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Phase-shift analysis of the backward rise of the elastic scattering angular distributions

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A phase-shift analysis of the ^{28}Si(^{16}O,^{16}O) elastic scattering angular distributions measured between 0 and 180° c.m. shows that the backward rise of the angular distributions is due to small statistical fluctuations of the scattering matrix elements.

[NUCLEAR REACTIONS Phase-shift analysis of heavy ion elastic scattering. Statistical fluctuations of the scattering matrix.]

I. INTRODUCTION

In the last few years several attempts using either the optical model or the Regge pole parametrization have been performed to analyze elastic scattering angular distribution exhibiting a strong backward rise. The main defect of these various analyses based, more or less, on orbiting phenomena or quasimolecular states is the complete impossibility in reproducing the shape of the angular distributions in the intermediate angular region. Furthermore, the fits in the backward region up to 180° c.m. are often qualitative. We shall concentrate in this paper on the analysis of the ^{28}Si(^{16}O, ^{16}O) elastic scattering angular distributions measured at the MP tandem facility of Brookhaven National Laboratory, respectively, at 33, 41.4, 50, and 55 MeV incident energy.

In a first attempt the optical potentials used are very particular, some of them exhibiting a large volume transparency, the imaginary depth being 15 times smaller than the real one, which is unphysical.[1] Other potentials present a small parity dependent term in an energy dependent and surface transparent optical potential.[2] Other qualitative fits are produced by using only strong surface transparent potential.[3] On the other hand, Regge pole analyses produce slightly better fits in the very backward angle region but are still unable to reproduce the experimental data for the intermediate angle region.[4] One of the defects of all these models is that the physics underlying all these sets of parameters is unclear.

Most important of all, additional data show that the excitation function of the ^{28}Si(^{16}O, ^{16}O) elastic scattering at backward angle (180° c.m.) presents an erratic behavior with strong bumps in the energy range where the angular distributions have been measured. The presence of a fine structure was also evidenced, indicating that statistical fluctuations play an important role in the elastic scattering channel.[5] From this fact we consider that a phase shift analysis of the angular distributions will be meaningful. To the usual nuclear background amplitudes we had for each partial wave a fluctuating term due to statistical resonance effects in the composite system. It is hoped that the magnitudes of these fluctuating terms are small compared to the background values.

We shall present in Sec. II our model of elastic scattering phase shift analysis and in Sec. III the analysis of the ^{28}Si(^{16}O, ^{16}O) elastic scattering data.

II. ELASTIC SCATTERING PHASE SHIFT ANALYSIS METHOD

The amplitude of the elastic scattering is given by the usual formula[6]

$$f(\theta) = f_c(\theta) + \frac{i}{2k}\sum_{l=0}^{\infty}(2l+1)e^{2i\sigma_l}(1-S_l)P_l(\cos\theta),$$

where $f_c(\theta)$ is the Rutherford elastic scattering amplitude, the σ_l are the pure Coulomb phase shifts, and the elements S_l are the coefficients of reflection.

We consider that S_l is equal to a background component given by a standard phenomenological parametrization[6] plus a small fluctuating part δS_l due to "compound" nuclear elastic scattering resonances:

$$S_l = S_l^0 + \delta S_l,$$

with

$$S_l^0 = \{1+\exp[(l_g - l)/\Delta]\}^{-1} + i\mu\frac{\partial}{\partial l}\{1+\exp[(l_g - l)/\Delta]\}^{-1},$$

and

$$\delta S_l = a_l e^{i\varphi_l}, \quad 0 \leq \phi_l < 2\pi$$

In order to satisfy the unitarity condition $|S_l| \leq 1$, it had turned out in the present analysis that a_l is of the order of few hundredths.

The grazing wave l_g and width Δ are given by the usual semiclassical formulas[6] with obvious notations:

$$l_g = kR(1 - 2\eta/kR)^{\frac{1}{2}},$$

$$\Delta = kd(1-\eta/kR)(1-2\eta/kR)^{-\frac{1}{2}};$$

where η is the Sommerfeld parameter and $R = r_0(A_T^{1/3}+A_P^{1/3})$.

A code named ESPSA[7] has been built to determine the parameters which allow us to reproduce the BNL data. The diffractional model parameters r_0, d, and μ are fixed in order to reproduce only the forward angle Fresnel diffraction pattern; afterward, using the automatic search subroutine STEPIT[8], all statistical parameters a_l and ϕ_l are varied up till a best fit of the experimental data is achieved.

III. PHASE SHIFT ANALYSIS OF THE ELASTIC SCATTERING ANGULAR DISTRIBUTION OF THE $^{28}Si(^{16}O,^{16}O)$ REACTION

In Table I are listed the diffractional model parameters obtained by fitting at forward angles the Fresnel diffraction pattern of the $^{28}Si(^{16}O,^{16}O)$ elastic scattering angular distributions measured at 33, 41.4, 50, and 55 MeV incident energy. The diffusivity was kept constant to the value $d = 0.600$ fm and only μ and r_0 were adjusted. The value obtained for r_0 is very common and equal for heavy ions, as expected, to the radius given by the quarter point recipe of Blair,[9] equal also to the half absorption radius[9] and also equal to the sensitive radius defined by Satchler.[10]

In Figs. 1 and 2 are presented the fits of the angular distributions of the $^{28}Si(^{16}O,^{16}O)$ elastic scattering. The fits are currently absolutely perfect for the full angular range between 0 and 180° c.m. The dashed curves correspond to the background matrix elements S_l^0 which allow us to reproduce only the Fresnel diffraction pattern at forward angles.

In Table II are given the parameters related to this phase shift analysis. The quality of the fits is mainly determined by the fluctuation of

TABLE I. Diffractional model parameters $[R_0 = r_0(A_T^{1/3}+A_P^{1/3})]$.

E_{lab} (MeV)	r_0 (fm)	d (fm)	μ	χ^{2a}
33	1.6425	0.6000	2.1966	0.70
41.4	1.5773	0.6000	2.9774	0.55
50	1.5240	0.6000	3.7208	2.4
55	1.5123	0.6000	3.5751	5.6

[a] 10% error bars.

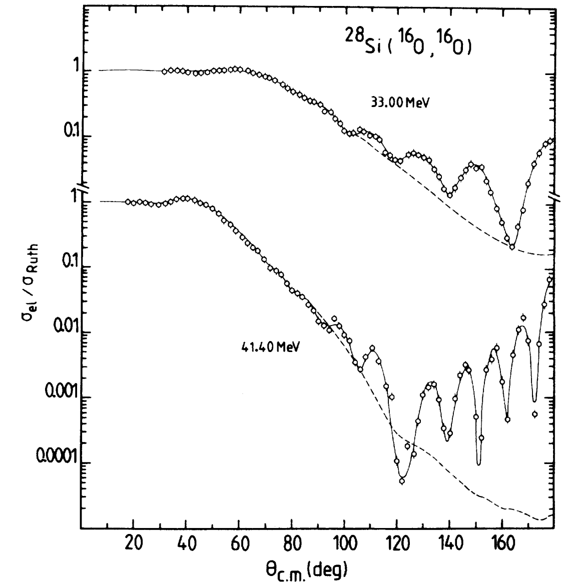

FIG. 1. Elastic scattering angular distributions of the $^{28}Si(^{16}O,^{16}O)$ reaction measured at 33 and 41.4 MeV incident laboratory energy. The solid curves are the best fits obtained with the scattering matrix elements $S_l = S_l^0 + \delta S_l$. On the other hand, the dashed curves correspond only to the background scattering matrix elements S_l^0.

the S matrix in the grazing wave region. The quantities l_i and l_f are, respectively, the lower and upper cutoff of the δS_l and correspond, respectively, to the value of the coefficient of reflection S_l^0 equal to 0.03 and 0.97. In fact, if we

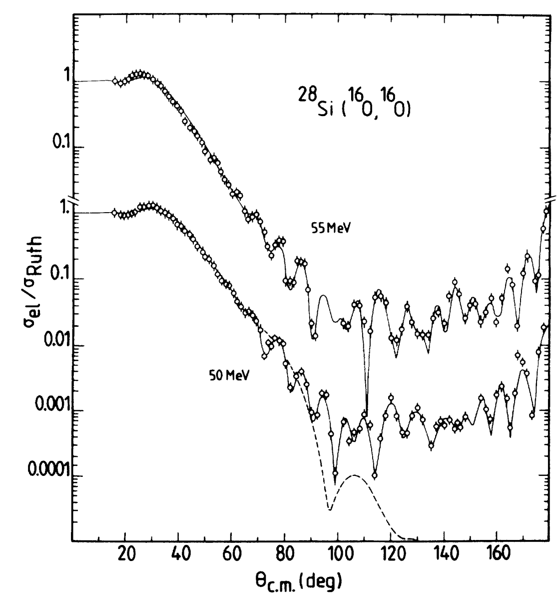

FIG. 2. Elastic scattering angular distributions of the $^{28}Si(^{16}O,^{16}O)$ reaction measured at 50 and 55 MeV incident laboratory energy. See caption of Fig. 1.

TABLE II. Fluctuating elastic scattering amplitudes.

E_{lab} (MeV)	l_i	l_f	l_g	$\sigma_{B.G.}$ (mb)	σ_{tot} (mb)	T (%)	$(\langle \lvert \delta S_l \rvert^2 \rangle)^{1/2}$	χ^2 [a]
33	1	23	11.2	673	640	4.9	0.0505	0.36
44.10	7	27	16.8	947	927	2.1	0.0130	1.93
50	9	30	20.7	1100	1064	2.6	0.0204	3.2
50	14	27	20.7	1100	1075	2.3	0.0209	5.4
55	10	32	22.9	1209	1186	0.6	0.0145	2.9
55	15	29	22.9	1209	1184	2.7	0.0211	4.36

[a] 10% error bars.

add lower l values the quality of the fits is still improved and the element $|\delta S_l|$ stays of the order of 0.01 indicating an extremely small volume transparency. Nevertheless, in the results presented here, we consider only a model which presents a small surface transparency. The grazing wave number is $l_g(S^0_{l_g} = 0.50)$. The quantities $\sigma_{B.G.}$ and σ_{Tot} are, respectively, the total reaction cross sections for the scattering matrix elements S^0_l and S_l. The difference indicates the degree of surface transparency of the model. The quantity T is this transparency and is given in %.

The mean square deviation $\langle |\delta S|^2 \rangle$ is given by the following relationship:

$$\langle |\delta S|^2 \rangle = \frac{1}{l_f - l_i + 1} \sum_{l=l_i}^{l_f} (|S^0_l| - |S_l|)^2 .$$

This quantity, as seen in Table II, has the tendency to decrease with the increase of incident energy, which is a normal behavior for a statistical model. The quantity χ^2 is the mean square deviation of the experimental point with respect of the calculated ones:

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\sigma^i_{expt} - \sigma^i_{theo}}{\Delta \sigma^i_{expt}} \right)^2 .$$

The error bars are equal to 10% for all the data points, N is the total number of points.

For the 50 and 55 MeV measurements, the calculations were repeated with a smaller grazing wave window. It can be seen from the χ^2 values of Table II that the quality of the fits are slightly deteriorated but still very good. The drawn curves in Figs. 1 and 2 correspond always to the best χ^2.

In Figs. 3 and 4, the background matrix elements S^0_l and the total matrix elements $S_l = S^0_l + \delta S_l$, are plotted in the complex Argand-Cauchy plane. The solid line is the analytical S^0_l curve. It is possible to see that the fluctuation δS_l is large at a low incident energy (33 MeV). Therefore, it may be possible at even lower incident energy to consider that we are dealing with real resonances. On the other hand, it seems that at higher incident energy

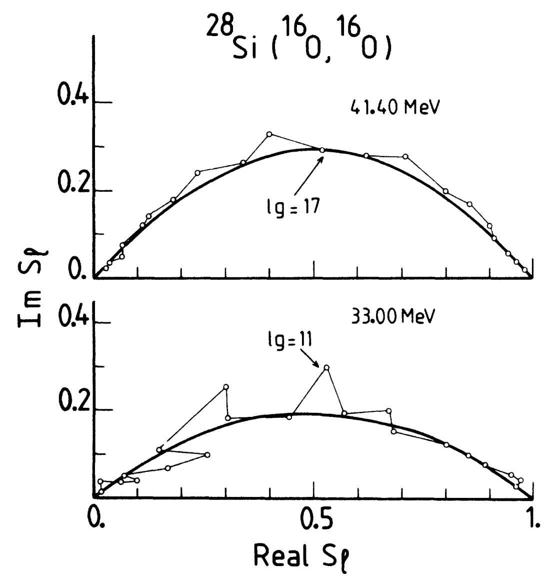

FIG. 3. Plot of the background S^0_l matrix elements (solid curve) and of S total matrix elements (circle) in the Argand-Cauchy plane; l_g is the grazing wave ($S^0_l = 0.50$).

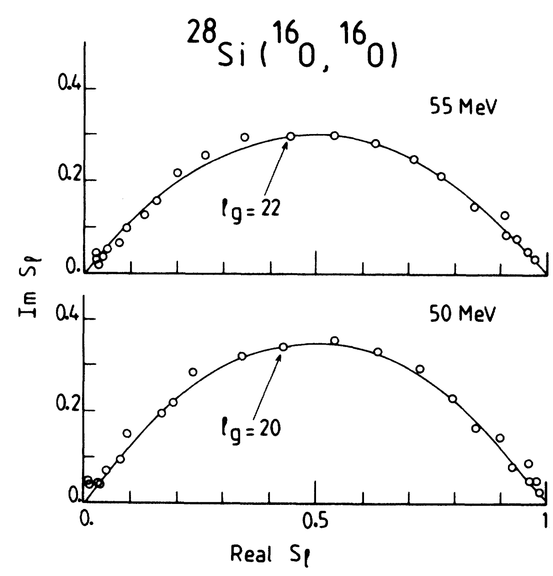

FIG. 4. Same as Fig. 3.

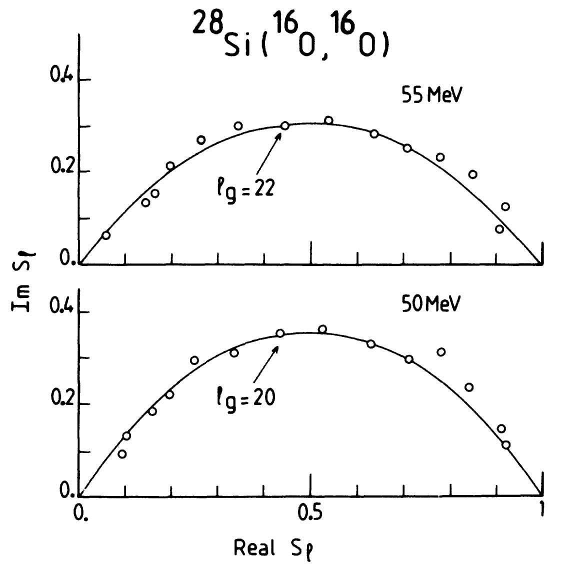

FIG. 5. Same as Fig. 3.

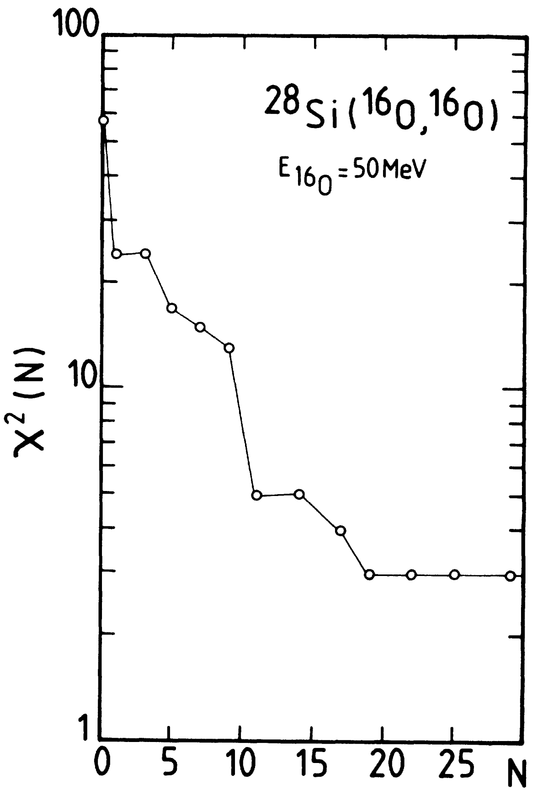

FIG. 6. Plot of the χ^2 value versus the number N of fluctuating terms δS_l of the scattering matrix.

the rise at backward angles of the elastic angular distributions is only due to statistical fluctuations of S matrix elements related to compound nucleus overlapping resonances. Nevertheless, that does not necessarily mean that a true compound nucleus is formed at so high an excitation energy but only a composite dinuclear system having a large level density. It would be interesting to have a theoretical model able to predict quantities of the kind of the $\langle |\delta S_l|^2 \rangle$ mean square deviation of the scattering matrix elements.

In Fig. 5 are also plotted the S matrix elements of the 50 and 55 MeV calculated angular distributions, but for a smaller l window. In all these calculations it has turned out that the phases ϕ_l of δS_l are almost random.

Furthermore, in Fig. 6, for the 50 MeV experimental data, are plotted the values of the χ^2 versus the number N of fluctuating terms δS_l; they are two parameters a_l and ϕ_l for one fluctuating term δS_l. The value $N = 0$ corresponds, for the best fit, only to the background matrix elements S_l^0. The number of fluctuating terms δS_l were added symmetrically with respect to the grazing wave $l_g = 20$. We can notice immediately that the χ^2 value decreases drastically as the number of fluctuating terms increases. This fact also establishes that many fluctuating terms contribute to the backward rise of the angular distributions.

IV. CONCLUSION

At least for the present $^{28}Si(^{16}O, ^{16}O)$ elastic data it has turned out that :

(i) Excellent fits for the full range of angles can be obtained by phase shift analysis. The small fluctuation of scattering matrix elements explains the backward rise of the angular distributions in agreement with the erratic structure seen in the excitation function experiments

(ii) It is erroneous, by fitting angular distributions at very backward angle by a $|P_l(\cos\theta)|^2$ function, to assign a given spin to a bump seen in the excitation function and call it resonance.

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