Neutral current neutrino reactions and weak charged current processes in deuterium

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Cross sections for the neutral current neutrino reactions $\nu + {}^2\mathrm{H} \rightarrow p + n + \nu$ and $\bar{\nu} + {}^2\mathrm{H} \rightarrow p + n + \bar{\nu}$ are calculated by the use of form factors obtained from pion-photoproduction data by means of SU(2) commutation relations via the elementary particle model. The results are extrapolated from threshold to 150 MeV in the incident neutrino energy. Charged current cross sections for the reactions $\nu_e + {}^2\mathrm{H} \rightarrow p + p + e^-$ and $\nu_\mu + {}^2\mathrm{H} \rightarrow p + p + \mu^-$ are also obtained. Averaged cross sections for the reaction $\nu_n + {}^2\mathrm{H} \rightarrow p + p + e^-$ from neutrinos obtained from muon decay and for the reactions $\bar{\nu}_e + {}^2\mathrm{H} \rightarrow n + n + e^+$ and $\bar{\nu}_e + {}^2\mathrm{H} \rightarrow n + p + \bar{\nu}_e$ from neutrinos obtained from the Savannah River reactor are also obtained and compared with recent experiments.

[NUCLEAR REACTIONS Neutral and charged current neutrino disintegration cross sections of deuterium ${}^2\mathrm{H}(\nu,\nu)np$, ${}^2\mathrm{H}(\bar{\nu},\bar{\nu})np$, ${}^2\mathrm{H}(\nu,\mu^-)pp$, ${}^2\mathrm{H}(\nu,e^-)pp$, ${}^2\mathrm{H}(\bar{\nu},e^+)nn$ are calculated using the elementary particle model.]

I. INTRODUCTION

Neutral current neutrino reactions in deuterium at low to intermediate energies remain an interesting test of the Weinberg-Salam model[1] because they effectively allow the isolation of the axial component of the weak neutral current.[2] As in the charged current case the neutral current matrix element is entirely dominated by the axial current form factor. Thus one can study a single part of the weak neutral current.

In this paper we present a calculation which makes use of pion-photoproduction data[3] from the reaction $\gamma + {}^2\mathrm{H} \rightarrow n + n + \pi^+$ and the soft-pion theorems[4] to obtain the weak axial current form factor. Thus we are able to reach the LAMPF energy range without great extrapolation. This form factor is then used along with weak vector-current form factors obtained from electron scattering data to calculate neutral and charged current cross sections for the reactions $\nu + {}^2\mathrm{H} \rightarrow p + n + \nu$, $\bar{\nu} + {}^2\mathrm{H} \rightarrow p + n + \bar{\nu}$, $\nu_e + {}^2\mathrm{H} \rightarrow p + p + e^-$, and $\nu_\mu + {}^2\mathrm{H} \rightarrow p + p + \mu^-$, respectively.

The framework of the calculation is the elementary particle model.[5] In this model the initial and final nuclear states are treated as elementary particles of well defined spin and parity. Form factors describing the matrix elements are obtained from existing experimental data from related processes via SU(2) commutation relations, the conserved vector current hypothesis (CVC) and the partially conserved axial-vector current hypothesis (PCAC). Thus it is possible if sufficient experimental data exist to avoid the use of nuclear wave functions which are often difficult to determine but to which cross sections are sensitive.

In Sec. II of this paper we discuss the general form of the weak current matrix elements and the transition matrix element and display the form factors necessary to describe them. In Sec. III of this paper we calculate cross sections for the neutral current processes $\nu + {}^2\mathrm{H} \rightarrow p + n + \nu$ and $\bar{\nu} + {}^2\mathrm{H} \rightarrow p + n + \bar{\nu}$ and for the charged current processes $\nu_e + {}^2\mathrm{H} \rightarrow p + n + e^-$ and $\nu_\mu + {}^2\mathrm{H} \rightarrow p + n + \mu^-$. We also calculate the cross section for the reaction $\nu_e + {}^2\mathrm{H} \rightarrow p + p + e^-$ averaged over the electron neutrino spectrum from muon decay and the cross section for the reactions $\bar{\nu}_e + {}^2\mathrm{H} \rightarrow n + n + e^+$ and $\bar{\nu}_e + {}^2\mathrm{H} \rightarrow p + n + \nu_e$ averaged over the anti-electron neutrino spectrum from the Savannah River nuclear reactor. Finally in Sec. IV we discuss our results and compare them with recent experimental data.

II. WEAK CURRENT MATRIX ELEMENTS

The matrix element for the neutral current neutrino reaction $\nu + {}^2\mathrm{H} \rightarrow p + n + \nu$ and the charged current neutrino reaction $\nu_e + {}^2\mathrm{H} \rightarrow p + p + e^-$ are given, respectively, by

$$M(\nu + {}^2\mathrm{H} \rightarrow p + n + \nu) = \frac{1}{\sqrt{2}} G \langle pn | J_\lambda^N(0) | {}^2\mathrm{H}\rangle \bar{u}_\nu \gamma^\lambda (1-\gamma_5) u_\nu , \tag{1a}$$

$$M(\nu_e + {}^2\mathrm{H} \rightarrow p + p + e^-) = \frac{1}{\sqrt{2}} G \cos\theta_C \langle pp | J_\lambda(0) | {}^2\mathrm{H}\rangle \bar{u} \gamma^\lambda (1-\gamma_5) u , \tag{1b}$$

where

$$J_\lambda^N = J_\lambda^{(3)}(0) - \sin^2\theta_W J_\lambda^{(\mathrm{em})}(0) , \tag{2a}$$

$$J_\lambda^{(3)}(0) = V_\lambda^{(3)}(0) - A_\lambda^{(3)}(0) , \tag{2b}$$

$$J_\lambda = V_\lambda(0) - A_\lambda(0) , \tag{2c}$$

where $V_\lambda(0)$ and $V_\lambda^{(3)}(0)$ are the charge raising vector current and the third component of the isotriplet of vector currents, respectively. Similarly $A_\lambda(0)$ and $A_\lambda^{(3)}(0)$ are the charge raising axial current and the third component of the isotriplet of axial currents, respectively. Also θ_C and θ_W are the Cabibbo and Weinberg angles, respectively, and G is the weak coupling constant.[6]

Thus the problem of determining the two matrix elements, Eqs. (1a) and (1b), depends upon the determination of $\langle np|J_\lambda^N(0)|^2\mathrm{H}\rangle$ and $\langle pp|J_\lambda(0)|^2\mathrm{H}\rangle$. The structure of these matrix elements has been determined and elaborated by the author in previous papers[7-10]. We summarize the results:

$$\langle np|V_\lambda^{(3)}(0)|^2\mathrm{H}\rangle = \eta\bar{u}(p_1)\left(\frac{F_1^{(3)}}{M_d^2}\epsilon_{\lambda\nu\rho\sigma}\xi^\nu Q^\rho d^\sigma + \frac{F_2^{(3)}}{M_d}\gamma^\nu\epsilon_{\nu\rho\sigma\lambda}\xi^\rho q^\sigma\right)\gamma_5 v(p_2), \tag{3a}$$

$$\langle np|A_\lambda^{(3)}(0)|^2\mathrm{H}\rangle = \eta\bar{u}(p_1)(F_A^{(3)}\xi_\lambda + F_p^{(3)}\xi\cdot Qq_\lambda/M_d^2)\gamma_5 v(p_2), \tag{3b}$$

$$\langle pp|V_\lambda(0)|^2\mathrm{H}\rangle = \eta\bar{u}(p_1)\left(\frac{F_1}{M_d^2}\epsilon_{\lambda\nu\rho\sigma}\xi^\nu Q^\rho d^\sigma + \frac{F_2}{M_d}\gamma^\nu\epsilon_{\nu\rho\sigma\lambda}\xi^\rho q^\sigma\right)\gamma_5 v(p_2), \tag{3c}$$

$$\langle pp|A_\lambda(0)|^2\mathrm{H}\rangle = \eta\bar{u}(p_1)\left(F_A\xi_\lambda + F_P\frac{\xi\cdot Qq_\lambda}{M_d^2}\right)\gamma_5 v(p_2). \tag{3d}$$

Where ξ_μ is the deuteron polarization vector, $p_{1\mu}$ and $p_{2\mu}$ are the nucleon four momenta, d_μ is the deuteron four momentum, and

$$Q_\mu = (p_1+p_2)_\mu, \quad P_\mu = (p_1-p_2)_\mu, \quad q_\mu = (Q-d)_\mu, \tag{4}$$

$$\eta = m^2/E_1E_2^{1/2}(2\pi)^{-1/2}(2d_0)^{-1/2}, \tag{5}$$

where m is the nucleon mass and M_d is the deuteron mass.

The form factors F_1, F_2, $F_1^{(3)}$, $F_2^{(3)}$, F_A, F_P, and $F_A^{(3)}$, $F_P^{(3)}$ are related through the usual SU(2) current commutation relations,

$$[I^{(i)}, J_\mu^{(j)}(0)] = i\epsilon_{ijk}J_\mu^{(k)}(0), \tag{6}$$

so that, for example,

$$\langle pp|J\mu(0)|^2\mathrm{H}\rangle = -\langle pp|[I^+, J_\mu^{(3)}]|^2\mathrm{H}\rangle = -\sqrt{2}\langle np|J_\mu^{(3)}|^2\mathrm{H}\rangle. \tag{7}$$

Clearly if we had started from $\langle nn|J_\mu^+(0)|^2\mathrm{H}\rangle$ we would have obtained, using $J_\mu^+(0) = [I^-, J_\mu^{(3)}]$,

$$\langle nn|J_\mu^+(0)|^2\mathrm{H}\rangle = \sqrt{2}\langle np|J_\mu^{(3)}|^2\mathrm{H}\rangle, \tag{8}$$

so that from Eqs. (3a)–(3d), (8), and (9),

$$F_A^{(3)} = \pm F_A/\sqrt{2}, \quad F_P^{(3)} = \pm F_P/\sqrt{2}, \quad F_1^{(3)} = \pm F_1/\sqrt{2}, \quad F_2^{(3)} = \pm F_2/\sqrt{2}. \tag{9}$$

Thus if the charged current form factors can be obtained, the neutral current form factors are immediately obtainable. Furthermore the form factors F_A and F_P are related by PCAC,[11]

$$F_P = -M_d^2F_A/(q^2-m_\pi^2), \tag{10}$$

so that it is necessary to obtain only F_A, F_1, and F_2.

Pion-photoproduction data from the reaction $\gamma + {}^2\mathrm{H} \to n+n+\pi^+$ can be used to obtain F_A by means of the Kroll-Ruderman theorem.[4] This has been done by the author in a previous paper.[12] By making use of PCAC in the form

$$[\partial_\mu \pm iea_\mu(x)]A^{\mu(\pm)}(x) = \sqrt{2}m_\pi^2 f_\pi\Phi_\pi^\pm(x), \tag{11}$$

where $a_\mu(x)$ is the photon field and f_π is the pion decay constant. This leads to the relation

$$ie\epsilon_\mu\frac{\langle nn|A^{\mu+}(0)|d\rangle}{\sqrt{2}} \simeq \langle nn\pi^+|\gamma d\rangle[(2\pi)^3 2k_0]^{1/2}f_\pi \tag{12}$$

in the soft pion limit and with Eq. (10) and Eq. (3d) allows us to obtain F_A from pion photoproduction data.[13] The results are $|F_A|^2 = |\mathfrak{F}_A|^2 f_A^2$, where

$$|\mathfrak{F}_A|^2 = \frac{(3.61\times10^1 + 6.13\times10^{-1}q_0)}{[(q_0-0.11)^2+6.76\times10^{-2}]}\times\{1.0-\exp[-6.7\times10^{-7}(q^2+1.6\times10^4)^2]\} \times\{1.0+1.57\times10^2\exp[-9.49\times10^{-10}(q^2+0.97\times10^5)^2]\} \times\{1.0-1.71\exp[-2.83\times10^{10}(q^2+1.05\times10^5)^2]\} \tag{13}$$

with $f_A = (1.0-q^2/M_A^2)^{-2}$ and $M_A = 0.912$ GeV. A word should be said about this fit. Because we are relating one set of data (charged current data) to another set of data (neutral current data), it is merely necessary that the fit be accurate. No uniqueness is implied.

The vector part of the weak charged current is obtainable from electrodisintegration[14] and photodisintegration[15] data available from the reactions $e+{}^2\mathrm{H}\to p+n+e'$ and $\gamma+{}^2\mathrm{H}\to p+n$. We note that, at low energies, the outgoing n and p are in an $I=1$, $I_2=0$ state so that matrix elements of $J_\mu^{(\mathrm{em})}$ are approximately equal to those of $V_\mu^{(3)}$ in that range. Because we have no way of generally isolating the isoscalar contributions to J^{em} but since we have data at 180° which isolates[16] $V_\mu^{(3)}$, we can determine only the latter. We therefore assume that the matrix elements of $V_\mu^{(3)}$ and J_μ^{em} are approximately equal. In any case in our energy range the contributions from $J_\mu^{(\mathrm{em})}$ (or $V_\mu^{(3)}$) to neutrino interactions are small.

The forms of F_1 and F_2 have been previously found.[7] Only the combination F_1-F_2 occurs and the results are

$$|F_1-F_2|^2=[f_1(q^2)]^2(\mathfrak{F}_1-\mathfrak{F}_2)^2, \tag{14a}$$

$$f_1(q^2)=(1-q^2/M_v^2),\quad M_v=0.84\ \mathrm{GeV}, \tag{14b}$$

$$|\mathfrak{F}_1-\mathfrak{F}_2|^2=\frac{\sqrt{2}(1.05+1.41\times10^{-4}qd)}{(5.21\times10^{-4}qd-2.26)^2+1.8}\left[1-\frac{1}{M_d^4}\left(0.75q^2-q\cdot d-\frac{gd}{M_d}\right)^2\right]R(q^2,\cos\theta). \tag{14c}$$

$$R(q^2,\cos\theta)=[1.0+2.9\cos^2\theta+q^2(1.73\times10^{-5}+5.02\cos^2\theta)+q^4(3.27-10^{-9}+9.48\times10^{-9}\cos^2\theta)]$$
$$\times\frac{1+0.12\exp[-9.8\times10^{-9}(q^2+0.02\times10^6)^2]}{1+9.90\times10^2+2.2\times10^3[1-\exp(-5.5\times10^{-12}q^4)]q^4}, \tag{14d}$$

where θ is the angle between $\vec{p}_1$ and $\vec{q}$.

By the use of Eqs. (9), (10), (12), (14a)–(14d), and the value $\sin^2\theta_W=0.30$ we can now determine either directly or indirectly the matrix elements of the currents Eqs. (3a)–(3d). We are therefore ready to determine the cross sections for the neutral and charged current neutrino reactions in deuterium.

III. CROSS SECTIONS FOR NEUTRAL AND CHARGED CURRENT NEUTRINO REACTIONS IN DEUTERIUM

Transition matrix elements squared for the reactions $\nu+{}^2\mathrm{H}\to p+n+\nu$ and $\nu+{}^2\mathrm{H}\to p+p+e^-$ are given by

$$|M(\nu+{}^2\mathrm{H}\to p+n+\nu)|^2=\frac{2}{3m_\nu{}^2m^2}\Big\{(\nu\cdot\nu'\vec{q}^{\,2}-\vec{\nu}\cdot\vec{q}\,\vec{\nu}\cdot\vec{q})(1-\sin^2\theta_W)^2|F_1^{(3)}-F_2^{(3)}|^2+|F_A^{(3)}|^2(p_1\cdot p_2+m^2)$$
$$\times\Big[(3\nu\nu'-\vec{\nu}\cdot\vec{\nu}')+2\,\frac{(-q\cdot\nu'\vec{q}\cdot\vec{\nu}-q\cdot\nu\vec{q}\cdot\vec{\nu}'+\nu\cdot\nu\vec{Q}^2}{q-m_\pi{}^2}$$
$$+\frac{\vec{Q}^2)}{(q^2-m_\pi{}^2)^2}(2q\cdot\nu'q\cdot\nu-\nu\cdot\nu'q^2)\Big]$$
$$+(1-\sin^2\theta_W)|F_A^{(3)}||F_1^{(3)}-F_2^{(3)}|(p_1\cdot p_2+m^2)\frac{\nu\cdot\nu'(\nu+\nu')}{M_d}\Big\} \tag{15a}$$

and

$$|M(\nu+{}^2H\to p+p+\bar{e})|^2=\frac{2}{3m_em_\nu m^2}\Big\{[\vec{Q}^2(q\cdot e+m_e{}^2)-(\vec{Q}\cdot\vec{e}\,)^2](F_1-F_2)^2+(p_1\cdot p_2+m^2)F_A{}^2$$
$$\times\Big[3e\cdot\nu+2\vec{e}\cdot\vec{\nu}+\frac{2\vec{Q}^2m_eE_\nu-q\cdot e\vec{Q}\cdot\vec{\nu}-q\cdot\nu\vec{Q}\cdot\vec{e}}{(q^2-m_\pi{}^2)}+\frac{\vec{Q}^2(q\cdot eq\cdot\nu-q^2\nu\cdot e)}{(q^2-m_\pi{}^2)^2}\Big]$$
$$+(p_1\cdot p_2+m^2)|F_A||F_1-F_2|\,\nu\cdot e\,\frac{(\nu+e_0)}{M_d}\Big\}, \tag{15b}$$

where in both expressions the sign of the last term on the right changes if there is an incident antineutrino.

Equation (15a) leads to the cross sections shown in Fig. 1. In Fig. 2 we plot the charged current neutrino reaction [obtained from Eq. (15b)]. A word must be said about the expected accuracy of these results. Corrections[17] to Eq. (12) are of the order of m_π/m_i or about 7–8% in the case of deuterium. For the nucleon case it is higher or of the size $2m_\pi/m_p$ (30%). For our case we would expect corrections in the range of 20% which should be tolerable under present experimental conditions.

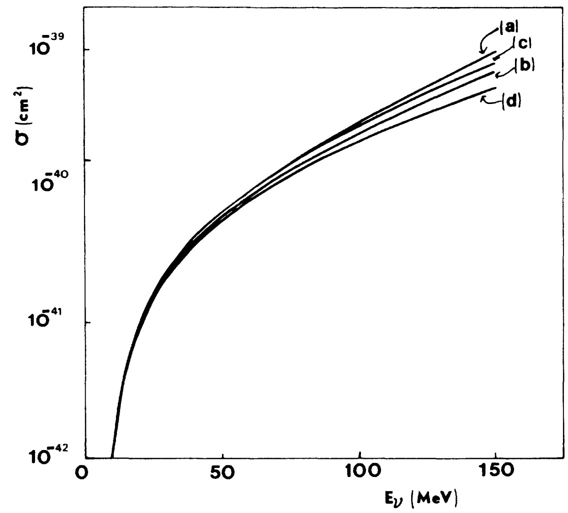

FIG. 1. Plot of cross sections for the reactions $\nu + {}^2H \to p + n + \nu$ and $\bar{\nu} + {}^2H \to p + n + \bar{\nu}$. Curve (a) refers to the calculation for the former and curve (b) refers to the calculation for the latter present here. Curves (c) and (d) are calculations for the former and latter, respectively, done by Ali and Dominguez.

We also calculate the value for $\langle\sigma(\nu^2H \to ppe^-)\rangle$ and $\langle\sigma(\nu^2H \to pn\nu)\rangle$, where the average is over the spectrum of electron neutrinos obtained from muon decay at rest. The results are

$$\langle\sigma(\nu^2H \to ppe^-)\rangle = 0.286\times 10^{-40}\ \mathrm{cm}^2\,, \qquad (16a)$$

$$\langle\sigma(\nu^2H \to pn\nu)\rangle = 0.305\times 10^{-40}\ \mathrm{cm}^2\,. \qquad (16b)$$

In addition we extrapolate to the region near threshold and obtain the cross sections for the reactions $\bar{\nu}_e + {}^2H \to n + n + e^+$ and $\bar{\nu}_e + {}^2H \to n + p + \bar{\nu}_e$ using the form factors obtained here. The results are shown in Figs. 3 and 4, respectively. In Fig. 5 we plot the cross section for the reaction $\nu_\mu + {}^2H \to p + p + u^-$ from threshold to 150 MeV incident neutrino energy. Finally, we obtain the cross sections $\langle\sigma(\bar{\nu}_e{}^2H \to nne^+)\rangle$ and $\langle\sigma(\bar{\nu}_e{}^2H \to pn\bar{\nu}_e)\rangle$ averaged over the $\bar{\nu}_e$ spectrum of the Savannah River reactor.[25] Our results are

$$\langle\sigma(\bar{\nu}_e{}^2H \to nne^+)\rangle = 2.35\times 10^{-45}\ \mathrm{cm}^2\,, \qquad (17a)$$

$$\langle\sigma(\bar{\nu}_e{}^2H \to pn\bar{\nu}_e)\rangle = 3.71\times 10^{-45}\ \mathrm{cm}^2\,, \qquad (17b)$$

where again we have used the form factors obtained in Sec. II.

IV. CONCLUSION

There are several theoretical calculations and a small but growing body of experimental data with which the results presented in Sec. III may be compared. In Fig. 1 we show the results of an impulse approximation calculation of $\nu + {}^2H \to p + n + \nu$ by Ali and Dominguez.[18] The results are in good agreement when the expected accuracies of the calculations are taken into account.

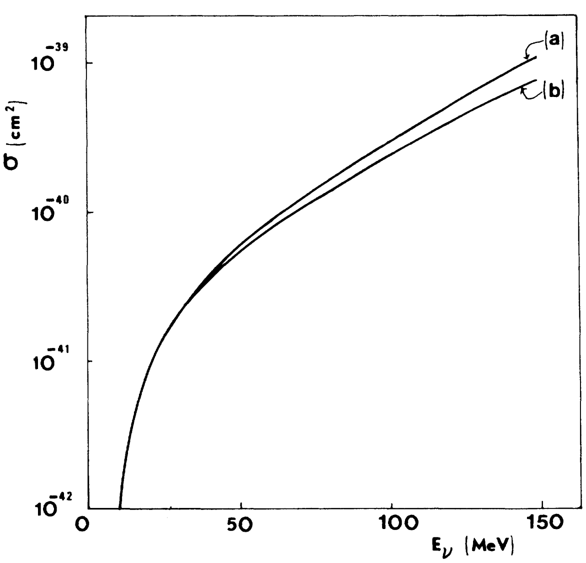

FIG. 2. Plot of cross sections for the reactions $\nu_e + {}^2H \to p + p + e^-$, curves (a), and $\bar{\nu}_e + {}^2H \to n + n + e^+$, curve (b).

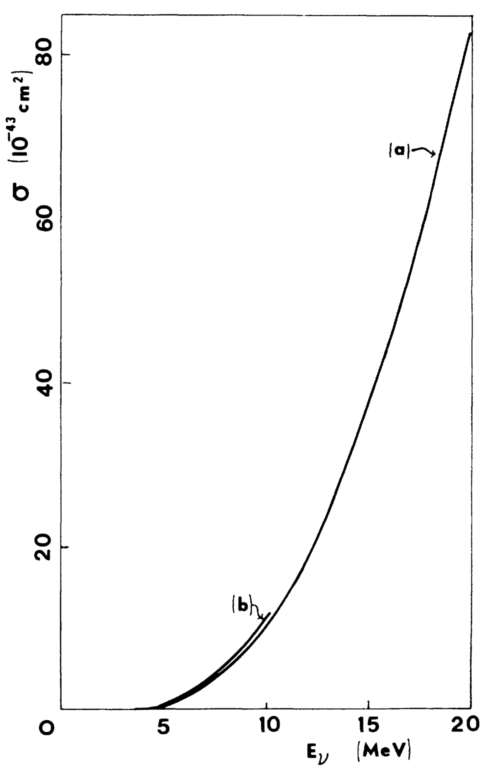

FIG. 3. Plot of the cross section for the reaction $\bar{\nu}_e + {}^2H \to n + n + e^+$ near threshold. Curve (a) refers to the calculation presented here and curve (b) refers to a calculation by Weneser.

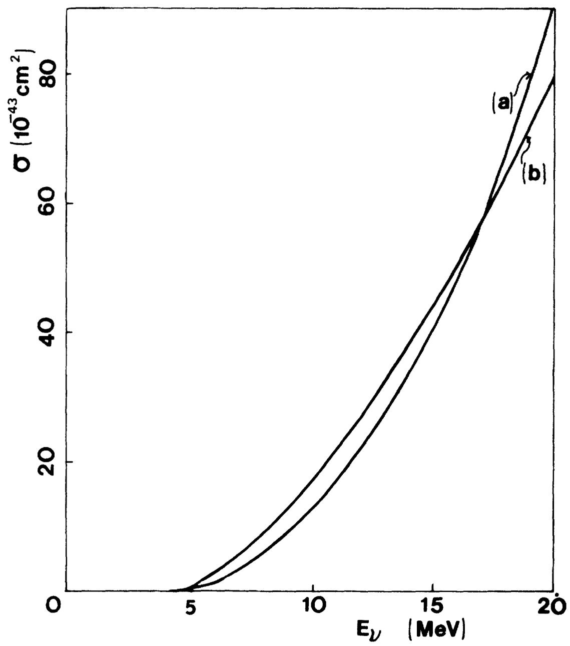

FIG. 4. Plot of the cross section for the reaction $\bar{\nu}_e + {}^2\mathrm{H} p + n + \bar{\nu}_e$ near threshold. Curve (a) refers to the calculation presented here and curve (b) refers to a calculation by Singh.

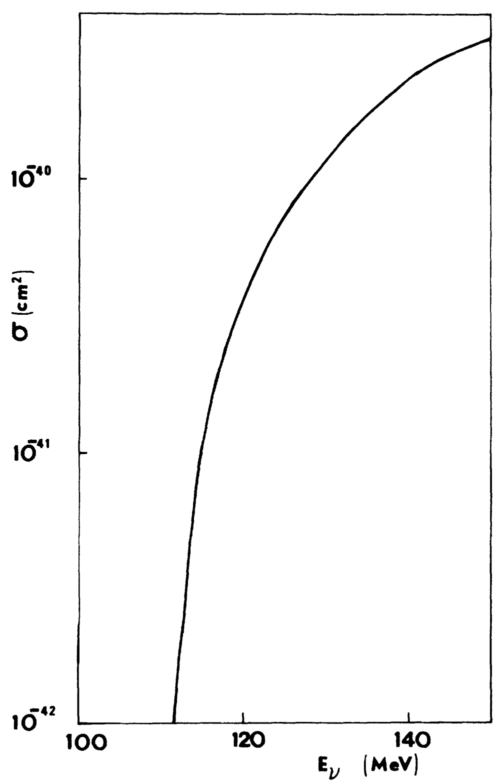

FIG. 5. Plot of the cross section for the reaction $\nu_\mu + {}^2\mathrm{H} \to p + p + \mu^-$.

Recently some data for the charged current reaction $\nu_e + {}^2\mathrm{H} \to p + p + e^-$ at LAMPF[19,20] energies have become available in the form $\langle\sigma(\nu_e{}^2\mathrm{H} \to pp\bar{e})\rangle = (0.52 \pm 0.18) \times 10^{-40}$ cm^2. We must reduce this number by a factor[21] of 2 for comparison with the result of Eq. (17a). When this is done there is very close agreement, $(0.26 \pm 0.09) \times 10^{-40}$ cm^2 for the experimental number versus 0.286×10^{-40} cm^2 for the calculated number. Because this charged current experimental result can be used to modify the form factors presented here, one could now obtain theoretical results for the neutral current process in a very direct way independent of detailed assumptions of nuclear structure.

In Fig. 3 and 4 we have extrapolated our results to the region near threshold. We also plot in Fig. 3 results obtained theoretically[22] by Weneser for the reaction $\bar{\nu}_e + {}^2\mathrm{H} \to n + n + e^+$, and find very close agreement with our own results. In Fig. 4 we also plot the results obtained[23] by Singh, for example; again there is reasonable agreement in spite of the fact that we are fairly far from the neutrino energy range for which we expect our form factors to be valid.

There have also been two experiments[24,25] at the Savannah River reactor in which cross sections for the reaction $\bar{\nu}_e + {}^2\mathrm{H} \to n + n + e^+$ averaged over the neutrino spectrum have been obtained. The results are $(1.5 \pm 0.4) \times 10^{-45}$ cm^2 for the most recent experiment and $(3.0 \pm 1.5) \times 10^{-45}$ cm^2 for the earlier one, both in reasonable agreement with the results presented here. In addition, a measurement[26] for the neutral current cross section $\bar{\nu}_e + {}^2\mathrm{H} \to p + n + \bar{\nu}_e$ averaged over the same neutrino spectrum has been made; the result $(3.89 \pm 0.09) \times 10^{-45}$ cm^2) is again in good agreement with the results presented here.

Thus at present there appears to be reasonable agreement at low energies between theoretical calculations based on the Weinberg-Salam model and experiment, although it would be desirable to improve the accuracy of both. It would be particularly interesting to have an intermediate energy neutral current neutrino experiment. If run with muon neutrinos, it would be possible to avoid a charged current background below 110 MeV as can be seen from Fig. 5. Furthermore, if accurate cross sectional data as a function of energy should become available for any of the charged current neutrino reactions at intermediate energy, it should be possible to produce calculations for the neutral current reaction in a largely nuclear model independent way.

Finally it has recently been suggested by Reines[27] that the physical neutrinos are combinations of base states and therefore oscillate. He has calculated the quantity

$$\frac{(\sigma_{exp}/\sigma_{th})_{charged}}{(\sigma_{exp}/\sigma_{th})_{neutral}}$$

and obtained a value of 0.43 ± 0.17. Using the data presented here we obtain a value of approximately 0.7 but with an error of over 20% and thus consistent with the Reines result. No similar effect is noted in the LAMPF data. Obviously this question must be examined with the greatest care.

ACKNOWLEDGMENTS

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[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[2]This fact has been noted by many authors, including, for example, Yu. V. Gapanov and I. V. Tyutin, Zh. Eksp. T Teor. Fiz. 47, 1826 (1964) [Sov. Phys.—JETP 20, 1231 (1965)] and A. Ali and C. A. Dominguez, Phys. Rev. D 12, 3673 (1975).

[3]G. Audit *et al.*, Phys. Rev. C 16, 1517 (1977); E. C. Booth *et al.*, Phys. Lett. 66B, 236 (1977).

[4]N. M. Kroll and M. A. Ruderman, Phys. Rev. 94, (1954) 233; D. Griffiths and C. W. Kim, Nucl. Phys. B6, 49 (1968).

[5]For details of the elementary-particle model see, for example, C. W. Kim and H. Primakoff, Phys. Rev. 139, B1447 (1965); 139, B566 (1965). For other processes see, for example, J. Frazier and C. W. Kim, Phys. Rev. 177, 2560 (1969); C. W. Kim and S. L. Mintz, Phys. Lett. 31B, 503 (1970).

[6]We use the values $G=1.025\times10^{-5}/m_p^2$, $\cos\theta_C=0.98$, and $\sin^2\theta_W=0.30$.

[7]S. L. Mintz, Phys. Rev. D 18, 3158 (1978).

[8]S. L. Mintz, Phys. Rev. D 13, 639 (1976).

[9]S. L. Mintz, Phys. Rev. D 10, 3017 (1974).

[10]S. L. Mintz, Phys. Rev. D 8, 2946 (1973).

[11]Y. Nambu, Phys. Rev. Lett. 4, 380 (1960).

[12]S. L. Mintz, Phys. Rev. C 22, 2507 (1980).

[13]See Ref. 3.

[14]G. A. Peterson and W. C. Barber, Phys. Rev. 128, 812 (1962); L. Katz, G. Ricca, T. E. Drake, and H. S. Kaplan, Phys. Lett. 28B, 114 (1968); K. M. Hanson, J. R. Dunning, Jr., M. Goitein, T. Kirk, L. E. Price, and R. Wilson, Phys. Rev. D 8, 753 (1973).

[15]E. A. Whalin, B. D. Schriever, and A. D. Hanson, Phys. Phys. Rev. 101, 377 (1956); D. R. Dixon and K. C. Bandtel, *ibid.* 104, 1730 (1956); J. A. Galey, *ibid.* 117, 763 (1960).

[16]J. Goldemberg and C. Schaerf, Phys. Lett. 20, 193 (1966).

[17]See Ref. 4 and also G. W. Gaffney, Phys. Rev. 161, 1599 (1967).

[18]A. Ali and C. A. Dominguez, Phys. Rev. D 12, 3673 (1975).

[19]S. E. Willis *et al.*, Phys. Rev. Lett. 44, 522 (1980).

[20]We take note of an earlier calculation of this cross section by J. S. O'Connell, Los Alamos Scientific Laboratory Report No. LA-5175-MS, 1973, yielding $\langle\sigma\rangle=0.48\times10^{-40}$ cm^2.

[21]Dr. R. L. Burman, private communication. This point deserves an additional comment. Some variation exists in the literature regarding the definition of total cross sections involving identical particles. In this particular calculation we have followed the discussion of Bjorken and Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964) (p. 135) and put in an additional factor of $\frac{1}{2}$ in total cross section not present in the differential cross sections. Thus our charged and neutral cross sections in this energy range are essentially the same size instead of the charged cross section being larger by a factor of 2. The use of this factor of 2 is not at all universal and is sometimes the cause of some confusion.

[22]J. Weneser, Phys. Rev. 105, 1335 (1957).

[23]S. K. Singh, Phys. Rev. D 11, 2602 (1975). See also T. Ahrens and T. P. Lang, Phys. Rev. C 3, 979 (1971) and the first authors in Ref. 2.

[24]T. L. Jenkins, F. E. Kinard, and F. Reines, Phys. Rev. Rev. 185, 1599 (1969).

[25]E. Pasierb *et al.*, Phys. Rev. Lett. 43, 96 (1979).

[26]F. T. Avignone III and L. H. Hopkins, Neutrino—78, edited by E. C. Fowler (Purdue University Press, W. Lafayette, Indiana, 1978).

[27]F. Reines, Bull. Am. Phys. Sec. 25, 570 (1980).