Neutron densities from K^+ meson scattering

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A theoretical study documenting the probing sensitivity of the K^+ meson to the neutron distribution is reported for ^{40}Ca and ^{48}Ca isotopes. Combining Hartree-Fock and phenomenological neutron densities with two independent empirical sets of kaon-nucleon amplitudes, different first-order microscopic kaon-nucleus optical potentials are constructed and compared. The sensitivity of predicted elastic cross sections to choice in potential is found to be significant and is dominated by the uncertainty in the K^+-neutron interaction. For relative cross section differences between Ca isotopes, however, the sensitivity to the neutron distribution is roughly comparable to the uncertainty introduced by the KN interaction, suggesting that preliminary K^+-nucleus measurements may provide improved neutron information about isotopic density differences. Conditions and prescriptions for extracting such information are discussed.

[NUCLEAR REACTIONS Kaon-nucleus elastic scattering. Calculated $\sigma(\theta,E)$ for ^{40}Ca and ^{48}Ca. First-order multiple scattering optical potential and impulse approximation. Hartree-Fock and empirical neutron densities. Neutron distribution sensitivity study.]

I. INTRODUCTION

It is somewhat surprising that almost fifty years after the discovery[1] that nuclei contain neutrons, precision neutron structure information is still lacking. To be sure, over this time period our knowledge of neutron distributions, form factors, transition densities, etc., has substantially grown, especially from recent results using high energy protons.[2,3] Quantitatively, however, it does not yet compare in accuracy and detail to corresponding proton information provided by electron scattering.[4]

Improved neutron structure information is of tantamount importance in understanding several unresolved topics which are fundamental to nuclear physics: state-of-the-art many-body calculations, the binding energy of nuclear matter, many-body forces, quark and meson internal degrees of freedom, nuclear compressibility and the existence of giant monopole resonances, neutron transition densities and particle-hole states, neutron skin effects, and the charge independence of the strong interaction. Even the Nolen-Schiffer anomaly,[5] which concerns Coulomb energy shifts and arises in part from reliance on less accurate Hartree-Fock neutron densities,[6] could presumably also be removed. Clearly, a long-standing documented need exists for obtaining significantly improved neutron information.

Progress in this direction has certainly been limited by the lack of a good neutron probe, one paralleling the electron's role in sampling the charge distribution. Nucleons, pions, and alpha particles do provide information. However, such results are somewhat model dependent as the reaction mechanism and strong absorption attending these projectiles precludes a clean separation of structure and scattering treatments. What is needed is a weakly interacting hadronic probe for which, like the electron, first-order theories are appropriate.

A logical candidate for a probe of neutron structure is, of course, the K^+ meson (kaon) which continues to attract increased attention and investigations.[7-19] The present paper continues this trend by studying the feasibility of extracting more accurate neutron densities using K^+-nucleus elastic scattering. This work extends the calculations of Ref. 8, which established the sensitivity of the K^+ to the nuclear matter distribution, by comparing cross sections computed with Hartree-Fock and phenomenological neutron densities. These comparitive calculations are then repeated utilizing different sets of KN amplitudes to meaningfully assess the sensitivity to the uncertainty in the KN interaction. Accordingly, parts of the investigation also complement previous surveys[7,8,12,14,18] of the sensitivity of K^+ nucleus scattering to the KN amplitudes.

The current study focuses not only on obtaining improved neutron information, but also on the possibility of deducing new, detailed features of the neutron density that would be difficult or impossible to obtain with other projectiles. In particular, two frequently raised, but important issues are addressed: (1) Can kaon scattering tell us more about the nuclear interior? (2) Is it possible to extract more than just two density parameters (i.e., a radius and a diffuseness) from a scattering analysis? The results presented below suggest an affirmative answer to both questions provided, however, that the K^+-neutron amplitude is more reliably known and that the first-order multiple scattering optical potential, on which this analysis is based, is sufficiently accurate.

This paper, of which portions have been previously reported,[17] is divided into four parts. Section II describes the theoretical aspects and establishes the framework that will be used to assess the neutron probing capability of the kaon. In Sec. III the results of several calculations are presented and scattering sensitivity is discussed. Finally, the conclusions are summarized in Sec. IV.

II. DETAILS OF THE ANALYSIS

At issue is the sensitivity of theoretical K^+-nucleus scattering calculations to uncertainties or, more precisely, variations in the target neutron density. As previously outlined in detail,[14] the elastic cross section was obtained by solving[20] the Klein-Gordon equation with a factorized, first-order multiple scattering optical potential U given by

$$U = At\rho. \tag{1}$$

Here ρ is the matter density, with unit norm, for A nucleons. In deriving Eq. (1) use has also been made of the impulse approximation in which the exact many-body kaon-nucleon (KN) transition amplitude is replaced by the free two-body amplitude t.

To properly assess scattering sensitivity to the neutron distribution the theoretical optical potential was decomposed into neutron and proton contributions involving separate densities ρ_n, ρ_p and transition amplitudes t_{Kn}, t_{Kp}, respectively. Equation (1) then takes the form

$$U = Zt_{Kp}\rho_p + Nt_{Kn}\rho_n, \tag{2}$$

which is appropriate for a nucleus having Z protons and N neutrons. In this study ρ_p was assumed to be known for each target and was therefore not varied. Using the results of electron scattering studies[4] of the charge distribution a parabolic Fermi shape was adopted entailing three parameters w, c, and a:

$$\rho_p(r) = \rho_0[1 + w(r^2/c^2)]/(1 + e^{(r-c)/a}). \tag{3}$$

Varying the neutron density was accomplished by using two distinctly different mathematical forms. One, an empirical choice,[3] was identical in form to Eq. (3). The other, a theoretical density, was constructed using Hartree-Fock wave functions. Roughly speaking, the difference between these two forms reflects the existing uncertainty in the neutron density which, among other goals, one would like to reduce.

The present work examines the isotopes ^{40}Ca and ^{48}Ca and focuses on the neutron isotopic differences. Table I lists the proton and neutron density parameters, used in Eq. (3), for each nucleus. To provide realistic scattering predictions, these parameters were taken from phenomenological electron[4] and high energy proton[3] elastic scattering analysis. The Hartree-Fock neutron density was calculated in a harmonic oscillator basis using the expansion coefficients provided by Negele's Hartree-Fock investigation.[21] Although more recent and improved structure calculations exist, Negele's wave functions remain representative and suffice for investigating the sensitivity of the K^+ as a probe.

The empirical and Hartree-Fock neutron densities differ both at the nuclear surface and in the nuclear interior. This is shown in Fig. 1 where the isotopic neutron density difference $\Delta\rho_n = \rho_n^{48} - \rho_n^{40}$ is plotted as a function of r. Table II gives the corresponding rms radii r_n^{40} and r_n^{48} and isotopic radius difference $\Delta_n = r_n^{48} - r_n^{40}$, which have been calculated from the empirical and theoretical densities.

Finally, to maintain the proper perspective, another important uncertainty besides the neutron density enters this calculation and is also examined. This is the KN interaction which, unfortunately, is not precisely known. To determine what effect this introduces, two different empirical sets of KN amplitudes were used: those measured and extracted by the BGRT group[22] and those provided by Martin[23] which are currently believed[24,25] to be more accurate. In the calculations reported below only s and p wave amplitudes, uncorrected for Fermi motion, are included. Table III gives the major s wave contribution from each set of amplitudes to the effective $K^+-{}^{40}$Ca proton and neutron optical model well depths at three different energies. Notice that the predominant uncertainty is in the K^+-neutron optical potential which stems from an imprecise knowl-

TABLE I. Phenomenological density parameters. The proton parameters c_p, a_p, and w_p were determined from electron scattering (Ref. 4). These same proton parameters were used in obtaining the neutron parameters c_n, a_n, and w_n by fitting high energy proton elastic scattering data (Ref. 3). All values, except w, are given in fm.

Nucleus	c_p	a_p	$w_p=w_n$	c_n	a_n
^{40}Ca	3.68	0.59	−0.102	3.60	0.59
^{48}Ca	3.74	0.53	−0.030	3.85	0.57

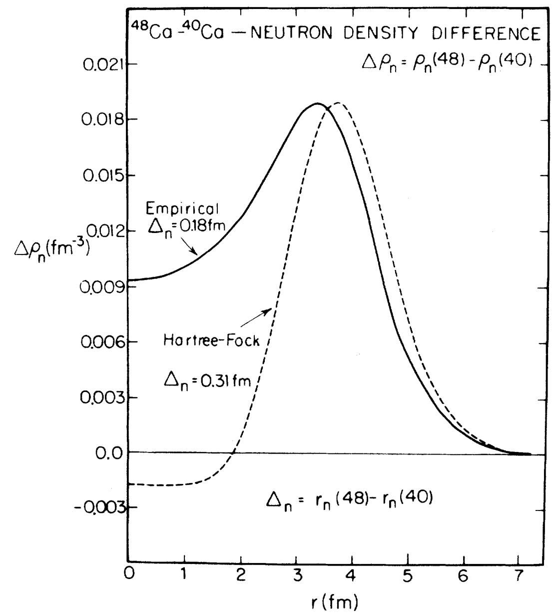

FIG. 1. Theoretical and phenomenologic neutron density differences between ^{48}Ca and ^{40}Ca.

edge of the K^+-neutron (isospin = 0) component of the elementary interaction. The K^+-proton (isospin = 1) part is much better known. It is of special interest to compare the effect of this uncertainty with that produced by varying the neutron density to determine the limits on extracting more accurate neutron structure information.

III. RESULTS

The major new results emerging from this sensitivity survey study are summarized in Figs. 2–10. The absolute effects from varying the *KN* interaction and neutron density are given in Figs. 2 and 3 for ^{40}Ca and ^{48}Ca targets, respectively. In these figures (and also Figs. 4–6) the solid curve represents the calculation with Martin amplitudes and empirical neutron densities. This curve should be compared to the dashed (dotted) line to assess the K^+ sensitivity to uncertainties in neutron density (*KN* interaction). An examination of Figs. 2 and 3 leads to the inescapable conclu-

TABLE II. Neutron rms radii in fm.

	^{40}Ca	^{48}Ca	Δ_n
Empirical	3.45	3.63	0.18
Hartree-Fock	3.37	3.68	0.31

sion that the uncertainty in the *KN* interaction dominates, implying that preliminary analyses of absolute K^+-nucleus cross sections should initially yield more information about the *KN* force that ρ_n.

It should be stressed again, however, that the Martin amplitudes are preferred over BGRT and that the true uncertainty in the *KN* interaction is in actuality not nearly as large as Table III and Figs. 1 and 2 suggest.[25] Nevertheless, assuming for the moment that the interaction uncertainty is this large, it is still possible to learn more about ρ_n, provided we calculate and measure the relative isotopic cross section difference $D(\theta)$

$$D(\theta)=\frac{\sigma^{40}(\theta)-\sigma^{48}(\theta)}{\sigma^{40}(\theta)+\sigma^{48}(\theta)}, \qquad (4)$$

which is commonly used in electron scattering. If the reaction mechanism and structure are weakly coupled, as for example in electron scattering, $D(\theta)$ is predominantly a function of target properties. Figures 4, 5, and 6 display $D(\theta)$ at three different energies for the same three curves in Figs. 2 and 3. Not suprisingly, the weakly interacting K^+ exhibits significant "decoupling" as the much smaller absolute effect produced by varying ρ_n is now comparable to the much larger absolute effect from changing the *KN* interaction. Presumably, any further reduction in the uncertainty of the *KN* force will immediately render the isotopic density difference as the major unknown entering the analysis. This would permit a direct opportunity to learn more about $\Delta\rho_n$.

As mentioned above, an important motivating aspect of kaon scattering is the possibility of extracting more structure information than merely the radius and tail of the neutron density. To determine if, in fact, the kaon is sensitive to more subtle details of the nuclear interior, two

TABLE III. $K^+-{}^{40}$Ca real and imaginary optical potential well depths from elementary, *s*-wave amplitudes. All values are in MeV. Note real part is repulsive.

E_{LAB}	Proton		Neutron	
	Martin	BGRT	Martin	BGRT
84	$16.53-5.25i$	$15.76-4.74i$	$10.05-2.85i$	$5.84-2.72i$
250	$11.95-7.17i$	$11.27-5.81i$	$7.96-3.98i$	$0.72-5.43i$
446	$8.35-7.24i$	$7.79-5.57i$	$6.49-4.13i$	$0.0\ -7.67i$

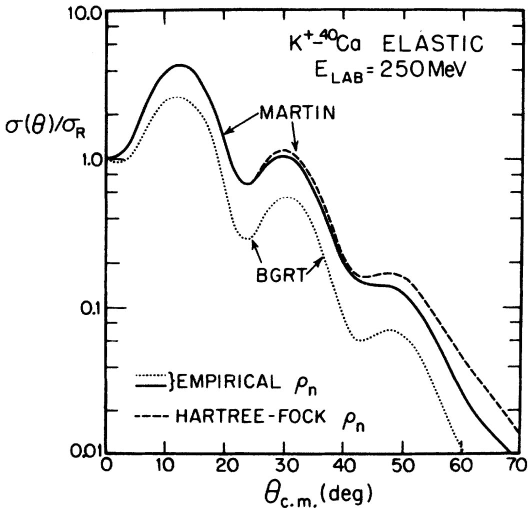

FIG. 2. Ratio of elastic to Rutherford scattering for K^+ scattering on ^{40}Ca. Differences between the three theoretical curves represent absolute effects from varying the neutron density and the KN interaction. In Figs. 2–5, the labels Martin and BGRT refer, respectively, to calculations using Martin and BGRT phenomenological KN amplitudes.

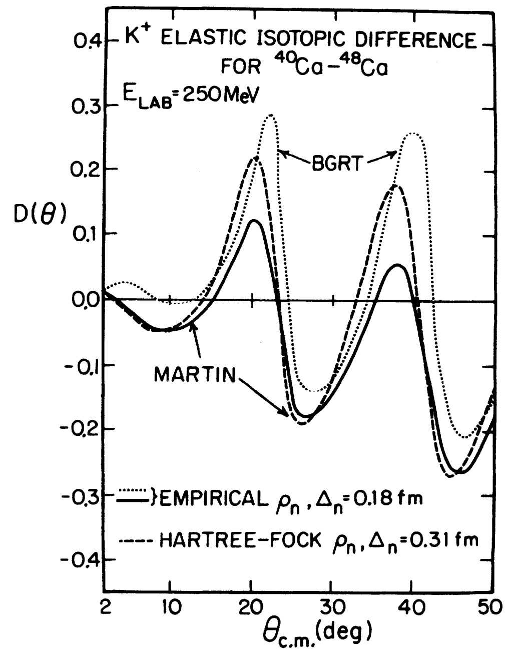

FIG. 4. Ratio of elastic cross section differences between ^{40}Ca and ^{48}Ca for the three different calculations plotted in Figs. 2 and 3. Differences between curves represent relative effects from varying the neutron density and KN interaction.

additional investigations were conducted.

The first examined the extent to which the kaon is sensitive to inner neutron core structure. Figure 7 illustrates theoretical ^{48}Ca – ^{40}Ca neutron density differences. The dashed line is simply the difference between ^{48}Ca and ^{40}Ca Hartree-Fock densities (same as in Fig. 1). This curve represents the expected rearrangement of the ^{40}Ca core when 8 neutrons are added to form ^{48}Ca and predicts, although not necessarily correctly, that the interior of ^{40}Ca is denser than ^{48}Ca. The other curve (solid line) is simply the $1f_{7/2}$ orbital Hartree-Fock density which is equivalent to assuming that the inner neutron cores of ^{40}Ca and ^{48}Ca are identical (no core rearrangement). The kaon elastic scattering calculations reflecting these two density differences are plotted in Figs. 8 and 9 for laboratory energies of 84 and 250 MeV, respectively. The sensitivity to core rearrange-

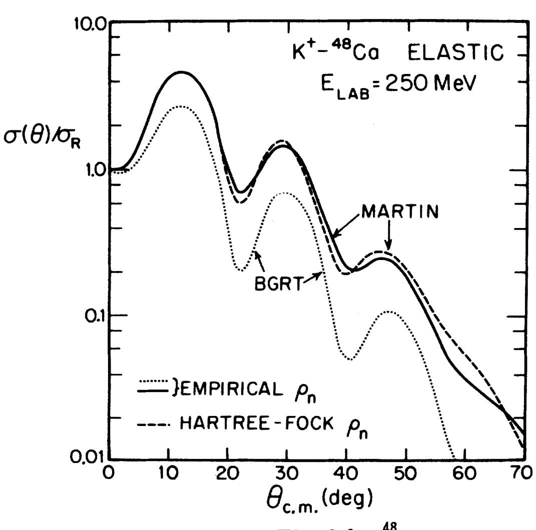

FIG. 3. Same as Fig. 2 for ^{48}Ca.

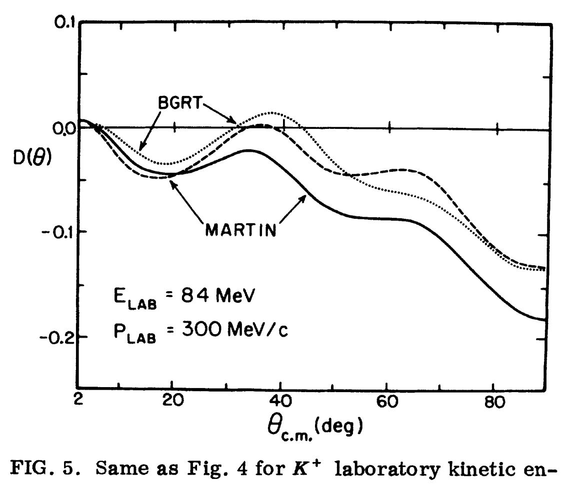

FIG. 5. Same as Fig. 4 for K^+ laboratory kinetic energy of 84 MeV.

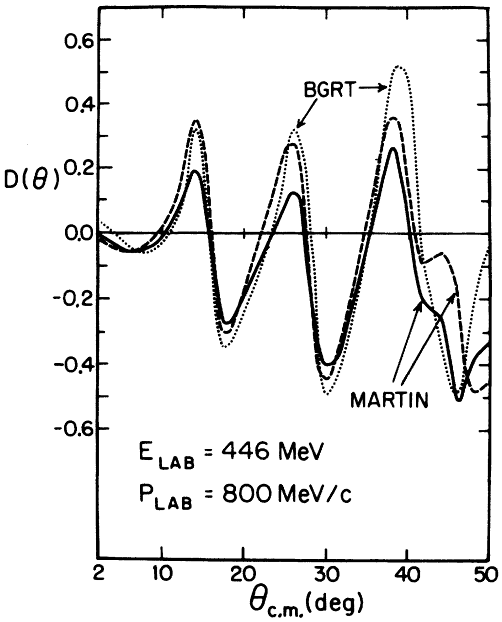

FIG. 6. Same as Fig. 4 for K^+ laboratory kinetic energy of 446 MeV.

ment effects is indeed significant.

The second analysis involved a comparison between elastic scattering cross sections in which the neutron density was varied in an artificial

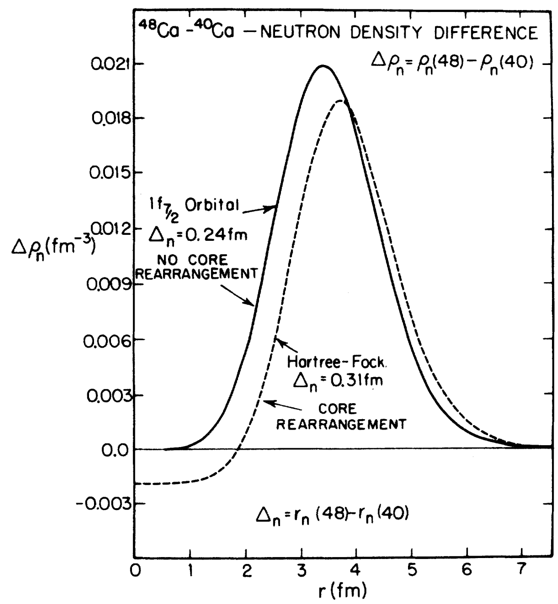

FIG. 7. Theoretical neutron density differences between ^{48}Ca and ^{40}Ca. Dashed curve, representing core rearrangement, is Hartree-Fock prediction constructed from Ref. 21. Solid curve is constructed from ^{40}Ca $1f_{7/2}$ Hartree-Fock orbital wave functions only and is equivalent to the assumption of no neutron core rearrangement.

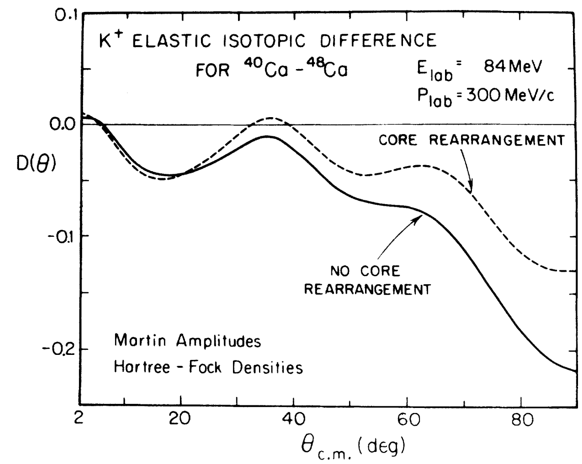

FIG. 8. Ratio of elastic cross section differences between ^{40}Ca and ^{48}Ca using the two density differences plotted in Fig. 7. Difference between curves represents kaon scattering sensitivity to inner neutron core structure.

but specific fashion. For convenience the parabolic Fermi form [Eq. (3)] was used with fixed diffuseness of $a = 0.59$ fm. Figure 10 shows the effect on the $K^+ - {}^{40}$Ca cross section produced by changing the remaining two parameters w and c. The solid and dotted curves represent the cross section variation, roughly 30 to 50%, accompanying a 20% change in the radius parameter

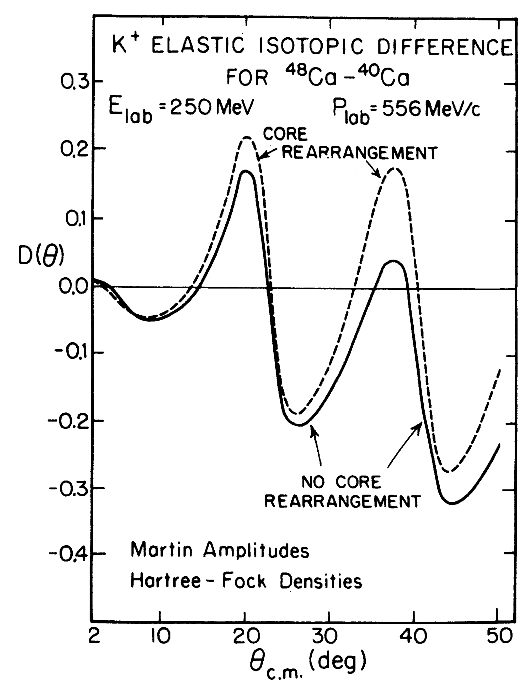

FIG. 9. Same as Fig. 8 for K^+ laboratory kinetic energy of 250 MeV.

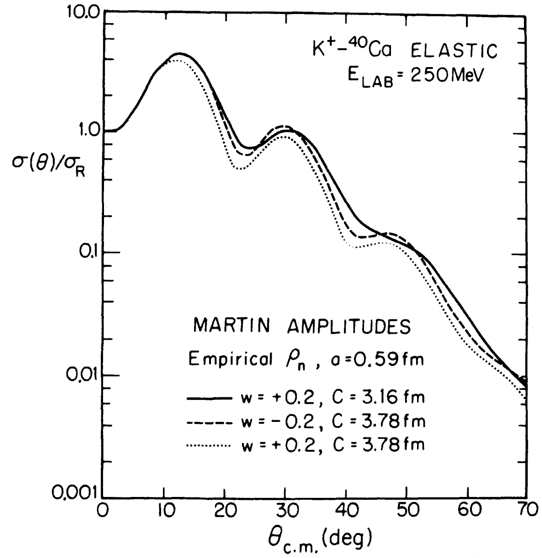

FIG. 10. Absolute effects from varying two parameters of a three-parameter Fermi neutron density. Solid and dashed curves are for neutron densities that yield an identical rms radius of r_n = 3.45 fm. Dotted curve is for a neutron density with r_n = 3.80 fm.

c above. While this does indeed represent strong sensitivity, one should correctly constrain the variation by requiring that each density yield an identical rms radius. This is not the case for the densities used to obtain the solid and dotted curves as their rms radii differ by 10%. To achieve equal neutron rms radii the remaining parameter w was also adjusted to a new value of $w = -0.2$ giving the dashed line in Fig. 10. Although this does reduce the cross section difference (compare solid and dashed curves with equal rms radii), sufficient variation still exists to learn more about higher moments of the neutron density. Recall that in the plane-wave Born approximation (reasonable for the K^+) the scattering amplitude reduces to the Fourier transform of the density which can also be represented by an infinite expansion in powers of q, the momentum transfer, with density moments as coefficients. As the momentum transfer increases, corresponding to larger scattering angles for fixed E_{LAB} higher density moments influence the cross section. This behavior can be seen in Fig. 10 where all cross sections agree at small angles ($q \to 0$) but diverge at large angles. For increasing angle (q) the first deviation is between solid and dotted curves, which as mentioned above, differ by 10% in the first moment (rms radius). This difference continues to escalate as the second nonzero moment (expectation value of r^4) becomes important at about 20° where now the solid and dashed curves also differ. The relative difference between various curves is governed of course by the magnitude of the difference between neutron moments. For the second neutron moment the solid and dotted curves differ by about 8% while the solid and dashed differ by only 2%. The small size of the latter is significant and testifies to the kaon's sensitivity as a neutron probe.

IV. CONCLUSIONS

The major contribution of this work has been to document, through survey calculations, the K^+ scattering sensitivity to uncertainties in the target neutron distribution. Based upon these calculations, this study concludes that measurements of relative isotopic elastic cross section differences can yield improved neutron information. Absolute cross section measurements, on the other hand, should provide more insight into the K^+-neutron interaction, which is currently the largest uncertainty entering this analysis. This uncertainty should be substantially reduced by new KN and K-deuteron phase shift analyses which include more abundant and accurate scattering and polarization data. Until such analyses, which are in progress,[26] are completed, a workable scenario would be to measure, to within 20% accuracy, absolute elastic K^+-nucleus scattering from different isotopes and then to phenomenologically analyze this data as a function of only the K^+-neutron real and imaginary well depths. With the KN interaction constrained, one could then analyze the isotopic difference $D(\theta)$ which now, presumably, would be predominantly sensitive to details of the neutron density.

It is important to stress that the utility of the K^+ as a neutron structure probe cannot be fully realized unless two conditions are satisfied. The first condition, as discussed above, is that the K^+-neutron amplitude must be more reliably known. The other condition is that corrections to the impulse approximated optical potential must be small. For the energy region used in the present work, effects from nucleon (Fermi) motion[7,14,18], different KN off-shell (momentum dependence) treatments [18,27], and kinematical (angle) transformation of KN amplitudes to different nuclear frames[27] have all been calculated and generally produce less than 10% corrections to the absolute K^+-nucleus cross section. Similarly, it has been estimated[8] that second-order effects from long-range Pauli correlations are 15% or less for s wave and negligible for p wave contributions to the K^+-nucleus optical potential. All of these refinements produce much smaller effects than those introduced by uncertainties in the K^+-neutron amplitude. Consequently, they should induce even smaller variations in $D(\theta)$, which is much more sensitive to differences in isotopic structure than to reaction mechanism (optical model) corrections. Further studies, however, on the adequacy of the first-order, impulse approximated optical model would still be useful.

Finally, this study also predicts that accurate measurements of kaon elastic scattering can provide new structure information about the nuclear interior. Studying the cross section as a function of momentum transfer permits extracting more than just two neutron density parameters. Knowing the KN interaction, it should, in principle, be possible to determine the neutron form factor, and therefore the density, in an almost model-independent fashion. Fundamentally, this would permit resolving a large number of important issues, some of which date back almost one-half of a century.

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