the neutrino. This procedure is straightforward and it consists in using Eqs. (B1) and (B2) for β_1 and β_2, Eq. (9) of I for α, Ref. 17 for P_N, and Ref. 19 for P_L, with the standard angular momentum algebra. The relations in Eq. (6) are independent of nuclear models and muon capture couplings. It seems that these can be derived on more general grounds by using helicity formalism and the rotational invariance of the problem, as shown by Bernabeu.[20] We wish to point out that we have obtained these independently by starting from the explicit muon capture Hamiltonian, deriving the complete expressions for β_1 and β_2, and comparing them with those for α, P_N, and P_L.

A few remarks about the relations in Eq. (6) are now in order. The first relation in Eq. (6) provides an estimate of the average recoil nuclear polarization of the intermediate nucleus in the sequence (3). For the process (1), using the measured[2] value of β_1, the $^{28}Al^*(1^+; 2202$ keV) average recoil polarization turns out to be ~ 0.6533, which can be verified by an independent measurement. The second relation in Eq. (6) and the first, yield

$$P_L = -\tfrac{2}{3}(\beta_1 + \beta_2)\,, \tag{7}$$

which gives for a pure Gamow-Teller transition ($\beta_1 = 0; \beta_2 = 1$), $P_L = -\frac{2}{3}$. However, upon using the measured[2] values of β_1 and β_2, we find the longitudinal polarization of $^{28}Al^*(1+; 2202$ keV) to be $\sim -(0.7599 \pm 0.085)$, which when compared to the ideal value $-\frac{2}{3}$, indicates that the strong interaction induced effects could enhance $|P_L|$ by about 15%. The third relation in Eq. (6) shows that among the three correlation coefficients only two are linearly independent. This relation is found to be satisfied by the experimental values of α, β_1, and β_2 by the William and Mary group[2] within the quoted experimental uncertainties. Further the relations in Eq. (6) provide bounds on β_1 and β_2. Using the bounds on P_L (0 and -1) by Bernabeu[21] on the basis of time reversal invariance and the bounds on P_N ($-\frac{1}{3}$ and $\frac{2}{3}$) by Rao *et al.*,[22] we obtain

$$0 < \beta_1 < 1.5\,, \quad -1.5 < \beta_2 < 1.5\,. \tag{8}$$

Similar bounds have been obtained by Oziewicz.[23] The measured values[2] of β_1 and β_2 for the process (1) satisfy these bounds. Now we proceed to give the numerical results of β_1 and β_2, and study their dependence on the induced pseudoscalar coupling constant.

IV. NUMERICAL RESULTS AND DISCUSSION

Equations (B1) and (B2) give the γ-neutrino angular correlation coefficients β_1 and β_2, respectively, taking into account the relativistic terms and higher order partial waves of the neutrino. They involve nuclear reduced matrix elements which are evaluated using the particle-hole model of Donnelly and Walker,[14] the details of which were discussed in I. We use the following numerical values for the muon capture coupling constants $g_V(0) = 0.987G$; $g_A(0) = -1.25 g_V(0)$, $g_M(0) = 3.7 g_V(0)$, $g_S(0) = 0$, and $G = 1.02 \times 10^{-5}/M^2$, where M is the proton mass. Since g_P and g_T occur in the linear combination $(g_P + g_T)$, and since g_T could possibly exist in muon capture although recent experiments[24] show its absence in beta decay, we study the behavior of β_1 and β_2 on $(g_P + g_T)$. The effects of meson exchange corrections are also studied and it is found that these corrections are negligible in α, β_1, and β_2. The results will be discussed at the end of this section.

The numerical results of β_1 and β_2 are given in Table I along with those of α (Ref. 25) for comparison. In this table, models I and II correspond to the results without and with the relativistic terms. It is clear from the results that these

TABLE I. Numerical values of γ-neutrino angular correlation coefficients α, β_1, and β_2 defined in Eq. (3). It is to be noted that the relation $1+\alpha=\beta_1+\beta_2$ is satisfied for the complete set of numerical values, almost exactly showing the correctness of the numerical computation of these coefficients. Models I and II are without and with nucleon-momentum dependent terms.

	α		β_1		β_2	
$(g_P+g_T)/g_A$	Model I	Model II	Model I	Model II	Model I	Model II
−10.0	−0.077 99	0.024 41	0.004 42	0.005 81	0.917 59	1.018 50
−7.5	−0.020 30	0.084 65	0.000 32	0.009 62	0.979 38	1.075 00
−5.0	−0.039 34	0.145 66	0.001 19	0.019 69	1.038 10	1.125 90
−2.5	0.100 26	0.206 45	0.007 99	0.036 94	1.092 20	1.169 50
0	0.161 60	0.265 81	0.021 70	0.062 23	1.139 80	1.203 50
2.5	0.222 21	0.322 31	0.043 25	0.096 29	1.178 90	1.226 00
5.0	0.280 76	0.374 35	0.073 49	0.139 63	1.207 20	1.234 70
7.5	0.335 69	0.420 26	0.113 09	0.192 43	1.224 60	1.227 80
10.0	0.385 31	0.458 39	0.162 43	0.254 49	1.222 80	1.203 80
12.5	0.427 89	0.487 21	0.221 50	0.325 14	1.206 30	1.162 00
15.0	0.461 78	0.505 52	0.289 86	0.403 27	1.171 90	1.102 20
17.5	0.485 53	0.512 48	0.366 56	0.487 35	1.118 90	1.025 10
20.0	0.498 10	0.507 80	0.450 18	0.575 51	1.047 90	0.932 28
22.5	0.498 90	0.491 66	0.538 93	0.665 69	0.959 97	0.825 97
25.0	0.487 89	0.464 82	0.630 69	0.755 81	0.857 19	0.709 00
27.5	0.465 58	0.428 40	0.723 29	0.843 89	0.742 29	0.584 51
30.0	0.432 98	0.383 89	0.814 56	0.928 21	0.618 42	0.455 67

terms significantly contribute to α and β_1 but not so much to β_2. For example, with $g_P=7.5g_A$ (PCAC estimate) and $g_T=0$, these correction terms enhance α and β_1 by 25% and 70%, respectively. One of the purposes of displaying the numerical values of α, β_1, and β_2 in the form of a table is to exhibit the fact that these values satisfy almost exactly the relation $1+\alpha=\beta_1+\beta_2$ for a wide range of values of (g_P+g_T) indicating the correctness of the numerical computations.

In Table II, we present the numerical values of α, β_1, and β_2 for the PCAC estimate of g_P with $g_T=0$, in the Fujii-Primakoff approximation (FPA)—models I and II along with the earlier theoretical results[4] and the measured values.[2] It is seen that our results are in reasonably good agreement with the experiment. The disagreement between earlier calculation[4] and the experiment is to be noted. In order to analyze the dependence of correlation coefficients on nuclear models we proceed as follows. Firstly, in FPA one neglects relativistic terms and considers only the S-wave neutrino and the coefficients are independent of nuclear models. Model I is obtained by including higher order partial waves for the neutrino but neglecting relativistic terms, while model II includes both and hence is complete. From Table II, a comparison between the results of α, β_1, and β_2 in FPA and model I shows that the higher order partial waves of the neutrino contribute significantly to α and β_1 but not so much to β_2. Secondly, a comparison between the results in models I and II reveals the fact that the relativistic terms contribute significantly to α and β_1 but not so much to

TABLE II. Comparison of α, β_1, and β_2 in FPA, other theoretical estimates, in models I and II (see Table I for captions) along with experimental data for the PCAC estimate of g_P with $g_T=0$.

γ-neutrino angular correlation coefficients	Ciechanawicz's values as given by Mukhopadhyay (Ref. 20)	FPA	Model I	Model II	Experiment of Miller *et al.* (Ref. 2)
α	0.4	0.2925	0.3357	0.4203	0.15 ± 0.25 0.29 ± 0.30
β_1	0.88	0.0809	0.11309	0.19243	0.02 ± 0.03
β_2	0.53	1.2115	1.2246	1.2278	1.12 ± 0.10

β_2. Thirdly, comparing the results in FPA and model II, we observe that α and β_1 are very sensitive to the nuclear wave functions while β_2 is relatively insensitive. We have also calculated these coefficients in the independent particle model for the nucleus and found that β_2 is nuclear model insensitive, while α and β_1 depend very much on them. This contradicts the conclusion of Popov *et al.*[6] who have claimed that all the coefficients are nearly nuclear model insensitive. Our analysis shows that β_2, being free from nuclear wave function uncertainties to a large extent, can be used to obtain a value for g_P as in the case of recoil nuclear polarization.[17,29] That is, there exists another observable in nuclear muon capture which can provide reliable information about g_P. We have also investigated the dependence of β_2 on g_M and found that it is very insensitive to g_M over a wide range of values for g_M. It is rather fortunate that this important observable, namely β_2, involves fewer experimental uncertainties than α and β_1 in the measurement[2] for process (1). Before proceeding to obtain a value for (g_P+g_T) in the impulse approximation, we shall examine the meson exchange effects briefly. Kubodera, Delorme, and Rho[26] have shown for axial current that the time component of the meson exchange amplitude goes like $O(1)$ while the space component like $O(p/M)$, to be compared with $O(p/M)$ and $O(1)$, respectively, for the single particle operator (in impulse approximation) where p is nucleon momentum. Thus for muon capture transitions dominated by the space part of the axial current in impulse approximation [allowed Gamow-Teller transition such as the process (1)], the meson exchange current contribution will be $O(p/M)$, while for muon capture transitions dominated by the time part of the axial current in impulse approximation (forbidden transition such as $0^+\rightarrow 0^-$ in ^{16}O), the meson exchange current contribution will be $O(1)$ and hence significant. The importance of such exchange corrections to the $0^+\rightarrow 0^-$ transition in ^{16}O has been studied by Guichon *et al.*[5] However, detailed calculations including core deformation of ^{16}O by Guichon and Samour[27] and Koshigiri, Ohtsubo, and Morita[28] indicate that the exchange currents are not found to be large. It is obvious from the above arguments that the exchange corrections to α, β_1, and β_2 in process (1), an allowed Gamow-Teller transition, will be qualitatively small. In order to obtain quantitative results, we include the time part of the mesonic exchange amplitude as a correction to the impulse approximation. This will enhance the time part of single particle axial current operator by $z\langle A_0\rangle$, where $\langle A_0\rangle$ is the matrix element of the time part of single particle axial current after nonrelativistic reduction and z is defined in Ref. 27. The numerical value of z seems to be uniform and qualitatively independent of the nucleus.[27,26] In order to obtain a quantitative estimate, we have chosen $z=1.2$ and 1.5, namely 20% and 50% meson exchange corrections to $\langle A_0\rangle$. For a representative value of $g_P=7.5g_A$ and $g_T=0$, β_2 turns out to be 1.2319 and 1.2394 to be compared with the impulse approximation ($z=1$) value 1.2278, showing that the exchange effects are very small. Similar numbers for α are 0.4341 and 0.4562 to be compared with 0.4203, the impulse approximation value, and for β_1 are 0.2022 and 0.2167 to be compared with the impulse approximation value 0.1924. These features are found for a wide range of values of (g_P+g_T), indicating that the meson exchange corrections to impulse approximation values of α, β_1, and β_2 are very small.

By comparing our calculations for β_2, which is measured[2] more accurately than α and β_1, we obtain

$$(g_P+g_T)=(13.5+3.5,\ -5.5)g_A\,,$$

a closer range for (g_P+g_T) than in I as aimed for in the Introduction. This value is to a large extent free from nuclear wave function uncertainties and the meson exchange corrections do not change the value much. This is in agreement with our analysis of ^{12}B(1^+; $g\cdot s$) recoil nuclear polarization[29] and with that of Kobayashi *et al.*[10] and in contradiction with that of Ciechanowicz[4] who finds $-4.9<(g_P+g_T)/g_A<1.2$. Our estimate, besides satisfying the PCAC value, indicates a remote possibility of the quenching of g_P in ^{28}Si. Using $g_P=7.5g_A$ we find $g_T=(6+3.5,\ -5.5)g_A$, in agreement with Kubodera *et al.*[30] and Parthasarathy and Waghmare.[31]

We summarize our results below:

(i) The correlation coefficient β_1 is found to be nuclear model sensitive similar to α in I, while β_2 is to a large extent free from nuclear wave function uncertainties.

(ii) The correlation coefficients are not affected much by the mesonic exchange corrections through the time part of the axial current.

(iii) A comparison of β_2 with experiment[2] yields $(g_P+g_T)=(13.5+3.5,\ -5.5)g_A$, in agreement with our earlier calculations.

(iv) With $g_P=7.5g_A$, $g_T=(6+3.5,\ -5.5)g_A$, agreeing with Refs. 30 and 31.

(v) The relations among α, β_1, and β_2 in Sec. III are independent of nuclear models and muon capture coupling constants. These provide estimates of P_N and P_L of ^{28}Al*(1^+; 2202 keV) which can be verified by future measurements.

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APPENDIX A

The additional terms in $(\rho_f)_{M_f M_f'}$ due to the muon polarization are

$$\sum_J \Bigg(Y_J^{M_J}(\hat{\nu})(\vec{\mathrm{p}}\cdot\hat{\nu}) \Bigg\{ \sum_{ll'} (i)^{l'-l}[l][l']\,g(l1J_f;l'1J_f)\big[G_A{}^2(-1)^{l'-J_f}C(ll'J;00)W(J_f1Jl';lJ_f) - G_P{}^2[J_f]^{-2}C(11J_f;00)C(l'1J_f;00)C(J_fJ_fJ;00)\big]$$

$$-2G_A\frac{g_V}{M}\sum_{ll'\lambda}\sqrt{2}(i)^{-l+l'+3}[l][l'][1][\lambda]C(l'1\lambda;00)C(l\lambda J;00)W(J_f1\lambda1;l'1)W(J_f\lambda J_fl;1J)$$

$$\times \mathcal{G}(l1J_f;l'1J_f0J_f) - 2G_P\frac{g_A}{M}\sum_{l\lambda}(i)^{-l+J_f+1}(-1)^{\lambda-J_f}[l][\lambda][J_f]^{-2}C(J_fJ_fJ;00)$$

$$\times C(l1J_f;00)\mathcal{G}(l1J_f;J_f1\lambda1J_f)\Bigg\}$$

$$+\sum_{\mathcal{L}}[Y_{\mathcal{L}}(\hat{\nu})\times Y_1(\hat{P})]_J^{M_J}[J][J_f]^{-1}(\tfrac{4}{3}\pi)^{1/2}\Bigg[2(G_P-G_A)G_A\sum_{ll'}(i)^{l'-l}[l][l']C(l1J_f;00)C(J_fl'\mathcal{L};00)$$

$$\times W(J_fl'J1;\mathcal{L}J_f)\,g(l1J_f;l'1J_f)$$

$$+2G_A\frac{g_A}{M}\sum_{l\lambda}(i)^{-l+J_f+1}[l][\lambda]C(lJ_f\mathcal{L};00)W(J_f1J_f\mathcal{L};lJ)g(l1J_f;J_f1\lambda1J_f)\Bigg]\Bigg)$$

$$\times(-1)^{M_f}\frac{[J_f]^2}{(4\pi)^{1/2}[J]}C(J_fJ_fJ;\,-M_fM_f'M_J)\,, \tag{A1}$$

where $(l1J_f;l'1J_f)$ and $\mathcal{G}(l1J_f;J_f1\lambda1J_f)$ are defined by Eqs. (3) and (4), respectively, in I.

Relations[8] used to combine spherical harmonics in Eq. (A1) and with that in Eq. (6) of I so as to reduce the form to Eq. (3) are

$$\int_0^{2\pi}(\vec{\mathrm{p}}\cdot\hat{\nu})P_J(\cos\theta_{\gamma\nu})d\phi_\nu = 2\pi\sum_L C(J1L;00)^2(\vec{\mathrm{p}}\cdot\hat{\gamma})P_L(\cos\theta_{\gamma\nu}), \tag{A2}$$

$$\int_0^{2\pi}(4\pi/3)^{1/2}Y_L{}^0(\hat{\gamma})[Y_{\mathcal{L}}(\hat{\nu})xY_1(\hat{p})]_L^0d\phi_\nu = \frac{[L][\ \ ]}{2}C(\mathcal{L}1L;00)(\vec{\mathrm{P}}\cdot\hat{\gamma})P_{\mathcal{L}}(\cos\theta_{\gamma\nu}). \tag{A3}$$

The complete expression for $I(\theta_{\gamma\nu})$using Eqs. (A1)–(A3) and Eqs. (2) and (6) of I, for process (1) is

$$I(\theta_{\gamma\nu}) = \frac{1}{6\pi}|a(M_1)|^2|\langle 0^+\|M1\|1^+\rangle|^2$$

$$\times\sum_J\Bigg(\sum_{ll'}(i)^{l'-l}[l][l']\,g(l11;l'11)$$

$$\times\Bigg\{P_J(\cos\theta_{\gamma\nu})(3G_A{}^2(-1)^{l'-1}C(ll'J;00)W(11Jl';l1)+(G_P{}^2-2G_PG_A)C(l11;00)C(l'11;00)$$

$$\times C(11J;00)+(\vec{\mathrm{P}}\cdot\hat{\gamma})[C(J1J+1;00)^2P_{J+1}(\cos\theta_{\gamma\nu})+\eta_JC(J1J-1;00)^2P_{J-1}(\cos\theta_{\gamma\nu})]$$

$$\times(3G_A{}^2(-1)^{l'-1}C(ll'J;00)W(11Jl';l1)-G_P{}^2C(l11;00)C(l'11;00)C(11J;00)$$

$$-2(G_A-G_P)G_A\sum_{\mathcal{L}}[1][\mathcal{L}]C(l11;00)C(l'1\mathcal{L};00)W(\mathcal{L}111;l'J)C(\mathcal{L}1J;00)P_{\mathcal{L}}(\cos\theta_{\gamma\nu})(\vec{\mathrm{P}}\cdot\hat{\gamma})\Bigg\}$$

$$+\sum_{l\lambda}(i)^{l-2}(-1)^{\lambda-1}[l][\lambda]C(l11;00)C(11J;00)\mathcal{G}(l11;11\lambda11)$$

$$\times\Big\{2(G_P-G_A)\frac{g_A}{M}P_J(\cos\theta_{\gamma\nu})-2G_P\frac{g_A}{M}[C(J1J+1;00)^2P_{J+1}(\cos\theta_{\gamma\nu})]$$

$$+\eta_J C(J1J-1;00)^2P_{J-1}(\cos\theta_{\gamma\nu})\Big\}(\vec{\mathrm{P}}\cdot\hat{\gamma})$$

$$+2G_A\frac{g_\nu}{M}\sum_{ll'\lambda}\sqrt{2}\,(i)^{l'-l+3}[l'][1]^3[l][\lambda]C(l'1\lambda;00)C(l\lambda J;00)W(11\lambda1;l'1)W(1\lambda1l;1J)$$

$$\times\mathcal{G}(l11;l'1101)\{P_J(\cos\theta_{\gamma\nu})+[C(J1J+1;00)^2P_{J+1}(\cos\theta_{\gamma\nu})+\eta_J C(J1J-1;00)^2$$

$$\times P_{J-1}(\cos\theta_{\gamma\nu})](\vec{\mathrm{P}}\cdot\hat{\gamma})\}$$

$$+2G_A\frac{g_A}{M}\sum_{l'\lambda\mathcal{L}}(i)^{l'-2}(-1)^{\lambda-\mathcal{L}}[\lambda][l'][1][\quad]C(dl^1\mathcal{L};00)W(1l'J1;1\mathcal{L})C(1\mathcal{L}J;00)\mathcal{G}(l'11;11\lambda11)$$

$$\times P_{\mathcal{L}}(\cos\theta_{\gamma\nu})(\vec{\mathrm{P}}\cdot\hat{\gamma})\Big)C(11J;1-1)\,, \qquad \text{(A4)}$$

where $\eta_J=1$ for $J>0$ and 0 for $J=0$.

APPENDIX B

The closed expression for β_1 in Eq. (2) is

$$\beta_1=\frac{1}{\sqrt{6}}\Big\{\sum_{ll'}(i)^{l'-l}[l][l']\mathcal{G}(l11;l'11)[3G_A^2(-1)^{l'-l}C(ll'2;00)W(112l';l1)-G_P^2(\tfrac{2}{3})^{1/2}C(l11;00)C(l'11;00)$$

$$+10(G_A-G_P)G_AC(l11;00)C(l'13;00)W(3111;l'2)]$$

$$+\sum_{l\lambda}(i)^{l-2}[l][\lambda]\mathcal{G}(l11;11\lambda11)\Big[2G_P\frac{g_A}{M}(-1)^\lambda(\tfrac{2}{3})^{1/2}C(l11;00)+10G_A\frac{g_A}{M}(-1)^\lambda C(1l3;00)W(1l21;13)\Big]$$

$$+2G_A\frac{g_\nu}{M}\sum_{ll'\lambda}\sqrt{2}\,(i)^{l'-l+3}[l][l'][\lambda][1]^3C(l1\lambda;00)W(1\lambda1l';12)W(11\lambda1;l1)\mathcal{G}(l'11;l1101)\Big\}\Big/\mathcal{D}\,,$$

where

$$\mathcal{D}=\sum_{ll'}(i)^{l'-l}\mathcal{G}(l11;l'11)\Big[-G_A^2\delta_{ll'}-(G_P^2-2G_PG_A)\frac{[l][l']}{3}C(l11;00)C(l'11;00)\Big]$$

$$-2(G_P-G_A)\frac{g_A}{M}\sum_{l\lambda}(i)^{l-2}(-1)^{\lambda-1}\frac{[l][\lambda]}{3}C(l11;00)\mathcal{G}(l11;11\lambda11)$$

$$+2G_A\frac{g_\nu}{M}\sum_{ll'}\sqrt{2}\,(i)^{l'-l+3}[l]\sqrt{3}\,C(l1l';00)W(11l'1;l1)\mathcal{G}(l'11;l1101)\,. \qquad \text{(B1)}$$

The closed expression for β_2 in Eq. (2) is

$$\beta_2=\Big[\sum_{ll'}(i)^{l'-l}\mathcal{G}(l11;l'11)\Big(-G_A^2\delta_{ll'}+\frac{[l][l']}{3}C(l11;00)C(l'11;00)$$

$$\times\Big\{G_P^2+2(G_A-G_P)G_A\Big[1-\frac{9}{\sqrt{6}}\Big(W(1111;l'2)(\tfrac{2}{3})^{1/2}+\frac{C(l'13;00)}{C(l'11;00)}W(3111;l'2)\tfrac{2}{3}\Big)\Big]\Big\}\Big)$$

$$+\sum_{l\lambda}(i)^{l-2}(-1)^{\lambda-1}\frac{[l][\lambda]}{3}C(l11;00)\mathcal{G}(l11;11\lambda11)$$

$$\times\Big(2G_P\frac{g_A}{M}-2G_A\frac{g_A}{M}\Big\{1-\frac{9}{\sqrt{6}}\Big[W(1l21;11)(\tfrac{2}{3})^{1/2}+\frac{C(1l3;00)}{C(l11;00)}W(1l21;13)\tfrac{2}{3}\Big]\Big\}\Big)$$

$$+6G_A\frac{g_\nu}{M}\sum_{ll'}\sqrt{2}\,(i)^{l'-l+3}[l]C(l1l';00)W(11l'1;l1)\mathcal{G}(l'11;l1101\Big]\Big/\mathcal{D}\,. \qquad \text{(B2)}$$

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