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Qualitative discussion of the YN interaction for Λ hypernuclei with $A \leq 4$

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Major difficulties associated with attempts to describe the YN (hyperon-nucleon) interaction in all Λ-hypernuclear systems having $A \leq 4$ by simple, effective Λp and Λn potentials are examined. The explicit A dependence of the YN interaction associated with each system is explored, and the necessity of using exact four-body theory (in the $A = 4$ system) to calculate small charge-symmetry-breaking effects in terms of ΛN potentials fitted to free ΛN scattering data is discussed.

[NUCLEAR STRUCTURE ΛN interactions; ${}^3_\Lambda$H, ${}^4_\Lambda$H, ${}^4_\Lambda$He; $\Lambda-\Sigma$ coupling.]

A detailed analysis of the binding energy data for the $A=3,4$ hypernuclei as well as low energy Λp scattering data (for $p_\Lambda = 145-290$ MeV/c) has been used by Deloff[1] to determine parameters for a phenomenological ΛN central-force potential of the form introduced by Herndon, Tang, and Dalitz.[2,3] A charge-symmetry-breaking (CSB) component was included in order to allow for binding energy differences in the ${}^4_\Lambda$He-${}^4_\Lambda$H ground and excited states. We wish to point out that, although the analysis of Ref. 1 is self-consistent, it may not be possible to make meaningful use of the deduced potentials. We shall indicate below how the coupling of the ΣN channel to the ΛN channel makes the effective ΛN potentials used in the Ref. 1 study a complex function of A. Since this A dependence was neglected in that analysis, the resulting potential parameters are not valid as a representation of the actual ΛN interaction. In addition, it has been previously demonstrated[4] that the *difference* between a true many-body calculation and an effective two-body calculation of the CSB binding energy in the $A=4$ ground states is of the *same* magnitude as that CSB binding energy itself. Therefore, the CSB binding energy differences are not properly reflected in the analysis of Ref. 1.

The phenomenological analysis of Ref. 1 is based upon the following spin decomposition of the effective ΛN central potential (neglecting for the moment any CSB difference between the Λp and Λn interactions):

$$\Lambda N:\ V_{YN} = \bar{V}^s_{\Lambda N},\ \bar{V}^t_{\Lambda N},$$

$${}^3_\Lambda\text{H}:\ V_{YN} = \tfrac{3}{4}\bar{V}^s_{\Lambda N} + \tfrac{1}{4}\bar{V}^t_{\Lambda N},$$

$${}^4_\Lambda\text{H}:\ V_{YN} = \tfrac{1}{2}\bar{V}^s_{\Lambda N} + \tfrac{1}{2}\bar{V}^t_{\Lambda N},$$

$${}^4_\Lambda\text{H}^*:\ V_{YN} = \tfrac{1}{6}\bar{V}^s_{\Lambda N} + \tfrac{5}{6}\bar{V}^t_{\Lambda N},$$

assuming that the singlet interaction is stronger than the triplet interaction.[2,3] Here, the YN subscript indicates that the potential describes the general hyperon-nucleon (ΛN-ΣN) interaction. Implicit in the above effective potential description is the assumption that the ΛN-ΣN coupling in the YN interaction is identical in each system regardless of the isospin; i.e., one has assumed that

$$V^i_{YN} = \begin{pmatrix} V^i_{\Lambda N} & V^i_{XN} \\ V^i_{XN} & V^i_{\Sigma N} \end{pmatrix},\quad i = s, t$$

can be represented by unique effective potentials $\bar{V}^i_{\Lambda N}$. Such is not the case.

Let us define the free interactions to be of the form

$$V^s_{YN} = \begin{pmatrix} V^s_{\Lambda N} & V^s_{XN} \\ V^s_{XN} & V^s_{\Sigma N} \end{pmatrix},\quad V^t_{YN} = \begin{pmatrix} V^t_{\Lambda N} & V^t_{XN} \\ V^t_{XN} & V^t_{\Sigma N} \end{pmatrix}.$$

[We note that ΛN elastic scattering is dominated by the triplet interaction, since $\sigma=(\sigma^s+3\sigma^t)/4$.] For the ${}^3_\Lambda$H system, where the np pair is restricted to be in the $S=1$, $T=0$ "deuteron" state, the potentials are

$$V^s_{YN}=\begin{pmatrix} V^s_{\Lambda N} & 0 \\ 0 & 0 \end{pmatrix},\quad V^t_{YN}=\begin{pmatrix} V^t_{\Lambda N} & 0 \\ 0 & 0 \end{pmatrix};$$

i.e., there is no ΣN coupling, unless one allows for np $T=1$ "excited" states in the formalism.[5] This is a consequence of the $T=0$ nature of the ${}^3_\Lambda$H ground state (the Λ and the deuteron each being $T=0$ objects); the Σ has $T=1$ and must couple to the $T=1$ singlet np state to produce a total $T=0$ system. For the $A=4$ hypernuclei, the $J^\pi=0^+$ ground state potentials are

$$V^s_{YN}=\begin{pmatrix} V^s_{\Lambda N} & -\frac{1}{3}V^s_{XN} \\ -\frac{1}{3}V^s_{XN} & V^s_{\Sigma N} \end{pmatrix},\quad V^t_{YN}=\begin{pmatrix} V^t_{\Lambda N} & V^t_{XN} \\ V^t_{XN} & V^t_{\Sigma N} \end{pmatrix}$$

and the $J^\pi=1^+$ excited state potentials are

$$V^s_{YN}=\begin{pmatrix} V^s_{\Lambda N} & V^s_{XN} \\ V^s_{XN} & V^s_{\Sigma N} \end{pmatrix},\quad V^t_{YN}=\begin{pmatrix} V^t_{\Lambda N} & \frac{1}{5}V^t_{XN} \\ \frac{1}{5}V^t_{XN} & V^t_{\Sigma N} \end{pmatrix}$$

(see, for example, Refs. 6 and 7). The singlet potential differs from the free interaction in the $A=4$ ground state and the triplet potential differs from the free interaction in the $A=4$ excited state. In each case the ΛN-ΣN coupling strength is reduced, weakening the YN interaction relative to its free strength. It is clear that, *in principle*, the YN interactions acting in each of the four systems (ΛN, ${}^3_\Lambda$H, ${}^4_\Lambda$H, and ${}^4_\Lambda$H*) cannot be represented by single, unique $\bar{V}^s_{\Lambda N}$ and $\bar{V}^t_{\Lambda N}$ effective potentials. In practice, one finds[8] that $V^s_{XN}\cong 0$, so that the effective potential representation of the free YN interactions is "reasonable" when dealing with the $A=4$ ground states, where $V^t_{YN}({}^4_\Lambda\mathrm{H})\equiv V^t_{YN}(\Lambda N\text{ scattering})$. However, the triplet interactions involved in the ${}^3_\Lambda$H and ${}^4_\Lambda$H* calculations differ from the free case, and the free effective triplet potential $\bar{V}^t_{\Lambda N}$ cannot be used in those calculations. The importance of including ΛN-ΣN coupling in calculations involving these nuclei has been noted previously (see, for example, Refs. 4, 6, 7, 9, and 10).

In the effort in Ref. 1 to parametrize the CSB part of the ΛN interaction, a CSB component was included in the effective ΛN potential. However, it has been demonstrated in Ref. 4 that a true four-body calculation may be required to correctly describe such small effects. The four-body CSB binding energy was found to be twice as large as that of an effective two-body calculation when the same V^{CSB} was used; this resulted from the fact that differences in the Λp and Λn effective ranges produced effects in the two calculations which were opposite in sign, in one case increasing the binding energy difference and the other case reducing it. In order to make this point clear, we have calculated the CSB binding energy for the $A=4$ ground state system using the exact four-body formalism outlined in Ref. 4 along with Λp and Λn interactions described by rank-one separable potentials whose parameters were determined from the scattering lengths and effective ranges quoted in Ref. 1 (see Table I). These low-energy ΛN scattering parameters were obtained for four different models in Ref. 1, and each model gave a CSB binding energy of about 0.35 MeV in that analysis. Our numerical calculation[11] shows instead the CSB binding energy to be about 0.62 MeV when an exact four-body theory is used; again, the simple model differs by almost a factor of 2. Thus, the CSB differences in the low-energy Λp and Λn scattering parameters quoted in Ref. 1 are not realistic.

TABLE I. Scattering lengths and effective ranges from the ΛN interaction models of Ref. 1 and in the appropriate charge combinations (Ref. 11) for ${}^4_\Lambda$H and ${}^4_\Lambda$He.

System	Spin	a (fm)	r (fm)	λ (fm^{-3})	β (fm^{-1})
Λn	s	−1.79	2.34		
	t	−1.00	2.94		
Λp	s	−2.54	2.00		
	t	−1.29	2.57		
${}^4_\Lambda$He	s	−1.99	2.23	0.405 38	1.8362
	t	−1.29	2.57	0.334 58	1.8275
${}^4_\Lambda$H	s	−2.23	2.11	0.451 91	1.8752
	t	−1.00	2.94	0.270 87	1.7833

In summary, it appears that one cannot make use of such simplifying assumptions as was done in Ref. 1 to extract meaningful representations of the ΛN interactions. In particular, ΛN-ΣN coupling should be included and one should utilize exact theory when dealing with small quantities such as the CSB energies in the $A=4$ system. In view of the flaws in the Ref. 1 analysis, great care should be exercised in using the potentials given there in further hypernuclear studies.

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[11]From Ref. 4 it is clear that for ${}^{4}_{\Lambda}\mathrm{He}$ one has a singlet interaction described approximately by $a^{-1} = (a_{\Lambda p}^{-1} + 2a_{\Lambda n}^{-1})/3$ and $r = (r_{\Lambda p} + 2r_{\Lambda n})/3$ as well as a triplet interaction which is identical to the Λp; for ${}^{4}_{\Lambda}\mathrm{H}$ the roles of Λp and Λn are interchanged.