PHYSICAL REVIEW C VOLUME 23, NUMBER 1 JANUARY 1981

Boson-exchange potentials and the nucleon potential energy in nuclear matter

M. R. Anastasio, L. S. Celenza, and C. M. Shakin
Department of Physics and Institute for Nuclear Theory, Brooklyn College of the City University of New York, Brooklyn, New York 11210
(Received 6 June 1980)

We report a calculation of the nucleon potential energy in nuclear matter based upon a one-boson-exchange potential which is obtained from a fit to the two-nucleon scattering data. We evaluate the potential energy in the Hartree-Fock approximation and also perform a calculation which includes the effects of nucleon-nucleon correlations. We find large correlation corrections and conclude that calculations making use of the Hartree (or Hartree-Fock) approximation require the use of *effective* coupling constants which differ from those determined in a fit to scattering data. These *effective* constants would include effects of correlations in an implicit fashion.

[NUCLEAR STRUCTURE Nucleon potential energy in nuclear matter; one-boson-exchange potential; validity of the Hartree-Fock approximation.]

In recent years there has been strong interest in developing a relativistic theory of nuclear matter and of finite nuclei.[1] These theories differ from more conventional theories in that they directly relate the potentials felt by a nucleon to the underlying process of meson exchange. These developments are interesting in that one may, in principle, be able to unify the meson-exchange theory of nuclear forces with theories of nuclear structure. To our knowledge, all attempts to construct a relativistic theory have used the Hartree or Hartree-Fock approximation.[1] Our goal in this work is to compare the values obtained for the nucleon potential energy in nuclear matter in the Hartree-Fock approximation and in an approximation which includes correlation effects. As we will show, correlation effects are extremely important and require the use of effective coupling constants if use is made of the Hartree (or Hartree-Fock) approximation in a relativistic theory of nuclear structure. Of course, the density dependence of such coupling constants would not be known.

In the relativistic theory there is a problem of *self-consistency* which does not exist in the nonrelativistic theory of nuclear matter. For example, the spinor representing the bound nucleon can be expanded in terms of the spinors which are positive and negative energy solutions of the Dirac equation without interactions. The amplitudes of these negative and positive energy spinors must be determined in a self-consistent manner. We have studied this question in other works[2,3] and have found large corrections to the saturation curve for nuclear matter when we include the negative energy spinors.[3] In this work, however, we investigate the leading term in the nucleon potential energy which follows from the use of only positive energy spinors for the nucleon wave function. This approximation is adequate at low density; however, there are significant corrections at $k_f \simeq 1.36$ fm^{-1}. A more complete analysis including the role of negative energy states in modifying the nucleon potential energy will be presented at a future time.

All our calculations are carried out in a momentum-space representation and may be summarized by reference to Fig. 1. In Fig. 1(a) we depict the terms calculated in the Hartree-Fock approximation. The wavy line represents the propagation of the exchanged meson: $\pi, \rho, \omega, \phi, \eta, \delta, \sigma$.[2] We use pseudovector coupling for the pion and the η meson. (The calculation in question requires an integral over the vector $\vec{q}$, where the maximum value of $|\vec{q}|$ is $k_F = 268$ MeV/c.) The second calculation we have made is indicated schematically in Fig. 1(b). There the large circle denotes a nucleon-nucleon reaction matrix calculated using the one-boson-exchange (OBE) potential of Ref. 4. The reaction matrix is calculated taking into account the Pauli effects in the propagators of the particles in the intermediate states. The energies of the occupied states need to be specified in the construction of the reaction matrix and these energies are calculated self-consistently using standard techniques. The calculational procedure for constructing the reaction matrix is essentially that described in Ref. 3. [The reaction matrix is appropriately symmetrized in our calculations; however, the exchange term is not shown in Fig. 1(b)].

Before we describe the results of our calculation we present some formal considerations. Let M denote the solution of a two-body relativistic equation

$$M = K + KGM , \tag{1}$$

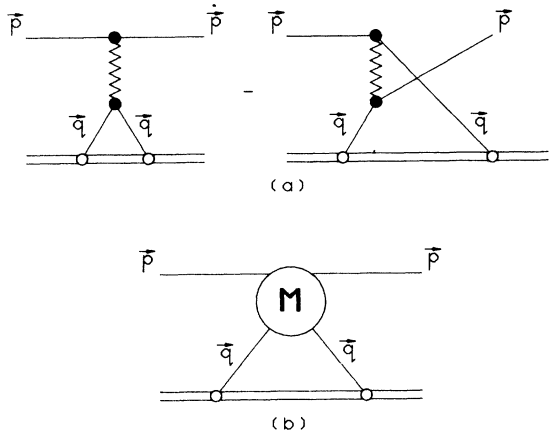

FIG. 1. (a) Feynman diagrams representing the nucleon self-energy in the Hartree-Fock approximation. The wavy line denotes the propagator for the exchanged mesons. The open circles are vertex functions which represent the amplitude for finding a particle of momentum $\vec{q}$ in the Fermi sea. To obtain the total self-energy one sums on the various types of exchanged mesons. (b) Feynman diagram for the nucleon self-energy in a relativistic Brueckner-Hartree-Fock approximation. The reaction matrix M is calculated with positive energy intermediate states and includes binding effects, Pauli principle restrictions, and dispersive corrections.

where G represents the Feynman propagator for the two intermediate nucleons. Using fairly standard techniques,[5,6] Eq. (1) may be rewritten as two equations

$$M = U + UgM \,, \tag{2}$$

$$U = K + K(G - g)U \,. \tag{3}$$

The choice of the propagator g depends on the problem at hand. In general, g has the same right-hand cut as G and is chosen so as to reduce the four-dimensional equation [Eq. (1)] to an equivalent three-dimensional equation. In our calculations the propagator g is chosen so as to allow only positive energy intermediate states. This propagator also includes dispersive effects and Pauli principle restrictions as noted above.

The *quasipotential* U is determined from the study of nucleon-nucleon scattering. In the OBE model[5] U describes the exchange of N different bosons so that we may write

$$U = \sum_{i=1}^{N} U_i \,. \tag{4}$$

Note that we may also write

$$M = \sum_{i=1}^{N} M_i \,, \tag{5}$$

where

$$M_i = U_i(1 + gM) \,. \tag{6}$$

The Hartree-Fock approximation consists of setting $M_i = U_i$ so that

$$M_{\mathrm{HF}} = \sum_{i=1}^{N} U_i \,. \tag{7}$$

For the purposes of this paper we define the potential energy of a nucleon as $V(\vec{p}) = [m/E(\vec{p})]\Sigma(\vec{p})$, where $\Sigma(\vec{p})$, the self-energy, is approximated by

$$\Sigma(\vec{p}) = \sum_{s'} \int \frac{d\vec{q}}{(2\pi)^3} \frac{m}{E(\vec{q})} \times \langle \bar{u}^{(s)}(\vec{p})\, \bar{u}^{(s')}(\vec{q}) \,|M(1 - P_{12})|\, u^{(s)}(\vec{p})\, u^{(s')}(\vec{q}) \rangle \,. \tag{8}$$

In Eq. (8) a trace over the spin variable s' and an integral over the Fermi sea is indicated. (Reference to the isospin trace is suppressed.) Further, P_{12} is the particle-exchange operator and the $u^{(s)}$ are Dirac spinors. The Hartree-Fock approximation for $\Sigma(\vec{p})$ consists in replacing M by U in Eq. (8).

In evaluating our expression for $\Sigma(\vec{p})$ we can use either the Hartree-Fock approximation [Eq. (7)] or a relativistic Brueckner-Hartree-Fock approximation [Eq. (5)]. The calculation implied by Eq. (7) is that depicted in Fig. 1(a). The results for the contribution to the nucleon potential from *each* of the exchanged mesons are given in Figs. 2 and 3. The solid lines in these figures represent the contributions calculated in the Hartree-Fock approximation. We note that for the nucleon-nucleon potential of Ref. 2, the total potential felt by a nucleon is *repulsive* in the Hartree-Fock approximation [see Fig. 4].

We now turn to a consideration of the role of correlations. We realize that once we include correlation effects it is not possible to separate the contributions of the individual mesons except in the manner implied by the use of Eq. (6). We recall that for the Hartree-Fock approximation, $M_i^{\mathrm{HF}} = U_i$. We also note that the inclusion of correlations leads to $M_i^{\mathrm{HF}} \to M_i = U_i\Omega$, where Ω is the wave matrix which introduces the correlation structure, i.e., $\Omega = 1 + gM$.

The dashed lines in Figs. 2 and 3 represent the contribution to the self-energy of the various M_i of Eq. (6). We see that for the ω, ϕ, σ, ρ, and δ mesons the inclusion of correlations significantly reduces the contributions of these mesons to the potential. As expected, this reduction is most important for the more massive mesons of this group, the σ meson contribution receiving the

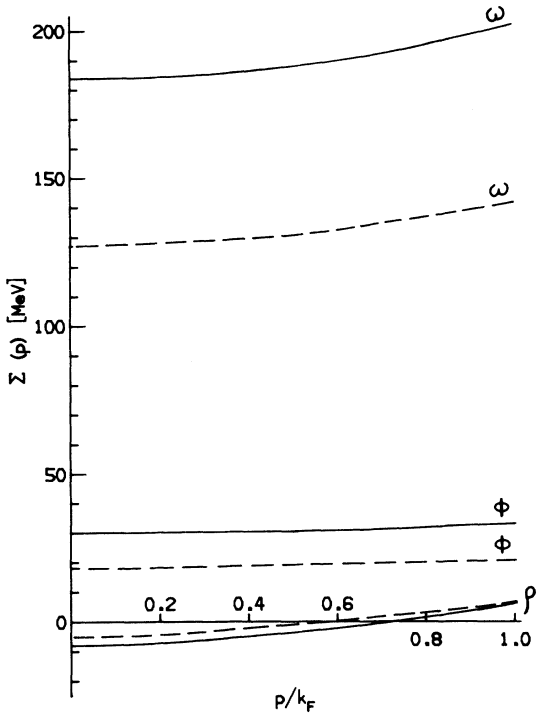

FIG. 2. Contributions of the ω, ρ, and σ mesons to the nucleon self-energy. The solid curve denotes the values obtained in the Hartree-Fock approximation. The dashed curves represent the contributions to the self-energy [Eq. (8)] of the reaction matrices defined in Eq. (6).

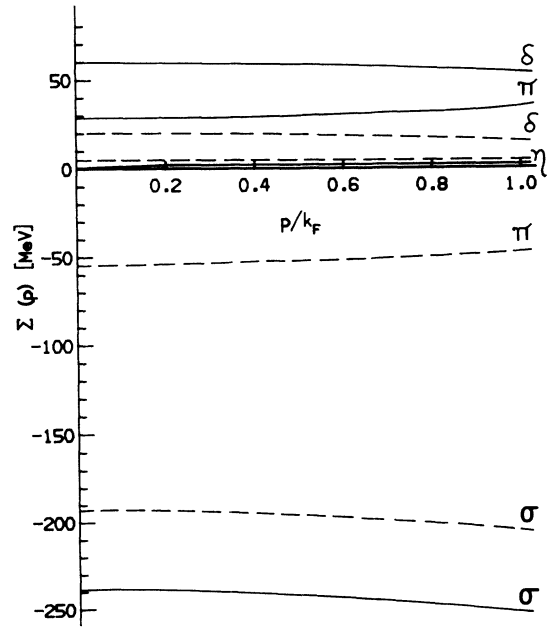

FIG. 3. Contributions of the π, δ, σ, and η mesons to the nucleon self-energy. (See caption to Fig. 2 and note the change of scale.)

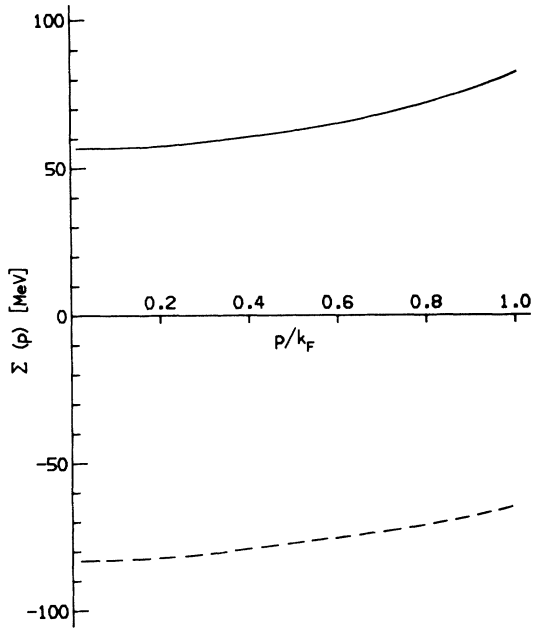

FIG. 4. The total self-energy calculated in the Hartree-Fock approximation (solid line) and in an approximation which includes correlation effects (dashed line).

smallest modification. Of course the very large correlation effects shown in Figs. 2 and 3 imply that the Hartree-Fock (or Hartree) approximation is not useful unless one introduces *effective* coupling constants.

It is of interest to note (see Fig. 3) that the contribution of the pion to the self-energy *changes sign* and is *enhanced* in magnitude when correlations are included. We believe that we can ascribe these effects to the tensor correlations present in the wave matrix Ω. The importance of such tensor correlations was discussed many years ago.[7] It was shown that the "long range" part of the tensor interaction when treated in second order gives rise to an *effective* attractive central force. Our study of correlation effects in the pion contribution to the potential energy also shows that the inclusion of correlations leads to a large attractive contribution to the nucleon potential. [Indeed, about 70% of $\Sigma(0)$ can be ascribed to pion exchange.] Further study of this matter is required for a complete understanding of the role of tensor correlations in modifying the calculation of pion-exchange processes.

We have studied the nucleon potential energy in two approximations and have determined that correlation effects are very large. We note that the contribution of the pion to the nucleon potential is zero in the Hartree approximation, repulsive in the Hartree-Fock approximation (see Fig. 3), and is strongly attractive in the relativistic Brueck-

ner-Hartree-Fock approximation used in this work. We may conclude that application of the Hartree or Hartree-Fock approximation for this problem requires the introduction of *effective* coupling constants that include correlation effects implicitly but whose density dependence is unknown.

In our future works we will extend our considerations to include a description of the negative energy components of the nucleon wave functions[2] and we will study the saturation properties of nuclear matter in our relativistic theory which emphasizes correlation effects.[3] We also hope to study the role of the negative energy components in modifying the nucleon potential energy.

This work was supported in part by the National Science Foundation and by the PSC-BHE Award Program of the City University of New York.

[1]J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974); S. A. Chin, *ibid.* 108, 301 (1977); M. Brittan, Phys. Lett. 79B, 27 (1978); L. D. Miller, Phys. Rev. Lett. 28, 1281 (1972); Phys. Rev. C 9, 537 (1974); J. V. Noble, *ibid.* 17, 2151 (1978); R. Brockman and W. Weise, *ibid.* 16, 1282 (1977); R. Brockman, *ibid.* 18, 1510 (1978); F. R. Serr and J. D. Walecka, Phys. Lett. 79B, 10 (1978); B. D. Serot, *ibid.* 86B, 146 (1979); M. Jaminon, C. Mahaux, and P. Rochus, Phys. Rev. Lett. 43, 1097 (1979); Phys. Rev. C 22, 2027 (1980).

[2]M. R. Anastasio, L. S. Celenza, and C. M. Shakin, Brooklyn College Report No. B.C.I.N.T. 80/052/99, Phys. Rev. C (to be published).

[3]M. R. Anastasio, L. S. Celenza, and C. M. Shakin, Brooklyn College Report No. B.C.I.N.T. 80/061/100, Phys. Rev. Lett. (to be published).

[4]The coupling constants, masses, and cutoff parameters are taken from the work of K. Holinde, K. Erkelenz, and R. Alzetta, Nucl. Phys. A194, 161 (1972); A198, 598 (1972) and are representative of their force designated as HEA.

[5]K. Erkelenz, Phys. Rep. C13, 191 (1974).

[6]J. J. de Swart and M. M. Nagels, Fortschr. Phys. 28, 215 (1978).

[7]T. T. S. Kuo and G. E. Brown, Phys. Lett. 18, 54 (1965); C. M. Shakin and Y. R. Waghmare, Phys. Rev. Lett. 16, 403 (1966); G. E. Brown, *Unified Theory of Nuclear Models and Forces* (North-Holland, Amsterdam, 1967), see Chap. 13.