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Analysis of π^- ^{4}He scattering and ^{4}He(π^-,n)^{3}H reactions at 290 MeV

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This study evaluates the amplitude for the pion absorption reaction ^{4}He(π^-,n)^{3}H to first order in the pion absorption operator for a pion laboratory kinetic energy of 290 MeV and the associated angular distribution of emitted neutrons is compared with recent experimental results. The pion absorption amplitude requires the full elastic scattering wave functions for the π^--^{4}He and n-^{3}H systems and the bulk of this study is the generation of the π^--^{4}He full elastic scattering wave function in the effective channel approach. The effective channel approach expresses the π^--^{4}He full elastic scattering state in terms of explicit elastic and inelastic channel wave functions. The input to the effective channel approach is a phenomenological πN separable interaction which satisfactorily accounts for the πN 33 resonance and a complex local Gaussian potential which, in conjunction with the separable interaction, produces the approximate spin and isospin averaged πN phase shifts and charge exchange cross section. A judicious choice of the form factor of the separable interaction allows an exact evaluation of the pion-nucleus potentials occurring in the application of the effective channel approach. These potentials are nonseparable and couple each partial wave to itself and to its two next neighbors as well as to the partial waves of the other channel. The complex local πN Gaussian potential also allows the analytic construction of the associated terms of the effective channel approach. The resulting two-channel system of integrodifferential equations is solved by iteration. The parameters of the πN potential are then varied to produce the observed total and inelastic π^--^{4}He cross sections and an angular distribution in reasonable accord with experiment to study pion absorption.

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[NUCLEAR REACTIONS π^--^{4}He elastic scattering, ^{4}He(π^-,n)^{3}H reaction at 290 MeV, effective channel approach.]

I. INTRODUCTION

Protons and pions as medium energy projectiles are useful probes of nuclei because their wavelengths are short enough to sense fine details of the nuclear structure. At the same time the complications of many-particle production are avoided in the medium energy range. A successful analysis of the scattering of those particles by nuclei can assist in the understanding of how the NN and πN amplitudes change when one nucleon is embedded in a nucleus, and so provide information about the dynamical properties of nuclei. The information gained in the analysis of elastic scattering is essential for an understanding of pion and proton induced reactions.

Either of two theoretical approaches[1,2] are commonly used to analyze the scattering of mesons or nucleons from nuclei. These approaches are based on Watson's multiple scattering theory[3] (MST) and Glauber's multiple diffraction theory[4] (MDT).

The MDT is a high energy approximation as originally developed and exploits the assumption that the projectile's trajectory is not appreciably altered during the scattering. In this virgin form all the inelastic excitation channels are treated by closure, and the MDT is not expected to be applicable at low energies or at large momentum transfer. Surprisingly, the various corrections[5-7] to the high energy form of the MDT appear to cancel each other and the uncorrected form has enjoyed a measure of success even at moderate energies.

Watson's MST, as modified by Kerman, McManus, and Thaler[8] relies on the careful construction of optical potentials to express all the effects associated with inelastic channels. To simplify the optical potential calculation, Feshbach and collaborators[9] expressed the problem in terms of a finite set of coupled equations. In this system of equations the driving term is the on-shell NN amplitude and the usual additional simplification is to assume the impulse approximation. Target excitations are treated in closure and are accounted for by one or more effective inelastic channels. Application of this form of the MST to p-^{4}He scattering provided the encouragement that information about correlation within the target nucleus could be discerned and that discrimination between the various off-shell extensions of the NN amplitude might be possible. That more definitive conclusions could not be drawn was caused by uncertainty in the input NN amplitudes and by the approximations made to facilitate the calculation.

The encouragement generated by the Feshbach application of the MST persuaded Hahn and Rule[10] to undertake an improvement of the theory by treating the inelastic channels more carefully and by shunning the impulse approximation. To avoid treating target excitations in closure they constructed an inelastic excitation pseudostate function with a single adjustable parameter determined by the projectile target total cross section. Unlike the usual distorted-wave Born approximation (DWBA) calculation, the effective channel approach (ECA) employed by Hahn and Rule begets an explicit elastic as well as inelastic wave function, which are both needed in a careful treatment of reaction processes. By generating an effective NN potential which reproduces the on-shell NN amplitude, the impulse approximation is avoided. What results can be described as a relativistic potential theory[11] and is justified as long as the production channels are closed, which is the case at the energies under investigation.

The ECA of Hahn and Rule has already been used to analyze p-^{4}He elastic scattering[10] for protons with laboratory kinetic energy ranging from 600 MeV to 1 GeV. Uncertainties in the calculation were small enough to allow discrimination between the 1974 Saclay angular distribution and that of the 1967 Brookhaven measurements.[1] The Saclay angular distribution was in reasonable accord with the ECA calculation, whereas the Brookhaven results could not be produced. Subsequent experimental measurements at UCLA-LBL and Argonne also agreed with the Saclay angular distribution.

MST has also been used to analyze pion-nucleus scattering, usually in the case of pion energies in the range of 100–300 MeV so that the πN 33 resonance can be assumed to dominate the pion-nucleus scattering.[1] The analysis of elastic scattering of pions with only first-order optical potentials has enjoyed much success in achieving agreement with observed angular distributions. There is reason to believe, however, that higher-order optical potentials are necessary to understand pion-nucleus angular distributions in the region of the second maximum, and without inclusion of these higher order terms agreement with experiment could be model dependent.[12] Besides the effects of higher-order optical potentials, questions persist about the kinematic (angle) transformation, resonance (Δ) propagation within the nucleus, and pion absorption during scattering.

The ECA used to study proton-nucleus scattering has been modified to assist in the resolution of some of the above questions about pion-nucleus scattering. This study treats pion scattering at energies above resonance ranging from 260–310 MeV, so that the rapid variation of the πN amplitude associated with the nearby 33 resonance must be confronted. The often used technique for dealing with this rapid energy variation is to introduce a separable isobar model interaction with a Yukawa form factor. Use of the Yukawa form factor precludes exact solution of the pion-nucleus scattering problem in that the associated optical potentials must be determined numerically, introducing uncertainty about the kinematic (angle) transformation. If a Guassian rather than Yukawa form factor is used, the integrations required to generate the optical potentials of the ECA can be accomplished exactly and the kinematic transformation can be investigated more carefully. This study makes such a treatment of pion-nucleus scattering.

The study of the elastic scattering of pions and nucleons by nuclei assists in the understanding of the elementary interactions and the resulting nuclear structure, but experimental uncertainties can mask details which must be understood because elastic scattering can be dominated by the lowest order density and the single particle structure, and be only weakly dependent on the off-shell behavior of the elementary interaction. Some of the information that could be gained by a study of elastic scattering is more readily sampled by a study of reaction processes. The analysis of these processes imposes additional constraints on the wave functions describing elastic scattering. The operator generating the transition between the two scattering states can be sensitive to parts of the scattering wave functions which do not strongly influence a description of the elastic scattering process.[13] The pion production and absorption processes involve a large momentum transfer, so that a reasonable suspicion is that these processes occur with the involvement of several nucleons and so should be sensitive to two nucleon correlations much more so than the elastic scattering process.

II. REVIEW OF THE EFFECT CHANNEL APPROACH

To describe elastic scattering, the ECA (Ref. 10) determines a system of two coupled differential equations beginning with the assumption that the elastic scattering wave function can be written as

$$\Psi(\vec{r}_0,\vec{r}) = u(\vec{r}_0)\psi_0(\vec{r}) + w(\vec{r}_0)\phi(\vec{r}_0,\vec{r}), \qquad (2.1)$$

with $w(\vec{r}_0)\phi(\vec{r}_0,\vec{r})$ called the effective inelastic channel wave function, $\vec{r}_0$ being the relative coordinate of the incident particle and the center of mass of the target, and $\vec{r}$ representing all the coordinates of the target particles as measured from the center of mass of the target.

The Hamiltonian of the projectile-target system is

$$H(\vec{r}_0,\vec{r}) = T(\vec{r}_0) + H_T(\vec{r}) + V(\vec{r}_0,\vec{r}), \qquad (2.2)$$

with $V(\vec{r}_0,\vec{r})$ being a sum of two-body interactions $V=\sum_{i=1}^{N} v(\vec{r}_0-\vec{r}_i)$. If the unperturbed wave functions of the target are defined by

$$H_T(\vec{r})\psi_n(\vec{r})=E_n\psi_n(\vec{r}), \tag{2.3}$$

then the total scattering wave function can be expanded as

$$\Psi(\vec{r}_0,\vec{r})=u_0(\vec{r}_0)\psi_0(\vec{r})+\sum_{n=1}^{\infty} u_n(\vec{r}_0)\psi_n(\vec{r}). \tag{2.4}$$

If the target is initially in its ground state $\psi_0(\vec{r})$, then the projection operator of the elastic component of the total scattering wave function is

$$P=|\psi_0\rangle\langle\psi_0|, \tag{2.5}$$

and with $Q=1-P$ the total scattering wave function may be expeditiously written as $\Psi=P\Psi+Q\Psi$. The ECA replaces the sum of terms in Q space by an *effective inelastic channel wave function* $\phi(\vec{r}_0,\vec{r})w(\vec{r}_0)$.

A reasonable and still convenient form for $\phi(\vec{r}_0,\vec{r})$ is[10]

$$\phi(\vec{r}_0,\vec{r})=[\hat{N}(r_0)]^{-1}Q\hat{V}(\vec{r}_0,\vec{r})P\psi_0(\vec{r}), \tag{2.6}$$

with $\hat{N}(r_0)$ being an r_0-dependent normalization so that

$$\int \prod_{i=1}^{N} d\vec{r}_i\,\delta\left(\frac{1}{N}\sum_{k=1}^{N}\vec{r}_k\right)|\phi(\vec{r}_0,\vec{r})|^2=1$$

or

$$\langle\phi|\phi\rangle=1,$$

and then

$$\hat{N}(r_0)=\{\langle\psi_0|\hat{V}Q\hat{V}|\psi_0\rangle\}^{1/2}. \tag{2.7}$$

The quantity $\hat{V}$ can be taken to be of the same form but not identical to V.

The specific coupled equations that embody the ECA are generated by an approximation of the inelastic channel propagator suitable for higher energy fast collisions.[14] If $\Psi=P\Psi+Q\Psi$, where the projection operators P and Q are as above, then $H\Psi=E\Psi$ becomes the coupled scattering equations[15]

$$\begin{aligned} P(H-E)P\Psi&=-PVQ\Psi,\\ Q(H-E)Q\Psi&=-QVP\Psi, \end{aligned} \tag{2.8}$$

appropriate to the description of elastic scattering from a target initially in its ground state. These equations are formally uncoupled by defining the Green's function

$$G^Q=[Q(E+i\epsilon-H)Q]^{-1}, \tag{2.9}$$

so that $Q\Psi=G^QVP\Psi$, and there results

$$P(H-E+VG^QV)P\Psi=0. \tag{2.10}$$

Now all complication resides in the determination of G^Q and the simplest approximation is closure,[2] in which

$$G^Q\to\frac{Q}{E-\overline{E}_c}, \tag{2.11}$$

where $\overline{E}_c$ is expected to be complex when the Q channels are open and could be energy dependent. This replacement of the entire QHQ by the number $\overline{E}_c$ is a rather drastic approximation in most instances. The approximation is improved for high energy fast collisions by taking[15]

$$G^Q\to\frac{Q}{E-T(r_0)-\overline{V}(r_0)-\overline{E}_F+i\epsilon}, \tag{2.12}$$

which amounts to the replacement

$$Q(H_T+V-E_0)Q\to\overline{V}(r_0)+\overline{E}_F-E_0.$$

This approximation rids G^Q of the internal target coordinates r so that only a two body scattering problem for G^Q remains. The target provides a distortion $\overline{V}(r_0)$ and an energy shift $\overline{E}_F$ during the collision so that this approximation is more plausible for high energy fast collisions than closure.

With this approximation of G^Q, the coupled scattering equations become

$$\begin{aligned} (T+V_{00}-E_0')u_0(\vec{r}_0)&=-V_{0\phi}w(\vec{r}_0),\\ (T+\overline{V}-E_\phi')w(\vec{r}_0)&=-V_{\phi0}u_0(\vec{r}_0), \end{aligned} \tag{2.13}$$

where each term is explicitly calculable. The elastic channel potential is $V_{00}=\langle\psi_0|V|\psi_0\rangle$ and the associated energy is $E_0'=E-E_0$, with E_0 as in $H_T\psi_0=E_0\psi_0$. The coupling potentials are $V_{\phi0}=\langle\phi|V|\psi_0\rangle$ and $V_{0\phi}=\langle\psi_0|V|\phi\rangle$, which are identical if V is a sum of local two-particle interactions. The coupling potentials represent the effects of two-particle correlations within the target.[15,16]

The effective channel potential includes four contributions: $\overline{V}=V_{\phi\phi}(r_0)+J_{\phi\phi}(r_0)+E_F(r_0)-\overline{E}_F$. The first term expresses up to three particle correlations. The term $J_{\phi\phi}$ exists because the incident particle cannot propagate without exciting the target, i.e., $T(r_0)$ does not commute with $\phi(\vec{r}_0,\vec{r})$, so that $J_{\phi\phi}=\langle\phi|T|\phi\rangle$.

Finally, the energy $E_F(r_0)=\langle\phi|H_T-E_0|\phi\rangle$ and becomes, asymptotically, $\overline{E}_F=\lim_{r_0\to\infty}E_F(r_0)$, so that $E_F'=E-\overline{E}_F$ is the energy associated with scattering in the effective channel. As is clear from the discussion above, our approach does not *a priori* introduce a model for the target He, such as He $=p+t$. We have in fact explicitly avoided precisely such approximations in order to study the kinematic effects and the center of mass correlations.

III. APPLICATION OF THE EFFECTIVE CHANNEL APPROACH TO π^--^{4}He ELASTIC SCATTERING

A. πN potential parametrization

In view of the proximity of the 33 resonance[17] to the energy at which pion absorption is to be studied, an explicit inclusion of this πN p-wave resonance is essential, and in customary fashion is represented by use of a separable nonlocal interaction

$$v^R = |f\rangle \frac{1}{E - E_R} \langle f| \,, \tag{3.1}$$

where the Gaussian form factor

$$f(\vec{r}) = \sqrt{\gamma}\, \vec{r}\cdot\hat{k}\, e^{-b_R r^2} \,, \tag{3.2}$$

where $\hat{k}$ is arbitrary, and to facilitate the calculation is chosen to be a unit vector in the direction of the incident pion beam. This form factor violates time reversal symmetry except for forward scattering, but does allow an exact treatment of the angle transformation of the π^--^{4}He problem.

Obviously, the form (3.2) is very restrictive, especially when we have chosen the arbitrary $\hat{k}$ vector in the direction of the incident pion. In general, a more flexible form of f with both θ and φ dependence should be used. However, for the present purpose of crude analysis for a preliminary study, the form (3.2) should provide a useful model. Besides, the more general case can be added later, together with other important modifications.

This interaction is input to a Schrödinger-like equation that incorporates relativistic kinematics[18]

$$(H_0 - E)|u\rangle = -|f\rangle \frac{1}{E - E_R} \langle f|u\rangle \,, \tag{3.3}$$

where

$$E = \frac{1}{2\mu} \frac{\epsilon^2 - m_\pi^2}{1 + \epsilon/m_N}$$

and

$$\epsilon = \{m_N^2 + m_\pi^2 + 2m_N(T_\pi + m_\pi)\}^{1/2} - m_N$$

and μ is the reduced πN mass.

Notice that the usual method of producing a Schrödinger equation from a relativistic equation with interaction results in the corresponding Schrödinger potential

$$V(r) = \frac{\epsilon}{\mu} \frac{1}{1 + \epsilon/m_N} \tilde{V}(r) \equiv \Lambda_{\pi N} \hat{V}(r) \,, \tag{3.4}$$

so that the potential parametrization made here is that for $V(r)$ and not for the potential $\tilde{V}(r)$ that appears in the relativistic equation. The decision to parametrize $V(r)$ so as to have energy dependence given only by the energy denominator $E - E_R$ was made simply because better fits to the reported[19] πN $l=1$, $I=\frac{3}{2}$, and $J=\frac{3}{2}$ phase shifts δ_{33} are possible in this model in which the parameters γ, E_R, and b_R are taken to be independent of energy. This decision requires a ratio of relativistic kinematic factors $\Lambda_{\pi\alpha}/\Lambda_{\pi N}$ when producing the corresponding π^--^{4}He potentials.

The separable interaction allows an exact solution. If u_i is the incoming wave and G is the free Green's function satisfying outgoing wave boundary conditions, then

$$|u\rangle = |u_i\rangle + |G|f\rangle\{E - E_R - \langle f|G|f\rangle\}^{-1}\langle f|u_i\rangle \,, \tag{3.5}$$

so that the elastic scattering amplitude is

$$\mathfrak{F}(\theta) = -\frac{\mu}{2\pi\hbar^2}\langle u_f|f\rangle\{E - E_R - \langle f|G|f\rangle\}^{-1}\langle f|u_i\rangle \,, \tag{3.6}$$

where u_f is the plane wave $e^{i\vec{k}'\cdot\vec{r}}$, with $|\vec{k}| = |\vec{k}'|$ for elastic scattering and $\hat{k}\cdot\hat{k}' = \cos\theta$.

For the Gaussian form factor chosen, all indicated integrals needed to evaluate the scattering amplitude can be done *analytically*. The shift operator, proportional to the strength γ, is

$$\langle f|G|f\rangle = -\frac{\mu}{\hbar^2}\gamma\frac{1}{24}\left(\frac{\pi}{b_R}\right)^{3/2} \times\left\{\frac{\sqrt{2}}{b_R}\left[1 + \frac{k^2}{b_R}\Phi\left(1; \frac{1}{2}; -\frac{k^2}{2b_R}\right)\right] + i\left(\frac{\pi}{b_R}\right)^{1/2}\left(\frac{k}{b_R}\right)^3 e^{-k^2/2b_R}\right\} \,, \tag{3.7}$$

where

$$\Phi(\alpha;\gamma;z) = \sum_{P=0}^{\infty}\frac{(\alpha)_p}{(\gamma)_p}\frac{Z^p}{P!}$$

is the degenerate hypergeometric function. Then

$$\langle u_f|f\rangle = -\sqrt{\gamma}\,\frac{ik}{2b_R}\left(\frac{\pi}{b_R}\right)^{3/2} e^{-k^2/4b_R}\cos\theta \text{ and}$$
$$\langle u_i|f\rangle = -\sqrt{\gamma}\,\frac{ik}{2b_R}\left(\frac{\pi}{b_R}\right)^{3/2} e^{-k^2/4b_R} \,. \tag{3.8}$$

The normalization is such that the elastic differential scattering cross section is given by $d\sigma/d\Omega = |\mathfrak{F}(\theta)|^2$ and $\sigma_t = (4\pi/k)\,\mathrm{Im}\mathfrak{F}(0°)$.

The three parameters of the form factor $f(\vec{r})$ are chosen so as to produce the δ_{33} over a range of pion energies. For the Gaussian form factor, as b_R is increased the integral $\langle f|G|f\rangle/\gamma$ decreases rapidly. Since the observed 33 resonance is elastic, so that energy denominators of the $E - E_R + i\Gamma/2$ are precluded, the entire width of the resonance, observed to be about 90 MeV, must be produced by the shift operator. So as b_R is increased, the strength γ must increase to maintain the rather broad width of the resonance. As γ is increased, the energy parameter E_R must also increase, as it is constrained by the requirement that the interaction produce a resonance at the experimentally observed energy $E^R = E_R + \mathrm{Re}\langle f|G|f\rangle$.

To parametrize the πN form factor, the range b_R was varied arbitrarily and the energy parameter E_R and strength γ were adjusted so as to produce the δ_{33} at the resonance energy T_R and at $T_G = 270$ MeV. Table I gives the values of E_R and γ for a range of b_R with $T_R = 193$ MeV.

Best fits to the reported δ_{33} for $T_\pi < T_R$ occur for $b_R = 0.9$ fm^{-2} if T_G is chosen to be 270 MeV. For $T_\pi > T_R$ and $T_G = 270$ MeV, the reported δ_{33} is most successfully produced for $b_R = 1.3$ fm^{-2}. Unfortunately, for $b_R > 1.0$ fm^{-2} the energy denominator occurring in the π^--^{4}He problem becomes too small to iterate for solution with $T_\pi = 290$ MeV. So the present study must be restricted to values of $b_R < 1.0$ fm^{-2} until techniques other than the iteration procedure adopted here are used. Figure 1 shows the variation with energy of the δ_{33} for $T_R = 193$ and $T_G = 270$ MeV and for $b_R = 0.6$, 0.8, and 1.0 fm^{-2}.

In addition to the separable πN interaction accounting for the 33 resonance and effecting only the $l = 1$ partial wave, a local complex Gaussian potential is used to account for the nonresonant πN interaction effecting the other partial waves and to generate the spin and isospin averaged inelastic cross section associated with charge exchange. This potential will be written as $\tilde{v}^L(r) = \Lambda_{\pi N}\tilde{v}_0^L e^{-b_L r^2}$, where $\Lambda_{\pi N}$ is the relativistic kinematic factor. The range of this local potential is arbitrarily chosen to be the same as that of the separable πN interaction. The πN phase shifts for $l > 2$ are uncertain and for $l = 1$ are dominated by δ_{33}.

The charge exchange cross section $\sigma(\pi^- p \to \pi^0 n)$ at $T_\pi = 290$ MeV is observed to be[17] 19.02 ± 1.0 mb. For the purpose of describing π^--^{4}He elastic

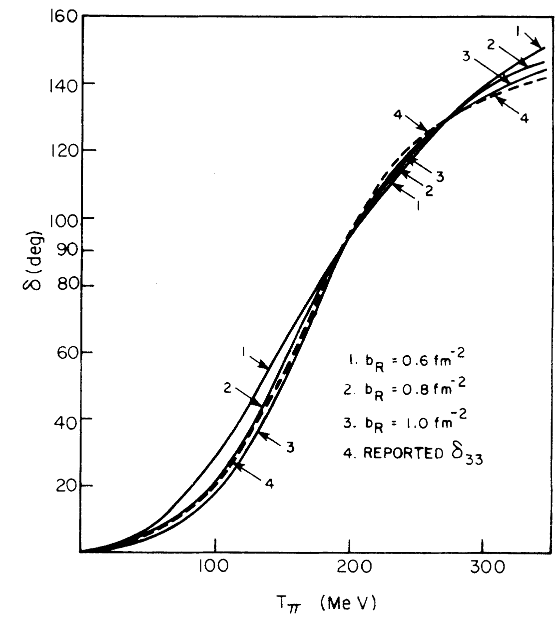

FIG. 1. Fits to the reported δ_{33} for $T_R = 193$ MeV and $T_G = 270$ MeV for various ranges of the πN separable interaction.

scattering, the charge exchange reaction must be regarded as an inelastic process and so is represented by the imaginary part of the local Gaussian potential. In the case of π^--^{4}He scattering the charge exchange reaction can occur only when the π^- interacts with a proton of the ^{4}He nucleus. So the strength of the imaginary part of the local πN Gaussian potential is adjusted to produce only 9.5 ± 1.0 mb. Table II presents the values of the parameters of the local πN Gaussian potential.

B. Calculation of the potentials descriptive of π^--^{4}He elastic scattering

The dynamical equation used to generate the state corresponding to elastic scattering is $H\Psi = E\Psi$, with the operator H and energy E of this equation adjusted so as to incorporate relativistic kinematics.[18] Thus

$$H(\vec{r}_0, \vec{r}) = T(\vec{r}_0) + H_T(\vec{r}) + V(\vec{r}_0, \vec{r}), \quad (3.9)$$

where T is the kinetic energy operator for the pion, H_T is the target Hamiltonian, and V is the

TABLE I. Parameters for the separable πN interaction.

b_R (fm^{-2})	γ (MeV^2 − fm^{-5})	E_R (MeV)
0.6	0.9783×10^4	257
0.8	0.2973×10^5	281
1.0	0.7447×10^5	303
1.2	0.1657×10^6	325
1.4	0.3334×10^6	349
1.6	0.6202×10^6	373

TABLE II. Parameters for the local πN Gaussian potential at $T_\pi = 290$ MeV.

b_L (fm^{-2})	Re $\tilde{v}_0^L$ (MeV)	Im $\tilde{v}_0^L$ (MeV)
0.8	-65 ± 5	-20 ± 2

pion-target interaction so that $V=\Lambda_{\pi\alpha}\sum_{i=1}^{4}\tilde{v}(\vec{r}_0-\vec{r}_i)$. If M is the mass of ^{4}He, then the total center of mass energy is

$$W=\{M^2+m_\pi{}^2+2M(T_\pi+m_\pi)\}^{1/2},$$

where T_π is the kinetic energy of the pion in the laboratory reference frame. Defining $\epsilon=W-M$, the relativistically corrected operators are

$$\begin{aligned} T&=-\frac{\hbar^2}{2\mu_{\pi\alpha}}\nabla_{r_0}{}^2,\\ H_T&=\frac{\epsilon}{\mu_{\pi\alpha}}[1+\epsilon/M]^{-1}\tilde{H}_T,\\ V&=\frac{\epsilon}{\mu_{\pi\alpha}}[1+\epsilon/M]^{-1}\tilde{V}\equiv\Lambda_{\pi\alpha}\tilde{V},\\ E&=\frac{1}{2\mu_{\pi\alpha}}[1+\epsilon/M]^{-1}(\epsilon^2-m_\pi{}^2), \end{aligned}\tag{3.10}$$

where V includes the interaction of the pion with each of the target nucleons by both the local complex πN Gaussian potential and the separable πN interaction accounting for the 33 πN resonance. As such, V is the sum of a local potential and a nonlocal operator. This dichotomy is expressed by writing

$$V=V^L+V^R.\tag{3.11}$$

The relativistically corrected separable πN interaction was assumed to be of the form $v^R=|f\rangle\{E-E_R\}^{-1}\langle f|$, with no multiplying relativistic kinematic factor. So the uncorrected separable πN interaction is given by $\tilde{v}^R=v^R/\Lambda_{\pi N}$ with

$$\Lambda_{\pi N}\equiv\frac{\epsilon_{\pi N}}{\mu_{\pi N}}\{1+\epsilon_{\pi N}/m_N\}^{-1}.$$

Now $\tilde{V}^R=\sum_{i=1}^{4}\tilde{v}^R(\vec{r}_0-\vec{r}_i)$ and $V=\Lambda_{\pi\alpha}\tilde{V}$, so that for the π-^{4}He problem the relativistically corrected nonlocal operator is

$$V^R=\Lambda_{\pi\alpha}\Lambda_{\pi N}{}^{-1}\sum_{i=1}^{4}v^R(\vec{r}_0-\vec{r}_i).\tag{3.12}$$

The point of view adopted in this study is that the range b_R, strength γ, and energy parameter E_R of the separable πN interaction are independent of energy and are input to the π^--^{4}He problem *without* modification. However, the variable energy of the energy denominator is different from that of the πN problem because the reference frame is now the π^--^{4}He barycentric system.[20,21] This variable energy $E_{D_{\pi\alpha}}$ is not the π^--^{4}He barycentric energy because $E_{D_{\pi\alpha}}$ should be a πN relative energy and not a π^--^{4}He relative energy. That is, the interaction $v^R(\vec{r}_0-\vec{r}_i)$ is for a πN and not for a π^--^{4}He system. So the variable energy of the energy denominator in the π-^{4}He problem is taken to be

$$E_{D_{\pi\alpha}}=\frac{1}{2\mu_{\pi N}}\{1+\epsilon_{\pi\alpha}/m_N\}^{-1}(\epsilon_{\pi\alpha}{}^2-m_\pi{}^2).\tag{3.13}$$

Then the π^--^{4}He nonlocal operator appearing in the Schrödinger-like equation is

$$V^R=\Lambda_{\pi\alpha}\Lambda_{\pi N}{}^{-1}\sum_{i=1}^{4}|f(\vec{r}_0-\vec{r}_i)\rangle\times\{E_{D_{\pi\alpha}}-E_R\}^{-1}\langle f(\vec{r}_0{}'-\vec{r}_i{}')|,\tag{3.14}$$

with $\vec{r}_0$ serving as the pion coordinate and measured from the center of mass of ^{4}He.

The elastic channel potential V_{00} and the coupling potentials $V_{0\phi}$ and $V_{\phi 0}$ of the ECA are functions only of the pion coordinate $\vec{r}_0$ and are derived from the potential $V(\vec{r}_0,\vec{r})$ according to[22-24]

$$\begin{aligned} V_{00}|u\rangle&=\langle\psi_0|V|\psi_0\rangle|u\rangle,\\ V_{0\phi}|w\rangle&=\langle\psi_0|V|\phi\rangle|w\rangle,\quad\text{and}\\ V_{\phi 0}|u\rangle&=\langle\phi|V|\psi_0\rangle|u\rangle. \end{aligned}\tag{3.15}$$

The effect of the inelastic channel potential $\overline{V}$ will be simulated by using V_{00}.

The functions ψ_0 and ϕ are, respectively, the ground state and effective inelastic channel wave functions of ^{4}He as described above.

The objective is to accomplish all integrations analytically except a final radial integration over the pion relative coordinate, with the pion wave function assumed to be expandable in partial waves as

$$u(\vec{r}_0)=\sum_{l=0}^{\infty}i^l(2l+1)P_l(\cos\Theta)\frac{u_l(r)}{kr}.\tag{3.16}$$

Executing the coordinate transformations

$$\begin{aligned} &\vec{r}_1=\vec{t}_1-\mu_0\vec{s}_1\text{ and }\vec{r}_0=\vec{t}_1+\delta_0\vec{s}_1,\text{ or}\\ &\vec{t}_1=\delta_0\vec{r}_1+\mu_0\vec{r}_0\text{ and }\vec{s}_1=\vec{r}_0-\vec{r}_1, \end{aligned}\tag{3.17}$$

with $\delta_0\equiv m_N\{m_N+m_\pi\}^{-1}$ and $\mu_0\equiv 1-\delta_0$, and performing the integrations over $\vec{r}_2$, $\vec{r}_3$, and $\vec{r}_4$ to give the single particle density

$$\rho(r_1)=\int\prod_{i=2}^{4}d\vec{r}_i|\psi_0|^2\delta\Big(\frac{1}{4}\sum_{i=1}^{4}\vec{r}_i\Big),\tag{3.18}$$

there results

$$V_{00}^R|u\rangle=N^R\int d\vec{r}_1\rho(r_1)f(\vec{r}_0-\vec{r}_1)\{E_{D_{\pi\alpha}}-E_R\}^{-1}e^{\nu/2r_1{}^2}\times\int d\vec{s}_1f(\vec{s}_1)e^{-\nu/2(\vec{t}_1-\mu_0\vec{s}_1)^2}u(\vec{t}_1+\delta_0\vec{s}_1).\tag{3.19}$$

Finally, under the transformation $\vec{x}\equiv\vec{t}_1+\delta_0\vec{s}_1$ the potential becomes

$$V^R|u\rangle = a_8\tilde{N}^R e^{-a_7 r^2}\int d\vec{x}\, u(\vec{x})e^{-a_5 x^2}e^{a_6\vec{r}_0\cdot\vec{x}} \times\{a_1 r_{0_z}{}^2 + a_2 x_z{}^2 + a_3 + a_4 r_{0_z} x_z\}. \qquad (3.20)$$

Notice that if $e^{a_6\vec{r}_0\cdot\vec{x}}$ is replaced by a constant, this potential becomes a sum of nonlocal separable potentials and effects only the first three partial waves. In this approximation the two channel problem can be solved analytically. The constants $a_1 \ldots a_8$ are functions of δ_0 and ν.

To obtain exact solutions by iteration, $u(\vec{x})$ must be expanded in partial waves and the integral over $|\vec{x}|$ must be done numerically during the iteration scheme. Writing

$$R_l^{00}(r) \equiv -\frac{kr}{2i^l} a_8\tilde{N}^R e^{-a_7 r^2}\sum_{n=0}^{\infty} i^n(2n+1)\int_0^{\infty} dx u_n(x) x e^{-a_5 x^2} A_{ln}(r,x), \qquad (3.21)$$

the only quantity remaining to be evaluated before solution by iteration is

$$A_{ln}(r,x) = \int_{-1}^{1} d\alpha P_l(\alpha)\int_{-1}^{1} d\mu P_n(\mu)\int_0^{2\pi} d\phi e^{a_6\vec{r}\cdot\vec{x}}\{a_1 r^2\alpha^2 + a_2 x^2\mu^2 + a_3 + a_1 r x\alpha\mu\}. \qquad (3.22)$$

Using[25]

$$e^{\vec{r}\cdot\vec{x}} = \sum_{n=0}^{\infty}(2n+1)i_n(rx)P_n(\cos\zeta) = 4\pi\sum_{n=0}^{\infty} i_n(rx)\sum_{m=-n}^{n} Y_{nm}(\Omega_{\hat{r}})Y_{nm}^*(\Omega_{\hat{x}}), \qquad (3.23)$$

with $\vec{r}\cdot\vec{x} = rx\cos\zeta$, and so the integral[26]

$$\int_{-1}^{2} d\alpha\int_{-1}^{2} d\mu\int_0^{2\pi} d\phi\, P_l(\alpha)P_n(\mu)e^{a\vec{r}\cdot\vec{x}} = \frac{8\pi}{2l+1}\delta_{l,n} i_l(arx), \qquad (3.24)$$

where the $i_l(arx)$ are modified spherical Bessel functions, the desired integration for A_{ln} is easily determined to be

$$A_{ln}(rx) = 8\pi\left[a_1 r^2 i_n(z)f(l,n) + a_2 x^2 i_l(z)f(n,l) + a_3\frac{1}{2l+1}i_l(z)\delta_{l,n} + a_4 rx\{i_{l+1}(z)g_+(l,n) + i_{l-1}(z)g_-(l,n)\}\right], \qquad (3.25)$$

with

$$f(l,n) = \frac{1}{2l+1}\frac{1}{2n+1}\left[\frac{l+1}{2l+3}\{(l+2)\delta_{n,l+2} + (l+1)\delta_{n,l}\} + \frac{l}{2l-1}\{l\delta_{l,n} + (l-1)\delta_{n,l-2}\}\right]$$

and

$$g_+(l,n) = \frac{1}{2l+1}\frac{1}{2n+1}\frac{l+1}{2l+3}\times\{(l+1)\delta_{l,n} + n\delta_{l,n-2}\}$$

and

$$g_-(l,n) = \frac{1}{2l+1}\frac{1}{2n+1}\frac{l}{2l-1}\times\{(n+1)\delta_{l,n+2} + l\delta_{l,n}\},$$

and using $z = a_6 rx$ as the argument of the modified spherical Bessel functions.

So with a form factor $f(\vec{r}) = \sqrt{\gamma}\,\vec{r}\cdot\hat{k}e^{-b_R r^2}$, as inspection of the result for A_{ln} reveals, any given partial wave is coupled to the two next adjacent partial waves. For example, solution for the d wave requires knowledge of the s and g partial waves.

The bulk of the work to express the nonlocal potential operators in a form suitable for iteration is now done. The results for the nonlocal coupling potentials can be written down by inspection. The $V_{0\Phi}$ nonlocal coupling potential operator is

$$\begin{aligned} V_{0\Phi}^R|w\rangle &= \langle\psi_0|V^R|\phi\rangle|w\rangle \\ &= \langle\psi_0|V^R\hat{N}^{-1}Q\hat{V}P|\psi_0\rangle|w\rangle \\ &= \langle\psi_0|V^R\hat{N}^{-1}\hat{V}|\psi_0\rangle|w\rangle - \langle\psi_0|V\hat{N}^{-1}|\psi_0\rangle \\ &\quad\times\langle\psi_0|\hat{V}|\psi_0\rangle|w\rangle, \end{aligned} \qquad (3.26)$$

where $\hat{V}$ is taken to be a real local potential of Gaussian form. The strength of $\hat{V}$ is arbitrary because the normalization $\hat{N}$ is also proportional to this strength. The normalization $\hat{N}$ is a function only of the magnitude of the π^--^{4}He relative coordinate. The two terms into which $V_{0\Phi}^R|w\rangle$ decomposes are each able to be cast in the same form as the single term for $V_{00}^R|u\rangle$. The r_0-dependent $\hat{N}$ is incorporated into the results for $V_{00}^R|u\rangle$ simply by making the replacement $u_l(r_0) \to w_l(r_0)\hat{N}^{-1}(r_0)$.

C. Abating the πN constraints

To vary the strengths of the πN local and separable potentials, regardless of constraints imposed by πN scattering, is tantamount to an admission of the occurrence of off-shell processes not taken into account by the πN potential.

A pion can be absorbed by a nucleon and the result can propagate as a Δ particle (Δ propagation), or as a nucleon (pion absorption), and then emit a pion of the same variety as was absorbed. If the pion is being scattered by a free nucleon then both of these processes have been taken into account; the former by the separable πN interaction and the latter by the local Gaussian πN potential. However, if the nucleon scattering the pion is embedded in a nucleus then both of these processes have been only approximately taken into account.

To further study the pion absorption process the parameters of the πN energy independent local and energy dependent separable potentials are varied to produce π^--^{4}He angular distributions and cross sections in reasonable agreement with experiment over a range of energies. The associated π^--^{4}He elastic scattering wave functions do not suffer from nonuniqueness to the same extent as wave functions generated by fitting experiment at a single energy and can be profitably used to evaluate the pion absorption amplitude.

D. Results for π^--^{4}He elastic scattering

The pion absorption amplitude will be calculated for a pion laboratory kinetic energy of $T_\pi = 290$ MeV and compared with the experimental results at that energy. The pion absorption amplitude calculation requires the π^--^{4}He scattering wave function at $T_\pi = 290$ MeV.

Elastic π^--^{4}He scattering has been experimentally studied[27] for pion laboratory kinetic energies ranging from 110 to 260 MeV. Angular distributions, total and inelastic cross sections, and the real part of the π^--^{4}He forward scattering amplitude have been measured. Additionally, the π^--^{4}He total cross section at $T_\pi = 290$ MeV has been measured. Table III gives the experimental results for π^--^{4}He elastic scattering at $T_\pi = 260$ MeV. Extrapolating these experimental results provides the π^--^{4}He cross sections and forward scattering amplitudes at $T_\pi = 290$ MeV as Table III presents. The angular distribution at $T_\pi = 290$ MeV was not measured, but the measured angular distributions at the energies ranging up to 260 MeV have one common feature that would probably be characteristic at $T_\pi = 290$ MeV. The first minimum is nearly independent of energy from 110 to 260 MeV and occurs at about $\Theta_{\text{c.m.}} = 75°$. The optical point at $T_\pi = 290$ MeV can be generated from the measured total cross section and the ratio of the real to imaginary part of the forward scattering amplitude according to

$$\frac{d\sigma}{d\Omega}(\Theta = 0°) = \left\{\frac{k\sigma_t}{4\pi}\right\}^2 (1+\rho^2). \tag{3.27}$$

So for $T_\pi = 290$ MeV with ρ as given in Table III, the optical point falls in the range

$$93\text{mb/sr} \le \frac{d\sigma}{d\Omega}(\Theta = 0°) \le 134 \text{ mb/sr}.$$

On the basis of the trend of the angular distributions from 180 to 260 MeV, the large angle scattering at 290 MeV should be about the same or somewhat less than that at 260 MeV.

The results of three different kinds of calculations will be presented. The calculation will be called *nonlocal* if the π^--^{4}He interaction V results from only the separable πN interaction

$$V = \Lambda_{\pi\alpha}\Lambda_{\pi N}^{-1}\sum_{i=1}^{4} v^R(\vec{r}_i - \vec{r}_0) = V^R, \tag{3.28}$$

with

$$v^R = |f\rangle\{E_{D_{\pi\alpha}} - E_R\}^{-1}\langle f|$$

and $f(\vec{r}) = \sqrt{\gamma}\,\vec{r}\cdot\hat{k}e^{-b_R r^2}$ as described above. The standard paramters for v^R are chosen to be $b_R = 0.8$ fm^{-2} and E_R and γ as fixed by the πN 33 phase shifts to be $E_R = 280$ MeV and $\gamma = 0.2937 \times 10^5$ MeV – fm^{-5}.

If the π^--^{4}He potential V results from the separable πN interaction and the complex local Gaussian potential with the strengths and range of the latter adjusted to generate the spin isospin averaged $l=0$ phase shift, then the calculation will be called *constrained*. In this case the π^--^{4}He potential can be written as

TABLE III. Experimental results for π^--^{4}He elastic scattering of $T_\pi = 260$ MeV and extrapolated value at 290 MeV.

T_π (MeV)	σ_t [mb]	σ_i [mb]	m [fm]	$\frac{d\sigma}{d\Omega}(0°)$ [mb/sr]	1st min
260	235	155	-1.0	100	75°
290	215	140 ± 5	$-1.0 \pm \frac{1}{2}$	114 ± 2	75°

$$V=\Lambda_{\pi\alpha}\Lambda_{\pi N}^{-1}\sum_{i=1}^{4} v^R(\vec{r}_i-\vec{r}_0)+\Lambda_{\pi\alpha}\sum_{i=1}^{4}\tilde{v}^L(\vec{r}_i-\vec{r}_0)\,, \tag{3.29}$$

with $\tilde{v}^L(r)=\tilde{v}_0^L e^{-b_L r^2}$. The standard set of parameters of the local πN potential $\tilde{v}^L$ are chosen to be $\tilde{v}_0^L=(-65-20i)$ MeV and $b_L=0.8$ fm^{-2}.

An *unconstrained* calculation will be one in which the strength of the local πN potential and the parameters of the πN separable interaction are varied to produce the observed characteristics of π^--^{4}He elastic scattering at $T_\pi=290$ MeV without regard for the spin isospin averaged $l=0$ phase shift. In such a calculation the π^--^{4}He potential is written as

$$V=\Lambda_{\pi\alpha}\Lambda_{\pi N}^{-1}\sum_{i=1}^{4} v_u^R(\vec{r}_i-\vec{r}_0)+\Lambda_{\pi\alpha}\sum_{i=1}^{4}\tilde{v}_u^L(\vec{r}_i-\vec{r}_0)\,, \tag{3.30}$$

with

$$\tilde{v}_u^L=\tilde{v}_{0_u}^L e^{-b_L r^2} \tag{3.31a}$$

and

$$v_u^R=|f\rangle\left\{E_{D_{\pi\alpha}}-E_R^u+i\frac{\Gamma}{2}\right\}^{-1}\langle f|\,, \tag{3.31b}$$

with $f(\vec{r})=\sqrt{\gamma}\,\vec{r}\cdot\hat{k}e^{-b_R r^2}$. The standard strength of $\tilde{v}_u{}^L$ is chosen to be $\tilde{v}_{0_u}{}^L=(-30-40i)$ MeV, and the standard parameter set of the separable interaction in the unconstrained calculation is $E_R^u=260$ MeV and $\Gamma=200$ MeV, and with b_R and γ as in the nonlocal calculation.

In each kind of calculation the range of the local potential $\hat{v}=\hat{v}_0 e^{-\hat{b}r^2}$ associated with the π^--^{4}He local potential $\hat{V}$ used to generate the inelastic channel pseudostate function $\phi=\hat{N}^{-1}Q\hat{V}P\psi_0$ is taken to be $\hat{b}=0.8$ fm^{-2}. As noted above, the strength $\hat{v}_0$ is arbitrary because $\hat{N}$ is proportional to that strength. Finally, as measured from e^--^{4}He scattering, the exponent of the single particle density $\rho(r)=(4\nu)/(3\pi)^{3/2}e^{-4/3\nu r^L}$ is taken to be $\nu=0.57$ fm^{-2}.

The angular distributions and other pertinent results associated with these three different calculations are presented in Figs. 3–5 in which, for comparison, the angular distribution as measured at $T_\pi=260$ MeV is shown.

The angular distributions resulting from the single and two channel nonlocal calculations predict much more large angle scattering than would be expected on the basis of the experimental studies at $T_\pi=260$ MeV and below. The addition of the absorption potentials in the constrained and unconstrained calculations significantly decreases this large angle scattering although it is probably still too large. The primary effect of increasing the real part of the local potentials in the constrained and unconstrained calculation is to increase the inelastic cross section and to pull in the first minimum. Increasing the range $\hat{b}$ of the potential $\hat{v}$ has the predictable effect of squelching the real part of the forward scattering amplitude and at the same time allowing more interaction so that σ_t and correspondingly $\mathrm{Im}\mathfrak{F}(0°)$ increase, with the result that ρ decreases.

The usual effect of the second channel is to increase the large angle scattering. However, this scattering is already large in the single channel calculations. This situation is perhaps traceable to the use of the separable πN form factor $f(\vec{r})=\sqrt{\gamma}\,\vec{r}\cdot\hat{k}e^{-b_R r^2}$ with $\hat{k}$ arbitrarily taken to be in the direction of the incident pion beam in violation of time reversal symmetry.

In the unconstrained calculation the addition of the width Γ to the π^--^{4}He energy denominator $E_{D_{\pi\alpha}}-E_R+i\Gamma/2$ has the effect of softening the target so that the large angle scattering is further decreased as well as providing an energy dependence of the π^--^{4}He scattering features more closely aligned with experiment.

The strength of the local potential $\tilde{v}^L$ of the constrained calculation is not well defined and a de-

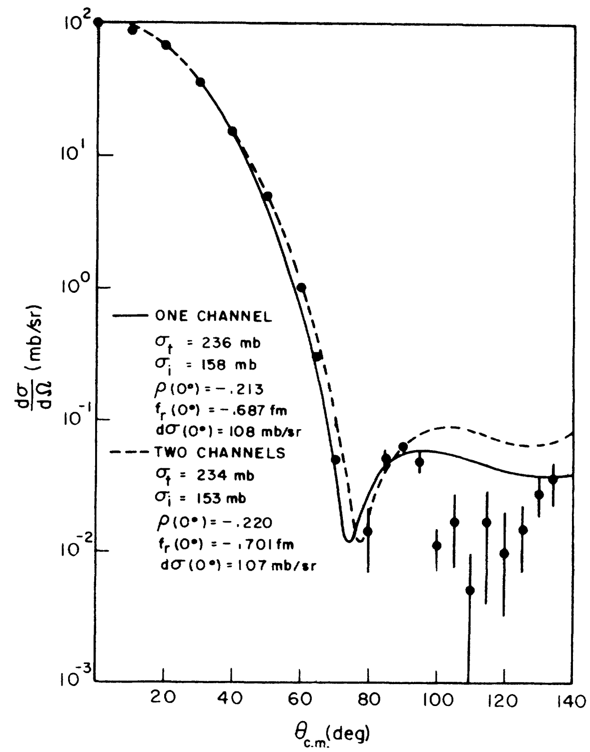

FIG. 2. Angular distribution for π^--^{4}He elastic scattering at $T_\pi=260$ MeV resulting from unconstrained calculation. The experimental points are for $T_\pi=260$ MeV.

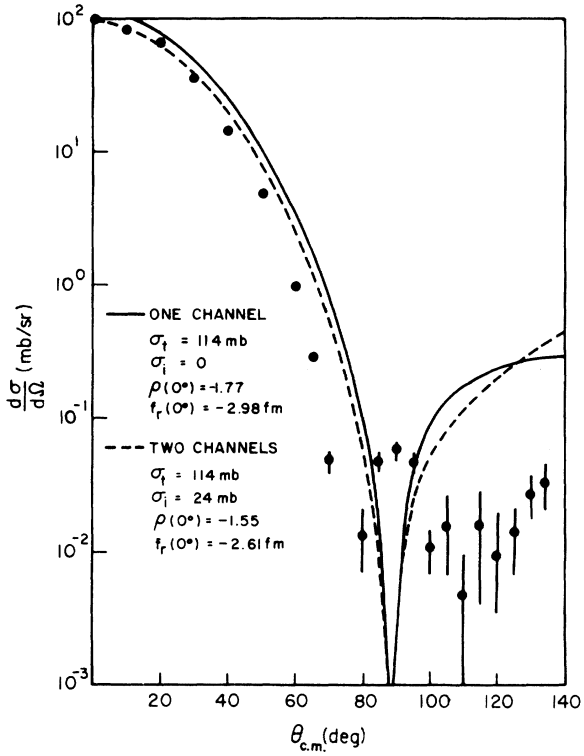

FIG. 3. Angular distribution for π^--^{4}He elastic scattering at $T_\pi = 290$ MeV resulting from nonlocal calculation. The experimental points are for $T_\pi = 260$ MeV.

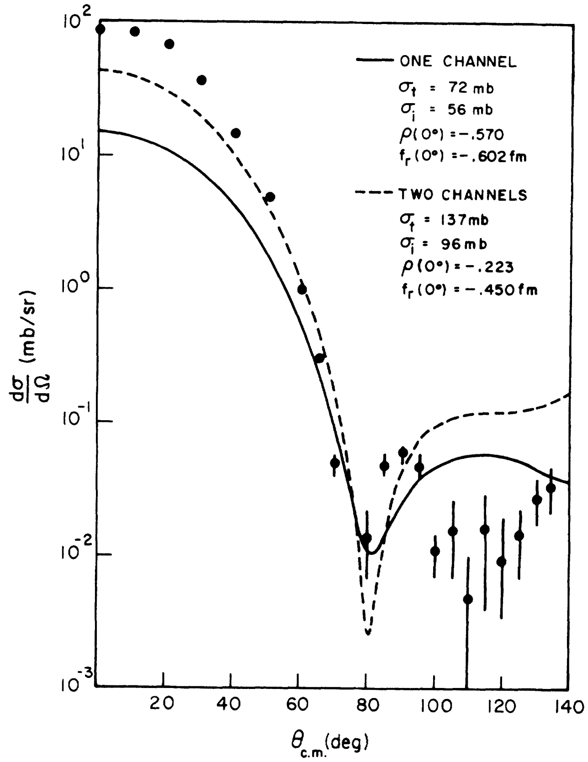

FIG. 4. Angular distribution for π^--^{4}He elastic scattering at $T_\pi = 290$ MeV resulting from πN constrained calculation. The experimental points are for $T_\pi = 260$ MeV.

crease in the real part of this strength by forty percent will still produce reasonable πN results but will increase $d\sigma(0°)$ for the π^--^{4}He problem to about 80 mb/sr.

Each of the three kinds of calculations can be made with the coupling potentials set to zero, and a "one channel calculation" results. These angular distributions are also presented in Figs. 2–5. In the one channel calculation the inelastic component of the π^--^{4}He scattering wave function is identically zero and the elastic component differs from the result of a two channel calculation. The one channel calculation includes the coupling among the partial waves of the elastic component.

In the two channel calculation each partial wave of the elastic component is coupled to itself, to its two next neighbors, and to three partial waves of the inelastic component.

Caution should be exercised so as not to confuse the results of these one and two channel calculations to similarly entitled calculations of the pion absorption amplitude as described in Sec. 5. In the latter case the so-called single channel calculation is made by neglecting the contribution of the inelastic component of the π^--^{4}He scattering wave function to the pion absorption amplitude. However, the elastic component of the π^--^{4}He wave function used in the single channel pion absorption calculation is the elastic component of a two channel π^--^{4}He elastic scattering calculation.

IV. GENERATION OF THE n-^{3}H ELASTIC SCATTERING STATE

Insufficient experimental information precludes application of the ECA to the n-^{3}H elastic scattering problem at the energy associated with the pion absorption reaction being studied.[9] The angular distributions and cross sections related to the n-^{3}H scattering problems have not been measured. The n-^{3}H problem is thus treated in single channel approximation using a Woods-Saxon potential in conjunction with a very short range Gaussian local potential to describe the large angle behavior. The Coulomb interaction is neglected and charge-independence of the strong interaction is assumed so that the potential parameters are adjusted to agree with the p-^{3}He elastic scattering angular distribution at $T_n = 582$ MeV. This potential is then applied to n-^{3}H elastic scattering at $T_n = 556$ MeV and the resulting wave functions are then used to study the pion absorption reaction ^{4}He(π^-,n)^{3}H at $T_\pi = 290$ MeV.[28,29]

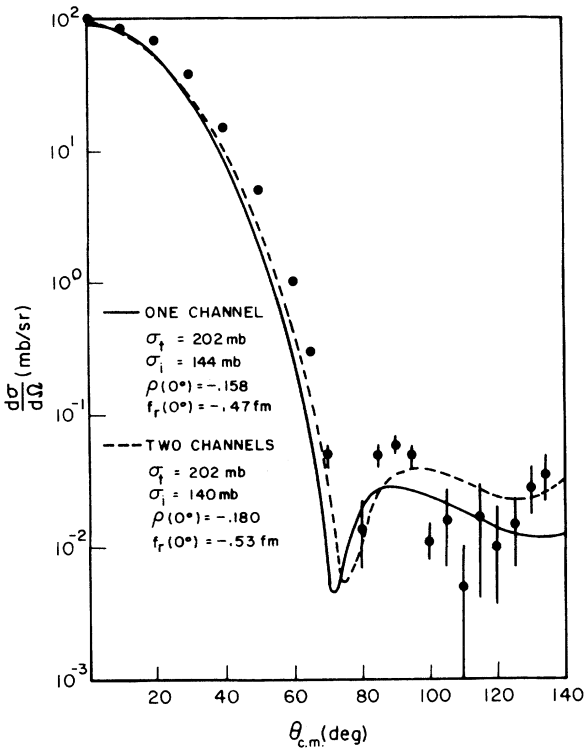

FIG. 5. Angular distribution for π^--^{4}He elastic scattering at $T_\pi = 290$ MeV resulting from unconstrained calculation. The experimental points are for $T_\pi = 260$ MeV.

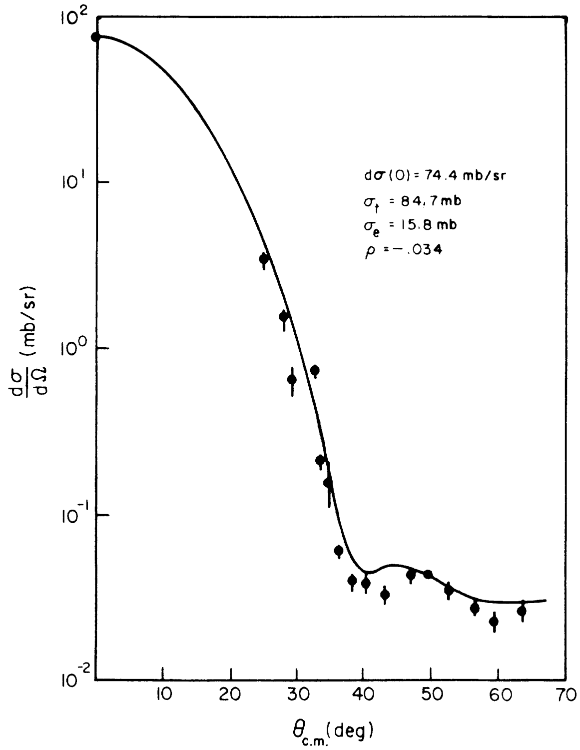

FIG. 6. Angular distribution for n-^{3}He elastic scattering for $T_n = 582$ MeV. The data are as in Ref. 22 for p-^{3}He elastic scattering with $T_p = 582$ MeV. The optical point is generated from p-^{4}He elastic scattering data of Ref. 20.

The Woods-Saxon potential is written as

$$V_F(r) = \hat{V}_F\{1 + e^{-w/d}\}\{1 + e^{(r-w)/d}\}^{-1}, \tag{4.1}$$

with w being the radius of the potential at half maximum and where d is a measure of the surface thickness. The local Gaussian potential is written as

$$V_G(r) = \hat{V}_G e^{-ar^2} \tag{4.2}$$

and a plausible fit to the p-^{3}He elastic angular distribution at the proton laboratory kinetic energy $T_p = 582$ MeV results with $\hat{V}_F = (-2 - 50i)$ MeV, $w = 1.45$ fm, and $d = 0.29$ fm corresponding to a surface thickness of 1.27 fm for the Fermi potential and with $\hat{V}_G = (250 - 100i)$ MeV and $a = 6.0$ fm^{-2} of the Gaussian core potential. This parametrization also produces a total cross section $\sigma_t = 85$ mb or about $\frac{3}{4}$ of the total cross section of the more often studied p-^{4}He scattering system and a ratio of the real to imaginary part of the forward scattering amplitude $\rho = -0.034$ also in agreement with what would be expected from studies of p-^{4}He elastic scattering. The n-^{3}H potential and resulting differential scattering cross section are shown in Fig. 6.

In evaluating the amplitude for the pion absorption reaction ^{4}He$(\pi^-, n)^3$H an explicit expression for the ^{3}H ground state is needed. So as to be able to perform the integrations involved in the evaluation of the pion absorption amplitude, the ground state for ^{3}H is parametrized as the Gaussian

$$\psi_t(\vec{r}_1, \vec{r}_2, \vec{r}_3) = C_3 \sum_{i=1}^{3} e^{-\alpha/2\vec{r}_i^{\,2}}, \tag{4.3}$$

with the normalization C_3 defined by

$$\int \prod_{i=1}^{3} d\vec{r}_i\, |\psi_t|^2 \delta\left(\frac{1}{3}\sum_{i=1}^{3} \vec{r}_i\right) = 1.$$

In customary fashion, the Dirac delta function is included in the integration over the bound nucleon coordinates so as to incorporate the constraint that $\sum_{i=1}^{3} \vec{r}_i = 0$ so that the $\vec{r}_i$ are measured from the center of mass of ^{3}H and are not independent. The normalization C_3 is easily found to be

$$C_3 = 27^{-1/4}\left(\frac{\alpha}{\pi}\right)^{3/2} \tag{4.4}$$

and, from electron-^{3}He scattering studies, the exponent[20] $\alpha = 0.45$ fm^{-2}.

V. FORMULATION FOR THE DESCRIPTION OF PION ABSORPTION

The final stage of the present study is the expression of the amplitude for the reaction $^4\mathrm{He}(\pi^-, n)^3\mathrm{H}$ in terms of a radial integral over the partial waves of the n-^{3}H scattering function with the radial component of the pion production operator acting on the partial waves of the π^--^{4}He scattering function. That is, from the general expression for the transition amplitude $M_{fi} = \langle f|H'|i\rangle$ with $|i\rangle$ representing the elastic scattering state of the π^--^{4}He system, and $|f\rangle$ representing the same for the n-^{3}H system and H' being the pion absorption operator, all indicated integrations must be accomplished except that over a final radial coordinate so that M_{fi} is then expressible in the form

$$M_{fi} = \sum_{l=0}^{\infty} A_l(\Theta) \int_0^{\infty} dr \mathfrak{M}^l_{fi}(r) , \tag{5.1}$$

where $\mathfrak{M}^l{}_{fi}$ depends on the partial waves of the neutron and pion scattering wave functions and Θ is the scattering angle.

To begin, the static form of the pion absorption operator[24] is written as

$$H' = iF_{\pi N}\vec{\sigma} \cdot \vec{\nabla}_\pi \vec{\tau} \cdot \vec{\phi} , \tag{5.2}$$

where, for the particular reaction considered,

$$\vec{\phi} = E_\pi^{-1/2}\hat{\phi} = (2E_\pi)^{-1/2}(\phi_x - i\phi_y) = (2E_\pi)^{-1/2}(1, -i, 0) \tag{5.3}$$

so that $\vec{\phi}$ includes the relativistic weighing of $E_\pi^{-1/2}$ and then

$$F_{\pi N}{}^2 = \frac{4\pi(\hbar c)^5}{(mc^2)^2} f^2$$

with $f^2 = 0.080 \pm 0.005$. The spatial part of the pion field is imbedded in the initial state $|i\rangle$ and is normalized by unity as is the continuum neutron wave function of the final state $|f\rangle$.

Now the π^--^{4}He elastic scattering state is represented by

$$|i\rangle \equiv \Psi_i = \{u_\pi(\vec{r}_0)\psi_\alpha(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) + \phi(\vec{r}_0, \vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)w_\pi(\vec{r}_0)\}|I=0, S=0\rangle , \tag{5.4}$$

where $|I=0, S=0\rangle$ is the isospin and spin structure of the target ^{4}He and all unprimed coordinates are measured relative to the ^{4}He center of mass.

The state $|f\rangle$ is the elastic scattering state of n-^{3}H and is written as

$$|f\rangle_{M_s' m_s} \equiv \Psi_f^{M_s' m_s} = u_N(\vec{r}_1{}')\psi_t(r_2', r_3', r_4')\,|I' = \tfrac{1}{2}M_I' = \tfrac{1}{2}S' = \tfrac{1}{2}M_s'\rangle\,|\tfrac{1}{2} - \tfrac{1}{2}\rangle\,|\tfrac{1}{2} m_s\rangle \tag{5.5}$$

so that this state has total isospin 1 as it must for isospin to have been conserved in the course of absorption.

To compute the inner products of the absorption amplitude in spin space and isospin space use $\tau \cdot \hat{\phi} = 1/\sqrt{2}(\tau_1 - i\tau_2) = \sqrt{2}\tau_-$ with $\tau_-\binom{1}{0} = \binom{0}{1}$ and write

$$\mathfrak{M}^{M_s' m_s} = \langle \tfrac{1}{2} m_s|\langle \tfrac{1}{2} - \tfrac{1}{2}|\langle \tfrac{1}{2} - \tfrac{1}{2}\tfrac{1}{2}M_s'|\vec{\sigma} \cdot \vec{\nabla}_\pi \tau_-|I=0, S=0\rangle . \tag{5.6}$$

Now define $\sigma \cdot \nabla_\pi = \sigma^+ d_- + \sigma^- d_+ + \sigma^0 d_0$ with $d_\pm = \nabla_{\pi_x} \pm i\nabla_{\pi_y}$ and $d_0 = \nabla_{\pi_z}$ and with σ^+ and σ^- as the nucleon spin raising and lowering operators so that, e.g., $\sigma^+|\tfrac{1}{2} - \tfrac{1}{2}\rangle = |\tfrac{1}{2}\tfrac{1}{2}\rangle$.

The inner products in configuration space remain to be evaluated. To integrate over the coordinates of the 2, 3, and 4 nucleons, the overlap of ψ_t with ψ_α must be determined. It is expressed in terms of coordinates measured from the center of mass of ^{4}He by

$$I_{00}(r_1) = \int \sum_{i=1}^{4} d\vec{r}_i \psi_t^*(\vec{r}_{234}'[\vec{r}_{1234}])\psi_\alpha(\vec{r}_{1234})\delta\left(\tfrac{1}{4}\sum_{i=1}^{4}\vec{r}_i\right) , \tag{5.7}$$

with $\vec{r}_{234}'[\vec{r}_{1234}]$ being the expression of the primed in terms of the unprimed coordinates, with the latter measured from the center of mass of ^{4}He. This transformation is accomplished by reference to Fig. 7. Using $\vec{X}_4 = \frac{1}{4}\sum_{i=1}^4 \vec{\rho}_i$ and $\vec{X}_3 = \frac{1}{3}\sum_{i=2}^4 \vec{\rho}_i$, where the $\vec{\rho}_i$ are measured from the origin of an arbitrary reference frame, the transformation is

$$\vec{r}_i' = \vec{r}_i + \tfrac{1}{3}\vec{r}_1 \text{ for } i = 1, 2, 3, 4 . \tag{5.8}$$

The associated contribution to the absorption amplitude is

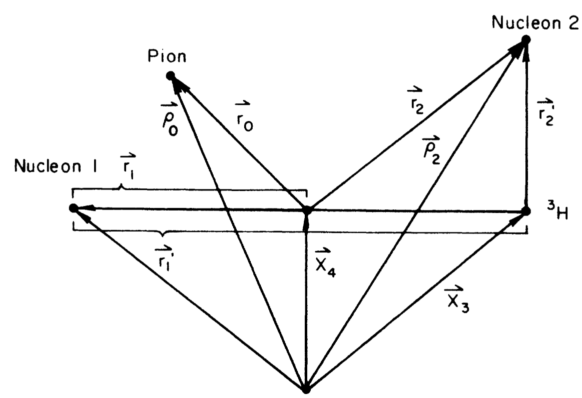

FIG. 7. Depiction of the various coordinates used in the calculation of the absorption amplitude.

$$_{M_s'm_s}\langle f|H'|i\rangle = i\,\frac{F_{\pi N}}{\sqrt{E_\pi}}\,\hat{I}_{00}\sqrt{2}\int d\vec{r}_1 u_N^*(\vec{r}_1{}')\times[\mathfrak{M}^{M_s' m_s} u_\pi]e^{-Ar_1^2}\,, \tag{5.9}$$

where $\mathfrak{M}^{M_s'm_s}u_\pi$ is determined by evaluating it first in pion coordinates $\vec{r}_0$ and then evaluating the result at the position of nucleon 1 so that $\vec{r}_0 \to \vec{r}_1$. Finally, the outgoing neutron wave $u_N^*(\vec{r}_1{}')$ must be expressed as a function of coordinates measured from the center of mass of ^{4}He so that $\vec{r}_1{}' \to \frac{4}{3}\vec{r}_1$.

Then the amplitude for the transition from the single initial state to the final state characterized by the quantum numbers $M_s' = \frac{1}{2}$ and $m_s = -\frac{1}{2}$ is given by

$$_{+-}\langle f|H'|i\rangle_0 = i\,\frac{F_{\pi N}}{\sqrt{E_\pi}}\sqrt{2}\int d\vec{r}_1 u_N^*(\vec{r}_1{}')[-\tfrac{1}{2}d_0 u_\pi]I_{00}(r_1)\,. \tag{5.10}$$

The nucleon wave function $u_N^*(\vec{r}_1{}')$ is expanded as

$$u_N(\vec{r}_1{}') = \sum_{l=0}^{\infty} i^l 4\pi \sum_{m=-l}^{l} Y_{lm}(\Omega_\Theta)\,Y_{lm}^*(\Omega_\theta)\,\frac{u_{N_l}(r_1')}{k_N r_1'} \tag{5.11}$$

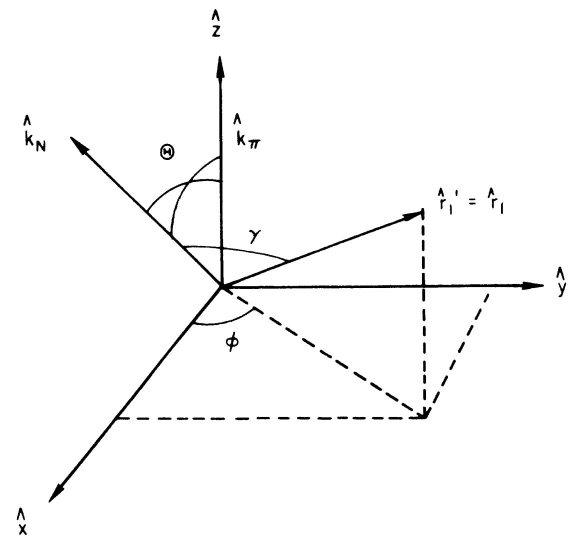

FIG. 8. Illustration of the orientation of the coordinate system used in the evaluation of the plane wave Born approximation of the absorption amplitude.

by reference to Fig. 8, so that Θ is the scattering angle. Thus the above transition amplitude becomes

$$\begin{aligned}
_{+-}\langle f|H'|i\rangle_0 &= -i\,\frac{F_{\pi N}}{\sqrt{E_\pi}}\,\frac{1}{\sqrt{2}}\sum_{lm} i^l 4\pi Y_{lm}(\Omega_\Theta)\int d\vec{r}\;\hat{I}_{00}e^{-Ar^2}Y_{lm}^*(\Omega_\theta)\,\frac{u_{N_l}^*(r')}{k_N r'}\\
&\quad\times\sum_{l'} i^{l'}\sqrt{4\pi}\left\{\frac{l'+1}{(2l'+3)^{1/2}}\left(\frac{d}{dr}-\frac{l'}{r}\right)\frac{u_{\pi L'}(r)}{k_\pi r}\,Y_{l'+1,0}\Omega_\theta)\right.\\
&\qquad\left.-\frac{l'}{(2l'-1)^{1/2}}\left(\frac{d}{dr}+\frac{l'+1}{r}\right)\frac{u_{\pi l'}(r)}{k_\pi r}\,Y_{l'+1,0}(\Omega_\theta)\right\}\\
&= -i\,\frac{F_{\pi N}}{\sqrt{E_\pi}}\,\frac{1}{\sqrt{2}}\sum_{l,l'} i^{l'-L}\,4\pi\left(\frac{3}{4}\right)\frac{1}{k_\pi k_N}\,P_l(\cos\Theta)\hat{I}_{00}\\
&\quad\times\int_0^\infty dr e^{-Ar^2}u_{N_l}(r')\left\{[l'^2\delta_{l'-1,l}-l^2\delta_{l'+1,l}]\,\frac{u_{\pi l'}(r)}{r}+[l'\delta_{l'-1,l}+l\delta_{l'+1,l}]\,\frac{d}{dr}u_{\pi l'}(r)\right\}
\end{aligned} \tag{5.12}$$

and all that remains is the radial integration involving the numerically computed partial waves u_{N_l} and $u_{\pi l'}$.

To complete the evaluation of the amplitude to the state characterized by the quantum numbers $M_s' = \frac{1}{2}$ and $m_s = -\frac{1}{2}$ the term $_{+-}\langle f|H'|i\rangle_\phi$ must be added to $_{+-}\langle f|H'|i\rangle_0$ and a prerequisite for the determination of the former is the evaluation of the overlap

$$I_{0\phi}(\vec{r}_1) = \int\prod_{i=2}^{4} d\vec{r}_i\,\psi_t(\vec{r}_{234}'[\vec{r}_{1234}])\phi(\vec{r}_0,\vec{r}_{1234})\delta\left(\frac{1}{4}\sum_{i=1}^{4}\vec{r}_i\right)\,, \tag{5.13}$$

with

$$\begin{aligned}
\phi(\vec{r}_0,\vec{r}_{1234}) &= \hat{N}^{-1}(r_0)Q\hat{V}P\psi_\alpha(\vec{r}_{1234})\\
&= \hat{N}^{-1}(r_0)\left\{\sum_{i=1}^{4}\hat{v}_0 e^{-\hat{b}(r_0-r_i)^2} - 4\hat{v}_0\left(\frac{4\nu}{4\nu+3\hat{b}}\right)^{3/2}e^{-4\hat{b}\nu r_0^2/(4\nu+3\hat{b})}\right\}\psi_\alpha(\vec{r}_{1234})\,.
\end{aligned} \tag{5.14}$$

Under the transformation $\vec{r}_i{}' = \vec{r}_i + \frac{1}{3}\vec{r}_1$ for $i = 1, 2, 3, 4$ and with $\vec{r}_0 = \vec{r}_1$ this overlap is found to be

$$I_{0\phi}(r_1)=\hat{N}^{-1}(r_1)4^3\left\{\frac{\pi}{\alpha+\nu}\right\}^{3/2}C_3C_4\hat{v}_0\, e^{-(2\nu/3)r_1^2}\left[\left(\frac{4}{3}\right)^{3/2}\left\{\frac{\pi}{\alpha+\nu}\right\}^{3/2}\left\{1-4\left(\frac{4\nu}{4\nu+3\hat{b}}\right)^{3/2}e^{-[4\hat{b}\nu/(4\nu+3\hat{b})]r_1^2}\right\}\right.$$
$$+3e^{-\hat{b}r_1^2}\left\{\frac{\pi}{(3/4)(\alpha+\nu)+\hat{b}}\right\}^{3/2}$$
$$\left.\times\exp\left\{\frac{\hat{b}^2r_1^2}{(3/4)(\alpha+\nu)+\hat{b}}\right\}\cdot\exp\left\{-\frac{7}{12}\frac{\hat{b}(\alpha+\nu)r_1^2}{(3/4)(\alpha+\nu)+\hat{b}}\right\}\right]$$
$$\equiv\hat{N}^{-1}(r_1)\hat{I}_{0\phi}\left[\left\{\frac{\pi}{\alpha+\nu}\right\}^{3/2}\left(\frac{4}{3}\right)^{3/2}\{1+f_{0\phi}(r_1)\}+g_{0\phi}(r_1)\right]e^{-Ar_1^2} \tag{5.15}$$

and is a function only of the magnitude of $\vec{r}_1$ so that the angular integrations accomplished for ${}_{M_s'm_s}\langle f|H'|i\rangle_0$ are the same as those for ${}_{M_s'm_s}\langle f|H'|i\rangle_{0\phi}$. This latter term can be written down by inspection using the replacements that

$$u_\pi(\vec{r})\to w_\pi(\vec{r})\text{ and }\hat{I}_{00}\to\hat{I}_{0\phi}\hat{N}^{-1}(r_1)\left[\left\{\frac{\pi}{\alpha+\nu}\right\}^{3/2}\left(\frac{4}{3}\right)^{3/2}\{1+f_{0\phi}(r_1)\}\right\}+g_{0\phi}(r_1)\Big], \tag{5.16}$$

with $\hat{I}_{0\phi}$, $f_{0\phi}$, $g_{0\phi}$, and A as Eq. (5.15) defines. For the case that $M_s'=\frac{1}{2}$ and $m_s=-\frac{1}{2}$ this prescription provides

$$_{+-}\langle f|H'|i\rangle_\phi=-i\frac{F_{\pi N}}{\sqrt{E_\pi}}\frac{1}{\sqrt{2}}\frac{3}{4}\frac{1}{k_\pi k_N}\sum_{l,l'}i^{l'-l}4\pi P_l(\cos\Theta)$$
$$\times\int_0^\infty dr\,\hat{I}_{0\phi}e^{-Ar^2}\hat{N}^{-1}(r)\left[\left\{\frac{\pi}{\alpha+\nu}\right\}^{3/2}\left(\frac{4}{3}\right)^{3/2}\{1+f_{0\phi}(r)\}+g_{0\phi}(r)\right]u_{N_l}(r')$$
$$\times\left\{[l'^2\delta_{l'-1,l}-l^2\delta_{l'+1,l}]\frac{u_{\pi_{l'}}(r)}{r}+[l'\delta_{l'-1,l}+l\delta_{l'+1,l}]\frac{d}{dr}u_{\pi_{l'}}(r)\right\}. \tag{5.17}$$

So it is not necessary to evaluate ${}_{M_s'm_s}\langle f|H'|i\rangle_\phi$ because all except the numerically performed radial integrations are identical to those calculated in evaluating ${}_{M_s'm_s}\langle f|H'|i\rangle_0$ and the form of the radial integration is able to be obtained by prescription from that for ${}_{M_s'm_s}\langle f|H'|i\rangle_0$. These two terms are combined to form the amplitude

$${}_{M_s'm_s}\langle f|H'|i\rangle\equiv{}_{M_s'm_s}M_{fi}={}_{M_s'm_s}\langle f|H'|i\rangle_0+{}_{M_s'm_s}\langle f|H'|i\rangle_\phi\,. \tag{5.18}$$

The remaining amplitudes to be evaluated are for $M_s'm_s=-+,++,$ and $--$ and by inspection ${}_{-+}\langle f|H'|i\rangle_0=-{}_{+-}\langle f|H'|i\rangle_0$. The last two amplitudes are

$${}_{++}\langle f|H'|i\rangle_0=i\frac{F_{\pi N}}{\sqrt{E_\pi}}\sqrt{2}\int d\vec{r}_1u_N^*(r_1')[-\tfrac{1}{2}d_-u_\pi]I_{00}(r_1)$$
$$=i\frac{F_{\pi N}}{\sqrt{E_\pi}}\frac{1}{\sqrt{2}}(4\pi)^{3/2}\frac{3}{4}\frac{1}{k_\pi k_N}\sum_{l,l'}i^{l'-l}Y_{l,-1}(\Omega_\Theta)$$
$$\times\int_0^\infty dr\,u_{N_l}(r)I_{00}(r)\left\{\frac{l(l+1)}{2l+1}\right\}^{1/2}\left[\left(-\frac{d}{dr}+\frac{l}{r}\right)u_{\pi_{l'}}(r)\delta_{l,l'+1}\right.$$
$$\left.+\left(\frac{d}{dr}+\frac{l-1}{r}\right)u_{\pi_{l'}}(r)\delta_{l,l'-1}\right] \tag{5.19}$$

and

$$
{}_{--}\langle f|H'|i\rangle_0 = i\frac{F_{\pi N}}{\sqrt{E_\pi}}\sqrt{2}\int d\vec{r}_1 u_N^*(\vec{r}_1')[\tfrac{1}{2}d_+u_\pi]I_{00}(r_1)
$$
$$
= i\frac{F_{\pi N}}{\sqrt{E_\pi}}\frac{1}{\sqrt{2}}(4\pi)^{3/2}\frac{3}{4}\frac{1}{k_\pi k_N}\sum_{l,l'} i^{l'-l}Y_{l,+1}(\Omega_\Theta)
$$
$$
\times\int_0^\infty dr u_{N_l}^*(r)I_{00}(r)\left\{\frac{l(l+1)}{2l+1}\right\}^{1/2}\left[\left(-\frac{d}{dr}+\frac{l}{r}\right)u_{\pi_{l'}}(r)\delta_{l,l'+1}+\left(\frac{d}{dr}+\frac{l-1}{r}\right)u_{\pi_{l'}}(r)\delta_{l,l'-1}\right] \quad (5.20)
$$

so that all amplitudes have been expressed in the desired form

$$
{}_{M_s'm_s}M_{fi} = \sum_{l=0}^{\infty} A_l(\Theta)\int_0^\infty dr_{M_s'm_s}\mathfrak{M}_{fi}^l(r)\,.
$$

The angular distribution for the absorption reaction is then given by

$$
\frac{d\sigma}{d\Omega} = \frac{1}{j_{\text{inc}}}\frac{2\pi}{\hbar}\sum_f |M_{fi}|^2\rho(E_f)\,. \quad (5.21)
$$

If the two particles of the final state are designated by 1 and 2, with 2 representing the residual nucleus, and the two particles of the initial state are indicated by 3 and 5, with 4 being the target nucleus, then the density of final states factor is

$$
\rho(E_f) = \frac{1}{(2\pi\hbar)^3}\frac{p_1E_1E_2}{c^2E_f}\,, \quad (5.22)
$$

where

$$
E_f = E_1 + E_2 = \{p_1^2c^2 + m_1^2c^4\}^{1/2} + \{p_1^2c^2 + m_2^2c^4\}^{1/2}
$$

in the barycentric reference frame so that $\vec{p}_1 = -\vec{p}_2$. Finally, if the scattering functions u_π and u_N are normalized to plane waves, then the incident flux is just the velocity of the incident wave

$$
j_{\text{inc}} = \frac{\hbar k_3 c^2}{E_3}\,, \quad (5.23)
$$

and so the angular distribution becomes

$$
\frac{d\sigma}{d\Omega} = \frac{E_3}{c^2\hbar k_3}\frac{2\pi}{\hbar}\frac{\hbar k_1E_1E_2}{c^2E_f}\frac{1}{(2\pi\hbar)^3}\sum_f |M_{fi}|^2\,. \quad (5.24)
$$

As the absorbing nucleus becomes increasingly massive the density of final states factor $\rho(E_f)$ $\to (2\pi\hbar)^{-3}p_1E_1/c^2$ so that Eq. (5.24) is, in this static limit, in accord with earlier calculations.[30]

VI. RESULTS FOR PION ABSORPTION

In the case that the outgoing neutron and incident pion wave functions are taken to be plane waves, the amplitude and angular distribution for pion absorption can be evaluated analytically.[30] Taking $\vec{k}_\pi = k_\pi\hat{z}$ and $\vec{k}_N$ to make angle Θ with $\hat{z}$ and angle ζ with $\vec{r}$, and rotating the axis with respect to which the integration is done so that $\vec{k}_N\cdot\vec{n}$ has no y component, the amplitude

$$
\langle f|H'|i\rangle_0 = -\frac{i}{\sqrt{2}}\frac{F_{\pi N}}{\sqrt{E_\pi}}\int d\vec{r}u_N^*(\vec{r}')[d_0u_\pi]I_{00}(r) \quad (6.1)
$$

with $d_0u_\pi = \partial/\partial z\, e^{ik_\pi z} = ik_\pi e^{ik_\pi z}$ and using $\tilde{k}_N \equiv \frac{4}{3}\tilde{k}_N$ and with $u_N(\vec{r}') = e^{i\tilde{k}_N r\cos\zeta}$ is readily evaluated in cylindrical coordinates to be

$$
{}_{+-}\langle f|H'|i\rangle_0 = \frac{1}{\sqrt{2}}\left\{\frac{\pi}{A}\right\}^{3/2} \times\frac{F_{\pi N}}{\sqrt{E_\pi}}\hat{I}_{00}k_\pi e^{-(\tilde{k}_N^2+k_\pi^2)/4A}e^{k_\pi\tilde{k}_N\cos\Theta/2A} \quad (6.2)
$$

with $A = \frac{4}{3}\nu/2$ as above. Expanding the angular dependent exponential in Legendre polynomials

$$
e^{k_\pi\tilde{k}_N\cos\Theta/2A} = \sum_{l=0}^{\infty}(2l+1)i_l(k_\pi\tilde{k}_N/2A)P_l(\cos\Theta) \quad (6.3)
$$

allows an expression for the contribution to the total amplitude from each free partial wave in terms of the modified spherical Bessel functions $i_l(k_\pi\tilde{k}_N/2A)$. This analytic zero order lth partial wave amplitude was used as a test of the accuracy of the numerical integration used to calculate the partial wave amplitudes when the full scattering states were used instead of plane waves. The integrand of the numerically computed radial integral oscillates with a period of $\lesssim 1$ fm and is damped by the Gaussian $e^{-4/3(\nu/2)r^2}$ with $\frac{4}{3}\nu/2 \cong 0.8$ fm^{-2} so that the integration must be continued out to $r \cong 4$ fm and cancellation occurs. The resulting numerically computed partial wave amplitudes and the corresponding analytic partial wave amplitudes agree to no better than 2% for $l \le 2$ but the agreement improves to better than within 0.5% for the higher partial waves and the summed amplitude suffers only the same inaccuracy as that of the low partial wave amplitudes or about 2%. For $T_\pi = 290$ MeV contributions to the summed amplitude from partial waves higher than $l \cong 15$ are negligible.

For plane waves the only other nonzero amplitude is $_{-+}\langle f|H'|i\rangle_0 = -_{+-}\langle f|H'|i\rangle_0$ so that the angular distribution in Born approximation for plane waves is given by

$$\frac{d\sigma}{d\Omega}=\frac{k_N}{k_\pi}\frac{E_N E_t}{E_N+E_t}E_\pi\frac{1}{(2\pi)^2}\frac{1}{(\hbar c)^4}[2|_{+-}\langle f|H'|i\rangle|^2] \tag{6.4}$$

or with $F_{\pi N}{}^2 = 4\pi(\hbar c)^5 f^2(m_\pi c^2)^{-2}$, where $f^2 = 0.080 \pm 0.005$, the angular distribution becomes

$$\frac{d\sigma}{d\Omega}=\frac{k_N}{k_\pi}\frac{E_N E_t}{E_N+E_t}E_\pi\frac{1}{(2\pi)^2}\frac{1}{(\hbar c)^4}\left[\frac{\pi}{A}\right]^3 \times\frac{F_{\pi N}{}^2}{E_\pi}\hat{I}_{00}{}^2 k_\pi{}^2 e^{-(\vec{k}_N-\vec{k}_\pi)^2/2A}\,. \tag{6.5}$$

The energy at which the n-^{3}H elastic scattering wave function is calculated incorporates a 20 MeV binding energy per nucleon for the ^{4}He nucleus and corresponds to a neutron laboratory kinetic energy $T_n = 556$ MeV.

Figure 9 is the angular distribution for the emitted neutrons in the case that plane waves are used for the n^3H and π^--^{4}He elastic scattering wave functions. The experimental results are indicated in Fig. 9 by a curve resulting from the more detailed experimental results of Fig. 10. Figure 9 also gives the neutron angular distribution resulting from a neglect of the transformation from the primed to the unprimed variables $\vec{r}_1' \to A/(A-1)\vec{r}_1$. The dramatic increase in the forward scattering from the neglect of the coordinate transformation demonstrates the need for a careful treatment of the recoil of the nucleus when free waves are used in a Born approximation of the absorption amplitude. Of course this effect decreases as the number of target nucleons increases but even for an absorbing nucleus of 16 nucleons the disparity caused by neglect of the coordinate transformation is an order of magnitude in the forward direction.

The angular distribution for neutrons emitted in the pion absorption reaction ^{4}He(π^-, n)^{3}H has been measured[28] for a pion laboratory kinetic energy of 290 MeV and the results of this measurement are presented in Fig. 10 as a gauge of the theoretical results which are now presented. In each calculation the angular distribution is calculated both with and without the contribution of the second

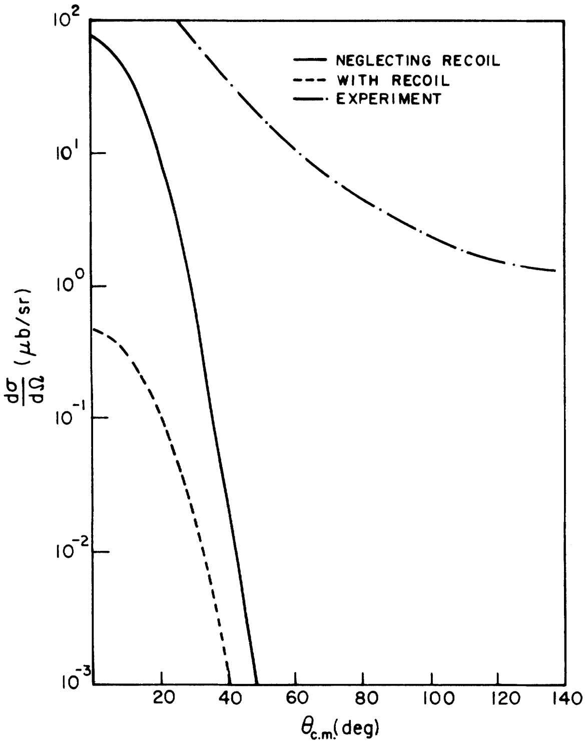

FIG. 9. Plane wave Born approximation of the pion absorption amplitude at $T_\pi = 290$ MeV.

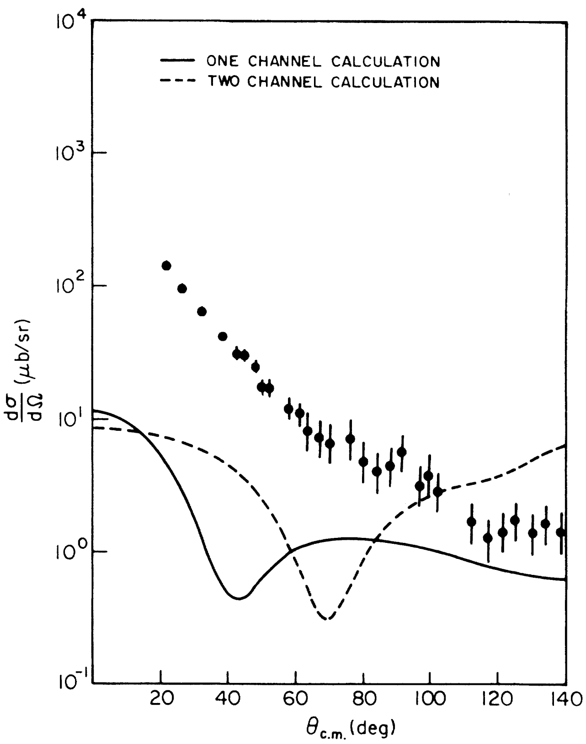

FIG. 10. Angular distribution of emitted neutrons resulting from nonlocal π^--^{4}He calculation at $T_\pi = 290$ MeV. The experimental points are as in Ref. 28 for $T_\pi = 290$ MeV.

channel included. These two different results will be called the two channel and single channel angular distributions.

The angular distributions resulting from the π^--^{4}He nonlocal, constrained, and unconstrained calculations are presented in Figs. 10–12. Figure 10 shows the single and two channel angular distribution generated with only the nonlocal potential. Figure 11 results from the πN constrained π^--^{4}He elastic scattering wave function and Fig. 12 is the angular distribution for the unconstrained π^--^{4}He elastic scattering wave function.

As most studies have determined, the calculated angular distribution is rather sensitive to the initial and final scattering states. For very small angles the contribution to the absorption amplitude by the second channel does not have great effect but the structure at large angles depends strongly on the contribution by the inelastic component of the π^--^{4}He elastic scattering state.

Experimental studies of this pion absorption reaction at $T_\pi = 200$ and 100 MeV produce angular distributions with a well marked dip at $\theta_{c.m.} \cong 70°$. This dip is thought to be suggestive of the dominance of the (3, 3) πN resonance[29] and not a diffractive effect because it is independent of the incident pion energy. The single channel angular distributions calculated with the πN constrained and unconstrained π^--^{4}He elastic scattering wave functions do produce a dip at $\theta_{c.m.} \cong 70°$ but the two channel angular distributions determined with these scattering states have a very pronounced dip at $\theta_{c.m.} \cong 54°$. When the π^--^{4}He elastic scattering states are generated solely with the nonlocal interaction the dip occurs at $\theta_{c.m.} \cong 70°$ only if the inelastic component of the π^--^{4}He scattering state is included.

VII. DISCUSSION

The great sensitivity of the pion absorption reaction to the elastic scattering wave functions used in its evaluation is a useful theoretical tool. Before this sensitivity can be used to advantage, however, the initial and final elastic scattering wave functions must be systematically generated. The explicit inclusion of Δ propagation and pion absorption is apparently a prerequisite to the systematic generation of the π^--^{4}He elastic scattering state. Inclusion of only the 33 resonance in the manner of this study has produced inelastic cross sections which are much smaller than is experimentally observed. Such a modification could be necessary to suppress the large back angle scat-

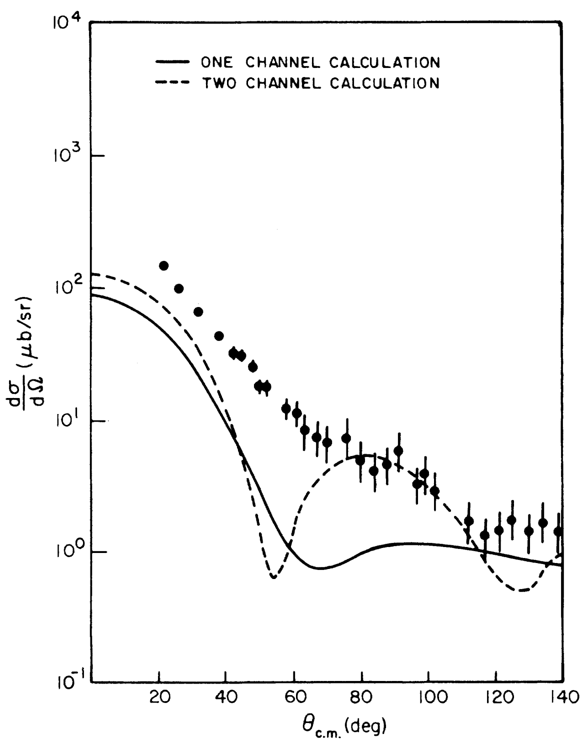

FIG. 11. Angular distribution of emitted neutrons resulting from constrained π^--^{4}He calculation compared with experiment for $T_\pi = 290$ MeV.

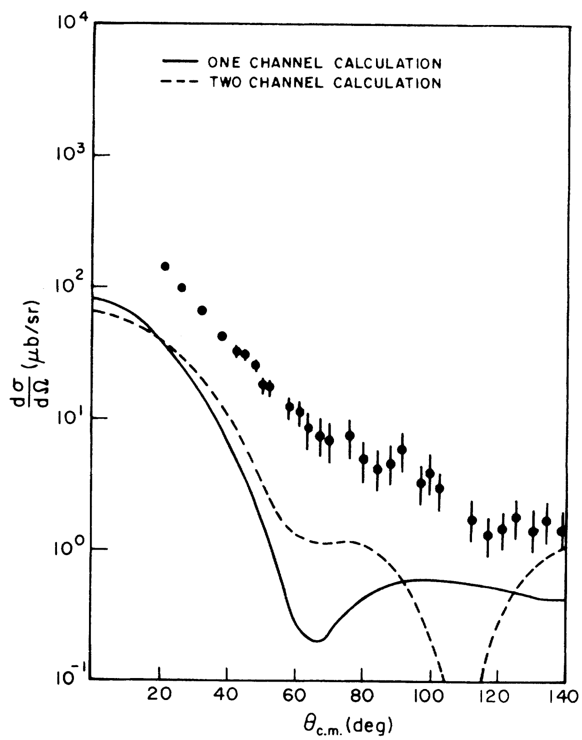

FIG. 12. Angular distribution of emitted neutrons resulting from unconstrained π^--^{4}He calculation compared with experiment for $T_\pi = 290$ MeV.

tering.

Nevertheless, valuable machinery now exists for probing features of N-nucleus and π-nucleus dynamics which only weakly effect elastic scattering angular distributions. The effective channel approach has been quite successful in describing N-nucleus elastic scattering,[10] and when additional n-^{3}He elastic scattering data become available, the final state wave function of the pion absorption amplitude will be more well defined. Then all study can be focused on the systematic determination of the π^--^{4}He elastic scattering wave function. The combined discrimination of the pion absorption amplitude and the elastic scattering angular distribution should reveal defects in the π^--^{4}He elastic scattering wave function.

The present work obviously requires much additional refinements before its result can be compared more meaningfully with experimental data. Thus: (i) improved parametrization of the πN parameters and solutions of the elastic problem across the 33 resonance region is very important. (ii) The Δ propagation inside the nucleus and the absorption effect should be treated more carefully, either by an additional effective interaction or by modifying the inelastic φ channel. The double counting problem between the v^R and this new term should be dealt with. (iii) The ECA treatment of the nt scattering requires additional experimental data, although the theory as it was applied to the p^4He problem earlier should be good enough. Effect of the inelastic component of the Ψ_{nt} on the pion absorption amplitude is yet to be estimated. Finally, (iv) the Galilean invariant term in the pion absorption amplitude should be evaluated incorporating the full $\Psi_{\pi\alpha}$ and Ψ_{nt}.

Note added in proof. Recently, an experimental data on $\pi^- + {}^4$He elastic scattering at 295 MeV was reported by J. Källne *et al.* [Phys. Rev. Lett. 45, 517, (1980)]. The general trends deduced earlier from the lower energy data persist, with a small dip near $\theta_{c.m.} = 75°$. A much deeper dip was seen also at $\theta_{c.m.} \approx 110°$.

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