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Elastic scattering of pions from ^{4}He and ^{16}O near 1 GeV

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The elastic scattering of pions from ^{4}He and ^{16}O is analyzed with a single-scattering optical potential, which is used in the solution of a relativistic integral scattering equation. Good agreement is obtained with available data near 1 GeV and choices are made for the nuclear and nucleon charge form factors and among the phase parameter sets at this energy.

[NUCLEAR REACTIONS Scattering theory, pions on ^{4}He and ^{16}O, $E=1$ GeV, calculate $\sigma(\theta)$, compare nuclear and nucleon form factors and phase shift sets.]

INTRODUCTION

The pion is currently one of the most widely used probes of nuclear structure. This is mainly true in the energy range which extends to slightly above the (3,3) resonance energy.[1,2] This energy range has received much attention because (i) the pion-nucleon scattering amplitudes are known with great precision in this range and (ii) there is available much pion-nuclear scattering data for a variety of nuclei over this entire energy range.

The same situation does not hold for higher energy pion-nuclear scattering. The available parametrizations of the pion-nucleon amplitudes begin to diverge from one another above the (3,3) resonance energy, and, even more critically, there is very little pion-nuclear scattering data at energies above 400 MeV.

In spite of these difficulties, the analysis of high energy pion-nuclear scattering can be very informative. Such studies would permit investigation of higher momentum components in the nuclear and nucleon form factors. These studies would also permit comparison of the various representations of the pion-nucleon amplitudes in an energy range for which the uniqueness achieved at lower energies does not exist.

In this analysis we solve the integral scattering equation in momentum space[3] with an optical potential derived from multiple scattering theory.[4] In view of the convergence of the multiple scattering expansion with increasing energy and the success of single scattering potentials near the (3,3) resonance, a single scattering potential can be used with confidence near 1 GeV. Such a potential requires knowledge of the π-N amplitudes and the nuclear matter distribution. The second quantity is obtained from the nuclear charge form factor and the nucleon form factor.

The scattering observables are calculated for the scattering of pions from helium and oxygen near 1 GeV and compared with experiment. The choices of phase parameter representation, and nuclear and nucleon form factors are made on the basis of this comparison.

CALCULATION

The single scattering optical potential in momentum space has the form

$$\langle \vec{k}|V|\vec{k}_0\rangle = A\langle \vec{k}|t|\vec{k}_0\rangle F(q)\,,$$

where A is the nucleon number of the target nucleus, t is the π-N scattering matrix averaged over the target nucleons, and F is the form factor of the target nucleus. $\vec{k}_0$ and $\vec{k}$ are the initial and final pion momenta in the π-nuclear c.m. frame. The Legendre component of this potential is obtained by numerical quadrature and is used in the solution of the integral equation for the partial wave component of the reaction matrix.

$$R_l(k,k_0)=V_l(k,k_0)-\frac{2}{\pi}P\int_0^\infty \frac{V_l(k,k')R_l(k',k_0)k'^2dk'}{E(k')-E(k_0)}\,,$$

where $E(k)=(k^2+m^2)^{1/2}+(k^2+M^2)^{1/2}$, and m,M are the masses of the pion and nucleus. This equation is solved numerically[5] for the on-shell partial wave amplitude, keeping thirty partial waves. The numerical procedure followed in the solution of the equation for R_l consists of approximating the principal part integral as a sum by numerical quadrature and solving the resultant system of equations by matrix inversion. The program has been fully tested[5] and found to be numerically accurate.

The π-N information is contained in the scattering matrix t, which may be expressed in terms of phase parameters or in a form dictated by the optical theorem.[6] In the case of phase parameter sets, we have employed the searches which are designated as CERN theoretical, Saclay, and Berkeley Boone, in the Herndon report,[7] as well as a search due to Davies.[8] We retain all the phase parameters given in these searches, usually to G or H partial waves. Parametrized expressions devised by Hahn[9] and Burgov *et al.*[10] were also investigated. The parametrized expressions give generally poorer fits to the data than are obtained using phase parameters.

The t amplitudes are transformed from the π-nucleon frame to the π-nuclear frame by the procedure described in, for example, Thomas and Landau.[2] The t amplitudes are extrapolated off-shell by treating them as functions of the Mandelstam variable $T = -|p' - p|^2$ only. We find at these energies that the π-nuclear scattering is largely dominated by the on-shell behavior of the t amplitudes, and the precise off-shell extension procedure has little effect on the calculated observables.

Nuclear structure information is included via the nuclear form factor F. This quantity is obtained by deconvoluting the nuclear charge form factor and the proton and neutron form factors, which are related by

$$F_{ch} = F_p F_{ch,p} + N/Z F_n F_{ch,n},$$

where all quantities are functions of the momentum transfer q. For the light nuclei studied here, $N = Z$ and

$$F_n \simeq F_p = F(q),$$

so that

$$F(q) = F_{ch}/(F_{ch,p} + F_{ch,n}).$$

Because the neutron has zero charge and electric quadrupole moment, the lowest power of q appearing in $F_{ch,n}$ is q^4, so that while most authors set $F_{ch,n}$ equal to zero, it can have a significant effect at large q. The nucleon form factors studied here are

$$F_{ch,p} = \exp(-a^2q^2/6) \quad \text{Gaussian}$$

$$= 1/(1 + a^2q^2/12)^2 \quad \text{exponential}$$

$$= 0.16 + \frac{0.29}{(1 + q^2/4.4)} + \frac{0.55}{(1 + q^2/8.3)} \quad \text{realistic},$$

$$F_{ch,n} = 0.26 + \frac{0.29}{(1 + q^2/4.4)} - \frac{0.55}{(1 + q^2/8.3)} \quad \text{realistic},$$

$$F_{ch,p} + F_{ch,n} = 0.42 + \frac{0.58}{(1 + q^2/4.4)} \quad \text{NPR}.$$

For the Gaussian and exponential form factors Hofstadter *et al.*[11] find $a = 0.81$ fm. The realistic form factor is due to Littauer *et al.*[12] and leads to a proton r.m.s. radius of 0.89 fm.

The nuclear charge form factor is obtained from electron scattering studies. The form factors studied here for ^{4}He are

$$F_{ch} = \exp(-q^2a^2/4) \quad \text{Gaussian},$$

$$F_{ch} = \{1 - (aq)^{12}\} \exp(-b^2q^2) \quad \text{modified Gaussian}.$$

The modified Gaussian is taken from Frosch *et al.*[13] The form factors studied here for ^{16}O are

$$F_{ch} = (1 - q^2 a_{c.m.}{}^2/8) \exp(-q^2 a_{ch}{}^2/4) \quad \text{EC},$$

$$F_{ch} = \frac{3}{x^3(1+3\alpha)} [\sin(x)(1 + \alpha x^2) - x\cos x] \times \exp(-\alpha x^2/2) \quad \text{AGP}.$$

The form EC is that designated as "corrected" in Ehrenberg *et al.*[14] The form AGP is found in Antonov *et al.*[15] with $x = qR$ and $\alpha = S^2/R^2$, S and R being the nuclear parameters.

RESULTS

We apply the calculation to the analysis of π^--^{4}He elastic scattering data[16-18] near 1120 MeV and to the π^--^{16}O data[10] at 890 MeV.

The π^--^{4}He data has been analyzed elsewhere.[18,9] We find with Liu and Shakin that an acceptable fit can be obtained only with a modified-Gaussian nuclear charge form factor. This reduces the variables affecting the fit to the nucleon form factors and the π-N amplitude representations.

The comparison of the nucleon form factors is shown in Fig. 1. These curves are calculated using the modified Gaussian nuclear charge form factor and the CERN Th phase parameter set. The G, E, and R plots all take the neutron form factor to be strictly zero while NPR uses the "realistic" form factors for both neutron and proton. As can be seen in Fig. 1, while the various nucleon form factors are readily distinguished, only the Gaussian can be eliminated on the basis of comparison with data. Additional large angle data is needed to choose among the remaining nucleon form factors. Since the realistic form factor asymptotically lies between the NPR and E calculations, it is used in our subsequent calculations. (This choice was also made in Ref. 18.)

The phase parameter representations of the t amplitudes are compared in Fig. 2, in which are compared CERN Th, Saclay, Davies, and Berkeley Boone. As can be seen, the various phase parameter sets can be readily distinguished. The Berkeley Boone set lies well below the data and may be eliminated as an acceptable set at this energy. The CERN Th, Saclay, and Davies sets

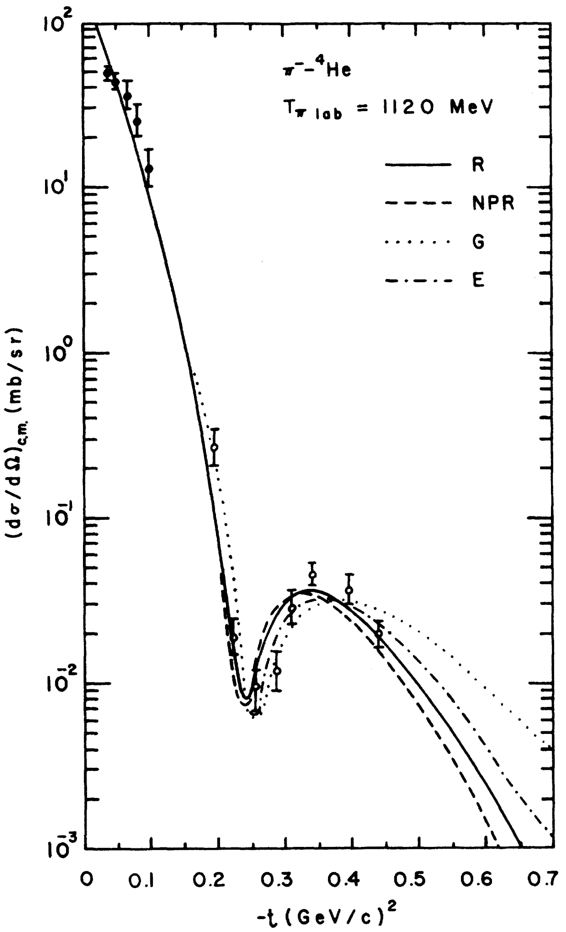

FIG. 1. Comparison of nucleon charge form factors for π^--^{4}He elastic scattering at 1120 MeV. *R* refers to realistic, NPR to combined neutron-proton realistic, *G* to Gaussian, and *E* to exponential. All curves calculated with a CERN Th phase parameter set and modified Gaussian nuclear charge form factor.

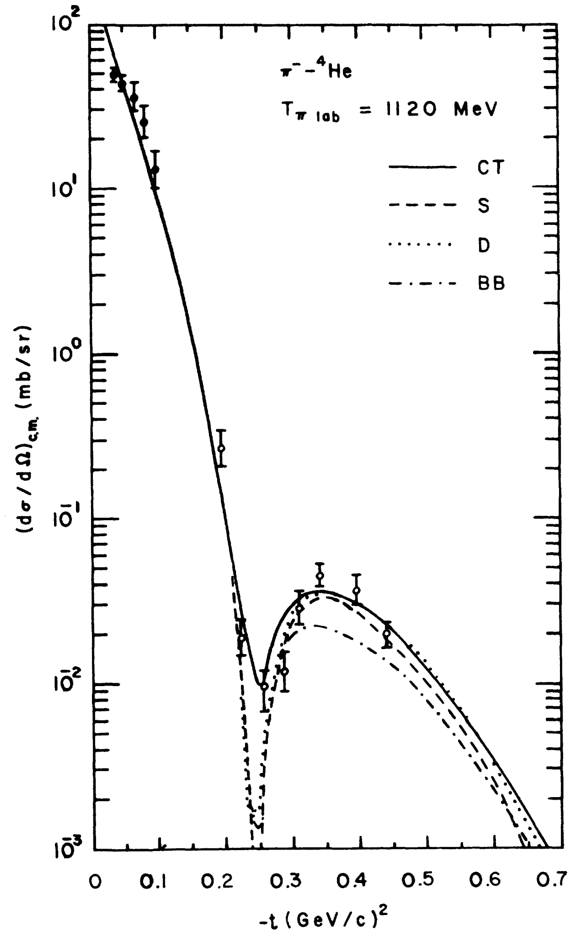

FIG. 2. Comparison of π-*N* phase parameter sets for π^--^{4}He elastic scattering at 1120 MeV. CT refers to CERN Th, *S* to Saclay, *D* to Davies, and BB to Berkeley Boone. All curves calculated with a realistic proton form factor and modified Gaussian nuclear form factor.

are readily distinguished in the region of the diffraction minimum, with CERN Th giving a rather shallow minimum, and Davies and Saclay leading to deep minima. Additional data in this region would be most helpful in distinguishing these phase parameter sets. We make a tentative choice of the CERN Th set, as it yields a minimum more consistent with the available data.

The π^--^{16}O data is that of Burgov *et al.*,[10] who obtained three data points at a scattering energy of 890 MeV. Figure 3 shows the analysis of this data comparing the EC and AGP nuclear charge form factors for ^{16}O and the *R* and *G* nucleon form factors, using the CERN Th phase parameter set. As can be seen, the EC nuclear charge form factor matches the data much more closely for both nucleon form factors than does the AGP. Figure 3 also shows that the *G* and *R* nucleon form factors, which were readily distinguished in the case of He, are barely separated in the case of ^{16}O. This is clearly due to the fact that while the nucleon size is significant compared to that of the ^{4}He nucleus, it is much less so in the case of ^{16}O. This indicates that for ^{16}O and larger nuclei, the choice of nucleon form factor is not very critical. Figure 4 shows the comparison of the phase parameter sets. As can be seen, the curves are well separated although the uncertainty in the data makes a choice among the sets difficult. We note however, that the Saclay and Davies plots run rather high through the error bars, while CERN Th lies closer to the data points. A refinement of this data as well as data near the diffraction minimum where the plots are well separated

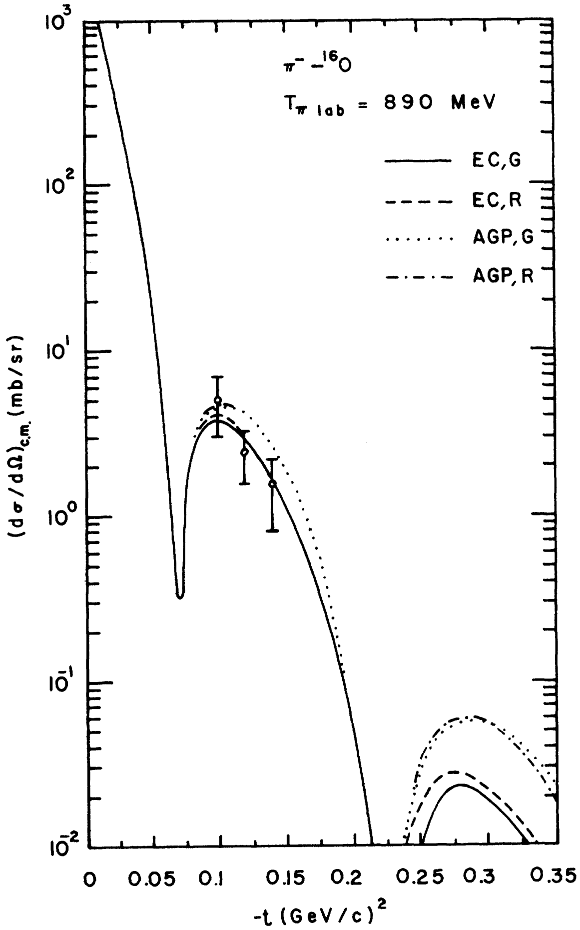

FIG. 3. Comparison of nuclear and nucleon form factors for π^--^{16}O elastic scattering at 890 MeV. Nuclear form factors are EC from Ref. 14 and AGP from Ref. 15. Proton form factors are Gaussian and realistic. Curves calculated with CERN Th phase parameter set.

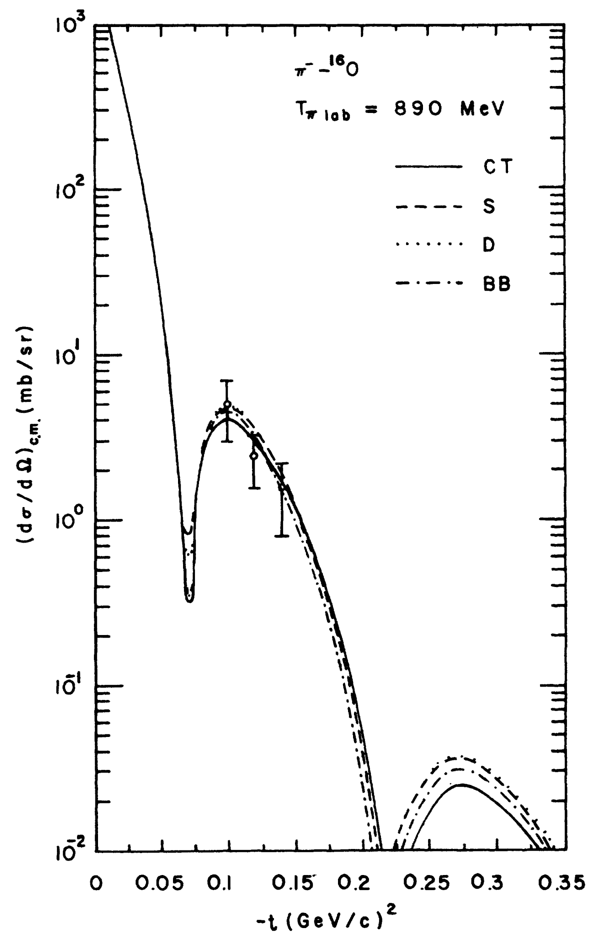

FIG. 4. Comparison of π-N phase parameter sets for π^--^{16}O elastic scattering at 890 MeV. CT refers to CERN Th, S to Saclay, D to Davies, and BB to Berkeley Boone. All curves calculated with a realistic proton form factors and modified Gaussian nuclear form factor.

would be most helpful in resolving the question of phase parameter sets.

CONCLUSION

We find from this work that the elastic scattering of 1 GeV pions from both ^{4}He and ^{16}O can be well fit with a single scattering optical potential used in a relativistic scattering equation. We are able to make a fairly unambiguous choice for the nuclear charge form factor for both ^{4}He and ^{16}O. This enables us to find the combination of nucleon form factor and phase parameter set which best fits both ^{4}He and ^{16}O data. In particular, the combination of the CERN Th phase parameter set and realistic nucleon form factor gives a good fit to the available data for both nuclei. Additional data for ^{4}He in the region of the diffraction minimum and the large angle region beyond the first diffraction maximum are needed to make a definitive choice for both phase parameter set and nucleon form factors. Finally, a more complete data set for ^{16}O would permit confirmation of the choices made on the basis of the ^{4}He data.

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