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Nucleus as a meson-baryon system

M. Gari
Institut für Theoretische Physik, Ruhr-Universität Bochum, Germany
and Max-Planck-Institut für Chemie, Kernphysik, Abteilung, Mainz, Germany

G. Niephaus and B. Sommer
Institut für Theoretische Physik, Ruhr-Universität Bochum, Germany
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We describe the nucleus by a system of mesons ($\pi,\rho,\ldots$) and baryons ($N,N^*,\Delta,\ldots$). Starting from a field theoretical Hamiltonian for the composite system we derive a Schrödinger equation in a subspace without mesons. In lowest order approximation we obtain a generalization of the one-boson-exchange formulation of the *NN* interaction for a general baryon-baryon interaction. We discuss two possible treatments of the many-body system: (i) nucleonic degrees of freedom only, and (ii) explicit inclusion of resonance degrees of freedom. In the case of nucleons and deltas the explicit form of the potentials is given together with a complete new fit of the Reid soft-core potential for the nucleonic part of the interaction. The coupled equations for the scattering states and the deuteron have been solved exactly using the complete baryon-baryon interaction in the one-boson-exchange limit. The deuteron $\Delta\Delta$ contribution is discussed in several limits of the general one-boson-exchange potential.

[NUCLEAR STRUCTURE Meson and resonance degrees of freedom, calculated *N*-*N* scattering, and deuteron properties.]

I. INTRODUCTION

The importance of meson and nucleon resonances in the nucleon-nucleon interaction has been recognized for a long time (see for example, Ref. 1-3). In the past especially the role of the $\Delta(33)$ resonance has been investigated. As this resonance was seen to be most important for the understanding of the intermediate range of the nucleon-nucleon force, one was tempted to treat the resonance degrees of freedom explicitly in the description of nuclei. During the last few years a considerable amount of work has been put into studies of the problems associated with such a treatment of the many body problem.[4-7]

Up to the present, essentially two different methods have been used to derive the baryon-baryon potentials. In the older one,[8] Feynman techniques are used to determine the *M*-matrices for the relevant processes. The potentials are then given by the Fourier transform of the nonrelativistic approximation of the *M* matrices. The second method for determining the transition part of the resonance potentials has been given by Durso *et al.*[9] The main idea of this method is to identify parts of a two-meson exchange graph with an intermediate resonance with a twice iterated one meson-exchange process. Such a procedure leads to a transition potential with a pion propagator which is different compared to the usual Feynman propagator. This then leads to a change in the strength (range) of the pion exchange potential. Essentially the strength of the potential is reduced this way. In the deuteron, for example, it has been shown that those potentials predict a very small Δ resonance probability[10,11]—too small to be measured at present. This shows that the identification of the potential is actually an essential point; however, it is not yet solved completely.

In the present paper we generalize our concept developed for the description of meson-exchange currents in external interactions with nuclei.[12] We describe the nucleus by a system of mesons and baryons, i.e., meson and nucleon resonances: π, $\rho,\ldots$ and N, N^*, $\Delta,\ldots$. Starting from a Schrödinger equation with a field-theoretical Hamiltonian for such a composite system of particles we apply a perturbation treatment in terms of the meson-baryon coupling. In lowest order approximation this corresponds to a generalization of the common concept of one-boson exchange (OBE) for the *N*-*N* interaction (Bonn potential,[2] for example), to the more general case of one-boson-exchange baryon-baryon interaction.

The general potential for the baryon-baryon interaction which we obtain is well defined in the present concept. The same is true for the meson-exchange currents concerning external interactions with the nucleus. In Sec. II we develop our concept of describing the nucleus by a meson-baryon system. In Sec. III we give an explicit application of our method to the determination of baryon-baryon potentials in the nonrelativistic limit involving only nucleons and deltas. Different approximations for the meson-propagators are discussed. Concerning the nucleon-nucleon interaction we took as a working concept the Reid-soft-core potential including a complete new fit to take care of the resonance degrees of freedom. The coupled equations for the scattering states and the deuteron have been solved exactly using the complete baryon-baryon interaction in the OBE limit. The deuteron $\Delta\Delta$ contribution is discussed in several limits of the general OBE potential.

II. THE NUCLEUS AS A MESON-BARYON SYSTEM

We describe the nucleus by a system of mesons and baryons (i.e., nucleons and nucleon resonances). For such a system the Schrödinger equation can be written as

$$H\Psi_i = (E_i + AM)\Psi_i \,, \tag{1}$$

where the Hamiltonian is given by

$$H = H_0 + H_I \,; \quad H_I = \sum_{i=1}^{A} H_I(i) \,. \tag{2}$$

Here H_0 denotes the operator of the kinetic energy and H_I the field theoretical baryon-baryon meson interaction operator between the particles of the system. $(E_i + AM)$ denotes the total energy of the system. A is the baryon-number of the nucleus and M the mass of a nucleon.

Defining by a_p, $a_p^\dagger$, α_p, $\alpha_p^\dagger$, b_q, $b_q^\dagger$ (where p,q denote the properties of the particles, i.e., momentum, spin, ...) operators which destroy and create nucleons, resonances, and mesons, respectively, the operators for kinetic and interaction energy can be expressed as

$$H_0 = \sum_{p,q} (E_p^N a_p^\dagger a_p + E_p^{N^*} \alpha_p^\dagger \alpha_p + \omega_q b_q^\dagger b_q) \,, \tag{3}$$

where the energies are given by

$$\begin{aligned} E_p^N &= \langle N(p)|H_0|N(p)\rangle = (p^2 + M^2)^{1/2} \,, \\ E_p^{N^*} &= \langle N^*(p)|H_0|N^*(p)\rangle = (p^2 + M^{*2})^{1/2} \,, \\ \omega_q &= \langle \mu(q)|H_0|\mu(q)\rangle = (q^2 + \mu^2)^{1/2} \,. \end{aligned} \tag{4}$$

M, M^*, and μ denote the masses of the nucleons, resonances, and mesons, respectively.

The interaction operator can be written as

$$\begin{aligned} H_I(i) = \sum_{\substack{\alpha,\beta,\gamma \\ \mu, N^*}} [&\tilde{\Gamma}_{N_\alpha \to N_\beta + \mu_\gamma}(i) b_\gamma^\dagger a_\beta^\dagger a_\alpha + \tilde{\Gamma}_{N_\alpha \to N_\beta^* + \mu_\gamma}(i) b_\gamma^\dagger \alpha_\beta^\dagger a_\alpha \\ &+ \tilde{\Gamma}_{N_\alpha^* \to N_\beta^* + \mu_\gamma}(i) b_\gamma^\dagger \alpha_\beta^\dagger \alpha_\alpha \\ &+ \tilde{\Gamma}_{N_\alpha^* \to N_\beta + \mu_\gamma}(i) b_\gamma^\dagger a_\beta^\dagger \alpha_\alpha] + \text{H.c.} \,, \end{aligned} \tag{5}$$

where the amplitudes $\tilde{\Gamma}_{B_\alpha \to B_\beta + \mu_\gamma}$ are defined by

$$\begin{aligned} \tilde{\Gamma}_{B_\alpha \to B_\beta + \mu_\gamma}(i) &= \langle B_\beta(p_\beta)\mu_\gamma(q_\gamma)|H_I(i)|B_\alpha(p_\alpha)\rangle \\ &= \langle B_\beta(p_\beta)\mu_\gamma(q_\gamma)|\Gamma_{B \to B+\mu}(i)|B_\alpha(p_\alpha)\rangle \,, \end{aligned} \tag{6}$$

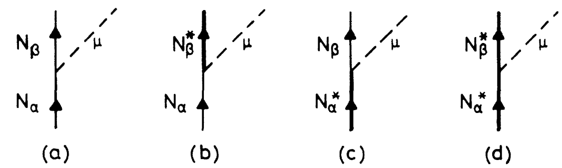

FIG. 1. Diagrammatical description of the $B_\alpha \to B_\beta + \mu$ amplitudes contributing to the interaction H_I Eq. (5).

where $p_\alpha, p_\beta, q_\gamma$ denote the momenta of the particles. B stands for nucleons (N) and resonances (N^*) (see Fig. 1).

As we are going to reduce Eq. (1) by a perturbation treatment[13] of lowest order in H_I, we directly consider physical masses and coupling constants. This is in accord with the widely used one-boson-exchange treatment for the nucleon-nucleon interaction which will be obtained as a special case of our more general treatment. The reason for considering physical masses and couplings at this stage is taken for practical purposes only, as it simplifies the discussion considerably. For the same reason we do not consider a presentation of the meson-baryon form factors, but rather take a phenomenological approach. We have to realize, however, that this is not necessary. In higher orders of H_I, however, one has to be wary about possible double counting problems.

Equation (1) describes a highly complicated system of mesons and baryons. Therefore it seems reasonable to consider a solution of the problem in a subspace.[13] Here one has several possibilities. The most common one is to consider a subspace of *the nucleons only*. This leads to the conventional treatment of nuclear properties by a Schrödinger equation for the nucleonic states. Another possibility is to consider a subspace of baryons, i.e., *nucleons and nucleon resonances*. Still another possibility, which has been considered in the past is to restrict on a subspace *without mesons*. This then leads to a coupled Schrödinger equation of baryons and antibaryons. Such an approach would be related to the approach of Gross who considered coupled equations for nucleons and antinucleons.[14]

In the present paper we are mainly interested in the connections of the first two mentioned approaches, namely, (i) subspace of *nucleons only* and (ii) subspace of *baryons*, i.e., nucleons and nucleon resonances. The formalism,[13] however, is general enough to allow for still other possibilities. The main aspect of the present treatment will be to see how both ways are related. Concerning the interaction of an external field with our system, the above treatment of the full Schrödinger equation leads to the introduction of meson-exchange currents. These are, of course, different in both approaches. For a discussion on this point we refer to Ref. 12.

Let us formulate the solution of Eq. (1) by dividing the total Hilbert space (baryons, antibaryons, and mesons) into two subspaces.[13] We introduce projection operators η and Λ corresponding to these two subspaces (later we shall discuss different physical possibilities for η and Λ):

$$\begin{aligned} &1=\eta+\Lambda, \quad \Lambda\eta=\eta\Lambda=0, \\ &\Lambda^2=\Lambda, \quad \eta^2=\eta, \quad [\eta, H_0]_-=0. \end{aligned} \tag{7}$$

According to the projection operators η and Λ we can separate our total state Ψ_i of Eq. (1) as follows:

$$\Psi_i=\eta\Psi_i+(1-\eta)\Psi_i=\eta\Psi_i+\Lambda\Psi_i \tag{8}$$

or

$$\Psi_i=\phi_i^{\,1}+\phi_i^{\,2} \quad .$$

Instead of Eq. (1) we now have a matrix equation for the states $\phi_i^{\,1}$ and $\phi_i^{\,2}$, namely,

$$\begin{pmatrix} \eta H\eta & \eta H\Lambda \\ \Lambda H\eta & \Lambda H\Lambda \end{pmatrix} \begin{pmatrix} \phi_i^{\,1} \\ \phi_i^{\,2} \end{pmatrix} = (E_i+AM) \begin{pmatrix} \phi_i^{\,1} \\ \phi_i^{\,2} \end{pmatrix} . \tag{9}$$

In principle, one can calculate the states $\phi_i^{\,1}$, $\phi_i^{\,2}$ and thereby Ψ_i on the basis of this matrix equation. However, the states $\phi_i^{\,1}$ and $\phi_i^{\,2}$ are still coupled. We decouple these states by performing a unitary transformation U which diagonalizes the Hamiltonian matrix of Eq. (9) (more details can be found in Ref. 13.):

$$\begin{pmatrix} \phi_i^{\,1} \\ \phi_i^{\,2} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \chi_i \\ \varphi_i \end{pmatrix} . \tag{10}$$

The matrix elements U_{ij} of U can be expressed by an operator F which connects space η with space Λ:

$$\begin{aligned} &F=\Lambda F\eta, \quad F^\dagger=\eta F^\dagger\Lambda, \\ &U_{11}=(1+F^\dagger F)^{-1/2}, \quad U_{12}=-F^\dagger U_{22}, \\ &U_{21}=FU_{11}, \quad U_{22}=(1+FF^\dagger)^{-1/2}, \end{aligned} \tag{11}$$

The operator F is determined by the diagonalization condition for the matrix of Eq. (9), i.e., F has to be a solution of the following equation:

$$\Lambda\left[H+[H,F]_- - FHF\right]\eta=0. \tag{12}$$

Note that F is independent of energy. A solution for F is given by

$$\begin{aligned} &F=\sum_{i=1}^{A} F_i+\sum_{i\neq j}F_{ij}+O(H_I^{\,3}), \\ &F_i=\frac{\Lambda}{e}H_I(i)\eta+\frac{\Lambda}{e}H_I(i)\frac{\Lambda}{e}H_I(i)\eta, \\ &F_{ij}=\frac{\Lambda}{e}H_I(i)\frac{\Lambda}{e}H_I(j)\eta, \end{aligned} \tag{13}$$

if we restrict ourselves to second order processes in H_I. The denominator e is given by

$$e=\epsilon-H_0, \tag{14a}$$

where ϵ denotes the starting energy. We choose the free energy of the asymptotically free particles, namely,

$$\epsilon=\sum_{n=1}^{A}(E_n^N+E_n^{N*})+\sum\omega_\mu . \tag{14b}$$

Note that this is a necessary boundary condition. For all cases under consideration in this paper we have

$$\epsilon=\sum_{n=1}^{A}E_n^N . \tag{14c}$$

The components χ_i and φ_i of the total state are determined by the following equations:

$$(\eta+F^\dagger F)^{-1/2}(\eta+F^\dagger)H(\eta+F)(\eta+F^\dagger F)^{-1/2}\chi_i=(E_i+AM)\chi_i , \tag{15}$$

$$(\Lambda+FF^\dagger)^{-1/2}(\Lambda-F)H(\Lambda-F^\dagger)(\Lambda+FF^\dagger)^{-1/2}\varphi_i = (E_i+AM)\varphi_i . \tag{16}$$

For states with no asymptotically free mesons, the component φ_i can be set equal to zero. The total state Ψ_i can then be expressed in terms of χ_i by

$$\Psi_i=(\eta+F)(\eta+F^\dagger F)^{-1/2}\chi_i . \tag{17a}$$

Up to second order in H_I we have

$$\Psi_i=(\eta+F-\tfrac{1}{2}F^\dagger F)\chi_i . \tag{17b}$$

This relation we shall use in the following. For other purposes than those considered in this paper, a solution of Eq. (16) might be appropriate. An example for such a case would be pion scattering.

In our further treatment we aim for a solution of Ψ_i on the basis of the relation Eqs. (15) and (17). We shall calculate the component χ_i by a differential equation and treat the operators F, $F^\dagger$ in an expansion in powers of the interaction H_I. As long as we restrict ourselves to the one-boson exchange limit, the second order solution for F given in Eq. (13) is sufficient.

A. Nonrelativistic approximation

In the following we shall consider the nonrelativistic approximation for two cases:

(I) projection η on nucleons only;

(II) projection η on nucleons and nucleon resonances.

In both cases we have the following approximation for the energy of the baryons:

$$\langle\alpha|H_0|\alpha\rangle=E_\alpha=(p_\alpha^{\,2}+M_\alpha^{\,2})^{1/2}\approx M_\alpha+p_\alpha^{\,2}/2M_\alpha . \tag{18a}$$

For the meson energy we take the full energy

$$\omega_q = (q^2 + \mu^2)^{1/2}. \tag{18b}$$

In terms of the operators F, $F^\dagger$ the equation of state, Eq. (15), can be expressed for both cases I and II by

$$(T + H_{intr} + V_{eff})\chi_i = E_i \chi_i. \tag{19}$$

T is the kinetic energy

$$T = \sum_{i=1}^{A} T_i, \quad T_i = \frac{p_i^2}{2M_i}, \tag{20}$$

and H_{intr} is the intrinsic Hamiltonian,

$$H_{intr} = \sum_{i=1}^{A} H_{intr}(i), \quad H_{intr}(i) = M_i - M. \tag{21}$$

(M is the nucleon mass.) The effective interaction V_{eff} in space η is given by

$$V_{eff} = (\eta + F^\dagger F)^{-1/2}(\eta + F^\dagger)H(\eta + F)(\eta + F^\dagger F)^{-1/2} - \eta H_0 \eta. \tag{22}$$

In general this is a many-body interaction (i.e., two-, three-,... body force).

For the cases I and II the operators F are different, therefore the effective interaction Eq. (22) takes a different form in these cases.

B. One boson exchange limit (OBE)

In the lowest order of meson-baryon interaction this can be summarized as follows.

Case I: *nucleons only* (Hilbert space $\mathcal{H}_\eta$ of nucleons only, conventional treatment).

The interaction is given by Eq. (22); $H_{intr} = 0$. In the lowest order in H_I the operator F has the form

$$F = \sum_{i=1}^{A} F_i,$$

$$F_i = \frac{\Lambda}{e} H_I(i)\eta$$

$$= \sum_{\mu, N^*} \left(\frac{\Lambda}{e} \Gamma_{N \to N+\mu}(i)\eta + \frac{\Lambda}{e} \Gamma_{N \to N^*+\mu}(i)\eta \right). \tag{23}$$

The summation includes the mesons (μ) and the nucleon resonances (N^*). In the one-boson exchange limit the second term in Eq. (23) does not contribute to the effective interaction V_{eff}. The interaction V_{eff} is given by

$$V_{eff} = V_{eff}^{NN} = \sum_{i<j} V_{ij}^{NN}(\text{OBE}), \tag{24}$$

$$V_{ij}^{NN}(\text{OBE}) = \sum_{\mu} \eta \Gamma_{N+\mu \to N}(i) \frac{\Lambda}{e} \Gamma_{N \to N+\mu}(j)\eta. \tag{25}$$

This is the common well-known OBE potential between nucleons. The OBE part contains no nucleon-resonance contributions explicitly (Fig. 2). Such contributions occur in the two-meson exchange part of the interaction. We discuss those contributions only diagrammatically (see Fig. 3). The explicit form of the two-meson-exchange part of the potential can be obtained from Eqs. (12) and (22).

Case II: *nucleons and isobars* (Hilbert space $\mathcal{H}_\eta$ of nucleons and resonances).

The interaction is given by Eq. (22). Note that $H_{intr} \neq 0$. In lowest order of meson-baryon inter-

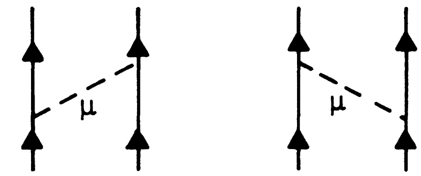

FIG. 2. Contributions to the effective nucleon-nucleon potential in the one-boson-exchange limit for case I, i.e., only nucleons in the subspace $\mathcal{H}_\eta$.

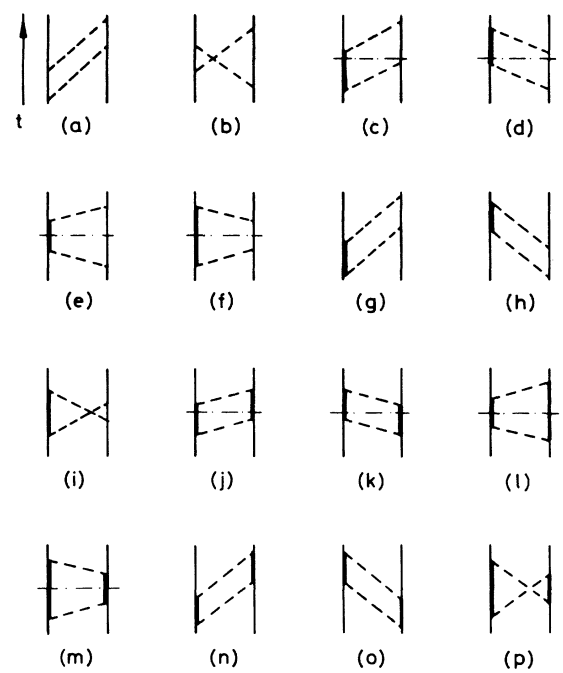

FIG. 3. Two-meson-exchange contributions to the effective nucleon-nucleon interaction for case I, only nucleons in the subspace $\mathcal{H}_\eta$. Not all contributions are shown but only the different types of processes. The total two-meson exchange potential is given by Eqs. (12) and (22). Note the different types of intermediate states. We have intermediate NN^* and N^*N^* contributions (c)–(f) and (j)–(m) as well as $NN^*\mu$ and $N^*N^*\mu$ contributions.

action the operator F is given as

$$F=\sum_{i=1}^{A}F_i\,, \tag{26}$$

$$\begin{aligned}F_i&=\frac{\Lambda}{e}H_I(i)\eta\\&=\sum_{\mu,\alpha,\beta}\left\{\frac{\Lambda}{e}\Gamma_{N_\alpha\to N_\beta+\mu}(i)\eta+\frac{\Lambda}{e}\Gamma_{N_\alpha\to N^*_\beta+\mu}(i)\eta\right.\\&\qquad\left.+\frac{\Lambda}{e}\Gamma_{N^*_\alpha\to N_\beta+\mu}(i)\eta+\frac{\Lambda}{e}\Gamma_{N^*_\alpha\to N^*_\beta+\mu}(i)\eta\right\}, \end{aligned}\tag{27}$$

or

$$F_i=\sum_{\mu,B,B'}\frac{\Lambda}{e}\Gamma_{B\to B'+\mu}(i)\eta\,, \tag{28}$$

where B,B' sums on nucleons and isobars.

Again the effective interaction is a sum of one-boson, two-boson,... exchange parts:

$$V_{\text{eff}}=V_{\text{eff}}^{BB'}=\sum_{i<j}V_{ij}^{BB'}(\text{OBE})+\cdots. \tag{29}$$

V_{ij} is given by (Fig. 4):

$$\begin{aligned}V_{ij}^{BB'}&=\sum_{\substack{B_\alpha B_\beta\\B'_\alpha B'_\beta}}V_{ij}^{B_\alpha B_\beta\to B'_\alpha B'_\beta}\\&=V_{ij}^{NN\to NN}+V_{ij}^{NN\to NN^*}+V_{ij}^{NN\to N^*N^*}+V_{ij}^{NN^*\to NN^*}\\&\quad+V_{ij}^{N^*N\to NN^*}+V_{ij}^{NN^*\to N^*N^*}+V_{ij}^{N^*N^*\to N^*N^*}\,.\end{aligned}\tag{30}$$

Note that compared to case I the nucleon-nucleon part of Eq. (30) ($V^{NN\to NN}$) is different. The reason is that in the present description contributions such as Figs. 3(c)–3(f) and 3(j)–3(m), which contribute to the two meson exchange part in case I, are now taken into account via the wave function.

The individual contributions to the effective baryon-baryon interaction [Eq. (30)] are given as follows:

$$\begin{aligned}V_{ij}^{B_\alpha B_\beta\to B'_\alpha B'_\beta}=\sum_\mu\Big(&\eta\Gamma_{B_\alpha+\mu\to B'_\alpha}(i)\frac{\Lambda}{e}\Gamma_{B_\beta\to B'_\beta+\mu}(j)\eta\\&+\eta\Gamma_{B_\beta+\mu\to B'_\beta}(j)\frac{\Lambda}{e}\Gamma_{B_\alpha\to B'_\alpha+\mu}(i)\eta\Big)\end{aligned}\tag{31}$$

with

$$e=\epsilon-H_0\,.$$

We see that this potential corresponds to a straightforward generalization of the OBE nucleon-nucleon potential to the case of a OBE baryon-baryon interaction. In the OBE limit the NN-$N\Delta$ and NN-$\Delta\Delta$ transition potentials are exactly those of Durso *et al.*[9] However, the potential Eq. (31) is more general in the sense that it describes *all* baryon-baryon transitions in the OBE limit,

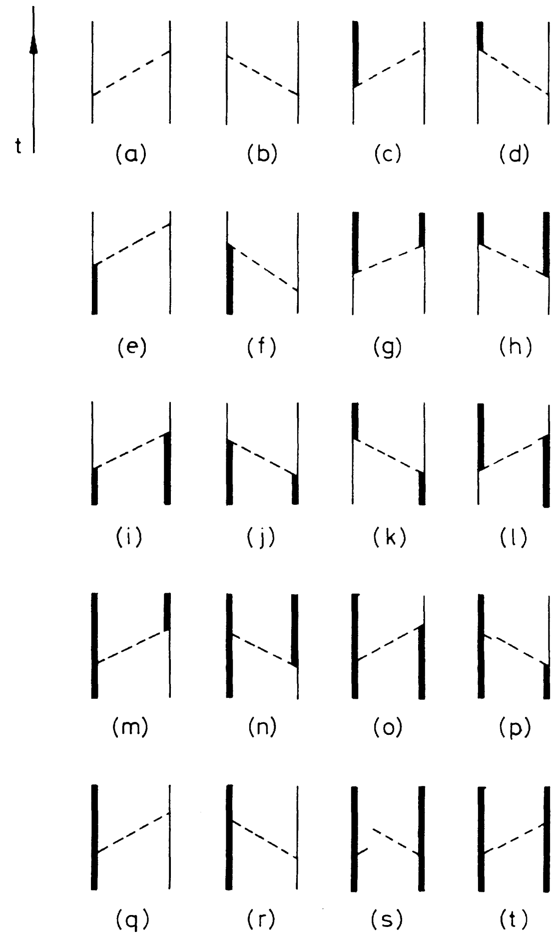

FIG. 4. One-boson-exchange contributions to the effective baryon-baryon potential in case II for a subspace $\mathcal{H}_\eta$ containing baryons only, i.e., nucleons and resonances, but no mesons.

i.e., not only the NN-$N\Delta$ and NN-$\Delta\Delta$ transition but also the transitions such as $N\Delta$-$N\Delta$, $N\Delta$-$\Delta\Delta$, $\Delta\Delta$-$\Delta\Delta$, ..., etc.

The two-boson-exchange part of the interaction is determined by Eq. (22), together with solutions of Eq. (12) up to third order in H_I. The individual contributions are shown diagrammatically in Fig. 5.

The above treatment of the eigenvalue problem Eq. (1) corresponds to a renormalized Tamm-Dancoff (TD) approach.[13] The χ_i are renormalized TD states ($\varphi\equiv 0$). As those states were obtained from the total states Ψ_i by a unitary transformation, the corresponding Hamiltonian Eq. (22) is Hermitian. Consequently, different states χ_i are mutually orthogonal and normalized according to

$$\langle\Psi_i|\Psi_j\rangle=\delta_{ij}=\langle\chi_i|U^\dagger U|\chi_j\rangle=\langle\chi_i|\chi_j\rangle\,. \tag{32}$$

In the following we shall discuss the connections

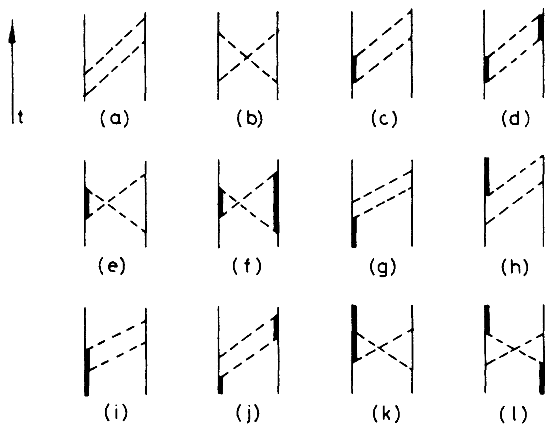

FIG. 5. Two-boson-exchange contributions to the effective baryon-baryon interaction of case II (subspace $\mathcal{H}_\eta$ containing baryons only, i.e., nucleons and resonances but no mesons). Only the different types of the contributions are shown. The total two-meson-exchange contribution is determined by Eqs. (12) and (22).

to the unrenormalized TD states.

The total solution of Eq. (1) had been expressed as

$$\Psi_i = \begin{pmatrix} \phi_i^1 \\ \phi_i^2 \end{pmatrix} = U \begin{pmatrix} \chi_i \\ \varphi_i \end{pmatrix} , \tag{33}$$

where U is the unitary transformation Eqs. (10) and (11). In the TD approach we have $\varphi_i \equiv 0$, therefore the connection between the states ϕ_i^1, ϕ_i^2, and χ_i are given by the following relation [see Eq. (11)]:

$$\begin{aligned} \phi_i^1 &= U_{11}\chi_i , \\ \phi_i^2 &= U_{21}\chi_i = F\phi_i^1 . \end{aligned} \tag{34}$$

In the unrenormalized TD approach, the total state is expressed in terms of ϕ_i^1 (and not in terms of the normalized states χ_i). In that case we have

$$\Psi_i = \phi_i^1 + \phi_i^2 = (1+F)\phi_i^1 . \tag{35}$$

Note that this transformation $\Psi \to \phi^1$ is not unitary. The relation Eq. (35) has to be compared with the one of Eq. (17) for the normalized states χ_i.

The equation of state for ϕ_i^1 is given by

$$\eta H(\eta + F)\phi_i^1 = (E_i + AM)\phi_i^1 \tag{36}$$

or

$$(T + H_{\text{intr}} + V_{\text{eff}})\phi_i^1 = E_i \phi_i^1 . \tag{37}$$

T and H_{intr} are the kinetic and intrinsic parts of the Hamiltonian [see Eqs. (20) and (21)]. The effective baryon-baryon interaction is given by

$$V_{\text{eff}} = \eta H_I F \eta , \tag{38}$$

where F is a solution of Eq. (12) as before.

This interaction has to be compared with the one of Eq. (22) for the renormalized state χ_i. In contrast to the interaction of Eq. (22) for the renormalized states we have now an interaction which is generally *not* Hermitian. The reason for that nonhermiticity can be found in the fact that the transformation between the total state Ψ_i and the unrenormalized state ϕ_i^1 is not unitary [compare Eq. (35)]. A direct consequence of the use of such a nonunitary transformation is the nonorthogonality of different states ϕ_i^1:

$$\begin{aligned} \langle \Psi_i | \Psi_j \rangle = \delta_{ij} &= \langle \phi_i^1 | (\eta + F^\dagger)(\eta + F) | \phi_j^1 \rangle \\ &= \langle \phi_i^1 | \phi_j^1 \rangle + \langle \phi_i^1 | F^\dagger F | \phi_j^1 \rangle . \end{aligned} \tag{39}$$

As the last term on the right-hand side is not equal to zero, the nonorthonormality is directly seen.

In order to avoid the problems arising from the use of non-Hermitian potentials and nonorthogonalities in the baryonic states we shall always use the renormalized amplitudes χ_i, i.e., we consider a calculation of the total state Ψ_i by the use of the unitary relation Eq. (17). Considering a phenomenological approach to the nucleon-nucleon part of the baryon-baryon interaction the use of the relation seems to be necessary.

C. External interactions and resonance probabilities

The interaction of our system with an external field[12,16] as well as the total resonance probability is defined through the total state Ψ_i and not by the states χ_i or ϕ_i^1. Denoting by O the operator of an external field, the interesting transition matrix is given by[12]

$$T_{fi} = \langle \Psi_f | O | \Psi_i \rangle \tag{40}$$

or, as we calculate the states Ψ_i by the relation Eq. (17),

$$\begin{aligned} T_{fi} &= \langle \chi_f | (\eta + F^\dagger - \tfrac{1}{2}F^\dagger F) O (\eta + F - \tfrac{1}{2}F^\dagger F) | \chi_i \rangle \\ &= \langle \chi_f | O_{\text{eff}} | \chi_i \rangle \end{aligned} \tag{41}$$

with

$$\begin{aligned} O_{\text{eff}} &= (\eta + F^\dagger - \tfrac{1}{2}F^\dagger F) O (\eta + F - \tfrac{1}{2}F^\dagger F) \\ &= O_{\text{eff}}(1) + O_{\text{eff}}(\text{many body}). \end{aligned} \tag{42}$$

O_{eff} denotes the effective interaction of the external field in space η. The many-body part of that operator defines what is usually called meson-exchange currents. For a discussion on this point we refer to Ref. 12.

Note that the effective interaction O_{eff} depends on how we calculate our total state Ψ_i. Choosing a different transformation we obtain different exchange currents. We exemplify this in the case of unrenormalized states ϕ_i^1. Choosing instead of Eq. (17) the nonunitary transformation of Eq. (35), the transition matrix T_{fi} is given by

$$T_{fi}=\langle\Psi_f|O|\Psi_i\rangle =\langle\phi_f^1|(\eta+F^\dagger)O(\eta+F)|\phi_i^1\rangle . \quad (43)$$

The effective operator of the external field is then given by

$$O_{\rm eff}=(\eta+F^\dagger)O(\eta+F) . \quad (44)$$

The differences of Eqs. (42) and (44) are given by the renormalization contribution

$$-\tfrac{1}{2}\{F^\dagger F, O\}_+ .$$

In the static limit of the baryons, for example, this contribution cancels exactly the recoil contribution contained in $F^\dagger OF$. In the case of unrenormalized states ϕ_i^1, where there is no renormalization contribution to $O_{\rm eff}$ the recoil contribution will fully survive. Of course the T matrix has to be independent of the choice of transformation. This means that the effect of the nonorthonormality of the states ϕ_i^1 is of the order of the recoil contribution. This contribution has been shown to be non-negligible in Ref. 15. To emphasize this point: In the case of considering renormalized amplitudes we have recoil and wave function renormalization contributions (they cancel in the static limit); in the case of the nonrenormalized states we have a recoil contribution which survives also in the static limit.

Similar to the case of external interactions, the resonance probability is *not* simply given by the states χ_i or ϕ_i^1 but by the total state Ψ_i.

Denoting by O a projection operator on certain resonances of interest, the probability for finding such resonances in our system is also given by the relations Eqs. (42) and (44). Exactly, as in the case of meson-exchange currents we obtain different expressions for that probability. In the present paper we use throughout renormalized states, so the expression for calculating resonance probabilities is the one of Eq. (42). Note that this probability is the total one, i.e., it also includes the contributions from the mesonic part of the total state.

III. APPLICATION

A. Baryon-baryon potentials involving nucleons and deltas

In this section we give the expressions for the baryon-baryon interaction Eq. (31) in coordinate space. We consider only nucleons and deltas (Δ 1236) and the exchange of π and ρ. The one pion exchange will be discussed in detail concerning several approximations of the pion propagator. For ρ exchange we give the final results for the total potential.

We assume the baryon-baryon-pion coupling to be of the form (Fig. 1)

$$\Gamma_{B_\alpha B_\beta \pi}=\frac{g_{B_\alpha B_\beta \pi}}{2M}\frac{1}{(2\omega)^{1/2}}(\vec{\sigma}_{B_\alpha B_\beta}\cdot\vec{q})\vec{\tau}_{B_\alpha B_\beta}\cdot\vec{\pi} \quad (45)$$

with

$$B_\alpha=N, \Delta , \quad \omega=(q^2+\mu^2)^{1/2} ,$$

where q denotes the momentum of the pion and $\vec{\pi}$ the pion state. The spin and isospin operators are defined by the following reduced matrix elements:

$$\begin{aligned}\langle\tfrac{1}{2}\|\sigma_{NN}\|\tfrac{1}{2}\rangle&=\langle\tfrac{1}{2}\|\tau_{NN}\|\tfrac{1}{2}\rangle=\sqrt{6} ,\\ \langle\tfrac{3}{2}\|\sigma_{\Delta N}\|\tfrac{1}{2}\rangle&=\langle\tfrac{3}{2}\|\tau_{\Delta N}\|\tfrac{1}{2}\rangle=2 ,\\ \langle\tfrac{3}{2}\|\sigma_{\Delta\Delta}\|\tfrac{3}{2}\rangle&=\langle\tfrac{3}{2}\|\tau_{\Delta\Delta}\|\tfrac{3}{2}\rangle=2\sqrt{15} .\end{aligned} \quad (46)$$

We discuss the explicit form of the baryon-baryon interaction in the static limit; i.e., we use the approximation $(p_i^2+m_i^2)^{1/2}\approx m_i$ for the in and out particles (nucleons and deltas). Within this approximation we obtain directly from Eqs. (31) and (45) the following form for the interaction $B_\alpha B_\beta \to B'_\alpha B'_\beta$ involving the exchange of a single pion:

$$V_\pi^{B_\alpha B_\beta\to B'_\alpha B'_\beta}=-g_{B_\alpha B'_\alpha \pi}g_{B_\beta B'_\beta \pi}\cdot(\vec{\sigma}_{B_\alpha B'_\alpha}\cdot\vec{q})(\vec{\sigma}_{B_\beta B'_\beta}\cdot\vec{q}) \times(\vec{\tau}_{B_\alpha B'_\alpha}\cdot\vec{\tau}_{B_\beta B'_\beta})f_{B_\alpha B_\beta\to B'_\alpha B'_\beta}(q) . \quad (47)$$

$B_\alpha, \ldots, B'_\beta$ denote nucleons or delta resonances. The propatator function $f(q)$ has the following form for the different processes $B_\alpha B_\beta \to B'_\alpha B'_\beta$. Note that these expressions are also valid for heavier meson exchange (compare Fig. 6):

$$f_{NN\to NN}=\frac{1}{\omega^2} , \quad (48)$$

$$f_{NN\to N\Delta}=\frac{1}{2\omega}\left(\frac{1}{\omega}+\frac{1}{\omega+\Delta}\right), \quad (49)$$

$$f_{NN\to\Delta\Delta}=\frac{1}{\omega}\frac{1}{\omega+\Delta}, \quad (50)$$

$$f_{N\Delta\to N\Delta}=\frac{1}{\omega}\frac{1}{\omega+\Delta}, \quad (51)$$

$$f_{N\Delta\to\Delta N}=\frac{1}{2\omega}\left(\frac{1}{\omega}+\frac{1}{\omega+2\Delta}\right), \quad (52)$$

$$f_{\Delta\Delta\to N\Delta}=\frac{1}{2\omega}\left(\frac{1}{\omega+\Delta}+\frac{1}{\omega+2\Delta}\right), \quad (53)$$

$$f_{\Delta\Delta\to\Delta\Delta}=\frac{1}{\omega}\frac{1}{\omega+2\Delta}. \quad (54)$$

Δ denotes the mass difference of delta and nucleon: $\Delta=M_\Delta-M$. In Fig. 6 we summarize the relation of the propagator function to the individual processes.

While the full potential Eq. (47) is valid only for the π (or φ) exchange, the propagator functions $f(q)$ [Eqs. (48)–(54)] are more general. They are

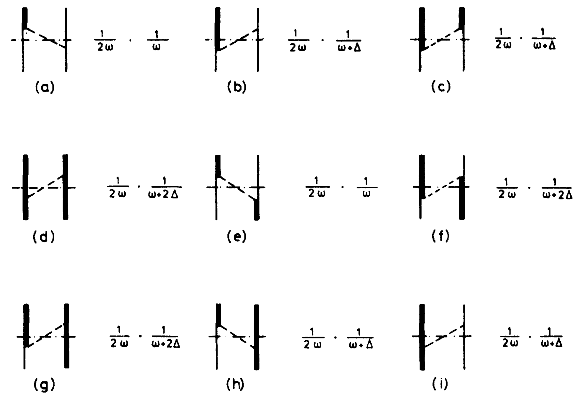

FIG. 6. Propagator functions $f_{B_\alpha B_\beta \to B_\alpha' B_\beta'}$ (static limit) for the different Δ-resonance contributions to the one-boson exchange potential of case II (Fig. 4). $\omega=(q^2+\mu^2)^{1/2}$ denotes the energy of the exchanged meson (π or ρ ...) and $\Delta=M_\Delta-M_N$ the mass difference between delta and the nucleon.

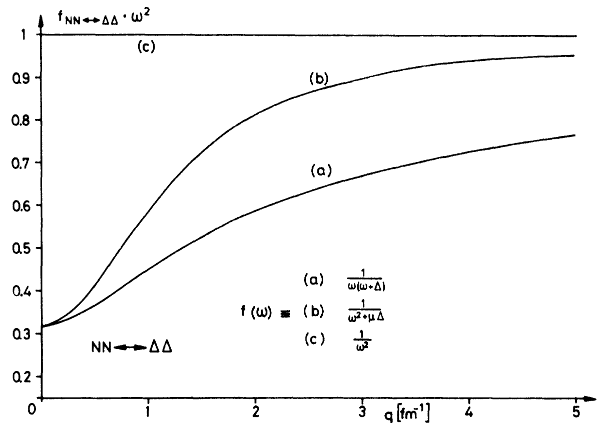

FIG. 7. Comparison of the different approximations of the propagator function $f(\omega)$ in the case of $NN \leftrightarrow \Delta\Delta$ interaction. Note that $f\times\omega^2$ is shown, $\omega=(q^2+\mu^2)^{1/2}$ is the energy and μ the mass of the exchanged meson (π, ρ, . . .) under consideration.

valid also for the exchange of the heavier mesons if ω denotes the energy of the exchanged particle.

We note that our formalism gives an easy prescription for the interaction between nucleons thereby creating a delta, however, also for the interaction between resonances. It is a simple matter to reproduce from Eq. (47) various potential approximations which are popular in the literature. In the following we shall discuss several baryon-baryon potentials in comparison with approximations which are common in the literature.

1. NN-ΔΔ

Let us first consider the transition potential NN-ΔΔ and discuss a series of approximations to Eq. (50).

$$f^{(a)}_{NN\leftrightarrow\Delta\Delta}=\frac{1}{\omega(\omega+\Delta)}, \tag{55}$$

$$f^{(b)}_{NN\leftrightarrow\Delta\Delta}=\frac{1}{\omega^2+\mu\Delta}, \tag{56}$$

$$f^{(c)}_{NN\leftrightarrow\Delta\Delta}=\frac{1}{\omega^2}. \tag{57}$$

Equation (55) is the exact expression of the propagator function in the static limit as given in Eq. (50). This is exactly the form derived by Durso *et al.*[9] in a perturbation approach to the two-pion exchange. [Compare the discussion following Eq. (32)]. The form Eq. (56) is an approximation to Eq. (55) as given by Durso *et al.* Approximation Eq. (57) arises from the neglect of the mass difference Δ between nucleon and resonance. This is the common OBE form.[8] Since the Fourier transformation for case (*a*) cannot be done easily by analytic integration, we evaluated the integrals numerically.

In Fig. 7 we discuss the different expressions relative to the usual OBE result. To this respect we plotted the function $f\times\omega^2$ for the expressions (*a*)–(*c*) of Eqs. (55)–(57). Curve (*c*), i.e., $\omega^2 f=(1/\omega^2)\times\omega^2$, corresponds to the Feynman OBE result. Curve (*b*) shows the quality of approximation Eq. (56). We realize that both approximations (*b*) and (*c*), respectively, are not too good.

In addition to the q dependence of the function f the potential Eq. (47) still contains a q^2 dependence from the baryon-meson couplings. This one has to realize when the explicit form of the potential is discussed. In order to see how the differences in the function $f_{B_\alpha B_\beta \to B_\alpha' B_\beta'}$ affect the total potential we give its form in coordinate space (see Figs. 8 and 9). As expected from Fig. 7 the approximations (*b*) and (*c*) are not reasonable.

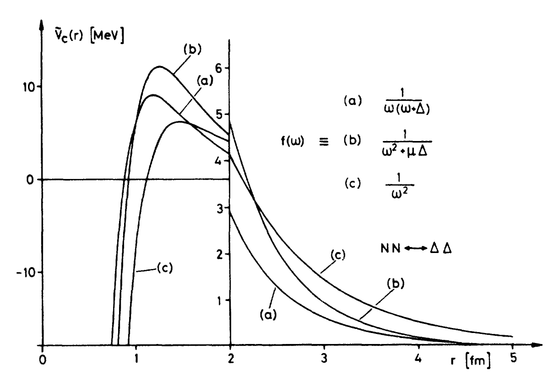

FIG. 8. Radial dependence $\tilde V_C(r)$ of the central part of the $NN \leftrightarrow \Delta\Delta$ interaction for the different approximations (*a*)–(*c*) in the propagator function $f(\omega)$. (π exchange.)

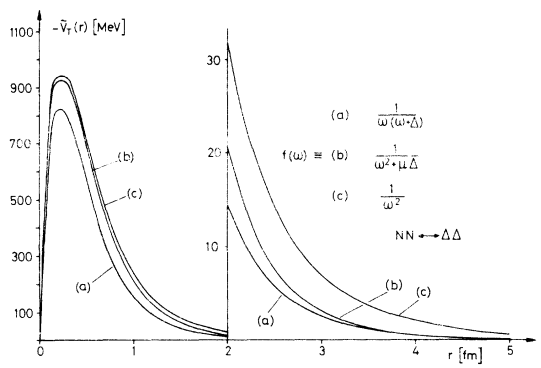

FIG. 9. Radial dependence $\tilde{V}_T(r)$ of the tensor part of the $NN \longleftrightarrow \Delta\Delta$ interaction for the different approximations (a)–(c) in the corresponding propagator function $f(\omega)$. (π exchange.)

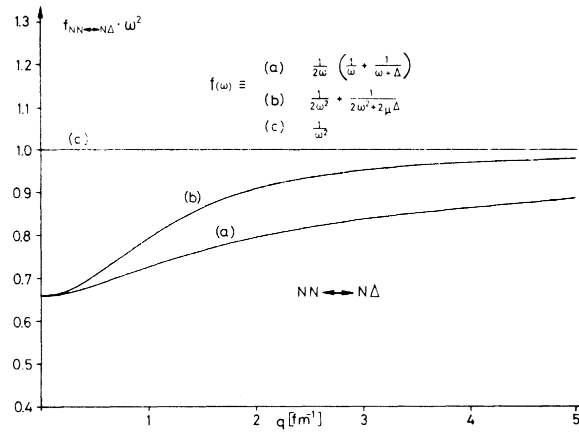

FIG. 10. Comparison of the different approximations of the propagator function $f(\omega)$ in the case of $NN \longleftrightarrow N\Delta$ interaction corresponding to the processes Figs. 6(a) and 6(b). Note that the curves are given for $f\times\omega^2$. $\omega = (q^2+\mu^2)^{1/2}$ denotes the energy and μ the mass of the exchanged particle ($\pi,\rho,\ldots$). $\Delta = M_\Delta - M$ is the mass difference between delta and nucleon.

2. NN-$N\Delta$

Concerning the transition potential NN-$N\Delta$ we also give the propagator function for the corresponding approximations to Eq. (49).

$$f^{(a)}_{NN-N\Delta} = \frac{1}{2\omega}\left(\frac{1}{\omega}+\frac{1}{\omega+\Delta}\right), \tag{58}$$

$$f^{(b)}_{NN-N\Delta} = \frac{1}{2\omega^2}+\frac{1}{2\omega^2+2\mu\Delta}, \tag{59}$$

$$f^{(c)}_{NN-N\Delta} = \frac{1}{\omega^2}. \tag{60}$$

Expression Eq. (58) is exactly the one given by Durso *et al.*[9] In Fig. 10 we show the approximate forms $f(q)$ Eqs. (59) and (60) together with the exact form Eq. (58). Again the approximations (b), (c) are not good. From our findings in the NN-$\Delta\Delta$ case we expect the same will hold for the corresponding potential in configuration space.

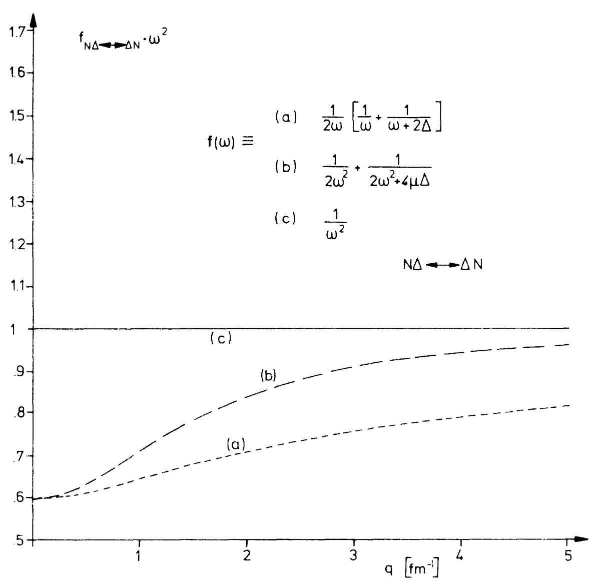

FIG. 11. Comparison of the different approximations of the propagator function $f(\omega)$ in the case of $N\Delta \longleftrightarrow \Delta N$ interaction corresponding to the processes Figs. 6(e) and 6(f). Note that the curves are given for $f\times\omega^2$. $\omega = (q^2+\mu^2)^{1/2}$ denotes the energy and μ the mass of the exchanged particle ($\pi,\rho,\ldots$). $\Delta = M_\Delta - M$ is the mass difference between delta and nucleon.

3. $N\Delta$-ΔN *and* $\Delta\Delta$-$\Delta\Delta$

For completeness we discuss the propagator function $f(q)$ also for the potentials $N\Delta$-ΔN and $\Delta\Delta$-$\Delta\Delta$ (see Figs. 11 and 12). Again we see that

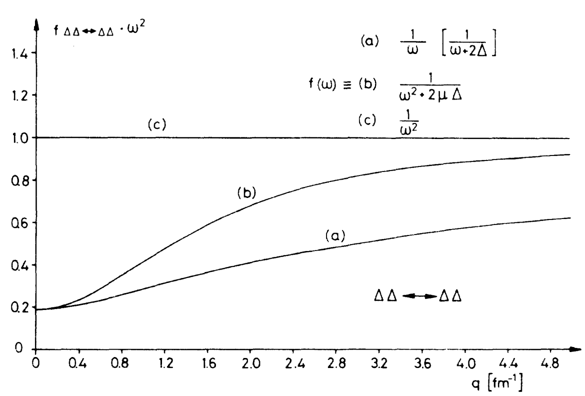

FIG. 12. Comparison of the different approximations of the propagator function $f(\omega)$ in the case of $\Delta\Delta \longleftrightarrow \Delta\Delta$ interaction corresponding to the process Fig. 6(d). Note that the curves are given for $f\times\omega^2$, where $\omega = (q^2+\mu^2)^{1/2}$ denotes the energy and μ the mass of the exchanged particle ($\pi,\rho,\ldots$). $\Delta = M_\Delta - M$ is the mass difference between delta and nucleon.

the neglect of the mass difference Δ in the propagator leads to very large deviations from the exact result. Again this approximation (*c*) is not usuable.

B. Explicit form of the potential $B_\alpha B_\beta \leftrightarrow B_\alpha' B_\beta'$

Taking into account vector mesons as well, we obtain the full baryon-baryon interaction in coordinate space by a Fourier transformation. The total potential can be summarized as follows:

$$V_{\text{eff}}^{BB'} = V^{NN}(\text{Reid modified}) + V_{\pi,\rho}^{NN-N\Delta} + V_{\pi,\rho}^{NN-\Delta\Delta} + V_{\pi,\rho}^{N\Delta-\Delta\Delta} + V_{\pi,\rho}^{N\Delta-N\Delta} + V_{\pi,\rho}^{\Delta\Delta-\Delta\Delta} + V_{\pi,\rho}^{N\Delta-\Delta N}. \tag{61}$$

As already mentioned, we use for the *NN* part of the interaction the form of the Reid soft core potential. In the case of an explicit treatment of the Δ degrees of freedom (case II), the nucleon-nucleon part of the potential has to be modified according to the phase shift and deuteron binding energy requirements. The explicit form of the parametrization of the Reid potential is given in the Appendix.

The explicit form of the resonance part of the potential reads

$$V_{\pi,\rho}^{B_\alpha B_\beta - B_\alpha' B_\beta'} = (\vec{\tau}^{\,1}_{B_\alpha B_\alpha'} \cdot \vec{\tau}^{\,2}_{B_\beta B_\beta'})\{(\vec{\sigma}^{\,1}_{B_\alpha B_\alpha'} \cdot \vec{\sigma}^{\,2}_{B_\beta B_\beta'})[V_C^{B_\alpha B_\alpha' B_\beta B_\beta',\pi}(r) + 2V_C^{B_\alpha B_\alpha' B_\beta B_\beta',\rho}(r)]$$
$$+ S_{12}^{B_\alpha B_\alpha' B_\beta B_\beta'}[V_T^{B_\alpha B_\alpha' B_\beta B_\beta',\pi}(r) - V_T^{B_\alpha B_\alpha' B_\beta B_\beta',\rho}(r)] + \alpha_1 V_S^{B_\alpha B_\alpha' B_\beta B_\beta',\rho}(r)\} + \alpha_2(1 \rightleftharpoons 2). \tag{62}$$

The functions V_C, V_T, and V_S are defined in the following way:

$$V_C^{B_\alpha B_\alpha' B_\beta B_\beta',\mu}(r) = -\frac{m_\mu}{3}\frac{2}{\pi}\frac{f_{B_\alpha B_\alpha'\mu} f_{B_\beta B_\beta'\mu}}{4\pi} \times \int_0^\infty dq \frac{q^4 j_0(qr)}{m_\mu^3} f_{B_\alpha B_\beta - B_\alpha' B_\beta'}(q^2) F_C^\mu(q^2,r), \tag{63}$$

$$V_T^{B_\alpha B_\alpha' B_\beta B_\beta',\mu}(r) = \frac{m_\mu}{3}\frac{2}{\pi}\frac{f_{B_\alpha B_\alpha'\mu} f_{B_\beta B_\beta'\mu}}{4\pi} \times \int_0^\infty dq \frac{q^4 j_2(qr)}{m_\mu^3} f_{B_\alpha B_\beta - B_\alpha' B_\beta'}(q^2) F_T^\mu(q^2), \tag{64}$$

$$V_S^{B_\alpha B_\alpha' B_\beta B_\beta',\mu}(r) = \frac{m_\mu}{3}\frac{2}{\pi}\frac{g_{B_\alpha B_\alpha'\mu} g_{B_\beta B_\beta'\mu}}{4\pi} \times \int_0^\infty dq \frac{q^2 j_0(qr)}{m_\mu} f_{B_\alpha B_\beta - B_\alpha' B_\beta'}(q^2) F_S^\mu(q^2), \tag{65}$$

with

$$S_{12}^{B_\alpha B_\alpha' B_\beta B_\beta'} = 3(\vec{\sigma}^{\,1}_{B_\alpha B_\alpha'} \cdot \hat{r})(\vec{\sigma}^{\,2}_{B_\beta B_\beta'} \cdot \hat{r}) - (\vec{\sigma}^{\,1}_{B_\alpha B_\alpha'} \cdot \vec{\sigma}^{\,2}_{B_\beta B_\beta'}).$$

TABLE I. Baryon-baryon-meson coupling constants used in this work.

$\frac{f_{NN\pi}^2}{4\pi}$	$\frac{f_{NN\rho}^2}{4\pi}$	$\frac{g_{NN\rho}^2}{4\pi}$	$\frac{f_{\Delta N\pi}^2}{4\pi}$	$\frac{f_{\Delta N\rho}^2}{4\pi}$	$\frac{f_{\Delta\Delta\pi}^2}{4\pi}$	$\frac{g_{\Delta\Delta\rho}^2}{4\pi}$	$\frac{f_{\Delta\Delta\rho}^2}{4\pi}$
0.077	4.5	0.55	0.35	13.0	0.003	0.55	0.18

F_C^μ, F_T^μ, F_S^μ denote form factors required for the regularization of the potential:

$$F_C^\mu(q^2,r) = (1 - e^{-\kappa r^2})\left(\frac{\Lambda_\mu^2 - m_\mu^2}{\Lambda_\mu^2 + q^2}\right)^2,$$
$$F_{\{T,S\}}^\mu(q^2) = \left(\frac{\Lambda_\mu^2 - m_\mu^2}{\Lambda_\mu^2 + q^2}\right)^2, \tag{66}$$

$(\Lambda_\pi = 5.315\ \text{fm}^{-1},\ \Lambda_\rho = 7.167\ \text{fm}^{-1},\ \kappa = 2.0\ \text{fm}^{-2})$.

The matrix elements of the spin and isospin operators are the usual ones and are summarized in the Appendix. The various meson-baryon coupling constants are given in Table I. The factors α_1 and α_2 occurring in Eq. (62) are summarized in Table II. The propagator functions $f_{B_\alpha B_\beta - B_\alpha' B_\beta'}(q^2)$ are those of Eqs. (48)–(54).

The two-nucleon problem has been solved in the coupled channel approach [Eq. (19)]. In the center-of-momentum frame the two-body wave function χ has the form

$$\chi = \chi^{jT}(\vec{r}) = \sum_n \sum_{L,S} \frac{U_{nLS}^{jT}(r)}{r} \mathcal{Q}\langle \hat{r} | n; [L(s_1 s_2)S]j; (t_1 t_2)T\rangle, \tag{67}$$

TABLE II. The factors α_1, α_2, β_1, β_2, γ_1, γ_2 of Eqs. (62), (A6), and (A7) for the different resonance potentials.

$B_1'B_2'$	B_1B_2	α_1	α_2	β_1	β_2	γ_1	γ_2
ΔΔ	*NN*	0	0	S'	4	$S+J$	$4\sqrt{30}$
ΔΔ	ΔΔ	1	0	$S'+1$	60	$S+J$	$60\sqrt{30}$
*N*Δ	*NN*	0	1	S'	$2\sqrt{6}$	$S+J$	$12\sqrt{5}$
ΔΔ	*N*Δ	0	1	S'	$4\sqrt{15}$	$S+J$	$60\sqrt{2}$
Δ*N*	*N*Δ	0	1	S'	4	$S+J+1$	$4\sqrt{30}$
*N*Δ	*N*Δ	1	1	S'	$6\sqrt{10}$	$S+J$	$60\sqrt{3}$

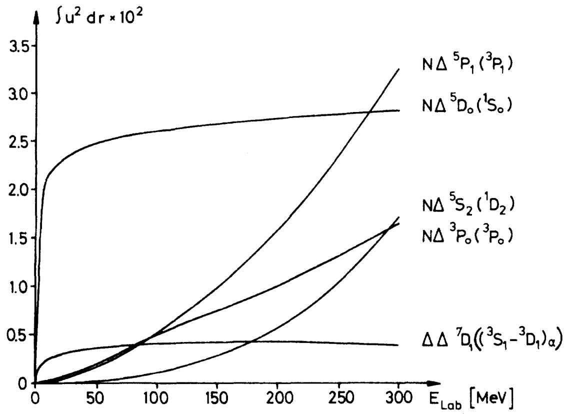

FIG. 13. Integral $N=\int_0^\infty dr\,[U^{jT}_{nLS}(r)]^2$ of different partial wave contributions to the NN scattering in dependence of scattering energy according to Eq. (72). In parentheses the corresponding nucleon-nucleon partial wave quantum numbers are given.

where $\mathfrak{a}$ is an antisymmetrization operator. The index n stands for the different possible baryon-baryon configurations (NN, $N\Delta$, $\Delta\Delta$).

With the potential Eq. (61) we obtain the following system of coupled differential equations from Eq. (19):

$$\left[\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}-2\mu_n(\Delta M_n-E)\right]U^{jT}_{nLS}(r)=2\mu_n\sum_m\sum_{L',S'}V^{jT}_{nLS,mL'S'}U^{jT}_{mL'S'}(r)\,. \quad (68)$$

μ_n denotes the reduced mass of the configuration n

$$\mu_n=\frac{M_n{}^1\cdot M_n{}^2}{M_n{}^1+M_n{}^2}\,;\quad \Delta M_n=M_n{}^1+M_n{}^2-2M \quad (69)$$

is the expectation value of the intrinsic Hamiltonian [Eq. (21)]. The matrix elements $V^{jT}_{nLS,mL'S'}$ of the interaction [Eq. (61)] are given in detail in the Appendix. The boundary conditions of Eq. (68) are different for scattering states and the deuteron. For scattering states they read

$$U^{jT}_{nLS}(r)\xrightarrow[r\to 0]{} r^{L+1}\ \text{for all } n;$$

$$U^{jT}_{nLS}(r)\xrightarrow[r\to\infty]{}\sin(kr-\tfrac{1}{2}L\pi+\delta_L)\ \text{for } n=1; \quad (70)$$

$$U^{jT}_{nLS}(r)\xrightarrow[r\to\infty]{} 0\ \text{for } n>1\,.$$

For the deuteron we have

$$U^{jT}_{nLS}(r)\xrightarrow[r\to 0]{} r^{L+1}\ \text{for all } n;$$

$$U^{jT}_{nLS}(r)\xrightarrow[r\to\infty]{}(M\cdot E_B)^{1/2}r\cdot Y_L[(M\cdot E_B)^{1/2}r]\ \text{for } n=1;$$

$$U^{jT}_{nLS}(r)\xrightarrow[r\to\infty]{} 0\ \text{for } n>1\,. \quad (71)$$

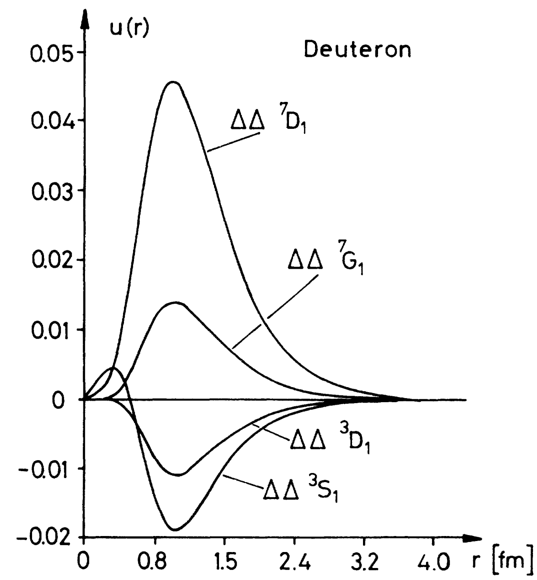

FIG. 14. Radial dependence $U(r)$ for the different $\Delta\Delta$ partial wave contributions to the deuteron state χ, calculated with the full propagator function.

As a numerical example for the contribution of the Δ states to the solution χ we give the values for the integrals of the partial wave contributions defined by the relation

$$N=\int_0^\infty dr[U^{jT}_{nLS}(r)]^2\,. \quad (72)$$

Note that the relation between the integral N and the total Δ probability is not trivial because of the relation between χ and the total solution Ψ Eq. (17b).

For the scattering states the integrals N for some partial waves are shown in Fig. 13 in dependence of the scattering energy (compare with Ref. 17). The deuteron $\Delta\Delta$ partial waves calculated with the full potential Eq. (61) are shown in Fig. 14.

In the case of the deuteron we discuss the different approximations to the propagator functions $f(q^2)$ corresponding to Figs. 7 and 12. In Table III we give the total integral N_t, i.e., the sum of the four $\Delta\Delta$ partial wave integrals. The total integral N_t for the full propagator functions corresponding to curve (*a*) in Figs. 7 and 12 has a value of $N_t=0.22\times10^{-2}$.

TABLE III. The total integrals N_t (= sum of the four $\Delta\Delta$ contributions) of the deuteron for the different approximations of the propagator function [Eqs. (55)–(57)].

	(*a*)	(*b*)	(*c*)
$N_t\times10^2$	0.22	0.43	0.54

The crude approximation (b) of Figs. 7 and 12 leads to a value of $N_t = 0.43 \times 10^{-2}$, which is larger than the exact value by a factor of 2 as expected from Figs. 7–9. This seems to be a general result also for other baryon-baryon potentials. The Feynman approach (c) leads to a value which is even larger than approach (b).

IV. SUMMARY

In the present paper we investigated nuclear properties by describing the nucleus by a meson-baryon system. Starting from a field theoretical Hamiltonian for such a composite system we formulated the eigenvalue problem in a subspace of baryons only. The total state of the system had been expressed in terms of such baryonic states. We have shown that there are different possibilities in describing the total state in terms of pure baryonic states. Explicitly we have considered two approaches: (i) reduction of the total state to a solution of the nucleonic component, and (ii) reduction of the total state to a solution of the coupled nucleon and nucleon resonance states. In the one-boson-exchange limit we have given the explicit expressions for the effective Hamilton operator in the subspaces (i) and (ii), respectively. Thereby, we have restricted our treatment to unitary transformations as such transformations lead to the conventional description of the nuclear states by a Hermitian Hamiltonian of the nuclear subspace. The difficulties which arise from a nonunitary transformation have been discussed. As an important point we have shown how the transition matrix of the interaction of an external field with our system has to be calculated. The same expressions apply for the resonance probabilities of our composite system. The exchange currents as well as the resonance probabilities are not only given by the baryonic states of the subspace but there are additional contributions according to the transformation into the subspace under consideration. This means that one has to be very careful in what is given as resonance probability.

In a practical application we considered the two-body system with the inclusion of Δ resonances. The deuteron properties as well as the N-N scattering have been calculated in the given frame with the use of the phenomenological Reid soft core potential for the pure nucleon-nucleon part. This potential has been adjusted to reproduce the N-N scattering data.

Starting from the general expressions for the baryon-baryon interactions we have discussed several approximations which have been used in the literature. As the different approximations to the full potential lead to substantial differences in the resonance probabilities, a consistent treatment is very important.

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APPENDIX

The modified Reid potential [V^{NN} (Reid modified) in Eq. (61)] has the following form:

$T=1$

$$V_{{}^1S_0}(x) = -h\frac{e^{-x}}{x} - 1650.6a_{11}\frac{e^{-4x}}{x} + 6484.2a_{12}\frac{e^{-7x}}{x}\,,$$

$$V_{{}^1D_2}(x) = -h\frac{e^{-x}}{x} - 12.322a_{21}\frac{e^{-2x}}{x} - 1112.6a_{22}\frac{e^{-4x}}{x} + 6484.2a_{23}\frac{e^{-7x}}{x}\,,$$

$$V_{{}^3P_0}(x) = -h\left[\left(1+\frac{4}{x}+\frac{4}{x^2}\right)e^{-x} - \left(\frac{16}{x}+\frac{4}{x^2}\right)e^{-4x}\right]\Big/x + 27.133a_{31}\frac{e^{-2x}}{x} - 790.74a_{32}\frac{e^{-4x}}{x} + 20662a_{33}\frac{e^{-7x}}{x}\,, \qquad \text{(A1)}$$

$$V_{{}^3P_1}(x) = h\left[\left(1+\frac{2}{x}+\frac{2}{x^2}\right)e^{-x} - \left(\frac{8}{x}+\frac{2}{x^2}\right)e^{-4x}\right]\Big/x - 100.0a_{41}\frac{e^{-2x}}{x} + 1000.0a_{42}\frac{e^{-4x}}{x} + 5000.0a_{43}\frac{e^{-6x}}{x}$$

$$V_{{}^3P_2-{}^3F_2}(x) = V_C(x) + V_T(x)\cdot S_{12} + V_{LS}(x)\vec{L}\cdot\vec{S}\,,$$

$$V_C(x) = \frac{h}{3}\frac{e^{-x}}{x} - 933.48a_{51}\frac{e^{-4x}}{x} + 4152.1a_{52}\frac{e^{-6x}}{x}\,,$$

$$V_T(x) = h\left[\left(\frac{1}{3}+\frac{1}{x}+\frac{1}{x^2}\right)e^{-x} - \left(\frac{4}{x}+\frac{1}{x^2}\right)e^{-4x}\right]\Big/x - 34.925a_{53}\frac{e^{-3x}}{x}$$

$$V_{LS}(x) = -2074.1a_{54}\frac{e^{-6x}}{x}\,.$$

$T=0$

$$V_{^1P_1}(x)=3h\frac{e^{-x}}{x}-634.39a_{61}\frac{e^{-2x}}{x}+2163.4a_{62}\frac{e^{-3x}}{x}\ ,$$

$$V_{^3D_2}(x)=-3h\left[\left(1+\frac{2}{x}+\frac{2}{x^2}\right)e^{-x}-\left(\frac{8}{x}+\frac{2}{x^2}\right)e^{-4x}\right]\Big/x-220.12a_{71}\frac{e^{-2x}}{x}+871.0a_{72}\frac{e^{-3x}}{x}\ ,$$

$$V_{^3S_1\text{-}^3D_1}(x)=V_C(x)+V_T(x)\cdot S_{12}+V_{LS}(x)\vec{\mathrm{L}}\cdot\vec{\mathrm{S}}\ ,$$

$$V_C(x)=-h\frac{e^{-x}}{x}+105.468a_{81}\frac{e^{-2x}}{x}-3187.8a_{82}\frac{e^{-4x}}{x}+9924.3a_{83}\frac{e^{-6x}}{x}\ ,$$

$$V_T(x)=-h\left[\left(1+\frac{3}{x}+\frac{3}{x^2}\right)e^{-x}-\left(\frac{12}{x}+\frac{3}{x^2}\right)e^{-4x}\right]\Big/x+351.77a_{84}\frac{e^{-4x}}{x}-1673.5a_{85}\frac{e^{-6x}}{x}\ ,$$

$$V_{LS}(x)=708.91a_{86}\frac{e^{-4x}}{x}-2713.1a_{87}\frac{e^{-6x}}{x}\ .$$

Here,

$$h=10.463\ \mathrm{MeV}\ ,$$

$$x=0.7\cdot r\ .$$

The parameters a_{ij} determined for the total potential Eq. (61) are given in Table (IV). The usual Reid-soft-core potential is obtained from Eq. (A1) with

$$\begin{aligned}a_{41}&=1.3525\ ,\\ a_{42}&=0.47281\ ,\\ a_{43}&=0\ ,\\ a_{ij}&=1\ \text{ in all other cases}\ .\end{aligned}\qquad\text{(A2)}$$

The matrix elements $V^{jT}_{nLS,mL'S'}$ of the interaction Eqs. (62)–(65) which occur in Eq. (68) are defined in the following way:

$$V^{jT}_{\Delta\Delta LS,\{^{NN}_{\Delta\Delta}\}L'S'}=\left\langle\Delta\Delta;[L(\tfrac{3}{2}\tfrac{3}{2})S]j;(\tfrac{3}{2}\tfrac{3}{2})T\,\middle|\,V^{BB'}_{\text{eff}}\,\middle|\begin{Bmatrix}NN\\ \Delta\Delta\end{Bmatrix};[L'(s_1's_2')S']j;(t_1't_2')T\right\rangle\tfrac{1}{4}[1-(-1)^{L+S+T}][1-(-1)^{L'+S'+T}]\qquad\text{(A3)}$$

with

$$s_1'=s_2'=t_1'=t_2'=\begin{Bmatrix}\frac{1}{2}\\ \frac{3}{2}\end{Bmatrix};$$

$$V^{jT}_{N\Delta LS,\{^{NN}_{\Delta\Delta}\}L'S'}=\left\langle N\Delta;[L(\tfrac{1}{2}\tfrac{3}{2})S]j;(\tfrac{1}{2}\tfrac{3}{2})T\,\middle|\,V^{BB'}_{\text{eff}}\,\middle|\begin{Bmatrix}NN\\ \Delta\Delta\end{Bmatrix};[L'(s_1's_2']S')j;(t_1't_2')T\right\rangle\tfrac{1}{2}\sqrt{2}[1-(-1)^{L'+S'+T}]\qquad\text{(A4)}$$

TABLE IV. The parameters a_{ij} of the modified Reid potential [Eq. (A1) of the Appendix].

$i \backslash j$	1	2	3	4	5	6	7	
1	0.599 50	0.724 21						$^1S_0$
2	−0.014 21	0.830 78	1.771 29					$^1D_2$
3	1.324 01	1.065 96	1.209 06					$^3P_0$
4	0.577 65	0.842 32	0.381 33					$^3P_1$
5	0.497 46	0.959 73	0.365 26	1.179 59				$^3P_2$-$^3F_2$
6	1.059 13	1.083 59						$^1P_1$
7	1.063 24	1.106 83						$^3D_2$
8	0.974 51	0.933 75	0.973 54	0.976 56	0.984 27	0.992 66	0.989 55	$^3S_1$-$^3D_1$

with

$$s_1' = s_2' = t_1' = t_2' = \begin{Bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{Bmatrix};$$

$$V^{jT}_{N\Delta LS, N\Delta L'S'} = \langle N\Delta; [L(\tfrac{1}{2}\tfrac{3}{2})S]j; (\tfrac{1}{2}\tfrac{3}{2})T \,|\, V^{BB'}_{\text{eff}} \,|\, N\Delta; [L'(\tfrac{1}{2}\tfrac{3}{2})S']j; (\tfrac{1}{2}\tfrac{3}{2})T\rangle + \langle \Delta N; [L(\tfrac{3}{2}\tfrac{1}{2})S]j; (\tfrac{3}{2}\tfrac{1}{2})T \,|\, V^{BB'}_{\text{eff}} \,|\, N\Delta; [L'(\tfrac{1}{2}\tfrac{3}{2})S']j; (\tfrac{1}{2}\tfrac{3}{2})T\rangle (-1)^{L+S+T+1} \quad \text{(A5)}$$

In the calculation of the right hand sides of Eqs. (A3)–(A5) the following expressions will occur:

$$\langle B_1' B_2'; (LS)j \,|\, (\vec{\sigma}^{\,1}_{B_1 B_1'} \cdot \vec{\sigma}^{\,2}_{B_2 B_2'}) \,|\, B_1 B_2; (L'S')j\rangle = (-1)^{\beta_1} \cdot \beta_2 \cdot \begin{Bmatrix} s_1 & s_2 & S \\ s_2' & s_1' & 1 \end{Bmatrix} \delta_{SS'} \delta_{LL'}, \quad \text{(A6)}$$

$$\langle B_1' B_2'; (LS)j \,|\, S_{12}^{B_1 B_1' B_2 B_2'} \,|\, B_1 B_2; (L'S')j\rangle = (-1)^{\gamma_1} \cdot \gamma_2 \hat{L}\hat{L}'\hat{S}\hat{S}' \begin{pmatrix} L & L' & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} L & S & j \\ S' & L' & 2 \end{Bmatrix} \times \begin{Bmatrix} s_1 & s_2 & S \\ s_1' & s_2' & S' \\ 1 & 1 & 2 \end{Bmatrix}. \quad \text{(A7)}$$

The factors β_1, β_2, γ_1, and γ_2 are listed in Table III.

For completeness we also give the baryon-baryon ρ-meson couplings which are necessary for the calculation of the potential Eqs. (62)–(65):

$$\Gamma_{BB'\rho} = -\frac{1}{\sqrt{2\omega}} \left[\frac{f_{BB'\rho}}{m_\rho} (\vec{\sigma}_{BB'} \times \vec{q}) \vec{\tau}_{BB'} \cdot \vec{\rho}_i + \delta_{BB'} g_{BB'\rho} \vec{\tau}_{BB'} \cdot \vec{\rho}_4 \right] \quad \text{(A8)}$$

with

$$\delta_{BB'} = \begin{cases} 1 & \text{for } BB' = NN, \Delta\Delta \\ 0 & \text{for } BB' = N\Delta . \end{cases}$$

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