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Comparative ${}^{4}_{\Lambda}$He-${}^{4}_{\Lambda}$H binding energy differences for four *YN* potential models

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Exact four-body calculations of the ${}^{4}_{\Lambda}$He-${}^{4}_{\Lambda}$H binding energy difference ΔB_Λ have been made using separable potential approximations to four of the *YN* potential models of Nagels, Rijken, and deSwart. The results indicate that two of the potential models are consistent with the *YN* charge symmetry breaking implied by the experimental ΔB_Λ.

[NUCLEAR STRUCTURE ${}^{4}_{\Lambda}$He, ${}^{4}_{\Lambda}$H, ΔB_Λ, exact four-body calculation.]

Nagels, Rijken, and deSwart have developed several meson theoretic potentials to describe the available nucleon-nucleon (*NN*) and hyperon-nucleon (*YN*) scattering data.[1-4] Mass differences in the isomultiplets, as well as symmetry breaking exchanges, were included in a combined analysis of *NN*, Λp, $\Sigma^+ p$, etc., data. We summarize the low energy scattering parameters (scattering lengths and effective ranges) for four of their models in Table I. In the calculations reported below, we have used rank-one, *s*-wave separable potentials designed to reproduce these low energy scattering parameters. We have recently reported binding energy estimates for the hypertrition using these potentials.[5] It was found that only model *A* appeared to overbind ${}^{3}_{\Lambda}$H so much as to be considered inconsistent with experiment.[6]

To further test these potential models, we have used separable potential approximations to the *YN* potentials to determine the ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H ground state energies. The latest experimental estimates of the Λ-separation energies for these $J^\pi = 0^+$ ground states are[6]

$$B_\Lambda({}^{4}_{\Lambda}\text{He}) = B({}^{4}_{\Lambda}\text{He}) - B({}^{3}\text{He}) \cong 2.42 \pm 0.04 \text{ MeV},$$

$$B_\Lambda({}^{4}_{\Lambda}\text{H}) = B({}^{4}_{\Lambda}\text{H}) - B({}^{3}\text{H}) \cong 2.08 \pm 0.06 \text{ MeV}.$$

Because we do not solve the complete set of tensor force equations (we treat the *YN* triplet potentials in a central force approximation and use the truncated *t* matrix approximation for the *NN* triplet force),[7] we consider the Λ-separation energy difference $\Delta B_\Lambda \cong 0.34 \pm 0.07$ MeV to be a better measure of model consistency. This ΔB_Λ reflects true charge symmetry breaking (CSB) in the *YN* interaction; simple considerations of Coulomb energies in the $A = 3$ and 4 nuclear systems suggest that ΔB_Λ^C, the additional Coulomb energy in ${}^{4}_{\Lambda}$He due to compression of the "^{3}He core," is small and of opposite sign.[8] It is this Coulomb corrected quantity $\Delta B_\Lambda \cong 0.36$ MeV that we estimate for each of the *YN* potential models defined by the low-energy scattering parameters in Table I.

The exact coupled two-variable integral equations that must be solved for the $A = 4$ hypernuclear problem when the *NN* and *YN* interactions are represented by separable potentials are described in detail in Ref. 7. The integral equations are solved numerically without resort to separable expansions of the kernels. The resulting solutions possess the characteristics of true few-body calculations: for an attractive potential with a negative scattering length, $|a| > |a'|$ implies that V is

TABLE I. The ΛN scattering lengths and effective ranges in fm for the *YN* potential models of Refs. 1–4.

Model	Ref.	$a^s_{\Lambda p}$	$r^s_{\Lambda p}$	$a^t_{\Lambda p}$	$r^t_{\Lambda p}$	$a^s_{\Lambda n}$	$r^s_{\Lambda n}$	$a^t_{\Lambda n}$	$r^t_{\Lambda n}$
A	1	−2.16	2.03	−1.32	2.31	−2.67	2.04	−1.02	2.55
B	2	−2.11	3.19	−1.88	3.16	−2.47	3.09	−1.66	3.33
D	3	−1.77	3.78	−2.06	3.18	−2.03	3.66	−1.84	3.32
F	4	−2.18	3.19	−1.93	3.35	−2.40	3.15	−1.84	3.37

TABLE II. Potential parametrizations and their low energy properties for those interactions appropriate to the $A=4$ hypernuclei.

Model	System	Spin	λ (fm^{-3})	β (fm^{-1})	a (fm)	r (fm)
A	$\Lambda N({}^{4}_{\Lambda}\mathrm{He})$	*s*	0.4787	1.8891	−2.48	2.04
		t	0.4348	1.9660	−1.32	2.31
	$\Lambda N({}^{4}_{\Lambda}\mathrm{H})$	*s*	0.4957	1.9217	−2.31	2.03
		t	0.3819	1.9608	−1.02	2.55
B	$\Lambda N({}^{4}_{\Lambda}\mathrm{He})$	*s*	0.1578	1.3634	−2.34	3.12
		t	0.1670	1.4229	−1.88	3.16
	$\Lambda N({}^{4}_{\Lambda}\mathrm{H})$	*s*	0.1532	1.3527	−2.32	3.16
		t	0.1542	1.4128	−1.66	3.33
D	$\Lambda N({}^{4}_{\Lambda}\mathrm{He})$	*s*	0.1099	1.2549	−1.94	3.70
		t	0.1581	1.3846	−2.06	3.18
	$\Lambda N({}^{4}_{\Lambda}\mathrm{H})$	*s*	0.1093	1.2607	−1.85	3.74
		t	0.1484	1.3785	−1.84	3.32
F	$\Lambda N({}^{4}_{\Lambda}\mathrm{He})$	*s*	0.1532	1.3527	−2.32	3.16
		t	0.1421	1.3531	−1.93	3.35
	$\Lambda N({}^{4}_{\Lambda}\mathrm{H})$	*s*	0.1525	1.3558	−2.25	3.18
		t	0.1428	1.3632	−1.84	3.37

more attractive than V' in two-body, three-body, and four-body calculations, whereas $r > r'$ implies that V is more attractive than V' in a two body calculation, but *less* attractive in three-body and four-body calculations. Even though this picture is an oversimplification in terms of scattering length and effective range, it is possible to understand ΔB_{Λ} from each of the models in Table I qualitatively in terms of the low energy scattering parameters of the various models.

In our numerical calculations, we assume that effective ΛN interactions $\overline{V}^{s,t}_{\Lambda N}$ (i.e., one channel ΛN potentials determined from the free ΛN scattering parameters) can be used to describe the coupled ΛN-ΣN hyperon-nucleon system. Fortunately, this can be justified for the $J^{\pi}=0^{+}$ ground state (such is not the case for the $J^{\pi}=1^{+}$ excited states), where the triplet interaction is unmodified from its free form

$$V^{t}_{YN}=\begin{bmatrix} V^{t}_{\Lambda N} & V^{t}_{XN} \\ V^{t}_{XN} & V^{t}_{\Sigma N}\end{bmatrix} \cong \overline{V}^{t}_{\Lambda N}\,.$$

Since $V^{s}_{XN}\cong 0$ in the singlet interaction,

$$V^{s}_{YN}=\begin{bmatrix} V^{s}_{\Lambda N} & -\frac{1}{3}V^{s}_{XN} \\ -\frac{1}{3}V^{s}_{XN} & V^{s}_{\Sigma N}\end{bmatrix} \cong \overline{V}^{s}_{\Lambda N}$$

is also a good approximation.[7] Thus the effects of Λ-Σ coupling upon the ΛN potential parameters, including charge symmetry breaking due to meson mixing, $\Sigma^{\pm,0}$ mass differences, etc., are taken into account implicitly, but there are no explicit Σ channels in the calculation.

The Λp and Λn potential averages appropriate to ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H are

$${}^{4}_{\Lambda}\mathrm{He:}\quad V^{t}_{\Lambda N}=V^{t}_{\Lambda p}\,,\quad V^{s}_{\Lambda N}=\tfrac{1}{3}V^{s}_{\Lambda p}+\tfrac{2}{3}V^{s}_{\Lambda n}\,,$$

$${}^{4}_{\Lambda}\mathrm{H:}\quad V^{t}_{\Lambda N}=V^{t}_{\Lambda n}\,,\quad V^{s}_{\Lambda N}=\tfrac{1}{3}V^{s}_{\Lambda n}+\tfrac{2}{3}V^{s}_{\Lambda p}\,.$$

Instead of using the two potential formula to obtain the required potentials, we used the excellent approximation of scattering length and effective range averages

$$a^{-1}_{\Lambda N}=\tfrac{1}{3}a^{-1}_{\Lambda p}+\tfrac{2}{3}a^{-1}_{\Lambda n}\,,$$

$$r_{\Lambda N}=\tfrac{1}{3}r_{\Lambda p}+\tfrac{2}{3}r_{\Lambda n}$$

to parametrize the ΛN singlet interaction, etc. The resulting potential parameters are listed in Table II, where we use the Yamaguchi[9] form

$$V^{i}_{\Lambda N}(k,k')=-\frac{\lambda_i}{2\mu}g_i(\vec{k})g_i(\vec{k}')\,.$$

The NN potential parameters[10,11] for the model calculations are listed in Table III; the triton binding energy is 7.05 MeV in the truncated t matrix approximation[12] which is only 7% below the complete model result.

The results of our ${}^{4}_{\Lambda}$He-${}^{4}_{\Lambda}$H binding energy difference calculations are tabulated in Table IV. Because the singlet potentials are averages of Λn and Λp potentials, most of the charge symmetry breaking results from the triplet interaction differences (see Table II). It is clear that differences between triplet scattering lengths and effective

TABLE III. The NN potential parameters and low energy properties for the separable interaction used in the $A=4$ hypernuclear calculations (Refs. 10 and 11).

Spin	λ (fm^{-3})	β (fm^{-1})	ξ	β_T (fm^{-1})	a (fm)	r (fm)
s	0.132 3	1.130	0		−17.0	2.84
t	0.142 97	1.2412	4.4949	1.9476	5.397	1.722
$B({}^{2}\mathrm{H})=2.225$ MeV						
$P_D({}^{2}\mathrm{H})=0.07$						
$B({}^{3}\mathrm{H})=7.59$ MeV						

TABLE IV. The ${}^4_\Lambda$He-${}^4_\Lambda$H binding energy difference ΔB_Λ for each of the YN model discussed in the text in the central potential approximation for the ΛN interaction.

Model	ΔB_Λ (MeV)
A	1.32
B	0.47
D	0.43
F	0.19

ranges for the ${}^4_\Lambda$He and ${}^4_\Lambda$H systems are very similar for models B and D. Thus one anticipates similar values of ΔB_Λ for models B and D, and these values are not inconsistent with experiment. Model A has an even larger difference in scattering length values ($\Delta a \sim -0.3$ fm vs -0.2 fm for models B and D) and effective range values ($\Delta r \sim -0.25$ fm vs -0.15 fm). Hence, ΔB_Λ for model A is expected to be larger than that for models B and D, as is the case; it is probably outside the limits set by the experimental values. The perhaps surprisingly large model A value of ΔB_Λ results from the small values of the effective ranges in that model, which produce large values of $B_\Lambda({}^4_\Lambda\text{He})$ and enhance CSB differences. We note that these small effective ranges of the model A singlet interactions are primarily responsible for the value of $B_\Lambda({}^3_\Lambda\text{H})$ being inconsistent with experiment.[5] It is clear from the effective ranges in Table I that model F is a much more charge symmetric model than models A, B, or D. In fact, the model F ${}^4_\Lambda$He and ${}^4_\Lambda$H scattering lengths and effective ranges in Table II show very little difference between the two singlet sets or the two triplet sets. Thus, one anticipates a small value of ΔB_Λ, one which is too small to be consistent with the experimental binding energy difference. Finally, we note that since we have used a central potential approximation to represent the ΛN triplet interaction, we have perhaps overestimated ΔB_Λ for each model[7]; however, the introduction of explicit ΛN-ΣN coupling should increase our ΔB_Λ estimates. Thus we have two compensating approximations in our triplet ΛN channels.

In summary, we have examined separable potential approximations to four of the hyperon-nucleon potential models of Nagels, Rijken, and deSwart[1-4] in an exact four-body calculation of ΔB_Λ for the ${}^4_\Lambda$He-${}^4_\Lambda$H isodoublet. We find model A, which overbinds ${}^3_\Lambda$H, to overestimate ΔB_Λ. Models B and D appear to be consistent with the experimental value of ΔB_Λ (and give reasonable ${}^3_\Lambda$H binding energies). We find model F, which is consistent with $B_\Lambda({}^3_\Lambda\text{H})$, to underestimate ΔB_Λ for the $A=4$ system; this result is understood in terms of the small differences between the singlet Λp and Λn scattering lengths and effective ranges in that model.

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