PHYSICAL REVIEW C VOLUME 23, NUMBER 6 JUNE 1981

Rotational and vibrational excitations in nuclear molecular spectra

K. A. Erb and D. A. Bromley
A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut
(Received 17 July 1980)

All the available data concerning barrier region resonances in the $^{12}C+{}^{12}C$ system have been examined within the framework of the molecular symmetry model suggested by Iachello. The model accounts very well for the energies and spins of the observed resonances in terms of vibrational and rotational excitations of a well-defined diatomic molecule.

[NUCLEAR REACTIONS $^{12}C+{}^{12}C$ barrier resonances $E_{c.m.}=3.1-13$ MeV; application of a molecular symmetry model.]

As ever more experimental data have become available concerning resonances in heavy-ion interactions, calculations and models[1–4] advanced to explain these phenomena have become correspondingly complex and early hopes for a very simple molecular explanation have become clouded and much less obviously realizable. At the same time, these resonances, for a long time considered isolated phenomena in only a very few heavy-ion systems, have been demonstrated to be a very general feature of heavy-ion interactions[5] and their understanding becomes correspondingly more important.

In this Communication we consider a particular subset of the resonance data—comprising some 38 narrow resonances in the Coulomb barrier region of the $^{12}C+{}^{12}C$ interaction—and demonstrate the existence of a remarkable regularity in the associated resonance spectrum. The pattern is characteristic of diatomic molecular phenomena. We concentrate on this system and energy region because it is by far the most thoroughly studied to date and thus the corresponding data are most nearly complete.

The narrow resonances appear prominently in the total reaction cross section, and this implies correspondingly enhanced $^{12}C+{}^{12}C$ partial widths[6,7] and suggests a two-body molecular interpretation. It is natural, then, to inquire whether the spectrum of resonances conforms to the pattern long familiar from studies of diatomic molecular phenomena.[8] For the latter, the energies of small and moderate vibrational and rotation excitations characterized by vibrational and angular momentum quantum numbers v and L, respectively, are well reproduced by the first few terms in the empirical expansion

$$E(v,L)=\sum_{m,n}A_{mn}(v+\tfrac{1}{2})^m L^n(L+1)^n \ . \qquad (1)$$

A similar expression emerges from a perturbation treatment of the anharmonic oscillator whenever the bonding potential, $V(r)$, can be represented adequately in the region of its minimum, at $r=r_0$, by means of an expansion

$$V(r)+V_0+\tfrac{1}{2}A(r-r_0)^2+B(r-r_0)^3+C(r-r_0)^4 \ . \qquad (2)$$

In the atomic situation the parameters of Eq. (2), and thus of Eq. (1), can be calculated in principle from a knowledge of the relevant interactions, and for simple molecules the results correspond approximately to an expansion of a Morse potential.

The nuclear case is considerably more complicated, partly because the molecular configurations are not stable, and partly because the primary binding mechanisms have not yet been unambiguously identified. Iachello has recently shown, however, that the energy level expression,

$$E(v,L)=-D+a(v+\tfrac{1}{2})-b(v+\tfrac{1}{2})^2+cL(L+1) \ , \qquad (3)$$

is appropriate for *any* molecular Hamiltonian having $U(4)\supset O(4)\supset O(3)$ symmetry, and has suggested that this dynamical symmetry may be a general feature of two-body molecular interactions.[9] It bears emphasis that Iachello's approach is based upon use of a spectrum generating algebra (SGA) focusing on the elementary quanta involved in the interaction and *not* on the interaction potential. It is our purpose in this Communication to reexamine the available data in the light of Iachello's suggestion.

In Table I we have collected *all* of the resonance candidates thus far identified in the $^{12}C+{}^{12}C$ system[6,7,10–17] which are correlated among several exit channels. We have found, and list in Table II, several sets of parameters, each of which, in conjunction with Eq. (3), provides an equivalently good overall reproduction of this resonance spectrum. The loci calculated with parameter set I are shown as the solid curves in Fig. 1. With only four parameters, we reproduce the energies of the 28 correlated resonances whose spins have been determined, with an

TABLE I. Resonances observed in the $^{12}C+^{12}C$ interaction below $E_{c.m.}=13$ MeV (Refs. 6, 7, and 10–17). The energies in parentheses denote resonances for which spins have not been measured; these are grouped arbitrarily according to the systematics discussed in the text.

	Correlated resonances			
$J=0$	$J=2$	$J=4$	$J=6$	$J=8$
(3.17)	(3.75)	(4.46)	(6.49)	9.65
(3.35)	4.62	5.96	7.55	9.84
4.25	4.88	6.85	8.86	10.30
5.80	5.00	(7.30)	9.05	10.63
	(5.37)	(7.45)	(9.33)	10.90
	5.64	7.71	9.98	11.38
	6.25	7.90	10.45	11.90
	6.63	8.26		12.36
	(7.05)	8.45		12.98

average rms deviation of 44 keV (Fig. 1). The remaining 10 resonances, of unknown spin, can also be accommodated comfortably within this scheme by arbitrarily assuming appropriate spin assignments, and these are indicated by means of the open circles in Fig. 1. (Only even values of spin and parity are allowed for this identical boson system.) Obviously, considerable interest attaches to the experimental determination of these missing assignments.

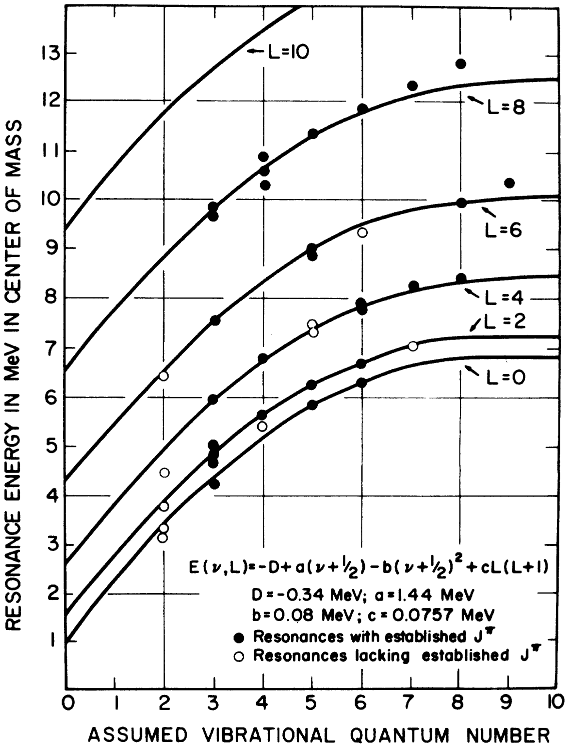

FIG. 1. Regularities in the $^{12}C+^{12}C$ Coulomb barrier resonance spectrum. The solid curves were calculated using an expression [Eq. (2), text] approximate for a diatomic molecule, with the parameters shown. The plotted points represent resonances (Refs. 6, 7, and 10–17 and Table I) for which spins have either been measured (filled circles) or assumed (open circles). The vibrational quantum number assignments are discussed in the text.

We have no means of deducing the absolute vibrational quantum numbers, v, of the observed resonances from the available data, but consideration of Eq. (1) shows that, for states of a given L, an arbitrary reassignment of v can be precisely compensated by corresponding changes in the parameters D and a, so as to regain the original spectrum. (The only qualification is that a decrease of v by one unit removes the lowest state of each vibrational band from the spectrum. Thus, we can conclude, for example, that $v\geqslant 2$ for the 6.28 MeV, 2^+ resonance.) Once the arbitrary assignment of v to a given state has been made, inspection of the differences between pairs of observed resonanced energies serves to determine the relative v assignments of the remaining states.

TABLE II. Three parameter sets which in conjunction with Eq. (1) (text) and the vibrational quantum assignments indicated in Fig. 1 provide essentially equivalent descriptions of the $^{12}C+^{12}C$ resonance spectrum.

Set	D (MeV)	a (MeV)	b (MeV)	c (MeV)
I	−0.34	1.44	0.08	0.076
II	−0.069	1.60	0.10	0.076
III	−0.87	1.23	0.06	0.076

The deduced value of the rotational parameter, $c=0.076$ MeV, corresponds physically to an intrinsic dumbbell configuration consisting of two, touching ^{12}C nuclei. This, in turn, implies an equilibrium separation of approximately 6.75 fm for the two ^{12}C molecular constituents in this simple model, and a corresponding minimum in the bonding potential at a radius far in excess of that implied by any $^{12}C+^{12}C$ optical potential.

Additional insight concerning this long range bonding mechanism, and the origin of the shape isomer implied by the molecular description adopted here, derives from the asymptotic behavior of the curves in Fig. 1. In analogy with the situation in atomic molecular phenomena, we note that the $L=0$ curve approaches a dissociation limit—approximately 7 MeV—with increasing vibrational excitation. Examination of Eq. (3) shows that the dissociation energy, E_d, is defined *without reference to the individual* v *as-*

signments[18] and would be precisely the same for any other internally consistent set of assignments. This energy does depend somewhat on the parameter set chosen, but we have been unable to vary E_d by more than ±0.5 MeV without destroying the quality of the fit to the data shown in Fig. 1. Thus, the dissociation energy of the $^{12}C-{}^{12}C$ molecule corresponds closely to the excitation energy at which the constituent nuclei become unstable to α decay ($E_x = 7.37$ MeV).

It is interesting to note in connection with the estimated dissociation energy, and equilibrium ^{12}C molecular separation distance, that even the earliest attempts[19] to reproduce the first experimental resonance data showed that the elastic channel optical potential alone could never yield the resonance spectrum; additional degrees of freedom associated with excitations of the interacting nuclei were therefore taken into consideration. Attention almost entirely has focused on the possible role of the 2^+ excitation at 4.43 MeV in ^{12}C. Within the framework of Iachello's approach, however, the inclusion of a 2^+ intrinsic excitation would, in priniciple, generate a much richer spectrum [more complex than O(4)], and while Fig. 1 does indicate some fragmentation, the overall pattern verifies the approximate validity of Eq. (3). This and the 7 MeV asymptote raise again the question of the 0^+ excited state at 7.66 MeV in ^{12}C and its possible role in molecular phenomena. This is not a new suggestion. It has long been recognized that this state has a quasilinear, three α particle structure[20] and Feshbach has suggested,[21] within the spirit of the Michaud-Vogt model of molecular interactions,[22] that this state could well play a significant role. It has tended to be neglected in most theoretical coupled channel studies because it appears only very weakly in $^{12}C+{}^{12}C$ inelastic scattering. But Wada *et al.*[14] have shown that the cross section leading to the $^{8}Be+{}^{16}O$ exit channel, to which the structure of the 7.66 MeV state would couple effectively, is relatively large in the Coulomb barrier region.

Figure 1 demands a broad range of new measurements; we have already alluded to the need for definite spin and parity assignments for *all* the resonances shown, and much more work on the $^{8}Be+{}^{16}O$ exit channel is essential. Further study of the Coulomb barrier region in other systems—a region that has been largely bypassed in the rush to higher energies—and particularly studies in nonidentical systems such as $^{12}C+{}^{16}O$ are important since in the latter the odd parity resonances are not precluded by parity considerations. And search for γ radiation cascades within the rotational bands having a given vibrational quantum number (vertically in Fig. 1), already underway at Yale, will be significant for nonidentical systems where $E1$ transitions involving Iachello's π boson can appear.

Obviously, too, it will be of fundamental importance to understand the relationship between the SGA spectrum of Fig. 1 and dynamical models of the molecular complex, and to probe their connection with the scattering problem. Any potential yielding the SGA spectrum must have a relatively long range in order to support the closely spaced vibrational states, and a minimum at a radius (~6.75 fm) where the Coulomb barrier occurs for $l=0$ motion in the elastic channel. Thus, the bonding potential, assuming one exists, is unlikely to be a component of the elastic channel optical potential. Among the possible configurations which could couple to the elastic channel to produce the resonance spectrum, we have mentioned the 4.44 MeV 2^+ and 7.66 MeV 0^+ states in ^{12}C; the extended three α particle structure of the latter, in particular, might lead to a potential of the appropriate form.

In this paper we have focused upon the $^{12}C+{}^{12}C$ Coulomb barrier resonances as a possible example of the O(4) dynamical symmetry predicted by Iachello as a general feature of two-body molecular systems. The results are encouraging and suggest a wealth of new experiments and theoretical questions. Obviously much more work remains to be done before definitive conclusions can be drawn but we believe that the spectrum generating algebra method has already shown substantial promise as an approach to coherent understanding of a very rapidly growing body of data in heavy-ion science.

We are very much indebted to F. Iachello for extensive discussions and for providing us with the preceding communication prior to its publication; and we are equally indebted to our many students and colleagues at Yale and elsewhere, who have made the measurements represented by Fig. 1 and Table I. This work was supported in part by U.S. DOE Contract No. DE-AC02-76ERO3074.

---

[1] B. Imanishi, Phys. Lett. 27B, 267 (1968); Nucl. Phys. A125, 33 (1969).

[2] W. Scheid, W. Greiner, and R. Lemmer, Phys. Rev. Lett. 25, 176 (1970); H. J. Fink, W. Scheid, and W. Greiner, Nucl. Phys. A188, 259 (1972); J. Y. Park, W. Greiner, and W. Scheid, Phys. Rev. C 16, 2276 (1977).

[3] T. Matsuse, Y. Kondo, and Y. Abe, Prog. Theor. Phys. 59, 1009 (1978); Y. Abe, Y. Kondo, and T. Matsuse, *ibid.* 59, 1393 (1978).

[4] N. Cindro, J. Phys. G 4, L23 (1978).

[5] Recent reviews may be found in the following: D. A. Bromley, in *Nuclear Molecular Phenomena,* edited by N.

Cindro (North-Holland, Amsterdam, 1978); R. G. Stokstad, S. Afr. J. Phys. 1, 191 (1978); and K. A. Erb, in *Future Directions in Studies of Nuclei Far From Stability,* edited by J. H. Hamilton *et al.* (North-Holland, Amsterdam, 1980), p. 311.

[6] K.-U. Kettner *et al.*, Phys. Rev. Lett. 38, 337 (1977).

[7] K. A. Erb *et al.*, Phys. Rev. C 22, 507 (1980).

[8] G. Herzberg, *Molecular Spectra and Molecular Structure,* 2nd ed. (Van Nostrand, Princeton, 1950).

[9] F. Iachello, Phys. Rev. C 23, 2778 (1981) (preceding paper).

[10] E. Almqvist, D. A. Bromley, J. A. Kuehner, and B. Whalen, Phys. Rev. 130, 1140 (1963).

[11] K. A. Erb *et al.*, Phys. Rev. Lett. 37, 670 (1976); S. K. Korotky *et al.*, Phys. Rev. C 20, 1014 (1979).

[12] Z. Basrak *et al.*, Phys. Lett. 65B, 119 (1976); J. Phys. Lett. 37, L131 (1976); H. Cocu *et al.*, J. Phys. (Paris) Lett. 38, L421 (1977).

[13] H. Voit *et al.*, Phys. Lett. 67B, 399 (1977); W. Galster *et al.*, Phys. Rev. C 15, 950 (1977).

[14] R. Wada, J. Schimizu, and K. Takimoto, Phys. Rev. Lett. 38, 1341 (1977).

[15] D. R. James and N. R. Fletcher, Phys. Rev. C 17, 2248 (1978).

[16] A. M. Sandorfi *et al.*, Phys. Rev. Lett. 40, 1248 (1978); A. M. Sandorfi and A. M. Nathan, *ibid.* 40, 1252 (1978); A. M. Sandorfi (priviate communication).

[17] G. Andritsopoulos *et al.*, Phys. Rev. C 21, 1648 (1980).

[18] Let E_1, E_2, E_3 be the energies of any three consecutive vibrational states ($E_1 < E_2 < E_3$). Then $2b = (E_2 - E_1) - (E_3 - E_2)$ and $E_d = (E_2 - E_1 - b)2/4b + E_2$, independent of v.

[19] D. A. Bromley, J. A. Kuehner, and E. Almqvist, Phys. Rev. Lett. 4, 365 (1960); E. W. Vogt and H. McManus, *ibid.* 4, 521 (1960).

[20] H. Morinaga, Phys. Lett. 21, 78 (1966); H. Horiuchi, K. Ikeda, and Y. Suzuki, Prog. Theor. Phys. 52, 89 (1972).

[21] H. Feshbach, J. Phys. (Paris) Colloq. 37, C5-177 (1976).

[22] G. Michaud and E. W. Vogt, Phys. Lett. 30B, 85 (1969); Phys. Rev. C 5, 350 (1972).