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Phenomenological Hamiltonian for pions, nucleons, and Δ isobars: Applications to the pion-deuteron system

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A phenomenological model of the interactions between pions, nucleons, and Δ isobars is constructed. The mass operator is defined on a Hilbert space made up of NN, $N\Delta$, and $NN\pi$ states with the following interaction mechanism: (1) a $\Delta \rightleftarrows N\pi$ vertex in the πN-P_{33} channel, (b) two-body $\pi N \not\rightarrow \pi N$ interactions in other πN channels, and (c) two-body interactions for $NN \rightarrow NN$, $N\Delta \rightarrow N\Delta$, and $NN \rightleftarrows N\Delta$ transitions. The model is Lorentz invariant and satisfies cluster separability. The interactions are parametrized by analytic separable forms with parameters determined by fitting the πN scattering phase shifts for $l \leq 1$ up to 300 MeV and NN scattering phase shifts for $l \leq 4$ up to 800 MeV. In fitting parameters and in the applications, nonrelativistic approximations are used for the baryons in the $N\Delta$ and $NN\pi$ channels. The model so determined gives a satisfactory description of pion absorption by deuterons and of elastic pion-deuteron scattering. Multiple rescattering of pions between N and Δ as well as NN interactions in $NN\pi$ intermediate states are found to be important in channels coupled to $N\Delta$ s waves. These effects enhance the cross sections for $\pi^+ + d \rightarrow pp$ by 40% in the resonance region. From the two-baryon Hamiltonian we construct a many-body Hamiltonian for nonrelativistic baryons.

[NUCLEAR REACTIONS A model of interactions between π, N, and Δ. Applications to the pion-deuteron system.]

I. INTRODUCTION

It is by now well recognized that intermediate energy nuclear reactions induced by pions are dominated by the mechanism of exciting the nucleon to the Δ isobar state. Many recent experimental[1] and theoretical[2-5] findings have indicated that both the scattering and the absorption of pions by nuclei are governed by the interactions between the Δ isobars and the nuclear medium. The excitation of intermediate states containing isobars is also known[6-8] to play a significant role in intermediate energy nucleon-nucleon collisions. Clearly, a quantum-mechanical many-body theory describing the interactions between pions, nucleons, and Δ isobars is needed for the study of intermediate energy nuclear reactions.

Any extension of the conventional nuclear many-body theory designed to include pions must provide a mechanism for pion production and absorption. If the basic pion-nucleon interaction is an $N \rightleftarrows N\pi$ vertex, as in the Chew-Low model or any field theory,[9] then the physical one-nucleon state involves an infinity of virtual pions and the composite (dressed) nature of the physical nucleon affects the many nucleon theory.

If, on the other hand, the basic pion nucleon interaction is a two-body interaction and a $\Delta \rightleftarrows N\pi$ vertex, then the physical nucleon is elementary and the many-body theory is considerably simpler. In such models, pion production and absorption are governed by an $NN \rightleftarrows N\Delta$ transition potential and the $\Delta \rightleftarrows N\pi$ vertex. While there are no virtual mesons in the physical nucleon of this model, nucleon-nucleon scattering involves, in principle, an infinity of virtual pions.

The purpose of this paper is twofold. First, such a model of the interactions between pions, nucleons, and Δ's is determined phenomenologically by fits to pion-nucleon and nucleon-nucleon scattering data. Second, as a first application of this model to pion-nucleus physics, the pion-deuteron system is investigated in some detail.

In our model, the Hilbert space of the states of the two nucleon system is

$$\mathcal{H} = \mathcal{H}_{NN} \oplus \mathcal{H}_{N\Delta} \oplus \mathcal{H}_{NN\pi} \oplus \mathcal{H}' ,$$

where

$$\mathcal{H}' = \mathcal{H}_{N\Delta\pi} \oplus \mathcal{H}_{\Delta\Delta} \oplus \mathcal{H}_{NN\pi\pi} \oplus \cdots .$$

We will assume that the interactions have no matrix elements from $\mathcal{H}_{NN}$ and $\mathcal{H}_{N\Delta}$ to $\mathcal{H}'$. The states in $\mathcal{H}_{NN\pi}$ are then "doorway states" for transitions to $\mathcal{H}'$. Components in $\mathcal{H}'$ can be eliminated by the usual projection technique. This procedure does not alter the Hamiltonian in $\mathcal{H}_{NN} \oplus \mathcal{H}_{N\Delta}$, but introduces an effective energy dependent interaction with domain and range in $\mathcal{H}_{NN\pi}$. For the present model we replace this energy dependent effective potential in $\mathcal{H}_{NN\pi}$ by an energy independent auxiliary NN potential designed to fit the NN scattering data below the pion threshold, and otherwise chosen for computational convenience. Such a model is, in principle, fully relativistic,[10] but in practice we

will treat only the pion and the motion of nucleons in $\mathcal{H}_{NN}$ relativistically.

Let us first briefly review the $NN\pi$ model of Ref. 10 (referred to as I hereafter). The models are constructed in the Hilbert space

$$\mathcal{H} = \mathcal{H}_N \otimes \mathcal{H}_{N\pi} \oplus \mathcal{H}_{NN}, \tag{1.1}$$

with

$$\mathcal{H}_{NN} = \mathcal{H}_N \otimes \mathcal{H}_N, \tag{1.2}$$

$$\mathcal{H}_{N\pi} = \mathcal{H}_N \otimes \mathcal{H}_\pi \oplus \mathcal{H}_\Delta, \tag{1.3}$$

where $\mathcal{H}_N$, $\mathcal{H}_\pi$, and $\mathcal{H}_\Delta$ denote, respectively, the one-particle Hilbert spaces of a nucleon, a pion, and a Δ isobar. Note that the space $\mathcal{H}_{N\pi}$ contains a two-particle πN subspace and a one-particle Δ subspace. Thus, the entire space $\mathcal{H}$ of the model involves three different channels: NN, $N\Delta$, and $NN\pi$. The relativistic invariance of the model is realized by expressing the generators of the Poincaré group in terms of the mass operator M, the intrinsic spin operator $\vec{j}$, the momentum $\vec{P}$, and the canonically conjugate position operator $\vec{X}$. In the case of a system of noninteracting particles, the expressions for these operators in terms of individual particle variables are well known.[11] Interactions are introduced by adding to the free mass operator an operator V which commutes with $\vec{P}$, $\vec{X}$, and $\vec{j}$ of the free particle system.

The interaction V is made up of three parts:

$$V = V_0 + V' + V''. \tag{1.4}$$

The two-body operator V_0 performs the transitions $NN \to NN$, $NN \to N\Delta$, and $N\Delta \to N\Delta$ and is defined to vanish on the three-body space $\mathcal{H}_{NN} \otimes \mathcal{H}_\pi$. The operator V' vanishes on $\mathcal{H}_{NN}$ and is constructed from operators v_{NN} and $v_{N\pi}$. The two-body interaction v_{NN} describes the deuteron bound state and nucleon-nucleon scattering below the pion production threshold. The interaction $v_{N\pi}$ is a $\Delta \rightleftharpoons N\pi$ vertex in the P_{33} channel and a two-body interaction in other πN channels.

The combined action of V_0 and V' results in pion production through the mechanism $NN \to N\Delta \to NN\pi$. The operator V'' may have nonvanishing matrix elements between $\mathcal{H}_{NN}$ and $\mathcal{H}_{NN} \otimes \mathcal{H}_\pi$, as well as diagonal matrix elements in $\mathcal{H}_{NN} \otimes \mathcal{H}_\pi$. It would be needed to account for π production not proceeding through the Δ and for possible three-body $NN\pi$ forces. In the present work, we shall let $V'' = 0$. This simplification does not prevent an adequate description of the physics, with the exception of pion production near threshold, which is therefore excluded from our considerations.

The model thus constructed is intended to describe the following processes (numbers in brackets indicate the energy region in which one expects the description to be adequate):

(a) $\pi N \to \pi N$ [0~300 MeV],

(b) $NN \to NN$ [0~800 MeV],

(c) $NN \to NN\pi$ [400–800 MeV],

(d) $NN \to \pi d$ [NN energy: 400–800 MeV],

(e) $\pi d \to \pi d$ [0~300 MeV].

In this paper, we accomplish this by finding a set of interactions v_{NN}, $v_{N\pi}$, and V_0 such that the features of these processes are accounted for. Convenient parametrizations are chosen for these interactions. In particular, all two-body interactions are taken to be of low-rank separable type with simple analytic form factors. It is clearly impractical to use the data for all processes listed above in order to determine the parameters. Therefore, the following strategy is employed. The two-body interactions in $NN\pi$ states are constructed first. The parameters of $v_{N\pi}$ (as well as the bare Δ mass) are determined by fits to πN scattering phase shifts. The parameters of v_{NN} are determined by fitting the NN phase shifts in the energy region where inelastic processes may be neglected. The operator V' is then completely determined, and only V_0 remains to be specified. This is accomplished by adjusting its parameters in such a way as to bring the NN phase shifts and inelasticities, as given by the full model, in agreement with the experimental values. Processes (c)–(e) can then be used as testing grounds for the model and as means of removing any remaining ambiguities in the parameters. In the present paper we consider only pion absorption on the deuteron and pion-deuteron elastic scattering, leaving pion production in NN collisions for future investigation.

It is perhaps useful to emphasize here that our approach is quite different in its foundation from most previous theoretical work[4,7,12-17] on the $NN\pi$ system. In particular, we do not assume an underlying field theory, and therefore there are no Bethe-Salpeter amplitudes. The question of the proper reduction[13] of the four-dimensional Bethe-Salpeter equation to three-dimensional form does not arise. Since our model does not contain an $N \rightleftharpoons N\pi$ vertex, the derivation of correct scattering equations is straightforward; no renormalization is called for and there is no danger of overcounting interactions. The Pauli principle is trivially satisfied by antisymmetrizing all states in the nucleon variables. Finally, since the scattering equations follow, through exact algebraic manipulations, from the time evolution of state vectors under the action of a self-adjoint Hamiltonian, they embody unitarity from the start.

The present work differs also from that of Refs.

4 and 6 in its purpose. No attempt is made at calculating NN scattering above the pion production threshold from independent input. Instead, empirical NN phase shifts and inelasticities are used to determine the completely phenomenological interaction V_0. Thus, we make no pretense of explaining the physics of the $NN\pi$ system in terms of other, more "fundamental" processes; we seek a set of interactions that correctly describe it and hope it can be used as a *starting point in many-body calculations*.

The construction of V described in I satisfies the requirements of Lorentz invariance and cluster separability[10] exactly. However, its numerical implementation without approximation is difficult because it involves taking square roots of complicated operators. In the calculations presented below, approximations are made based on the following assumptions: (i) All particle momenta in the space $\mathcal{H}_N \otimes \mathcal{H}_{N\pi}$ are small compared to baryon masses; and (ii) the strength of any two-body interaction in $\mathcal{H}_N \otimes \mathcal{H}_{N\pi}$ is small compared to the mass of the interacting cluster. In Sec. II, the construction of V is summarized and the approximations just mentioned are introduced and discussed.

The two-body interactions in $NN\pi$ states are constructed in Sec. III. The pion-nucleon interaction $v_{N\pi}$ is dealt with in Sec. III A, and the nucleon-nucleon interaction v_{NN} in Sec. III B.

Once $v_{N\pi}$ and v_{NN} are determined, the operator V' is completely known. One can then turn to the determination of V_0. For this purpose, the formulation of NN elastic scattering as given by the full model is needed. This is done in Sec. IV, drawing on the formal results of I. A set of parameters of V_0 obtained by fitting data for NN scattering both below and above the pion production threshold is presented. This section also contains a discussion of the role played by multiple pion rescattering between N and Δ and by NN interactions in $NN\pi$ intermediate states in the description of NN elastic scattering.

Sections V and VI are devoted to testing our model in the study of the π-d system. The reaction $\pi d \rightarrow NN$ is studied in Sec. V. The relevant scattering equations are written down and results for the total cross section as a function of energy and for the angular distributions at a few energies, obtained with the interactions determined in previous sections, are presented and compared to data. The importance, for this process, of pion multiple rescattering and of NN interactions in $NN\pi$ intermediate states is also discussed.

In Sec. VI, we turn to pion-deuteron elastic scattering. Practical scattering equations are derived using the results of I; total and differential cross sections, as well as polarizations, are calculated within our model. Comparison with the available data is made; special attention is paid to the influence of the coupling to the NN channel ("true" absorption).

In Sec. VII, we discuss the extension of the present work to a many-body system with $A > 2$. If all baryons are nonrelativistic, a second-quantized Hamiltonian can be written down, such that the corresponding scattering equations for the $A = 2$ case reduce, after suitable truncation, to those used in Secs. IV–VI. This Hamiltonian contains the interactions V_0 and $v_{N\pi}$, but not the interaction v_{NN}. From the equations for NN scattering, it can be seen that this last interaction is to be interpreted as an effective one, containing, in addition to the $NN \rightarrow NN$ part of V_0, the effects of the coupling to states with more than one pion or Δ. The motivation for parametrizing this interaction by a two-body potential fit to NN phase shifts below the pion production threshold is discussed. Thus, Sec. VII provides further insight in the nature of the space truncation inherent in the model constructed in the previous sections. Finally, our conclusions are summarized in Sec. VIII.

II. THE MODEL

In order to make the present paper reasonably self-contained and to define our notations, we devote Secs. II A and II B to a summary of the construction of our $NN\pi$ model. A discussion of the Lorentz invariance and cluster separability properties of this construction can be found in I. In Sec. III C, we introduce approximations made in actual calculations.

A. Pion-nucleon mass operator

The two-body πN problem is formulated in the space $\mathcal{H}_{N\pi}$ given by (1.3). In what follows, we shall frequently denote variables referring to the nucleon, the pion and the Δ by subscripts N, π, and Δ, respectively. Thus $\vec{p}_N$, $\vec{s}_N$, and $\vec{t}_N$ stand for the momentum, spin, and isospin of the nucleon. Let $\vec{P}_{N\pi}$ and $M^0_{N\pi}$ be the momentum and mass operators on $\mathcal{H}_{N\pi}$. Define

$$\vec{Q}_{N\pi} = \vec{P}_{N\pi} / M^0_{N\pi} , \tag{2.1}$$

$$\tilde{k} = L(\vec{Q}_{N\pi}) \tilde{p}_\pi , \tag{2.2}$$

$$\vec{x} = i \frac{\partial}{\partial \vec{k}} \tag{2.3}$$

$$\vec{\sigma}_\alpha = \mathcal{R}[\vec{p}_\alpha, \vec{Q}_{N\pi}] \vec{s}_\alpha , \quad \alpha \equiv N, \Delta \tag{2.4}$$

where $\tilde{k}$, $\tilde{p}_\pi$ are four-vectors and $L(\vec{Q})$ is the Lorentz transformation that transforms $\{\vec{Q}, (1+\vec{Q}^2)^{1/2}\}$ into $\{0, 0, 0, 1\}$. The Wigner rotation operator $\mathcal{R}[\vec{p}, \vec{Q}]$ is defined by the following decomposition[18]

of the Lorentz transformation $L(\vec{Q})$:

$$L(\vec{Q}) = L\left(\frac{\vec{p}'}{m}\right) \mathcal{R}[\vec{p}, \vec{Q}]\, L^{-1}\left(\frac{\vec{p}}{m}\right), \tag{2.5}$$

where

$$\tilde{p}' = L(\vec{Q})\,\tilde{p}\,, \quad \tilde{p}'^2 = \tilde{p}^2 = m^2\,.$$

The intrinsic angular momentum of the πN system is

$$\vec{j}_{N\pi} = \begin{cases} \vec{x}\times\vec{k} + \vec{\sigma}_N & \text{on } \mathcal{H}_N \otimes \mathcal{H}_\pi\,, \\ \vec{\sigma}_\Delta & \text{on } \mathcal{H}_\Delta\,. \end{cases} \tag{2.6}$$

The Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_\pi$ is spanned by functions $\psi(\vec{p}_{N\pi}, \vec{k}, \tau_\pi, \sigma_N, \tau_N)$ and the space $\mathcal{H}_\Delta$ by functions $\psi(\vec{P}_{N\pi}, \sigma_\Delta, \tau_\Delta)$, where $\sigma_N(\tau_N)$ stands for the z-axis projection of $\vec{\sigma}_N(\vec{t}_N)$. The πN mass operator $M_{N\pi}$ has matrix elements proportional to $\delta(\vec{P}'_{N\pi} - \vec{P}_{N\pi})$ and the matrix elements in the space of the intrinsic variables are

$$\begin{aligned}(\vec{k}', \nu'_\pi, \nu'_N|\, \hat{M}_{N\pi}\,|\nu_N, \nu_\pi, \vec{k}) &= W(\vec{k})\,\delta(\vec{k}' - \vec{k})\,\delta_{\nu'_N\nu_N}\delta_{\nu'_\pi\nu_\pi} \\ &\quad + (\vec{k}', \nu'_\pi, \nu'_N|\, \hat{v}_{N\pi}\,|\nu_N, \nu_\pi, \vec{k})\,, \end{aligned} \tag{2.7a}$$

$$(\nu'_\Delta|\, \hat{M}_{N\pi}\,|\nu_\Delta) = \delta_{\nu_\Delta\nu'_\Delta} m_\Delta\,, \tag{2.7b}$$

$$(\nu'_\Delta|\, \hat{M}_{N\pi}\,|\nu_N, \nu_\pi, \vec{k}) = (\nu'_\Delta|\, \hat{v}_{N\pi}\,|\nu_N, \nu_\pi, \vec{k})\,, \tag{2.7c}$$

where ν_N stands for $\{\sigma_N, \tau_N\}$ and

$$W(\vec{k}) = E_N(\vec{k}) + E_\pi(\vec{k})\,, \tag{2.8}$$

with $E_\alpha(\vec{k}) = [\vec{k}^2 + m_\alpha{}^2]^{1/2}$. The operator $\hat{v}_{N\pi}$ has the invariance properties

$$[\hat{v}_{N\pi}, \vec{j}_{N\pi}] = [\hat{v}_{N\pi}, \vec{t}_{N\pi}] = 0\,, \tag{2.9}$$

where $\vec{t}_{N\pi}$ is defined as $\vec{t}_N + \vec{t}_\pi$ on $\mathcal{H}_N \otimes \mathcal{H}_\pi$ and $\vec{t}_\Delta$ on $\mathcal{H}_\Delta$.

B. Mass operator for the $NN\pi$ system

As a preliminary step in the construction of the full mass operator for the $NN\pi$ system, we consider the nucleon-nucleon system in the space $\mathcal{H}_{NN}$ given by (1.2). We label the two nucleons a and b and denote by $\vec{P}_{ab}$ and M^0_{ab} the momentum and mass operator on $\mathcal{H}_{NN}$. We define

$$\vec{Q}_{ab} = \vec{P}_{ab}/M^0_{ab}\,, \tag{2.10}$$

$$\tilde{\kappa} = L(\vec{Q}_{ab})\,\tilde{p}_a\,, \tag{2.11}$$

$$\vec{x} = i\,\frac{\partial}{\partial \vec{k}}\,, \tag{2.12}$$

$$\vec{\sigma}_\alpha = \mathcal{R}[\vec{p}_\alpha, \vec{Q}_{ab}]\,\vec{s}_\alpha\,, \quad \alpha \equiv a, b\,, \tag{2.13}$$

$$\vec{j}_{ab} = \vec{x}\times\vec{\kappa} + \vec{\sigma}_a + \vec{\sigma}_b\,. \tag{2.14}$$

The Hilbert space $\mathcal{H}_{NN}$ is spanned by functions $\psi(\vec{P}_{ab}, \vec{\kappa}, \nu_a, \nu_b)$. The mass operator describing the deuteron and NN scattering at low energy has matrix elements proportional to $\delta(\vec{P}'_{ab} - \vec{P}_{ab})$ and the matrix elements in the restricted space of relative variables are

$$\begin{aligned}(\vec{\kappa}', \nu'_a, \nu'_b|\, \hat{M}_{ab}\,|\nu_b, \nu_a, \vec{\kappa}) &= 2E_N(\vec{\kappa})\,\delta(\vec{\kappa}' - \vec{\kappa})\,\delta_{\nu'_a\nu_a}\,\delta_{\nu'_b\nu_b} \\ &\quad + (\vec{\kappa}', \nu'_a, \nu'_b|\, \hat{v}_{NN}\,|\nu_b, \nu_a, \vec{\kappa})\,. \end{aligned} \tag{2.15}$$

The operator $\hat{v}_{NN}$ satisfies

$$[\hat{v}_{NN}, \vec{j}_{ab}] = [\hat{v}_{NN}, \vec{t}_{ab}] = 0\,, \tag{2.16}$$

where $\vec{t}_{ab} = \vec{t}_a + \vec{t}_b$.

The contribution $V_{ab;\pi}$ to the full mass operator, corresponding to NN interaction in $NN\pi$ states, can now be constructed as follows. Let

$$\vec{Q} = \vec{P}/M^0\,, \tag{2.17}$$

$$\tilde{q}_\pi = L(\vec{Q})\,\tilde{p}_\pi\,, \tag{2.18}$$

where $\vec{P}$ and M^0 are the free momentum and mass operators for the whole system. States of $\mathcal{H}_{NN} \otimes \mathcal{H}_\pi$ can be represented by functions $\Psi(\vec{P}, \vec{q}_\pi, \nu_\pi, \vec{\kappa}, \nu_a, \nu_b)$. We define the interaction operator $\mathcal{V}_{ab}$ by its kernel in this representation:

$$\begin{aligned}&(\vec{P}', \vec{q}'_\pi, \nu'_\pi, \vec{\kappa}', \nu'_a, \nu'_b|\, \mathcal{V}_{ab}\,|\nu_b, \nu_a, \vec{\kappa}, \nu_\pi, \vec{q}_\pi, \vec{P}) \\ &\quad = \delta(\vec{P}' - \vec{P})\,\delta(\vec{q}'_\pi - \vec{q}_\pi)\,\delta_{\nu'_\pi\nu_\pi}(\vec{\kappa}', \nu'_a, \nu'_b|\, \hat{v}_{NN}\,|\nu_b, \nu_a, \vec{\kappa})\,. \end{aligned} \tag{2.19}$$

Let

$$\mathfrak{M}_{ab} = M^0_{ab} + \mathcal{V}_{ab}\,. \tag{2.20}$$

The interaction $V_{ab;\pi}$ is given in terms of this mass operator by

$$V_{ab;\pi} = (\vec{q}_\pi{}^2 + \mathfrak{M}_{ab}{}^2)^{1/2} - (\vec{q}_\pi{}^2 + M_{ab}{}^{0^2})^{1/2}\,. \tag{2.21}$$

The operator $V_{b\pi;a}$ corresponding to interaction of the pion with nucleon b in the presence of nucleon a is constructed in a similar manner. Let

$$\tilde{q}_\alpha = L(\vec{Q})\,\tilde{p}_\alpha\,, \tag{2.22}$$

$$\vec{s}_\alpha = \mathcal{R}[\vec{p}_\alpha, \vec{Q}]\,\vec{s}_\alpha\,, \tag{2.23}$$

$$\vec{y}_\alpha = i\,\frac{\partial}{\partial \vec{q}_\alpha}\,, \quad \alpha \equiv a, \Delta\,. \tag{2.24}$$

States of $\mathcal{H}_{NN} \otimes \mathcal{H}_\pi$ can be represented by functions $\Psi(\vec{P}, \vec{q}_a, \mu_a, \vec{k}_{\pi b}, \nu_b, \nu_\pi)$ and states of $\mathcal{H}_{N\Delta}$ by functions $\Psi(\vec{P}, \vec{q}_a, \mu_a, \nu_\Delta)$, where μ_a stands for $\{S_a, \tau_a\}$. The interaction operator $\mathcal{V}_{b\pi}$, defined in this representation by

$$\begin{aligned}&(\vec{P}', \vec{q}'_a, \mu'_a, \vec{k}'_{\pi b}, \nu'_\pi, \nu'_b|\, \mathcal{V}_{b\pi}\,|\nu_b, \nu_\pi, \vec{k}_{\pi b}, \mu_a, \vec{q}_a, \vec{P}) \\ &\quad = \delta(\vec{P}' - \vec{P})\,\delta(\vec{q}'_a - \vec{q}_a)\,\delta_{\mu'_a\mu_a}(\vec{k}'_{\pi b}, \nu'_\pi, \nu'_b|\hat{v}_{N\pi}|\nu_b, \nu_\pi, \vec{k}_{\pi b})\,, \end{aligned} \tag{2.25a}$$

$$(\vec{P}', \vec{q}'_a, \mu'_a, \nu'_\Delta|\, \mathcal{V}_{b\pi}\,|\nu_\Delta, \mu_a, \vec{q}_a, \vec{P}) = 0\,, \tag{2.25b}$$

$$(\vec{P}', \vec{q}_a', \mu_a', \nu_\Delta' | v_{b\pi} | \nu_b, \nu_\pi, \vec{k}_{\pi b}, \mu_a, \vec{q}_a, \vec{P}) = \delta(\vec{P}' - \vec{P})\,\delta(\vec{q}_a' - \vec{q}_a)\,\delta_{\mu_a'\mu_a}(\nu_\Delta' | \hat{v}_{N\pi} | \nu_b, \nu_\pi, \vec{k}_{\pi b}) \tag{2.25c}$$

is used to construct the operator $V_{b\pi;a}$:

$$V_{b\pi;a} = (\vec{q}_a^{\,2} + \mathfrak{M}_{b\pi}^{\,2})^{1/2} - (\vec{q}_a^{\,2} + M_{b\pi}^{0\,2})^{1/2}, \tag{2.26}$$

where

$$\mathfrak{M}_{b\pi} = M_{b\pi}^0 + v_{b\pi}. \tag{2.27}$$

The operator V' is defined as

$$V' = V_{ab;\pi} + V_{a\pi;b} + V_{b\pi;a}. \tag{2.28}$$

The operator V_0 has nonvanishing matrix elements in $\mathcal{H}_{NN}$ and in $\mathcal{H}_{N\Delta}$, as well as matrix elements between states of $\mathcal{H}_{NN}$ and states of $\mathcal{H}_{N\Delta}$. These matrix elements are proportional to $\delta(\vec{P}' - \vec{P})$. The restricted operator $\hat{V}_0$ acting on the space of functions $\Psi(\vec{q}_a, \mu_a, \mu_b)$ and $\Psi(\vec{q}_N, \mu_N, \mu_\Delta)$ satisfies

$$[\hat{V}_0, \vec{j}_{BB}] = [\hat{V}_0, \vec{t}_{BB}] = 0, \tag{2.29}$$

where the total spin $\vec{j}_{BB}$ of the two-baryon system is given in terms of the $\vec{q}_\alpha$'s, $\vec{y}_\alpha$'s, and $\vec{S}_\alpha$'s by equations similar to (2.14). The total isospin of the two-baryon system is denoted by $\vec{t}_{BB}$. The full mass operator is given by

$$\mathfrak{M} = M^0 + V_0 + V'. \tag{2.30}$$

Scattering equations for the various amplitudes of interest can be formulated in terms of $\mathfrak{M}$ and the mass operator M_f describing free stable particles. Expressions for the scattering amplitudes corresponding to processes (a)–(e) listed in the Introduction have been presented in I and will be exploited in the subsequent sections of the present paper. However, before turning to this, it is helpful to introduce some simplifying approximations.

C. Approximate mass operator

The numerical implementation of the construction of $\mathfrak{M}$ outlined in Sec. II B is made difficult by the appearance of square roots of complicated operators. In the calculations of Secs. IV–VI, this problem is avoided by the use of the following approximations.

(i) We assume that, in the c.m. frame, all particle momenta in the space $\mathcal{H}_N \otimes \mathcal{H}_{N\pi}$ are small compared to the baryon masses. More specifically, if $\vec{q}$ denotes any particle momentum in that space, then terms that are formally of order $\vec{q}^{\,2}/m_N^{\,2}$ (or $\vec{q}^{\,2}/m_\Delta^{\,2}$) are dropped. The expressions (2.21) and (2.26) then become

$$V_{ab;\pi} = v_{ab} - \frac{\vec{q}_\pi^{\,2}}{2} M_{ab}^{0\,-1} v_{ab} \mathfrak{M}_{ab}^{-1}, \tag{2.31a}$$

$$V_{b\pi;a} = v_{b\pi} - \frac{\vec{q}_a^{\,2}}{2} M_{b\pi}^{0\,-1} v_{b\pi} \mathfrak{M}_{b\pi}^{-1}. \tag{2.31b}$$

Since any interacting pair in the model contains at least one baryon, it is clear from (2.5) that all Wigner rotations may be neglected at this level of approximation. It is therefore unnecessary to distinguish between the variables $\vec{\sigma}$ and $\vec{S}$. We shall refer to this approximation as the nonrelativistic baryon (NRB) approximation. It should be stressed that no approximation is made in the space $\mathcal{H}_{NN}$, where the nucleons are kept relativistic. On-energy-shell kinematics can be invoked to provide some qualitative justification for this approximation. The maximum c.m. kinetic energy of the NN system we shall be dealing with is about 400 MeV. In the $N\Delta$ system, roughly 300 MeV are taken up by the Δ mass leaving only 100 MeV in kinetic energy to be shared between the two baryons. Similarly, in πNN states, 140 MeV are converted into the pion mass, leaving 260 MeV in kinetic energy to be shared by the three particles. Therefore, in $N\Delta$ and πNN states, baryon kinetic energies should be small enough to allow a nonrelativistic treatment. Ultimately, of course, the validity of the NRB approximation must be established by calculating corrections. Sections II A and II B above provide a well defined framework in which to do this.

(ii) We further assume that

$$\| v_{ab} M_{ab}^{0\,-1} \| \ll 1, \quad \text{and} \quad \| v_{b\pi} M_{b\pi}^{0\,-1} \| \ll 1. \tag{2.32}$$

If (2.32) is valid, the second terms on the right-hand side of Eqs. (2.31) may be dropped and (2.28) becomes

$$V' = v_{ab} + v_{a\pi} + v_{b\pi}. \tag{2.33}$$

This approximation, which we shall call the small mass defect (SMD) approximation, rules out interactions with an excessively strong repulsive core.

III. TWO-BODY INTERACTIONS IN $NN\pi$ STATES

A. Pion-nucleon interaction

The Lippmann-Schwinger equation satisfied by the πN T matrix is given in I. Using the definitions and formulas of the Appendix, one easily obtains the partial wave expanded forms. For all but the P_{33} channel,

$$T_{N\pi}^{\gamma}(k', k; W_0) = v_{N\pi}^{\gamma}(k', k) + \int_0^\infty dk'' k''^2 \, \frac{v_{N\pi}^{\gamma}(k', k'') T_{N\pi}^{\gamma}(k''; k; W_0)}{W_0 - W(k'') + i\epsilon}, \tag{3.1}$$

where W_0 is the c.m. total energy and γ stands for the set of quantum numbers $\{l_\gamma, J_\gamma, T_\gamma\}$. For convenience, we assume separable two-body interactions:

TABLE I. Parameters of the πN interactions.

πN channel	F_1	k_1 (MeV/c)	F_2	k_2 (MeV/c)
S_{11}	−5.741	368.4		
S_{13}	9.217	962.8		
P_{11}	4.520	409.8	−3.240	522.5
P_{13}	0.6167	316.3		
P_{31}	1.687	300.9		
P_{33}	F_Δ=0.9783	k_Δ=358 MeV/c	m_Δ=1280 MeV	

$$v^{\gamma}_{N\pi}(k',k)=\sum_{i=1}^{2} f^{\gamma}_i(k')F^{\gamma}_i f^{\gamma}_i(k)\,. \tag{3.2}$$

Equation (3.1) is then readily solved. In the P_{33} channel, assuming that the only interaction is the $\Delta \rightleftharpoons N\pi$ vertex, the scattering equation can be easily solved, giving

$$T^{\gamma\Delta}_{N\pi}(k',k;W_0)=f_\Delta(k')[W_0-m_\Delta-\Sigma(W_0)+i\epsilon]^{-1}f_\Delta(k)\,, \tag{3.3}$$

where

$$\Sigma(W_0)=\int_0^\infty dk''k''^2\frac{f_\Delta^2(k'')}{W_0-W(k'')+i\epsilon}\,. \tag{3.4}$$

The form factors are parametrized as follows:

$$f_\Delta(k)=\frac{F_\Delta}{[2(m_N+m_\pi)]^{1/2}}\frac{k}{m_\pi}\left(\frac{k_\Delta^2}{k_\Delta^2+k^2}\right)^2, \tag{3.5}$$

$$f^{\gamma}_i(k)=\frac{2^{1/2}}{[2(m_N+m_\pi)]^{1/2}}\left(\frac{k}{m_\pi}\right)^{l_\gamma}e^{-(k/k^{\gamma}_i)^2}. \tag{3.6}$$

The πN scattering phase shifts can be calculated from the on-energy-shell T matrix using the standard formulas. The parameters m_Δ, F_Δ, and k_Δ for the P_{33} channel and F^{γ}_i, k^{γ}_i for other πN channels are adjusted to fit the CERN phase shifts[19] for energies $W_0 \lesssim 1300$ MeV. The resulting parameters are given in Table I and the corresponding phase shifts are shown in Fig. 1.

B. Nucleon-nucleon interaction

The T matrix corresponding to the interaction $\hat{v}_{NN}$ satisfies the Lippmann-Schwinger equation [see I and Eq (A4)]:

$$T^{\gamma'\gamma}_{NN}(\kappa',\kappa;E)$$

$$=v^{\gamma'\gamma}_{NN}(\kappa',\kappa)$$

$$+\sum_{\gamma''}\int_0^\infty d\kappa''\kappa''^2\frac{v^{\gamma'\gamma''}_{NN}(\kappa',\kappa'')\,T^{\gamma''\gamma}_{NN}(\kappa'',\kappa;E)}{E-2E_N(\kappa'')+i\epsilon}\,, \tag{3.7}$$

where E is the total c.m. energy. In order to simplify the calculations of the forthcoming sections, we again assume separable interactions:

$$v^{\gamma'\gamma}_{NN}(\kappa',\kappa)=\sum_{i=1}^{2} g^{\gamma'}_i(\kappa')\,G^{\gamma'\gamma}_i\,g^{\gamma}_i(\kappa)\,. \tag{3.8}$$

The following NN interactions in $NN\pi$ states are included in the calculations to be presented in Secs. IV–VI: $^3S_1$-$^3D_1$, $^1S_0$, $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$. Interactions in partial waves with orbital angular momentum $l \geq 2$ are expected to be relatively unimportant in the energy region of interest and are neglected. For all but the $^3S_1$-$^3D_1$ interaction, the form factors are parametrized as follows:

$$g^{\gamma}_i(\kappa)=\frac{\kappa^l(\kappa^{\gamma}_i)^{l+1}}{[\kappa^2+(\kappa^{\gamma}_i)^2]^{l+1}}\,. \tag{3.9}$$

The parameters $G^{\gamma'\gamma}_i$ and κ^{γ}_i, determined by fitting NN phase shifts[20] for laboratory energies below 400 MeV, are collected in Table II. The fits obtained with such simple interactions are quite adequate for our purpose. The $^3S_1$-$^3D_1$ interaction re-

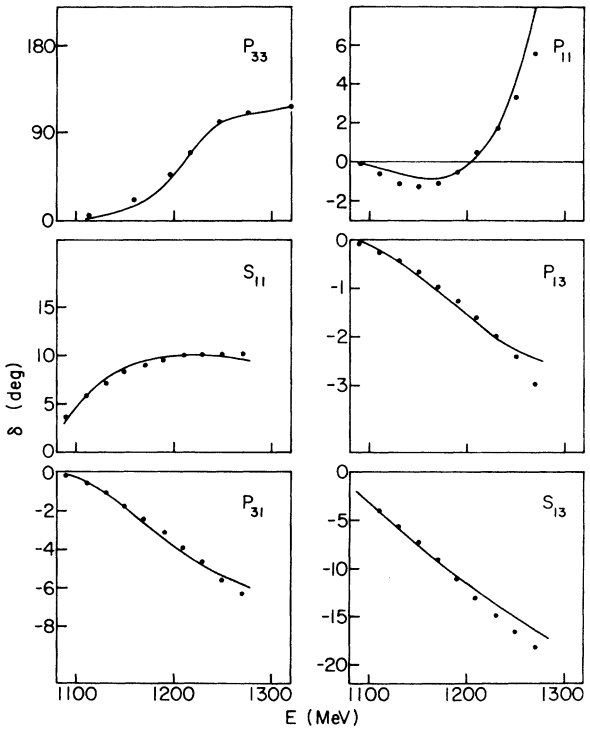

FIG. 1. Fits to the πN phase shifts of Ref. 19. The corresponding parameters are given in Table I.

TABLE II. Parameters of the NN interactions in $NN\pi$ states.

NN channel	G_1	κ_1 (MeV/c)	G_2	κ_2 (MeV/c)
$^1S_0$	−1.114	346.5	3.506	858.5
$^1P_1$	10.42	612.2		
$^3P_0$	−1.500	340.1	13.96	805.1
$^3P_1$	3.259	479.7		
$^3P_2$	−1.275	527.6		

quires more care, since a realistic deuteron wave function is required for calculations of the process $\pi d \rightleftarrows NN$. We have used the rank-two interaction of Ref. 21, which gives the same deuteron wave function and roughly the same low energy phase shifts as the Reid soft-core potential.

IV. NUCLEON-NUCLEON SCATTERING

Once the interactions $v_{N\pi}$ and $v_{N\nu}$ have been determined, operator V' can be constructed in the approximation (2.33) using (2.25) and (2.19). The determination of V_0 is then accomplished by fits to NN scattering data at energies up to 750 MeV. The relevant formal scattering equations have been given in I. In Sec. IV A, we make use of the approximations introduced in Sec. II to write these equations in explicit form suitable for computation. Numerical results are discussed in Sec. IV B.

A. Scattering equations

As shown in I, the c.m. NN elastic scattering amplitudes are given by matrix elements between NN states of an operator $\tilde{T}$ defined as follows. Let $\mathcal{P}$ be the projection operator onto $\mathcal{H}_{NN} \oplus \mathcal{H}_{N\Delta}$ and $\bar{\mathcal{P}} = 1 - \mathcal{P}$. The effective interaction obtained by projecting out the space $\mathcal{H}_{NN} \otimes \mathcal{H}_\pi$ may be written as

$$\mathcal{P} V \bar{R}(E) V \mathcal{P} = V_D(E) + V_C(E), \tag{4.1}$$

where

$$\bar{R}(E) = [E + i\epsilon - M^0 - \bar{V}]^{-1} \bar{\mathcal{P}}, \tag{4.2}$$

$$\bar{V} = \bar{\mathcal{P}} V \bar{\mathcal{P}}, \tag{4.3}$$

and the Δ self-interaction V_D is by definition that part of left-hand side of (4.1) that does not vanish when the nucleon in $\mathcal{H}_{N\Delta}$ is moved to infinity [see Eq. (71) of I]. The effective interaction V_C is then defined by (4.1). Let us separate the self-interaction V_D from other interactions by introducing the following operators

$$\tilde{V}(E) = V_0 + V_C(E), \tag{4.4}$$

$$\tilde{M}^0(E) = \mathcal{P} M^0 \mathcal{P} + V_D(E), \tag{4.5}$$

$$\tilde{R}(E) = [E + i\epsilon - \tilde{M}^0(E) - \tilde{V}(E)]^{-1}, \tag{4.6}$$

$$\tilde{R}^0(E) = [E + i\epsilon - \tilde{M}^0(E)]^{-1}. \tag{4.7}$$

Then, the scattering operator $\tilde{T}$ defined on $\mathcal{H}_{NN} \otimes \mathcal{H}_{N\Delta}$ can be written as

$$\tilde{T}(E) = \tilde{V}(E) + \tilde{V}(E) \tilde{R}(E) \tilde{V}(E). \tag{4.8}$$

In the case at hand ($V'' = 0$), the operator V_C vanishes on $\mathcal{H}_{NN}$. It therefore follows from (4.4) and (4.8) that the operator

$$T_0(E) = V_0 + V_0 \tilde{R}(E) V_0 \tag{4.9}$$

has the same matrix elements between NN states as $\tilde{T}$, and the NN elastic scattering amplitude is obtained by taking appropriate matrix elements of T_0. Defining

$$R_C(E) = [E + i\epsilon - \tilde{M}^0(E) - V_C(E)]^{-1}, \tag{4.10}$$

we may rewrite (4.9) in the form

$$T_0(E) = V_0 + V_0 R_C(E) T_0(E). \tag{4.11}$$

To solve Eqs. (4.8) or (4.11), it is useful to introduce scattering operators associated with the resolvents $\bar{R}$ and R_C:

$$\bar{T}(E) = \bar{V} + \bar{V} \bar{R}(E) \bar{V}, \tag{4.12}$$

$$T_C(E) = V_C(E) + V_C(E) R_C(E) V_C(E). \tag{4.13}$$

From (4.2), (4.7), and (4.10), it follows that these T matrices are solutions of the equations

$$\bar{T}(E) = \bar{V} + \bar{V} \bar{R}^0(E) \bar{T}(E), \tag{4.14}$$

$$T_C(E) = V_C(E) + V_C(E) \tilde{R}^0(E) T_C(E), \tag{4.15}$$

where

$$\bar{R}^0 = [E + i\epsilon - M^0]^{-1} \bar{\mathcal{P}}. \tag{4.16}$$

Also,

$$\bar{R}(E) = \bar{R}^0(E) + \bar{R}^0(E) \bar{T}(E) \bar{R}^0(E) \tag{4.17}$$

and

$$R_C(E) = \tilde{R}^0(E) + \tilde{R}^0(E) T_C(E) \tilde{R}^0(E). \tag{4.18}$$

The effective interactions V_D and V_C are obtained by substituting (4.17) into (4.1). Since we are assuming that the only πN interaction in the P_{33} channel is the $\Delta \rightleftarrows N\pi$ vertex, the second term of (4.17) does not contribute to V_D. Therefore, if we define

$$V_E(E) = \mathcal{P} V \bar{R}^0(E) V \mathcal{P} - V_D(E) \tag{4.19}$$

and

$$V_B(E) = \mathcal{P} V \bar{R}^0(E) \bar{T}(E) \bar{R}^0(E) V \mathcal{P}, \tag{4.20}$$

both V_E and V_B vanish when the nucleon in $\mathcal{H}_{N\Delta}$ is moved to infinity and, obviously,

$$V_C(E) = V_E(E) + V_B(E). \tag{4.21}$$

The only interaction contributing to V_D and V_E is the $\Delta \rightleftharpoons N\pi$ vertex. Using (2.33), (2.25), and (A3), together with the approximations introduced in Sec. II C, we get

$$(\vec{q}', \mu_N', \mu_\Delta' | V_D(E) | \mu_\Delta, \mu_N, \vec{q}) = \delta(\vec{q}' - \vec{q})\delta_{\mu_N'\mu_N}\delta_{\mu_\Delta'\mu_\Delta}\Sigma_{N\Delta}(q; E), \tag{4.22}$$

with

$$\Sigma_{N\Delta}(q; E) = \int_0^\infty dk\, k^2 f_\Delta^2(k)\left[E + i\epsilon - m_N - \frac{q^2}{2m_N} - \frac{q^2}{2M_{N\pi}(k)} - \frac{k^2}{2m_N} - M_{N\pi}(k)\right]^{-1}, \tag{4.23}$$

where $M_{N\pi}(k) = m_N + E_\pi(k)$. The $N\Delta$ propagator including the Δ self-interaction is therefore

$$(\vec{q}', \mu_N', \mu_\Delta' | \tilde{R}^0(E) | \mu_\Delta, \mu_N, \vec{q}) = \delta(\vec{q}' - \vec{q})\delta_{\mu_N'\mu_N}\delta_{\mu_\Delta'\mu_\Delta}\tilde{R}^0_{N\Delta}(q; E), \tag{4.24}$$

with

$$\tilde{R}^0_{N\Delta}(q; E) = \left[E + i\epsilon - M_{N\Delta} - \frac{q^2 M_{N\Delta}}{2m_N m_\Delta} - \Sigma_{N\Delta}(q; E)\right]^{-1}, \tag{4.25}$$

where $M_{N\Delta} = m_N + m_\Delta$.

The operator V_E is simply the one-pion exchange interaction between a nucleon and a Δ:

$$(\vec{q}', \mu_N', \mu_\Delta' | V_E(E) | \mu_\Delta, \mu_N, \vec{q}) = -J_{N\pi}^{-1/2}(\vec{q}', \vec{q}_\pi; \vec{P}'_{N\pi}, \vec{k}') \times \sum_{\nu_\pi} (\mu_\Delta' | \hat{v}_{N\pi} | \mu_N, \nu_\pi, \vec{k}) R^0(q', q, q_\pi; E) \times (\vec{k}', \nu_\pi', \mu_N' | \hat{v}_{N\pi} | \mu_\Delta) J_{N\pi}^{-1/2}(\vec{q}, \vec{q}_\pi; \vec{P}_{N\pi}, \vec{k}), \tag{4.26}$$

where $J_{N\pi}(\vec{q}, \vec{q}_\pi; \vec{P}_{N\pi}, \vec{k})$ stands for the Jacobian of the transformation $\{\vec{q}, \vec{q}_\pi\} \rightarrow \{\vec{P}_{N\pi}, \vec{k}\}$ defined by (2.2). Explicit formulas for such transformations can be found in Ref. 11; in the NRB approximation, they yield

$$\vec{k}' = \vec{q}_\pi + \frac{E_\pi(q_\pi)}{M_{N\pi}(q_\pi)}\vec{q}, \quad \vec{k} = \vec{q}_\pi + \frac{E_\pi(q_\pi)}{M_{N\pi}(q_\pi)}\vec{q}', \quad \vec{q}_\pi = -\vec{q} - \vec{q}', \tag{4.27}$$

$$J_{N\pi}(\vec{q}, \vec{q}_\pi; \vec{P}_{N\pi}, \vec{k}) = \frac{E_\pi(q_\pi)}{E_\pi(k)}, \tag{4.28}$$

$$R^0(q', q, q_\pi; E) = \left[E + i\epsilon - 2m_N - \frac{q^2 + q'^2}{2m_N} - E_\pi(q_\pi)\right]^{-1}. \tag{4.29}$$

The minus sign in front of (4.26) is easily seen to arise from the antisymmetry of the intermediate states in the nucleon variables.

In order to simplify the calculation of the effective interaction V_B, we introduce one additional approximation: We assume that all but the P_{33} πN interactions can be neglected in $NN\pi$ states. This approximation is justified by the relative weakness of nonresonant πN interactions. In the energy region just above the pion production threshold, we should expect the s-wave πN interaction to play a significant role. However, by setting the operator V'' of (1.4) equal to zero, we have already given up a reliable description of the coupling to the $NN\pi$ channel in that energy region. In this approximation, the only interaction contributing to $\overline{V}$ is v_{NN}. From (4.14), (4.3), (2.33), and (2.19), it follows that

$$(\vec{q}_\pi', \nu_\pi', \vec{k}', \mu_a', \mu_b' | \overline{T}(E) | \mu_b, \mu_a, \vec{k}, \nu_\pi, \vec{q}_\pi) = \delta(\vec{q}_\pi' - \vec{q}_\pi)\delta_{\nu_\pi'\nu_\pi}(\vec{k}', \mu_a', \mu_b' | \hat{T}_{NN}(\epsilon_\pi) | \mu_b, \mu_a, \vec{k}), \tag{4.30}$$

where

$$\epsilon_\pi = E - \frac{q_\pi^2}{4m_N} - E_\pi(q_\pi), \tag{4.31}$$

and $\hat{T}_{NN}(\epsilon_\pi)$ satisfies a Lippmann-Schwinger equation with $\hat{v}_{NN}$ as driving term and with the propagator

$$\overline{R}^0(\kappa, \epsilon_\pi) = \left(\epsilon_\pi + i\epsilon - 2m_N - \frac{\kappa^2}{m_N}\right)^{-1}. \tag{4.32}$$

The matrix elements of V_B are therefore given explicitly by

$$(\vec{q}', \mu_N', \mu_\Delta' | V_B(E) | \mu_\Delta, \mu_N, \vec{q}) = \tfrac{1}{2}\sum_{\nu_\pi \mu_N'' \mu_N'''} \int d^3q_\pi\, J_{N\pi}^{-1/2}(\vec{q}', \vec{q}_\pi; \vec{P}'_{N\pi}, \vec{k}') \times (\mu_\Delta' | \hat{v}_{N\pi} | \mu_N''', \nu_\pi, \vec{k}) \overline{R}^0(\kappa', \epsilon_\pi)(\vec{k}', \mu_N', \mu_N''' | \hat{T}_{NN}(\epsilon_\pi) | \mu_N'', \mu_N, \vec{k})_a \times \overline{R}^0(\kappa, \epsilon_\pi)(\vec{k}', \nu_\pi, \mu_N'' | \hat{v}_{N\pi} | \mu_\Delta) J_{N\pi}^{-1/2}(\vec{q}, \vec{q}_\pi; \vec{P}_{N\pi}, \vec{k}), \tag{4.33}$$

where

$$\vec{k}' = \vec{q}' + \frac{\vec{q}_r}{2}, \quad \vec{k} = \vec{q} + \frac{\vec{q}_r}{2} \tag{4.34}$$

and the subscript G is used to indicate that a matrix element is taken between properly antisymmetrized NN states.

A graphical representation of the equations for NN scattering, under the approximations mentioned above, is shown in Fig. 2. Partial-wave expansions of (4.26) and (4.33) are given in the Appendix. Once the effective interactions V_D, V_E, and V_B have been constructed, the $N\Delta$ T matrix T_C can be obtained by solving (4.15), which reads, in partial wave expanded form,

$$T_C^{\gamma',\gamma}(q',q;E) = V_C^{\gamma',\gamma}(q',q;E) + \sum_{\gamma''} \int_0^\infty dq''\, q''^2\, V_C^{\gamma',\gamma''}(q',q'';E) \times \tilde{R}^0_{N\Delta}(q'';E) T_C^{\gamma'',\gamma}(q'',q;E), \tag{4.35}$$

where the driving term is the sum of (A7) and (A17). The solutions of Eq. (4.35) are then used to evaluate R_C of (4.18). The last step is to solve (4.11) for T_0. We assume again a separable form for the interaction V_0:

$$V_0^{\rho',\rho}(q',q) = \sum_{i,j=1}^{N} h_i^{\rho'}(q') H_{i,j}^{\rho',\rho} h_j^{\rho}(q), \tag{4.36}$$

where the notation of (A5) is used and the symbol ρ stands for the set of superscripts $\{\gamma, \delta\}$. With

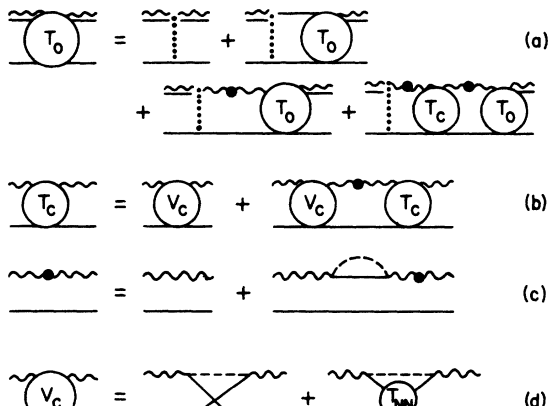

FIG. 2. Scattering equations for NN scattering. Solid, wiggly, and dashed lines stand for nucleons, deltas, and pions, respectively. A solid and a wiggly line appearing together indicate that the particle can be either a nucleon or a delta. The interaction V_0 is represented by a dotted line. Equation (a) stands for (4.11) and (4.18) of the main text, Eq. (b) stands for (4.15), Eq. (c) for (4.5) and (4.7), and Eq. (d) for (4.21), (4.26), and (4.33).

the help of (4.18), (4.24), and (4.36), the solution of (4.11) may be written in the form

$$T_0^{\rho',\rho}(q',q;E) = \sum_{i,j=1}^{N} h_i^{\rho'}(q') [K^{-1}(E)]_{i,j}^{\rho',\rho} h_j^{\rho}(q), \tag{4.37}$$

where K, a matrix whose row and column indices run over all sets $\{\rho, j\}$, is constructed as follows:

$$K = H^{-1} - N - Q, \tag{4.38}$$

where

$$N_{i,j}^{\rho',\rho}(E) = \delta_{\rho\rho'} \int_0^\infty dq\, q^2 h_i^{\rho}(q) R_\delta^0(q;E) h_j^{\rho}(q), \tag{4.39}$$

$$Q_{i,j}^{\rho',\rho}(E) = \delta_{\delta',2}\delta_{\delta,2} \int_0^\infty dq\, q^2 \times \int_0^\infty dq'\, q'^2 h_i^{\rho'}(q') R_2^0(q';E) \times T_C^{\gamma',\gamma}(q',q;E) R_2^0(q;E) h_j^{\rho}(q), \tag{4.40}$$

$$R_1^0(q;E) = [E + i\epsilon - 2E_N(q)]^{-1}, \quad R_2^0(q;E) = \tilde{R}^0_{N\Delta}(q,E). \tag{4.41}$$

The S matrix for NN elastic scattering is computed from T_0 in the usual way:

$$S^{\gamma',\gamma} = \delta_{\gamma',\gamma} - 2\pi i \rho_0 T_0^{\gamma',\gamma}(q_0,q_0;E), \tag{4.42}$$

with

$$E = 2E_N(q_0); \quad \rho_0 = \tfrac{1}{2} E q_0.$$

In practice, the following parametrization is adopted for the form factors of (4.36):

$$h_j^{\rho}(q) = \frac{q^l (q_j^{\rho})^{l+1}}{[q^2 + (q_j^{\rho})^2]^{l+1}} \tag{4.43}$$

and the equations derived above are used to determine the strength parameters $H_{i,j}^{\rho',\rho}$ and the range parameters q_i^{ρ} by fits to NN scattering data both above and below the pion production threshold.

B. Numerical results

Because it would require treating many angular momentum channels simultaneously, the determination of the parameters of V_0 directly from NN scattering observables would be very difficult. The task is greatly alleviated by relying on the phase-shift analysis of Ref. 20, which makes it possible to consider one channel at a time. We should acknowledge at the outset that the phase shifts and inelasticities are not at this time unambiguously determined, especially at the higher end of the energy region of interest. It is likely that, as more data for medium-energy NN scat-

tering accumulate, significant changes will occur in the output of the phase-shift analysis. The set of parameters we present below should therefore be considered as tentative. In any case, the experience gained in the present work should be useful, since many of the conclusions we reach concerning the importance of various effects are valid independent of the precise values of the phase shifts and inelasticities.

Before turning to the presentation of our results, we discuss briefly the techniques used in numerical calculations. The first step is to construct the effective $N\Delta$ interactions V_D, V_E, and V_B using (4.23) and (A6)–(A17) together with the results of Sec. III. For $E > 2m_N + m_\pi$, three-body branch cuts appear in the propagators of (4.23), (A8), and (A15). The usual rotation of contour method[22] is used to circumvent these singularities: the interaction matrices are constructed on a momentum grid defined on a ray in the complex plane, at an angle θ_R with the real axis. The integration over q_π in (A17) is performed over the same grid. Accordingly, Eq. (4.35) becomes a matrix equation over the complex momentum grid; its solution is obtained by matrix inversion. The energy-dependent matrices N and Q defined by (4.39) and (4.40) are computed by evaluating the corresponding integrals also over the complex grid. It is then simple to construct and invert the matrix K, and to obtain the NN scattering matrix from (4.37) and (4.42). The method of Ref. 22 can be used to determine the allowed range of values of θ_R; one finds[23] $0 < \theta_R < \pi/2$ (measured clockwise). In actual calculations, we have used values of θ_R ranging from 15° down to 10° as the energy varied from threshold to 800 MeV. The motivation for choosing these rather small values is explained in Sec. V, where we discuss calculations of pion absorption on the deuteron. The stability of the results with respect to changes in θ_R has been checked in a number of test cases.

The same grid was used for all momentum variables in the calculations. It was divided into four segments, over each of which a Gaussian mesh was set up. In the evaluation of the Q integrals, an accuracy of better than 1% was obtained in all cases studied with the following choice of segments and numbers of mesh points (the numbers are momenta in MeV/c): 0–200 (12 points); 200–600 (12 points); 600–1200 (4 points); 1200–∞ (8 points). The last segment was mapped into the interval $[-1, 1]$ according to $q = 1800(1+x)/(1+x) + 1200$. As will be seen below, the Q integrals are in many cases small compared to the N integrals, and an accuracy of 5–10% is then sufficient in their evaluation. This was achieved with the same choice of segments and with 8, 8, 4, and 4 mesh points, respectively. The corresponding accuracy in the evaluation of the N integrals was always better than 0.2%. The angular integrals (A8) and (A15) were also evaluated by Gaussian quadrature; a 1% accuracy was obtained with 20 mesh points.

In order to devise a workable strategy for the determination of the parameters of V_0, it is important to note that T_C does not depend on them. Therefore, the calculation of T_C which requires a large amount of computing time need not be repeated in the course of the parameter search. (This is the motivation for treating V_0 and V_C differently in the formulation of the scattering equations.) Storing the matrix elements of T_C for all the energy values used in the fits and evaluating the Q integrals at each step of the fitting procedure would still be too costly. We remark, however, that the only parameters of V_0 involved in the integrals N and Q are the ranges of the $N\Delta$ form factors. In view of this, the strategy used in the fits was to hold these ranges fixed and perform a search over all the coupling constants H as well as over the NN form factor cutoffs. The integrals N and Q may then be evaluated once and for all. The only operation that must be repeated at each step of the search is the inversion of the small matrix K.

Since the strength of the $NN \to N\Delta$ coupling is determined by fitting the inelasticity in NN scattering, the determination of V_0 is possible only for those angular momentum channels that exhibit inelasticity in the phase-shift analysis. Obviously, only $T = 1$ channels are considered in our study. The NN channels of interest are listed in Table III. Threshold behavior suggests that, in the energy region up to 800 MeV, most of the pion production in a given NN channel proceeds via the $N\Delta$ partial waves of lowest orbital angular momentum.

TABLE III. Partial wave decomposition of NN and $N\Delta$ systems.

NN	$N\Delta$	NN interactions in $NN\pi$ states
$^1S_0$	$^5D_0$	
$^3P_0$	$^3P_0$	$^3P_1$ $^1P_1$
$^3P_1$	$^3P_1$ $^5P_1$	$^3S_1$ $^3D_1$ $^3P_0$ $^3P_1$ $^3P_2$ $^1P_1$
$^3P_2$ $^3F_2$	$^3P_2$ $^5P_2$	$^1S_0$ $^3S_1$ $^3D_1$ $^1P_1$ $^3P_1$ $^3P_2$
$^1D_2$	$^5S_2$	$^3S_1$ $^3D_1$
$^3F_3$	$^5P_3$	$^3S_1$ $^3D_1$ $^3P_0$ $^3P_1$ $^3P_2$
$^1G_4$	$^5D_4$	
	$^3S_1$	$^1S_0$ $^3S_1$ $^3D_1$ $^1P_1$ $^3P_1$ $^3P_2$

Therefore, only those partial waves have been included in the calculations. This approximation is corroborated by the fact that only NN channels that couple to an $N\Delta$ configuration with $l<2$ exhibit inelasticity.

As mentioned in Sec. III, NN interactions v_{NN} in $NN\pi$ channels with $l_{NN}\geq 2$ (with the exception of $^3D_1$) are assumed negligible. Since including them all in $NN\pi$ channels with $l_{NN}\leq 1$ would require a large amount of computing time, it is worthwhile to investigate beforehand the relative importance of NN interactions in different partial waves. This was accomplished as follows. A set of parameters of V_0 was obtained, neglecting all interactions in the $NN\pi$ channel. The contribution of each NN partial wave in $NN\pi$ states was estimated by treating it as a perturbation in the following sense: The corresponding Q integrals were evaluated by replacing T_C by V_C. As is clear from (A17), different NN partial waves give additive contributions to Q in that approximation. It is therefore possible to estimate separately the importance of each NN partial wave in V_B for the calculation of the phase shifts and inelasticities by performing calculations for a few energies with and without the corresponding contribution to Q. This procedure relies on two assumptions. First, it is necessary that the parameters of V_0 determined by neglecting all NN interactions in $NN\pi$ states be sufficiently close to those resulting from a more accurate calculation. This was found to be the case at the crude level of accuracy needed for the estimates just discussed. Second, errors made by replacing T_C by V_C should be sufficiently small. They were found to be of order 10–20% at most in all instances.

The following conclusions emerge from this study. The $^5S_2$ $N\Delta$ partial wave is most affected by the $^3S_1$-$^3D_1$ NN interaction in $NN\pi$ states. The contributions to the Q integrals from NN ^3P waves are an order of magnitude smaller and lead to effects in the $^1D_2$ NN phase shift and inelasticity that are small compared to the error bars. Angular momentum conservation rules out the $^1S_0$ and $^1P_1$ NN interactions in this $N\Delta$ partial wave. The $^3P_0$ $N\Delta$ partial wave is affected negligibly by the $^1S_0$ NN interaction; the $^3P_1$ and $^1P_1$ NN interactions are more important and give effects of comparable magnitude. The $^3S_1$-$^3D_1$, $^3P_0$, and and $^3P_2$ NN interactions are excluded by selection rules. For the $^3P_1$-$^5P_1$, $^3P_2$-$^5P_2$, and $^5P_3$ $N\Delta$ partial waves, no NN interaction dominates the others, and therefore all those allowed by selection rules were kept in the final calculations. However, as will be seen shortly, their effects on NN phase shifts and inelasticities are quite small. In the $^5D_0$ and $^5D_4$ $N\Delta$ partial waves, the NN interactions in intermediate $NN\pi$ states are completely negligible. The situation is summarized in Table III, where the NN interactions in $NN\pi$ states kept in the final calculations are listed. This table also includes the $^3S_1$ $N\Delta$ interaction, which does not couple to the NN channel but does couple to the πd channel and will be needed in Sec. IV.

The fits obtained for all NN partial waves of interest are displayed in Fig. 3. The corresponding parameters of V_0 are presented in Table IV. The coupling constant matrix H was assumed to have the following structure:

$$H_{i,j}^{\gamma 1,\gamma 1}=\delta_{ij}H_i^{\gamma 1,\gamma 1},\quad i=1,2,3 \tag{4.44a}$$

$$H_{i,j}^{\gamma' 1,\gamma 1}=0,\quad \gamma'\neq\gamma \tag{4.44b}$$

$$H_{i,j}^{\gamma' 1,\gamma 2}=\delta_{i3}H_j^{\gamma' 1,\gamma 2},\quad j=1,2 \tag{4.44c}$$

$$H_{i,j}^{\gamma' 2,\gamma 2}=0\ . \tag{4.44d}$$

Note that (4.44b) means that we are neglecting tensor coupling between different NN partial waves. The only tensor coupled partial waves relevant to us are $^3P_2$ and $^3F_2$; in view of the small value of the mixing coefficient ($|\epsilon|<3^\circ$), the approximation is justified in this case. By assuming (4.44d), we have also discarded possible direct $N\Delta\to N\Delta$ interactions. This simplification does not impair significantly the description of NN scattering in our model. Allowing V_0 to have a nonzero $N\Delta\to N\Delta$ component did not result in significantly better fits.

The range parameter $q_j^{\gamma 2}$ are fixed at the values 250 MeV/c ($i=1$) and 700 MeV/c ($i=2$). This choice is arbitrary; it is hoped that the independent variation of the coupling constants $H_j^{\gamma 1,\gamma 2}$ ($j=1,2$) gives enough freedom to obtain an adequate dependence of V_0 on the off-shell $N\Delta$ momentum.

In the case of coupled $N\Delta$ partial waves (^3P-^5P), the number of coupling constants is too large for an unambiguous determination to be possible if no further constraint is imposed. This problem is avoided by requiring the ratio

$$R^{\gamma'\gamma''}(q_1,q_2)=V_0^{\gamma 1,\gamma' 2}(q_1,q_2)/V_0^{\gamma 1,\gamma'' 2}(q_1,q_2) \tag{4.45}$$

to have a fixed value for some typical momenta q_1 and q_2 (we chose $q_1=575$ MeV/c, $q_2=300$ MeV/c, roughly corresponding to an "on-shell" $NN\to N\Delta$ transition at 700 MeV). The value of $R^{\gamma'\gamma''}$ was taken to be that given by a static one-pion exchange interaction with a dipole vertex form factor of cutoff 1 GeV/c. The phase shifts and inelasticities in a given angular momentum channel are obviously independent of the overall sign of $V_0^{\gamma 1,\gamma 2}$ in that channel. The relative signs of this

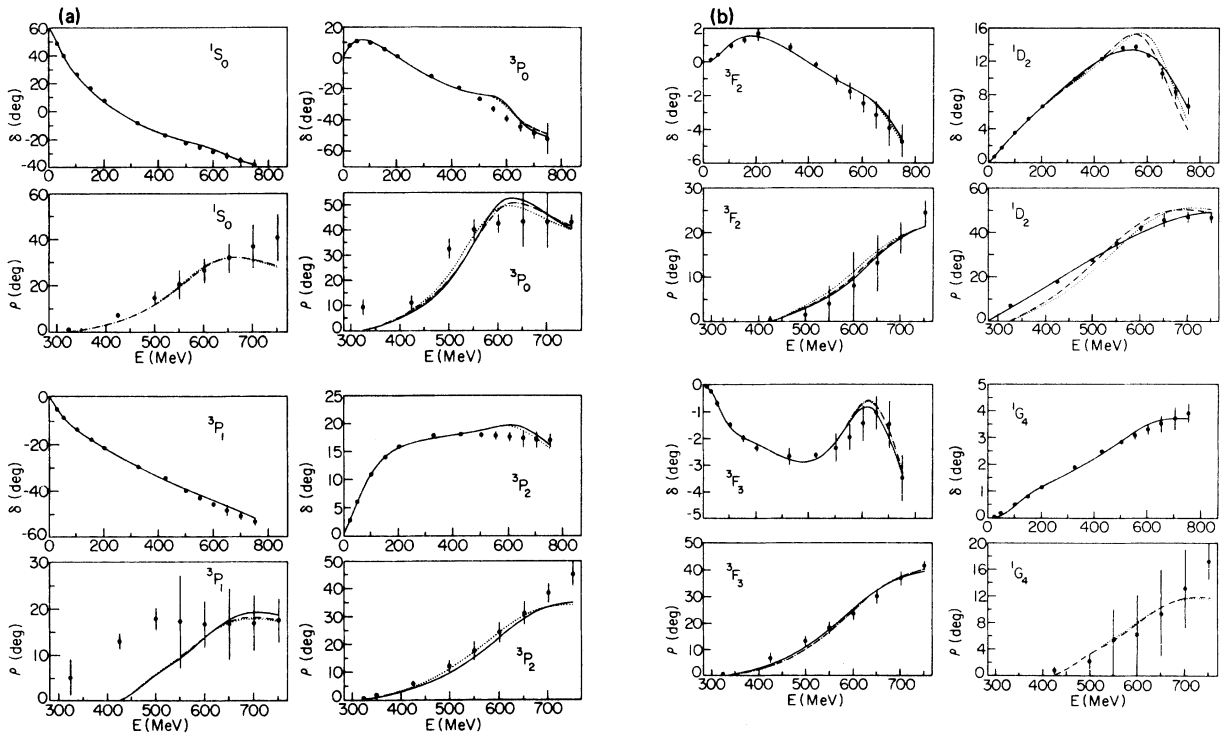

FIG. 3. Fits to the NN phase shifts and inelasticities of Ref. 20 ($\eta = \cos\rho$); the parameters of the corresponding interaction V_0 are given in Table IV. The horizontal scale gives the nucleon lab kinetic energy. The solid curves are the fits obtained when all interactions in $NN\pi$ states listed in Table III are included. The dot-dashed curves are obtained with the same V_0 but no NN interactions in $NN\pi$ states. The dotted curves correspond again to the same V_0, but $T_C = 0$ (see text). Curves not drawn are indistinguishable from those shown.

interaction in different angular momentum channels cannot therefore be determined from NN elastic scattering. These signs were assigned again by comparison with the one-pion exchange (OPE) transition potential. These recipes are clearly unsatisfactory; a more reliable determination would require a study of the kinematically complete reaction $NN \to NN\pi$.

As can be seen from Fig. 3 and Table IV, satisfactory fits can be obtained with relatively simple

TABLE IV. Parameters of V_0 (q_i is in MeV/c and H_i is unitless).

	$NN \to NN$						$NN \to N\Delta$	
	H_1	q_1	H_2	q_2	H_3	q_3	H_1	H_2
$^1S_0(NN) \rightleftarrows {}^5D_0(N\Delta)$	−2.565	423	27.54	911	0		−18.86	27.69
$^3P_0(NN) \rightleftarrows {}^3P_0(N\Delta)$	−0.0307	103	−3.103	401	42.11	750	−15.81	21.78
$^3P_1(NN) \rightleftarrows {}^3P_1(N\Delta)$	0.4043	268	28.42	795	0		3.965	−5.606
$^3P_1(NN) \rightleftarrows {}^5P_1(N\Delta)$	0.4043	268	28.42	795	0		−6.543	9.250
$^3P_2(NN) \rightleftarrows {}^3P_2(N\Delta)$	−0.2593	373	−0.2426	426	−1.176	827	1.105	−0.6772
$^3P_2(NN) \rightleftarrows {}^5P_2(N\Delta)$	−0.2593	373	−0.2426	426	−1.176	827	1.603	−0.9819
$^3F_2(NN) \rightleftarrows {}^3P_2(N\Delta)$	−1.547	340	200.6	1325	0		−11.56	−6.890
$^3F_2(NN) \rightleftarrows {}^5P_2(N\Delta)$	−1.547	340	200.6	1325	0		−11.56	−6.890
$^1D_2(NN) \rightleftarrows {}^5S_2(N\Delta)$	−0.2221	241	−1.191	529	0		3.753	−2.682
$^3F_3(NN) \rightleftarrows {}^5P_3(N\Delta)$	1.144	274	12.78	658	0		−6.389	−2.022
$^1G_4(NN) \rightleftarrows {}^5D_4(N\Delta)$	−3.471	359	−11.55	698	0		14.72	−27.73

interactions V_0. The only exception is the inelasticity in the $^3P_1$ partial wave, for which the values of $\rho(=\cos\eta)$ given by the phase-shift analysis for energies below 500 MeV seem too large to correspond to the production mechanism $NN \rightarrow N\Delta \rightarrow NN\pi$.

The influence of the $N\Delta$ effective interaction V_C on NN phase shifts and inelasticities is also shown in Fig. 3, where the results obtained by neglecting V_B but keeping V_E and by neglecting V_C altogether are compared to the more accurate results (the parameters of V_0 are those of Table IV in all calculations). The effects due to V_C are appreciable (compared to the present error bars) only for the $^1D_2$ NN partial wave, for which both V_B and V_E are important. Note that this NN partial wave is the only one that couples to an $N\Delta$ state with $l=0$. It is reasonable to expect that the significant role played by NN interactions in $NN\pi$ intermediate states in this partial wave is a general feature of the $NN\pi$ system, independent of the details of the model. This suggests that the discrepancy between the results of Ref. 7 and the phenomenological phase shifts might be due in part to the neglect of these interactions. It also sets a limit to the reliability of calculations of the imaginary parts of the NN phases in which these interactions are omitted.[6]

V. PION ABSORPTION ON THE DEUTERON

As a first test of the model constructed in the previous sections, we consider the reaction $\pi + d \rightarrow N + N$. In Sec. V A, the expressions used in the calculation of the transition amplitudes are derived, starting from the formal equations of I. Numerical results are presented in Sec. V B.

A. Transition amplitudes

As shown in I, the c.m. amplitude for $\pi + d \rightarrow N + N$ can be obtained by taking matrix elements, between appropriate initial and final states, of an operator $\mathcal{T}_d$ given by

$$\mathcal{T}_d(E) = [1 + U(E)\tilde{R}(E)]\, U_d(E), \tag{5.1}$$

with

$$U(E) = V + V\bar{R}(E)\, V - V_D(E), \tag{5.2}$$

$$U_d(E) = [1 + V\bar{R}(E)]\, V_d, \tag{5.3}$$

where V_D, $\bar{R}$, and $\tilde{R}$ have been defined by Eqs. (4.1)–(4.6) and

$$V_d = V - V_{ab,\pi}. \tag{5.4}$$

Recall that $V_{ab,\pi}$ is defined by Eq. (2.21). Under the assumptions introduced in Sec. IV, we may rewrite (5.1) in the form

$$\mathcal{T}_d(E) = V_0 \tilde{R}(E) V_d, \tag{5.5}$$

where the terms of (5.2) and (5.3) which do not contribute to the transition amplitude for $\pi d \rightarrow NN$ have been dropped. From (4.6), (4.10), (4.11), (4.18), and (2.33), it then follows that

$$\mathcal{T}_d(E) = T_0(E)[\bar{R}^0(E) + \tilde{R}^0(E) T_C(E) \bar{R}^0(E)][v_{a\pi} + v_{b\pi}], \tag{5.6}$$

where $v_{a\pi}$ and $v_{b\pi}$ are given by Eq. (2.25).

Let us define

$$(\vec{q}, \mu_N, \mu_\Delta | \mathcal{W} | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}) = \sqrt{2} \sum_{\mu'_N} J_{N\pi}^{-1/2}(\vec{q}, \vec{q}_{\pi_0}; \vec{P}_{N\pi}, \vec{k}) \times (\mu_\Delta | \hat{v}_{N\pi} | \mu'_N, \nu_\pi, \vec{k})\, (\mu_N, \mu'_N | \Phi | \vec{k}_d, \sigma_d), \tag{5.7}$$

where σ_d is the z-axis projection of the deuteron spin, $\Phi(\vec{k}_d, \sigma_d)$ is the deuteron wave function and $\vec{q}_{\pi_0}$ and $\vec{q}$ are, respectively, the relative momenta for the πd and $N\Delta$ systems. The momenta in (5.7) are related by the equations

$$\vec{k} = \vec{q}_{\pi_0} + \frac{E_\pi(q_{\pi_0})}{M_{N\pi}(q_{\pi_0})}\vec{q}, \tag{5.8}$$

$$\vec{k}_d = \vec{q} + \frac{\vec{q}_{\pi_0}}{2}. \tag{5.9}$$

Using (5.6) and (5.7), the amplitude for $\pi + d \rightarrow N + N$ in the c.m. frame can be written in the form

$$(\vec{q}_{N_0}, \mu_a, \mu_b | \mathcal{T}_d(E) | \sigma_d, \nu_\pi, \vec{q}_{\pi_0})_\alpha = \sum_{\xi_{N\Delta}} \int d^3q\, (\vec{q}_{N_0}, \mu_a, \mu_b | T_0(E) | \xi_{N\Delta}, \vec{q})_\alpha\, \tilde{R}^0_{N\Delta}(q; E)\, (\vec{q}, \xi_{N\Delta} | \tilde{\mathcal{W}}(E) | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}), \tag{5.10}$$

with

$$(\vec{q}, \xi_{N\Delta} | \tilde{\mathcal{W}}(E) | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}) = (\vec{q}, \xi_{N\Delta} | \mathcal{W} | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}) + \sum_{\xi'_{N\Delta}} \int d^3q'\, (\vec{q}, \xi_{N\Delta} | T_C(E) | \xi'_{N\Delta}, \vec{q}')\, \tilde{R}^0_{N\Delta}(q'; E)\, (\vec{q}', \xi'_{N\Delta} | \mathcal{W} | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}), \tag{5.11}$$

where $\xi_{N\Delta}$ stands for $\{\mu_N, \mu_\Delta\}$ and $\vec{q}_{N_0}$ is the relative momentum of the final NN state. Note that the antisymmetry of the intermediate $NN\pi$ states in the nucleon variables results in the factor $\sqrt{2}$ in (5.7). This

is expected since each one of the nucleons in the deuteron can be changed into a Δ. Equations (5.10) and (5.11) are shown pictorially in Fig. 4. The partial wave expanded form of (5.10) is defined by Eq. (A23). The multipole amplitudes of Eq. (A23) are given by

$$\mathcal{T}_d^{\gamma,\Gamma}(E) = \sum_{\gamma'} \sum_{i,j=1}^{N} h_i^{\gamma,1}(q_{N_0}) [K^{-1}(E)]_{i,j}^{\gamma,\gamma'} [X_j^{\gamma',\Gamma}(E) + Y_j^{\gamma',\Gamma}(E)] , \tag{5.12}$$

where the symbols γ and Γ stand for the set of quantum numbers necessary to specify an angular momentum channel of the NN and πd systems, respectively, and

$$X_j^{\gamma',\Gamma}(E) = \int_0^\infty dq\, q^2 h_j^{\gamma',2}(q) \tilde{R}_{N\Delta}^0(q; E) \mathcal{W}^{\gamma',\Gamma}(q, q_{\pi_0}) \tag{5.13}$$

$$Y_j^{\gamma',\Gamma}(E) = \sum_{\gamma''} \int_0^\infty dq\, q^2 \int_0^\infty dq'\, q'^2 h_j^{\gamma',2}(q) \tilde{R}_{N\Delta}^0(q; E)\, T_C^{\gamma',\gamma''}(q, q'; E) \tilde{R}_{N\Delta}^0(q'; E) \mathcal{W}^{\gamma'',\Gamma}(q', q_{\pi_0}) . \tag{5.14}$$

The differential cross section for $\pi^+ + d \to p + p$ is related to the transition amplitude in a standard fashion:

$$\frac{d\sigma}{d\Omega} = \frac{1}{3} \frac{(2\pi)^4 q_{N_0}}{q_{\pi_0}} \mu_{\pi d} \frac{E_N(q_{N_0})}{2} \sum_{\sigma_a \sigma_b \sigma_d} |(q_{N_0}, \sigma_a \tau_a, \sigma_b \tau_b | \mathcal{T}_d(E) | \sigma_d, \nu_\pi, \vec{q}_{\pi_0})_\alpha|^2 , \tag{5.15}$$

where $\tau_a = \tau_b = \frac{1}{2}$, $\nu_\pi = +1$, m_d is the deuteron mass, and

$$\mu_{\pi d} = m_d E_\pi(q_{\pi_0}) / [m_d + E_\pi(q_{\pi_0})] . \tag{5.16}$$

B. Numerical results

Using the formulas of Sec. V A we have calculated the total cross section for $\pi^+ + d \to p + p$ as a function of energy, as well as angular distributions for a few energy values. As in the case of NN scattering, most of the computing time went into the calculation of the $N\Delta$ T matrix T_C, which appears both in the Y integrals defined by (5.14) and in the constants K [see (5.12) and (4.38)–(4.41)]. For this reason, the calculations of the $NN \to NN$ and $\pi d \to NN$ amplitudes were performed simultaneously, using the complex momentum grid described in Sec. IV. This grid was also used for the calculation of the functions $\mathcal{W}$ given by (A21) and (A22). Because these involve the fixed momentum q_{π_0}, the maximum rotation angle allowed is smaller than $\pi/2$ and is a decreasing function of the energy.[22] This is the reason for choosing the rather small values of θ_R mentioned in Sec. IV. The accuracies achieved in the calculation of the amplitudes for $\pi d \to NN$ were similar to those for NN scattering amplitudes discussed above. Since the calculations for $NN \to NN$ and $\pi d \to NN$ were performed for the same set of energies (i.e., those for which the phase shifts and corresponding error bars are given by the phase shift analysis), it was necessary to interpolate the results to obtain the angular distributions at the energies for which data are available. The errors introduced by this procedure did not exceed 5%.

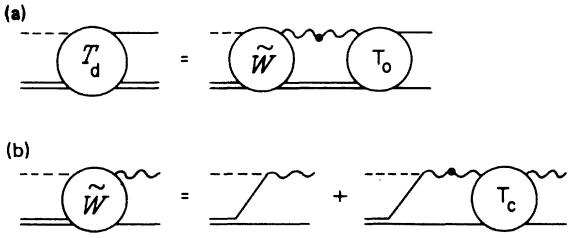

FIG. 4. Pictorial representation of the $\pi d \to NN$ transition amplitude. The double line represents the deuteron. Other symbols have the same meaning as in Fig. 2. Equation (a) corresponds to (5.10) and Eq. (b) to (5.11).

An analysis parallel to that described in Sec. IV was carried out to determine which NN interactions in $NN\pi$ states were to be included. It turns out that the conclusions reached in the case of NN scattering apply to pion absorption also. The NN interactions included in the calculations discussed below are those listed in Table III.

Results for the total absorption cross section, as a function of energy, are presented in Fig. 5. The interaction V_0 used in these calculations is that of Table IV. The shape and magnitude of the cross section in the resonance region are fairly well reproduced. For low pion energies ($T_\pi < 80$ MeV), s-wave πN interactions are expected to play a significant role[25]; it is therefore not surprising that the predictions of the present model fall below the data in that energy region. For energies beyond 150 MeV, our model overestimates the cross section by ~2 mb. A study of individual angular momentum channel contributions shows that most of the flux goes into the $^1D_2$ and $^3F_3$ NN partial waves. The contributions from all other partial waves add up to only a few percent. Figure

5 also shows the effect of leaving out the $^3F_3$ NN channel. The calculated total cross section is then in quite good agreement with the data. These results suggest that the contribution from the $^3F_3$ channel is overestimated. The existence of a dibaryon resonance[26] in that channel might provide an explanation of this discrepancy. The analysis of forward cone πd elastic scattering suggests rather small values for the partial decay widths of these resonances into the πd channel[26] and correspondingly large widths for decay into the $NN\pi$ channel. The existence of a competing mechanism for inelasticity in NN scattering would imply that the strength of the $^3F_3(NN)\rightarrow{}^5P_3(N\Delta)$ transition operator given in Table IV is too large. In view of the uncertainties in the phase-shift analysis and the present lack of knowledge concerning the nature of dibaryon resonances, this question remains open.

One of the important differences between the present calculation and others available in the literature[25] is the inclusion of nonstatic pion rescattering between N and Δ, as well as NN interactions in intermediate $NN\pi$ states. It is therefore of interest to examine the size of the error made by leaving these effects out. In the formalism above, this corresponds to neglecting T_C altogether, that is, to putting the constants Y and Q equal to zero [see (5.14) and (4.40)]. As shown in Fig. 5, the total cross section obtained in this approximation differs significantly from the result of the more complete calculation, especially, in the vicinity of the maximum, where the former underestimates the latter by about 40%. Comparison with the corresponding results obtained when the $^3F_3$ NN channel is left out indicates that the effects described by T_C are appreciable mainly in the 2^+ channel. Thus, the study of pion absorption on the deuteron leads to the same conclusion as the study of NN scattering: *pion exchange and NN interactions in intermediate $NN\pi$ states influence significantly $N\Delta$ configurations with orbital angular momentum equal to zero.*

As mentioned in Sec. IV, the relative signs of the $NN\rightarrow N\Delta$ transition operator for different angular momentum channels are left undetermined by the fits to NN elastic scattering. The total cross section for $\pi d\rightarrow NN$ is also unaffected by the choice of these signs. However, this is not true of the differential cross section. Since the prescription adopted in Sec. IV to fix these signs is rather unreliable, we expect that calculations of angular distributions performed with the interaction V_0 of Table IV will achieve only limited agreement with

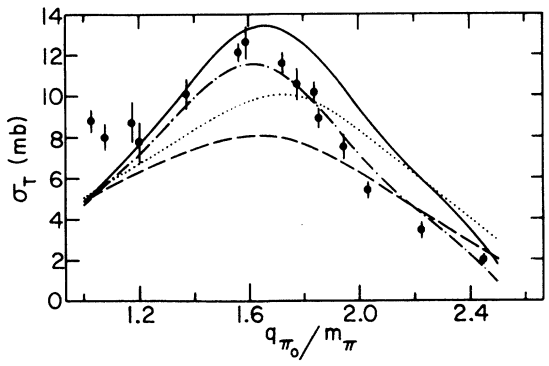

FIG. 5. Total cross section for $\pi + d\rightarrow NN$ as a function of the incident c.m. pion momentum. The interaction V_0 is that of Table IV. The data is taken from the compilation of Ref. 24. (a) Solid curve: obtained with all NN channels and all NN interactions in $NN\pi$ states listed in Table III; (b) Dot-dashed curve: same as (a) but the $^3F_3$ NN channel is left out; (c) Dotted curve: same as (a) but $T_C = 0$ (see text); and (d) Dashed curve: same as (b), but $T_C = 0$.

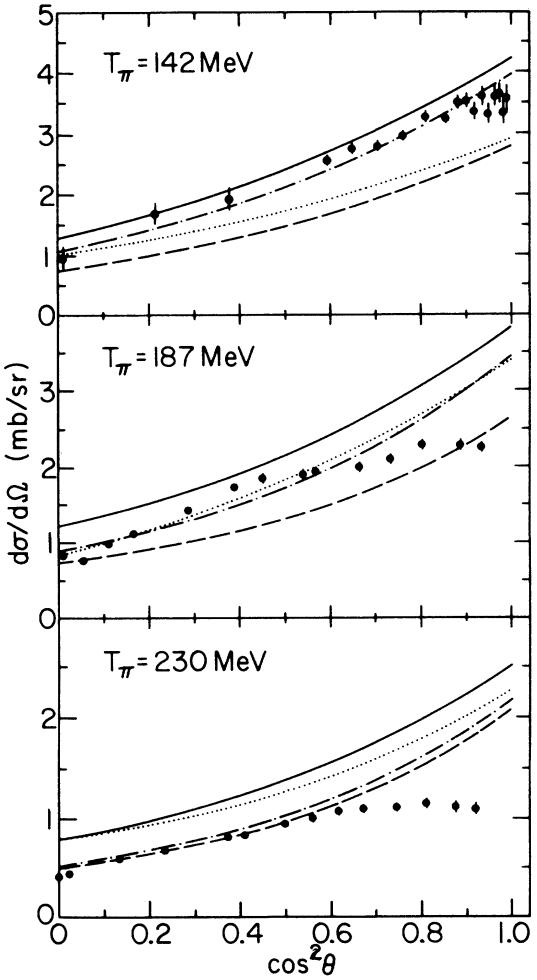

FIG. 6. Angular distributions for $\pi + d\rightarrow NN$. See caption of Fig. 5 for the description of each curve. The data are from Refs. 27 and 28.

the data. Indeed, Fig. 6 shows that the calculated angular distribution agrees reasonably well with experiment at $T_\pi = 142$ MeV but becomes considerably too forward peaked at higher energies. Concerning the role played by the $^3F_3$ NN channel and the importance of contributions associated with T_C, the comments made about the total cross section are essentially valid for the angular distribution also. Neither of these effects alters much the shape of the differential cross section, but they both influence its magnitude significantly.

A detailed comparison with all available data for differential cross sections and polarizations for this reaction would certainly be helpful in reducing the ambiguities in the interaction V_0. However, such a systematic study is beyond the scope of the present work.

VI. PION-DEUTERON ELASTIC SCATTERING

To further test the model, we study in this section the pion-deuteron elastic scattering. Our model provides a framework in which the influence of true pion absorption on pion-deuteron elastic scattering in the (3, 3) resonance region can be studied. This is the main topic of the present section. In Sec. VI A the results of I are used as a starting point to derive practical expressions for the relevant amplitudes. Numerical results for total cross sections, angular distributions, and polarizations are discussed in Sec. VI B.

A. Scattering amplitudes

The c.m. elastic π-d scattering amplitude was shown in I to be given by matrix elements, between appropriate initial and final states, of an operator $\mathcal{T}_{dd}$ which can be can be calculated from the formula

$$\mathcal{T}_{dd}(E) = U_{dd}(E) + V_d[1 + \bar{R}(E)V]\tilde{R}(E)[1 + V\bar{R}(E)]\,V_d, \tag{6.1}$$

where V_d is defined by (5.4) and

$$U_{dd}(E) = V_d + V_d\bar{R}(E)V_d. \tag{6.2}$$

The simplifying assumptions of Sec. IV allow us to rewrite Eq. (6.1) as

$$\mathcal{T}_{dd}(E) = V_d\tilde{R}(E)V_d, \tag{6.3}$$

where the terms of (6.1) that do not contribute to the elastic scattering amplitude have been dropped. Using the definitions (4.6) and (4.10), the amplitude may be broken up in two terms:

$$\mathcal{T}_{dd}(E) = \mathcal{T}_S(E) + \mathcal{T}_A(E), \tag{6.4}$$

where

$$\mathcal{T}_S(E) = V_d R_C(E) V_d, \tag{6.5}$$

$$\mathcal{T}_A(E) = V_d R_C(E) T_0(E) R_C(E) V_d. \tag{6.6}$$

Here, as in the previous section [see (5.6)],

$$V_d = v_{a\pi} + v_{b\pi}. \tag{6.7}$$

In the notation of (5.7) and (5.11), the matrix elements of (6.5) and (6.6) read

$$(\vec{q}'_{\pi_0}, \nu'_\pi, \sigma'_d|\mathcal{T}_S(E)|\sigma_d, \nu_\pi, \vec{q}_{\pi_0}) = \sum_{\xi_{N\Delta}\xi'_{N\Delta}} \int d^3q\, d^3q' (\vec{q}'_{\pi_0}, \nu'_\pi, \sigma'_d|w|\xi_{N\Delta}, \vec{q})(\vec{q}, \xi_{N\Delta}|R_C(E)|\xi'_{N\Delta}, \vec{q}') \times (\vec{q}', \xi'_{N\Delta}|w|\sigma_d, \nu_\pi, \vec{q}_{\pi_0}), \tag{6.8}$$

$$(\vec{q}'_{\pi_0}, \nu'_\pi, \sigma'_d|\mathcal{T}_A(E)|\sigma_d, \nu_\pi, \vec{q}_{\pi_0}) = \sum_{\xi_{N\Delta}\xi'_{N\Delta}} \int d^3q\, d^3q' (\vec{q}'_{\pi_0}, \nu'_\pi, \sigma'_d|\tilde{w}|\xi_{N\Delta}, \vec{q}) \times \tilde{R}^0_{N\Delta}(q; E)(\vec{q}\xi_{N\Delta}|T_0(E)|\xi'_{N\Delta}, \vec{q}')\tilde{R}^0_{N\Delta}(q'; E)(\vec{q}', \xi'_{N\Delta}|\tilde{w}|\sigma_d, \nu_\pi, \vec{q}_{\pi_0}). \tag{6.9}$$

A graphical representation of these equations is given in Fig. 7. Partial wave expansions are carried out in a standard fashion [see Eq. (A24)]. Using (4.18), (A20), and (4.37), we obtain the multipole amplitudes of Eq. (6.8)

$$\mathcal{T}_S^{\Gamma',\Gamma}(E) = \mathcal{T}_I^{\Gamma',\Gamma}(E) + \mathcal{T}_R^{\Gamma',\Gamma}(E), \tag{6.10}$$

where the symbol Γ has the same meaning as in Sec. V, and

$$\mathcal{T}_I^{\Gamma',\Gamma}(E) = \sum_\gamma \int_0^\infty dq\, q^2 w^{\Gamma',\gamma}(q_{\pi_0}, q)\tilde{R}^0_{N\Delta}(q; E)w^{\gamma,\Gamma}(q, q_{\pi_0}), \tag{6.11}$$

$$T_R^{\Gamma',\Gamma}(E) = \sum_{\gamma\gamma'} \int_0^\infty dq'\, q'^2 \int_0^\infty dq\, q^2 w^{\Gamma',\gamma'}(q_{\pi_0}, q')\tilde{R}^0_{N\Delta}(q'; E)T_C^{\gamma',\gamma}(q', q; E)\tilde{R}^0_{N\Delta}(q; E)w^{\gamma,\Gamma}(q, q_{\pi_0}). \tag{6.12}$$

The multipole amplitudes of Eq. (6.9) are

$$\mathcal{T}_A^{\Gamma',\Gamma}(E) = \sum_{\gamma\gamma'} \sum_{ij} [X_j^{\Gamma',\gamma'}(E) + Y_j^{\Gamma',\gamma'}(E)][K^{-1}(E)]^{\gamma'2,\gamma^2}_{j,i}[X_i^{\gamma,\Gamma}(E) + Y_i^{\gamma,\Gamma}(E)]. \tag{6.13}$$

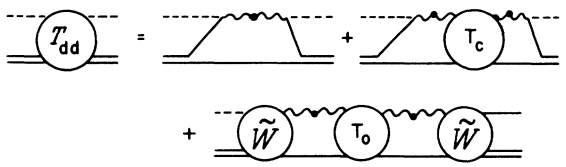

FIG. 7. Pictorial representation of the π-d elastic scattering amplitude. The symbols have the same meaning as in Figs. 2 and 4. This figure combines Eqs. (6.4), (6.8), and (6.9).

All quantities on the right-hand side of the above equations have been defined in Sec. V.

Clearly, if V_0 has no $N\Delta \to N\Delta$ matrix element (as was assumed in Secs IV and V), only those angular momentum channels which are coupled to the NN space contribute to (6.13) and $\mathcal{T}_A$ represents the contribution to the amplitude due to true pion absorption.

The differential cross section and polarizations are given by the standard formulas

$$\frac{d\sigma}{d\Omega} = \tfrac{1}{3}(4\pi^2\mu_{\pi d})^2 \,\mathrm{Tr}\left[T_{dd}(\theta)\,T_{dd}^{+}(\theta)\right], \tag{6.14}$$

$$t_{kq} = \frac{\mathrm{Tr}\left[T_{dd}(\theta)T_{dd}^{+}(\theta)\tau_{kq}\right]}{\mathrm{Tr}\left[T_{dd}(\theta)T_{dd}^{+}(\theta)\right]}\,, \tag{6.15}$$

where the Madison convention[29] is used and $T_{dd}(\theta)$ is defined as an operator in the deuteron spin space with matrix elements

$$(\sigma_d'|\,T_{dd}(\theta)\,|\sigma_d) = (\vec{q}_{\pi_0}', \nu_\pi', \sigma_d'|\,\mathcal{T}_S(E) + \mathcal{T}_A(E)\,|\sigma_d, \nu_\pi, \vec{q}_{\pi_0})\,, \tag{6.16}$$

where $\cos\theta = \hat{q}_{\pi_0}' \cdot \hat{q}_{\pi_0}$. The total cross section can be calculated using the optical theorem

$$\sigma_T = -\frac{4\pi^2\mu_{\pi d}}{3q_{\pi_0}}\,\mathrm{Im}\left[\sum_{\sigma_d}(\sigma_d|\,T_{dd}(\theta=0)\,|\sigma_d)\right] \tag{6.17}$$

It is straightforward to express (6.14), (6.15), and (6.17) in terms of multipole amplitudes.

B. Numerical results

Three-body calculations[30] of π-d elastic scattering have by now reached a high level of sophistication. Several detailed investigations of the sensitivity of the results to different choices of interactions, kinematics, etc., have been performed. It is not our purpose here to contribute one more such study or to produce an optimum calculation. Rather, we focus our attention on one specific question, namely that of the influence of true pion absorption on π-d elastic scattering observables. As in previous sections, we take into account only the $P_{33}\pi N$ interaction. Although nonresonant πN interactions are known to play some role in bringing the lower energy theoretical results into agreement with the data, they are not expected to affect the semiquantitative conclusions reached below.

In addition to the constants K, X, and Y, whose evaluation was described in previous sections, the amplitudes $\mathcal{T}_I$ and $\mathcal{T}_R$ defined by (6.11) and (6.12) must be calculated. Here again, most of the labor goes into calculating T_C. For this reason, the computations were performed in parallel with those of $NN \to NN$ and $\pi d \to NN$ amplitudes, using the same complex momentum grid and the same rotation angle. Interpolation in energy was again used to obtain the π-d elastic scattering observables at the energies for which data exist. The levels of accuracy achieved were comparable to those given in Secs. IV and V.

The results for π-d scattering observables presented below were obtained using the interaction V_0 of Table IV. Note that the ambiguities in the choice of relative signs of this interaction in different angular momentum channels are irrelevant here. The $N\Delta$ partial waves for which T_C was computed, as well as the NN interactions included in $NN\pi$ states, are those listed in Table III. In addition to the channels considered in previous sections, this list includes the $^3S_1$ $N\Delta$ channel which cannot couple to the NN space but does contribute to $\mathcal{T}_I$ and $\mathcal{T}_R$. On the other hand, the $^3P_0$ $N\Delta$ channel does not couple to π-d states. It was verified by sample calculations that T_C has negligible effect in higher $N\Delta$ partial waves.

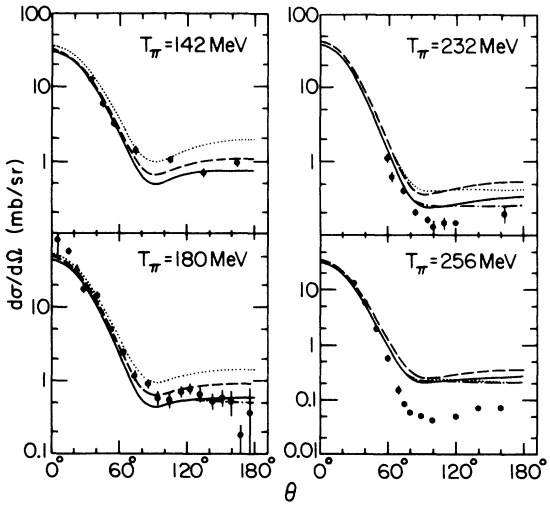

FIG. 8. Angular distributions for π-d elastic scattering. The data are from Refs. 31–34. (a) Solid curve: complete calculation (the interaction V_0 of Table IV is used); (b) Dot-dashed curve: same as (a) but the $^3F_3$ NN channel is left out [not shown when undistinguishable from (a)]; (c) Dashed curve: no true absorption, $\mathcal{T}_a = 0$; and (d) Dotted curve: single scattering only, $\mathcal{T}_R = \mathcal{T}_A = 0$.

However, many more partial waves are needed in the calculation of $\mathcal{T}_I$. Including all partial waves with total angular momentum $J < 10$ was found to give better than 1% accuracy at all energies considered.

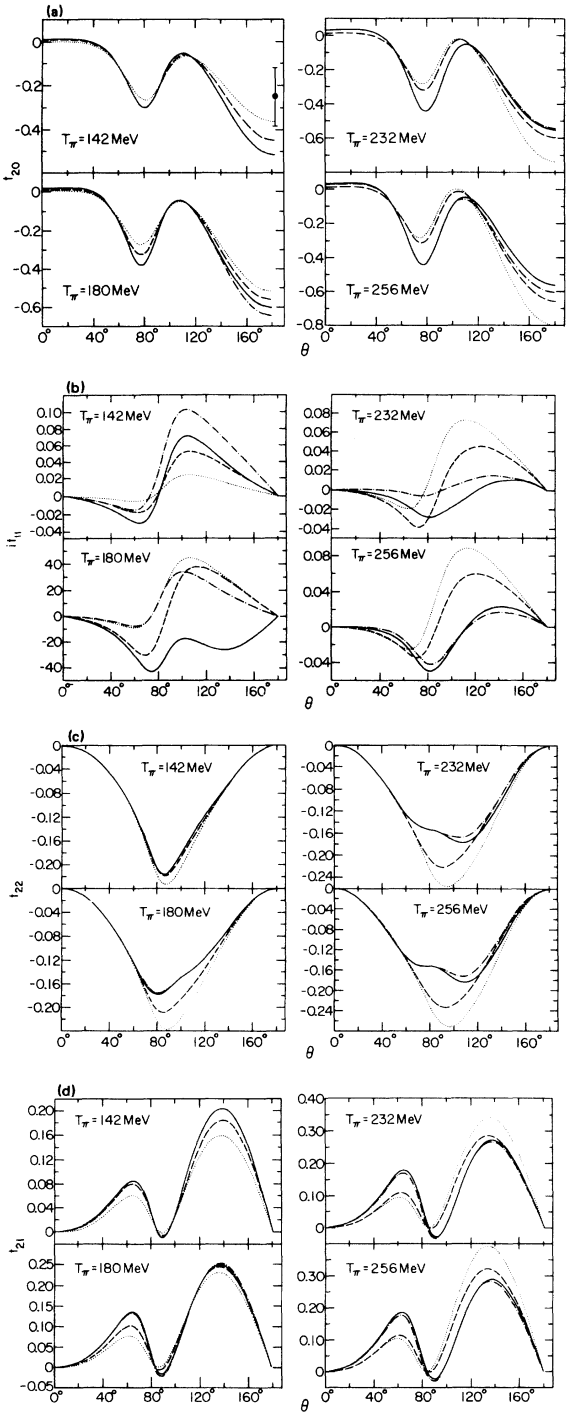

FIG. 9. Vector and tensor polarizations in π-d elastic scattering. The data are from Ref. 35. See caption of Fig. 8 for the description of each curve.

Angular distributions at four energy values are presented in Fig. 8. The results of the complete calculation are compared with those obtained when $\mathcal{T}_A$ is left out (no true absorption) and when both $\mathcal{T}_R$ and $\mathcal{T}_A$ are left out (single scattering only). The effects of $\mathcal{T}_R$ and $\mathcal{T}_A$ are rather small at forward angles, but not negligible at backward angles. At 142 and 180 MeV, both $\mathcal{T}_R$ and $\mathcal{T}_A$ contribute to lower the cross section in the backward hemisphere, thereby improving the agreement with data (note that nonresonant πN partial waves are known[4] to help fill up the minimum at 142 MeV). At 232 MeV, $\mathcal{T}_R$ plays a small role but $\mathcal{T}_A$ significantly lowers the back angle cross-section. At 256 MeV, this effect is still present but much too small to eliminate the disagreement with experiment. Figure 8 also shows the effect of leaving out the $^3F_3$ NN channel is the calculation of $\mathcal{T}_A$; the differences with the complete calculations are small.

Vector and tensor polarizations are displayed in Fig. 9. The vector polarization is considerably affected by both $\mathcal{T}_R$ and $\mathcal{T}_A$. Note, however, that it remains quite small and that nonresonant πN interactions are known to influence it considerably, so that our results are indicative only. Tensor polarizations are larger and less sensitive, although the effects of $\mathcal{T}_R$ and $\mathcal{T}_A$ are significant, especially on t_{22} near 90° and on t_{20} near 180°. Absorption does not help in bringing t_{20} (180°) at 142 MeV in agreement with the recently measured value.[35]

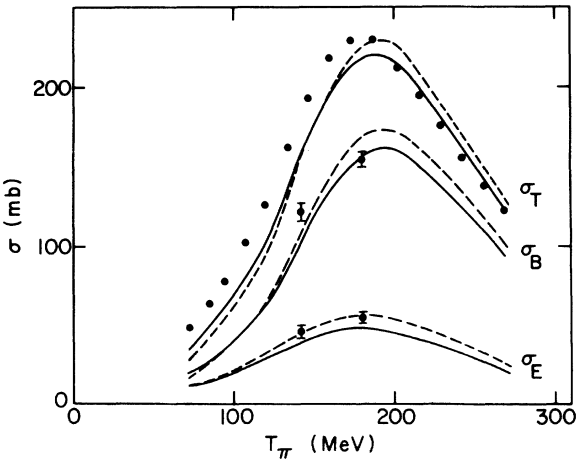

FIG. 10. Integrated breakup and elastic cross sections and total cross sections for π-d scattering. The data for σ_E are from Refs. 31 and 30. The data for σ_T are from Ref. 30. The data points for σ_B are obtained from data for σ_T, σ_E, and the absorption cross section (see Fig. 5). Solid curve: "true" absorption included. Dashed curve: no "true" absorption.

Finally, the total cross section, as well as the angle-integrated elastic and breakup cross sections are plotted against energy in Fig. 10. At energies around and above the resonance, the effect of the coupling to the NN channel is to decrease all three of these cross sections.[36] At lower energies, true absorption increases somewhat the total cross section. The total cross section is in fairly good agreement with the recent measurements, except at low energy where it is too small. Nonresonant πN partial waves are needed for an accurate description in that energy region.[4]

To conclude this section, we briefly compare our results with those published recently by Rinat *et al.*[4] These authors also take into account the coupling to the NN channel, though in a way rather different from ours. They rely on a truncated field theory picture including, in addition to the $\Delta \rightleftharpoons N\pi$ vertex, a P_{11} $N \rightleftharpoons N\pi$ vertex, as well as $\Delta \rightleftharpoons N\rho$ and $N \rightleftharpoons N\rho$ vertices. Most of their conclusions are qualitatively similar to ours, but significant quantitative differences exist. In particular, in their model, true pion absorption raises the back-angle differential cross section for energies above 200 MeV, and has very large effects on the tensor polarizations, especially at energies below the resonance. This suggests that these effects are due to absorption not proceeding through the Δ, which we have disregarded. In view of the many differences between the two calculations, it seems difficult to reach a definite conclusion on this matter. We remark that, in Ref. 4, no results are provided for NN elastic scattering and for the reaction $\pi d \to NN$. In fact, in this calculation, the NN T matrix at energies above the pion threshold (which is needed in the evaluation of the π-d elastic scattering amplitude) was generated by a Hermitian potential. This inconsistent procedure obviously endangers unitarity.[38] Clearly, more work is needed to settle quantitatively the question of the influence of true absorption on π-d elastic scattering.

VII. MANY-BODY HAMILTONIAN

The ultimate aim of the present work is the construction of a many-body Hamiltonian for use in calculations for systems with more than two nucleons. A fully relativistic generalization of our model Hamiltonian to any number of baryons is not available. However, in the approximation that all baryons are nonrelativistic, it is possible to write down a many-body Hamiltonian such that the corresponding scattering equations for $NN \to NN$, $\pi d \to NN$, and $\pi d \to \pi d$ can be approximated by those used in Secs. IV–VI. This Hamiltonian takes the form

$$H = H_0 + U\,, \tag{7.1}$$

where

$$H_0 = \sum_p E_N(p) b_p^\dagger b_p + \sum_p E_\Delta(p) \beta_p^\dagger \beta_p + \sum_k E_\pi(k) a_k^\dagger a_k\,, \tag{7.2}$$

$$\begin{aligned} U = {} & \tfrac{1}{2} \sum_{p_1' p_2' p_1 p_2} \langle p_1' p_2'; 1 | V_0 | p_1 p_2; 1\rangle b_{p_1'}^\dagger b_{p_2'}^\dagger b_{p_2} b_{p_1} + \frac{1}{\sqrt{2}} \Big(\sum_{p_1' p_2' p_1 p_2} \langle p_1' p_2'; 2 | V_0 | p_1 p_2; 1\rangle b_{p_1'}^\dagger \beta_{p_2'}^\dagger b_{p_2} b_{p_1} + \text{H.c.} \Big) \\ & + \sum_{p_1' p_2' p_1 p_2} \langle p_1' p_2'; 2 | V_0 | p_1 p_2; 2\rangle b_{p_1'}^\dagger \beta_{p_2'}^\dagger \beta_{p_2} b_{p_1} + \sum_{p_N' p_\pi' p_N p_\pi} \langle p_N' p_\pi' | V_{N\pi} | p_N p_\pi\rangle b_{p_N'}^\dagger a_{p_\pi'}^\dagger b_{p_N} a_{p_\pi} \\ & + \sum_{p_\Delta p_N p_\pi} \langle p_\Delta | V_{N\pi} | p_N p_\pi\rangle \beta_{p_\Delta}^\dagger b_{p_N} a_{p_\pi} + \text{H.c.}\,, \end{aligned} \tag{7.3}$$

where, e.g., p_N stands for $\{\vec{p}_N, \mu_N\}$; b_p, β_p, and a_k are annihilation operators for protons, deltas, and pions, respectively. The free baryon energies are given by the nonrelativistic forms. An index 1 is used to denote an NN state and an index 2 to denote an $N\Delta$ state. The last term of (7.3) is related to the $\Delta \rightleftharpoons N\pi$ vertex as follows:

$$\begin{aligned} \langle p_\Delta | V_{N\pi} | p_N p_\pi\rangle = {} & \delta(\vec{p}_\Delta - \vec{p}_N - \vec{p}_\pi) [E_\pi(k)/E_\pi(p_\pi)]^{1/2} \\ & \times \langle \mu_\Delta | \hat{v}_{N\pi} | \mu_N \mu_\pi \vec{k}\rangle\,, \end{aligned} \tag{7.4}$$

where $\vec{k}$ is defined in (4.27) and the origin of the kinematic factor is clear from (4.26) and (4.28).

Note that the interaction v_{NN} does not appear in this Hamiltonian. In order to illuminate the re-

lationship between this model and that discussed in the previous sections, as well as to clarify the origin of the interaction v_{NN}, we formulate nucleon-nucleon scattering using the Hamiltonian (7.1)–(7.3).

It is clear from its structure that this Hamiltonian generates from an NN state an infinite hierarchy of subspaces: $NN \to N\Delta \to NN\pi \to N\Delta\pi \to NN\pi\pi \to \cdots$. As before, let $\mathcal{P}$ and $\bar{\mathcal{P}}$ be projection operators onto $\mathcal{H}(NN)\oplus\mathcal{H}(N\Delta)$ and $\mathcal{H}(NN\pi)$, respectively. Denote by Q the projection operator onto the direct sum of all other subspaces ($Q = 1 - \mathcal{P} - \bar{\mathcal{P}}$). Starting from the Lippmann-Schwinger equation

$$T = U + UR^0T\,, \tag{7.5}$$

$$R = (E + i\epsilon - H^0)^{-1}\,, \tag{7.6}$$

and using standard projection operator techniques, one derives scattering equations which differ from (4.1)–(4.8) only in that the interaction $\bar{V}$ of (4.3) is replaced by an effective interaction

$$\bar{U} = \bar{\mathcal{P}}U\bar{\mathcal{P}} + \bar{\mathcal{P}}UR_{QQ}U\bar{\mathcal{P}}\,, \tag{7.7}$$

where

$$R_{QQ} = (E - H^0 - QUQ)^{-1}Q\,. \tag{7.8}$$

The second term of (7.7) can be broken up in two parts: a part in which the pion remains a spectator throughout ("disconnected" part) and a part in which the pion interacts ("connected" part). Typical contributions to each part are depicted in Fig. 11. In order to approximate (7.7) by (4.3), it is first necessary to neglect the connected part. From the structure of U, it is easy to see that Eqs. (7.7) and (7.8) then become

$$\begin{aligned}\bar{U} &= \bar{\mathcal{P}}U\bar{\mathcal{P}} + \bar{\mathcal{P}}UQ\tilde{R}^{(\pi)}QU\bar{\mathcal{P}} \\ &= \bar{\mathcal{P}}v_{N\pi}\bar{\mathcal{P}} + \bar{\mathcal{P}}v_{NN}^{(\pi)}(E)\bar{\mathcal{P}}\,,\end{aligned} \tag{7.9}$$

where the resolvent $\tilde{R}^{(\pi)}$ is given by equations identical to (4.1)–(4.6), except for the substitution $\bar{V} \to \bar{U}$ and the presence of an additional spectator pion everywhere. The last step of (7.9)

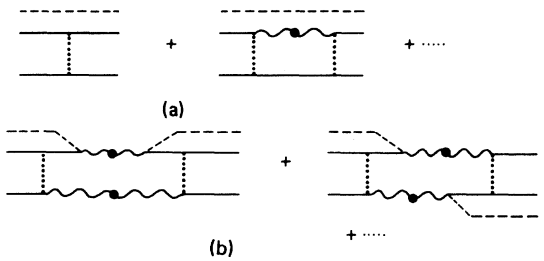

FIG. 11. Graphical representation of some contributions to the disconnected (a) and connected (b) parts of $\bar{U}$ [see Eq. (7.7)]. The various lines have the same meaning as in Fig. 2.

follows from the definition

$$\bar{\mathcal{P}}v_{NN}^{(\pi)}(E)\bar{\mathcal{P}} = \bar{\mathcal{P}}(V_0 + V_0Q\tilde{R}^{(\pi)}QV_0)\bar{\mathcal{P}}\,. \tag{7.10}$$

As the notation indicates, this interaction describes NN scattering in the presence of a spectator pion. This is easily seen by recasting (4.8) in the form of a Lippmann-Schwinger equation for the NN T matrix, with the effect of the coupling to $N\Delta$ states accounted for by an effective interaction. Let 1 label the NN space and 2 the $N\Delta$ space; one can rewrite Eq. (4.8) as

$$\tilde{T}_{11} = \tilde{U}_{11} + \tilde{U}_{11}R^0\tilde{T}_{11}\,, \tag{7.11}$$

with

$$\tilde{U}_{11} = (V_0)_{11} + (V_0)_{12}\tilde{R}_{22}(V_0)_{21}\,. \tag{7.12}$$

It is clear that the matrix elements of (7.10) between $NN\pi$ states differs from those of (7.12) (between NN states) only by the presence of a spectator pion.

At this stage, Eqs. (4.1), (4.2), (4.4)–(4.8), and (7.9)–(7.10) form a closed set of coupled equations such that NN scattering in the presence of a spectator π is calculated self-consistently. Although this is appealing in principle, it is hard to deal with in practice. If the parameters are determined in such a way that the effective interaction $\tilde{U}_{11}$ correctly describes NN scattering, the same is true of $v_{NN}^{(\pi)}(E)$. For the relevant values of E, the subenergy of the NN system in the presence of a pion does not exceed the pion production threshold. The interaction $v_{NN}^{(\pi)}(E)$ is therefore real and it is natural to parametrize it by a real energy independent two-body potential fit to NN phase shifts below the pion production threshold. This is the interaction v_{NN} which was introduced from the start in the construction of our $NN\pi$ model.

The approximation just described is expected to be superior to a strict space truncation, i.e., to entirely neglecting the coupling to the Q space. This is because, in those NN channels which couple to $N\Delta$ states, a large part of the effective NN interaction is due to this coupling, so that the second term of (7.10) is not a small correction to the first. We note, however, that for the most important NN interaction in $NN\pi$ states, namely $^3S_1$-$^3D_1$, $(v_0)_{11}$ and v_{NN} are identical and the space truncation is automatic. As seen in previous sections, other NN interactions in $NN\pi$ states have a small influence on observables and the use of v_{NN} to estimate their effects should be adequate.

VIII. CONCLUSIONS

A model has been presented in which the nucleon, the pion and the isobar are the basic degrees of

freedom. The parameters of the Hamiltonian are determined by fits to πN scattering phase shifts for $l \leq 1$ and energies up to 300 MeV, and fits to NN scattering phase shifts for $l \leq 4$ and energies up to 800 MeV. Since $N\Delta$ states are coupled to $NN\pi$ states the treatment of NN scattering involves the solution of three-body equations. Dynamic effects of the $NN\pi$ channel turn out to be important in $N\Delta$ s waves but not in higher partial waves. This means, in particular, that they play an important part in the calculation of $^1D_2$ NN phase shifts.

We have explored the consequences of the model so determined for elastic pion-deuteron scattering and for pion absorption by deuterons. Interactions in the $NN\pi$ channel contribute up to 40% to the absorption cross sections. The absorption channel influences significantly the elastic pion deuteron cross section.

In its present form our model is somewhat crude, but the results are encouraging. The model can and should be improved along the following lines.

(i) Comparison with experimental angular distributions and polarizations for the reaction $NN \to \pi d$ and $NN \to NN\pi$. Modifications of the model that may be required to account for these data.

(ii) Inclusion of nonresonant πN partial wave interactions as well as an $NN \to NN\pi$ interaction that accounts for low energy pion production.

(iii) Use of meson theory to constrain the form of the interactions and the values of parameters.

The goal of our approach to the $NN\pi$ system has been the construction of a well-defined Hamiltonian that accounts for the observed features of this system and can be generalized to a many-body Hamiltonian. A rigorous many-body generalization of the relativistic models of Ref. 10 is not available. However, if all baryons are nonrelativistic, a many-body Hamiltonian can be constructed such that, after well-defined approximations are made, the corresponding scattering equations for the $A = 2$ system are identical to those for our $NN\pi$ model. This Hamiltonian should be useful in theoretical studies of many medium-energy nuclear reactions of current interest, as well as in nuclear matter calculations.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix, we collect explicit formulas for the partial wave expansions mentioned in the main text. For any pair of particles (π_1, π_2), we define

$$\mathcal{Y}^{\gamma MM_T}_{\pi_1\pi_2}(\hat{q}) = \sum (l\lambda s\sigma | JM)(s_1\sigma_1 s_2\sigma_2 | s\sigma)(t_1\tau_1 t_2\tau_2 | TM_T) Y_{l\lambda}(\hat{q}) | s_1\sigma_1, t_1\tau_1; s_2\sigma_2, t_2\tau_2\rangle , \tag{A1}$$

where s_i and t_i denote the spin and isospin of particle π_i. The corresponding greek letters are used for z-axis projections. The symbol γ stands for the set of quantum numbers $\{l, s, J, T\}$. The summation in (A1) is over all magnetic quantum numbers except for M and M_T.

The partial wave expansion of the two-body πN interaction is

$$(\vec{k}', \nu'_\pi, \nu'_N | \hat{v}_{N\pi} | \nu_N, \nu_\pi, \vec{k}) = \sum_{MM_T} v^{\gamma}_{N\pi}(k', k) \langle \nu'_\pi, \nu'_N | \mathcal{Y}^{\gamma MM_T}_{N\pi}(\hat{k}') \mathcal{Y}^{\gamma MM_T}_{N\pi}(\hat{k}) | \nu_N, \nu_\pi \rangle . \tag{A2}$$

The πN T matrix is expanded in the same way. The matrix elements of the $\Delta \rightleftharpoons N\pi$ vertex may be written as

$$(\nu'_\Delta | \hat{v}_{N\pi} | \nu_N, \nu_\pi, \vec{k}) = f_\Delta(k) \mathcal{Y}^{\gamma_\Delta \sigma'_\Delta \tau'_\Delta}_{N\pi}(\hat{k}) | \nu_N, \nu_\pi \rangle , \tag{A3}$$

where $\gamma_\Delta \equiv \{1, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}\}$.

The partial wave expansions of baryon-baryon interactions (and T matrices) take the form

$$(\vec{k}', \nu'_a, \nu'_b | \hat{v}_{NN} | \nu_b, \nu_a, \vec{k}) = \sum_{\gamma'\gamma MM_T} v^{\gamma'\gamma}_{NN}(\kappa', \kappa) \langle \nu'_a, \nu'_b | \mathcal{Y}^{\gamma' MM_T}_{NN}(\hat{k}') \mathcal{Y}^{\gamma MM_T}_{NN}(\hat{k}) | \nu_b, \nu_a \rangle \tag{A4}$$

$$(\vec{q}', \mu'_1, \mu'_2; \delta' | V_0 | \delta; \mu_2, \mu_1, \vec{q}) = \sum_{\gamma\gamma' MM_T} V_0^{\gamma'\delta', \gamma\delta}(q', q) \langle \mu'_1, \mu'_2 | \mathcal{Y}^{\gamma' MM_T}_{\delta'}(\hat{q}') \mathcal{Y}^{\gamma MM_T}_{\delta}(\hat{q}) | \mu_2, \mu_1 \rangle , \tag{A5}$$

where a particle channel label δ has been introduced, such that $\delta = 1$ for an NN pair and $\delta = 2$ for an $N\Delta$ pair.

The partial wave expansion of the effective $N\Delta$ interactions (4.26) and (4.33) can be obtained by standard

methods from the above definitions, together with (4.27), (4.28), and (4.34). In order to write the results in compact form, it is useful to define a sum over angular momentum coupling coefficients as follows:

$$\mathcal{S}^{\Lambda,a',a}_{K',K} = \delta_{s_0 s_0'}(-1)^P \hat{t}\hat{t}'\hat{j}\hat{j}'\hat{s}\hat{s}'\hat{l}\hat{l}'\hat{L}\hat{L}'\hat{S}\hat{S}'\hat{\Lambda}^2\delta_{t_0 t_0'}\left[\frac{(2L'+1)!\,(2L+1)!}{(2a')!\,[2(L'-a')]!\,(2a)!\,[2(L-a)]!}\right]^{1/2}\begin{Bmatrix} t & t_1' & T \\ t' & t_1 & t_0 \end{Bmatrix}$$

$$\times\sum_{dd'}\hat{d}^2\hat{d}'^2\begin{pmatrix} L'-a' & a & d \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l & d & \Lambda \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} L-a & a' & d' \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} l' & d' & \Lambda \\ 0 & 0 & 0\end{pmatrix}$$

$$\times\sum_{e}\hat{e}^2\begin{Bmatrix} l' & s' & J \\ s & l & e\end{Bmatrix}\begin{Bmatrix} l' & e & l \\ d & \Lambda & d'\end{Bmatrix}\begin{Bmatrix} a & L-a & L \\ L'-a' & a' & L' \\ d & d' & e\end{Bmatrix}\begin{Bmatrix} j' & s_1 & s' & s \\ S' & s_0 & s_1' & L \\ L' & S & e & j\end{Bmatrix}, \tag{A6}$$

with

$$P = t_1' + t_1 + t' + t + l + \Lambda + s_1' + s_1 - s_0 - 2s' - J .$$

In (A6), the symbol K stands for the set of quantum numbers $\{[L(s_1 s_0)S]j;\ (t_1 t_0)t; ls\}$ and the 12-j coefficient is that defined by Ord-Smith.[39]

Expanding (4.26) in partial waves then yields

$$V_E^{\gamma',\gamma}(q',q;E) = -\sum_{a'=0}^{1}\sum_{a=0}^{1}\sum_{\Lambda}\mathcal{S}^{\Lambda,a',a}_{K'_\gamma,K_\gamma}A^{\Lambda,a',a}(q',q;E), \tag{A7}$$

where $K_\gamma \equiv \{[1(\frac{1}{2}\,0)\frac{1}{2}]\frac{3}{2}\,;(\frac{1}{2}\,1)\frac{3}{2}\,;ls\}$ and

$$A^{\Lambda,a',a}(q',q;E) = q'^{\,1-a+a'}q^{1-a'+a}\tfrac{1}{2}\int_{-1}^{+1}dx\,P_\Lambda(x)\beta(q_\pi)^{a+a'}\phi(k',q_\pi)R^0(q',q,q_\pi;E)\phi(k,q_\pi), \tag{A8}$$

where

$$x = \hat{q}\cdot\hat{q}', \tag{A9}$$

$$\phi(k,q_\pi) = \left[\frac{E_\pi(k)}{E_\pi(q_\pi)}\right]^{1/2}\frac{f_\Delta(k)}{k}, \tag{A10}$$

$$\beta(q_\pi) = \frac{m_N}{M_{N\pi}(q_\pi)}. \tag{A11}$$

In order to compute the effective $N\Delta$ interaction (4.33), it is first necessary to solve the scattering equation for $\hat{T}_{NN}(\epsilon_\pi)$. Using (3.8), this can be done straightforwardly, with the result

$$T^{\bar{\gamma}',\bar{\gamma}}_{NN}(\kappa',\kappa;\epsilon_\pi) = \sum_{ij} g_i^{\bar{\gamma}'}(\kappa')[\Gamma^{-1}(\epsilon_\pi)]^{\bar{\gamma}',\bar{\gamma}}_{i,j}g_j^{\bar{\gamma}}(\kappa), \tag{A12}$$

where

$$\Gamma(\epsilon_\pi)^{\bar{\gamma}',\bar{\gamma}}_{i,j} = (G^{-1})^{\bar{\gamma}',\bar{\gamma}}_{i}\delta_{ij} - \int_0^\infty d\kappa\,\kappa^2 g_i^{\bar{\gamma}'}(\kappa)\bar{R}^0(\kappa,\epsilon_\pi)g_j^{\bar{\gamma}}(\kappa). \tag{A13}$$

Let $K_\gamma \equiv \{[1(0\,\frac{1}{2})\frac{1}{2}]\frac{3}{2}\,;(1\,\frac{1}{2})\frac{3}{2}\,;ls\}$ and $\bar{K}_\gamma \equiv \{[\bar{l}(\frac{1}{2}\frac{1}{2})\bar{s}]\bar{J}\,;(\frac{1}{2}\frac{1}{2})\bar{T}\,;f\bar{J}\}$; define

$$C^{\gamma,\bar{\gamma}}_{f,j}(q,q_\pi) = \sum_{a=0}^{1}\sum_{\bar{a}=0}^{\bar{l}}\sum_{\Lambda}\mathcal{S}^{\Lambda,a,\bar{a}}_{K_\gamma\bar{K}_\gamma}B^{\Lambda,a,\bar{a}}_{\bar{\gamma},j}(q,q_\pi), \tag{A14}$$

where

$$B^{\Lambda,a,\bar{a}}_{\bar{\gamma},j}(q,q_\pi) = q^{\bar{l}-\bar{a}+a}q_\pi^{1-a+\bar{a}}\tfrac{1}{2}\int_{-1}^{+1}dx\,P_\Lambda(x)\left[\frac{E_\pi(q_\pi)}{M_{N\pi}(q_\pi)}\right]^a\frac{1}{2^a}\phi(k,q_\pi)\bar{R}^0(\kappa,\epsilon_\pi)\frac{g_j^{\bar{\gamma}}(\kappa)}{\kappa^{\bar{l}}}, \tag{A15}$$

$$x = \hat{q}\cdot\hat{q}_\pi. \tag{A16}$$

The multipoles of (4.33) are given by

$$V_B^{\gamma',\gamma}(q',q;E) = \sum_{\bar{\gamma}'\bar{\gamma}}[1-(-1)^{\bar{l}+\bar{s}+\bar{T}}]\sum_f\sum_{i,j}\int_0^\infty dq_\pi q_\pi^2 C^{\gamma',\bar{\gamma}'}_{f,i}(q',q_\pi)[\Gamma^{-1}(\epsilon_\pi)]^{\bar{\gamma}',\bar{\gamma}}_{i,j}C^{\gamma,\bar{\gamma}}_{f,j}(q,q_\pi). \tag{A17}$$

The deuteron wave function may be written in the form

$$\Phi(\vec{k}_d, \sigma_d) = \sum_{l_d=0,2} \phi_{l_d}(k_d) \mathcal{Y}_{NN}^{\gamma_d c_d}(\hat{k}_d) , \tag{A18}$$

where $\gamma_d \equiv \{l_d, 1, 1, 0\}$ and the normalization is

$$\sum_{l_d=0,2} \int_0^\infty dk\, k^2 |\phi_{l_d}(k)|^2 = 1 . \tag{A19}$$

The operator $\mathcal{W}$ defined in (5.7) may be expanded in partial waves as follows. Let $\Gamma \equiv \{L, 1, J, 1\}$ and $K_\Gamma \equiv \{[l_d(\frac{1}{2}\frac{1}{2})1]1(\frac{1}{2}\frac{1}{2})0; L\,1\}$. Then,

$$(\vec{q}, \mu_N, \mu_\Delta | \mathcal{W} | \sigma_d \nu_\pi \vec{q}_\pi) = \sum_{\substack{\gamma\Gamma \\ MM_T}} \mathcal{W}^{\gamma,\Gamma}(q, q_\pi)(\mu_N, \mu_\Delta | \mathcal{Y}_{N\Delta}^{\gamma MM_T}(\hat{q}) \mathcal{Y}_{\pi d}^{\Gamma MM_T}(\hat{q}_\pi^+) | \sigma_d, \nu_\pi) , \tag{A20}$$

with

$$\mathcal{W}^{\gamma,\Gamma}(q, q_\pi) = \sqrt{2} \sum_{l_d=0,2} \sum_{a'=0}^{1} \sum_{a=0}^{l_d} \sum_{\Lambda} \$_{K_\gamma, K_\Gamma}^{\Lambda, a', a} D_{l_d}^{\Lambda, a', a}(q, q_\pi) , \tag{A21}$$

$$D_{l_d}^{\Lambda, a', a}(q, q_\pi) = (\tfrac{1}{2})^a \left[\frac{E_\pi(q_\pi)}{M_{N\pi}(q_\pi)}\right]^{a'} q^{l_d - a + a'} q_\pi^{1-a'+a} \tfrac{1}{2} \int_{-1}^{+1} dx\, P_\Lambda(x) \phi(k, q_\pi) \frac{\phi_{l_d}(k_d)}{k_d^{l_d}} , \tag{A22}$$

where k and k_d are given in terms of q, q_π, and x by (5.8), (5.9), and (A16).

The amplitude for $\pi + d \to N + N$ is written as

$$(\vec{q}_{N_0}, \mu_a, \mu_b | \mathcal{T}_d(E) | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}) = \sum_{\substack{\gamma\Gamma \\ MM_T}} \mathcal{T}_d^{\gamma,\Gamma}(E)(\mu_a, \mu_b | \mathcal{Y}_{NN}^{\gamma MM_T}(\hat{q}_{N_0}) \mathcal{Y}_{\pi d}^{\Gamma MM_T}(\hat{q}_{\pi_0}^+) | \sigma_d, \nu_\pi) . \tag{A23}$$

The multipoles $\mathcal{T}_d^{\gamma,\Gamma}$ are given explicitly by Eqs. (5.11)–(5.13). Note that the NN states used in the definitions of the partial wave expansions (A4), (A5), and (A23) are not antisymmetrized. Properly antisymmetrized amplitudes are obtained trivially by taking appropriate linear combinations.

The multiple expansion of the πd elastic scattering amplitude is

$$(\vec{q}'_{\pi_0}, \nu'_\pi, \sigma'_d | \mathcal{T}_{dd}(E) | \sigma_d, \nu_\pi, \vec{q}_{\pi_0}) = \sum_{\substack{\Gamma'\Gamma \\ MM_T}} \mathcal{T}_{dd}^{\Gamma',\Gamma}(E)(\nu'_\pi, \sigma'_d | \mathcal{Y}_{\pi d}^{\Gamma' MM_T}(\hat{q}'_{\pi_0}) \mathcal{Y}_{\pi d}^{\Gamma MM_T}(\hat{q}_{\pi_0}) | \sigma_d, \nu_\pi) . \tag{A24}$$

Explicit formulas for the amplitudes $\mathcal{T}_{dd}^{\Gamma',\Gamma}$ are given in the main text [see Eqs. (6.8)–(6.13)].

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