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Method for determining neutron-neutron scattering parameters

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A new and model independent method is proposed as a means of extracting the neutron-neutron scattering parameters. This is done by exploiting the analyticity of reaction cross sections in the internal kinetic energy of a two particle subsystem in the final state.

[NUCLEAR REACTIONS Neutron-neutron final state interaction peak. Analyticity structure of cross sections. Empirical continuation.]

It is extremely difficult to study neutron-neutron scattering directly. All our knowledge on the scattering parameters comes from analyzing the final state interaction peaks of two neutrons in more complicated processes. One can extract the scattering parameters either by describing the shape using the Watson-Migdal formula or, in the case of a three nucleon system, by solving the Faddeev equations. (See Refs. 1 and 2, for instance.) In order to get reliable results it is necessary to have an adequate description of the whole complicated process in the studied region. Recent developments both in the techniques and in the understanding of the empirical continuation (or, using other terminology, analytic extrapolation) methods[3-6] make it possible to overcome this difficulty and to extract singularity characteristics and parameters connected with them by fitting the background terms phenomenologically rather than by attempting to describe them by some theoretical means.

Let us consider a reaction with at least two neutrons in the final state. One of the variables on which the cross section depends is the internal kinetic energy E_{nn} of the n-n subsystem. In the spirit of the Faddeev approach, the reaction amplitude contains the term given in Fig. 1. We assumed there a three particle final state, but it is arbitrary. Independent of the fact that in the initial state of their scattering the two neutrons are off-shell, the scattering amplitude t has a virtual bound state pole in the relative kinetic energy E_{nn} (or wave number k_{nn}, as they are on-shell) of the two final neutrons at $(E_{nn})^{1/2} = k_{nn} = i\cdot\mu_0$, $\mu_0 < 0$, $-\kappa_0^2 = \epsilon_{nn} \approx -140$ keV. Thus the reaction amplitude $T = T(\ldots, k_{nn})$ also has a pole there. In the physical region the reaction cross section σ is a bilinear function of the amplitudes and their complex conjugates $\sigma = \sigma(T, T^*)$. An analytic continuation of this quantity outside the physical region is possible[6] if one replaces $T^*(\ldots, k_{nn})$ by $T^*(\ldots, k_{nn}^*)$. The cross section defined in this way is an analytic function of k_{nn} and has two poles at $k_{nn} = \pm i\cdot\kappa_0$. Consequently, the function $(k_{nn}^2 - \epsilon_{nn})\sigma(k_{nn})$ is regular there. The position of the virtual bound state pole is not affected by the fact that the two neutrons are generated in the reaction itself, but the residue in it depends on the details. As a first step we propose to apply the methods of empirical continuation[5] to extract the *position* of the pole by analyzing directly measurable cross sections. Note that for the derivation of the Watson-Migdal formula very similar considerations are used,[1] but we did not assume that the formation amplitude of the n-n system is constant, nor did we apply the effective range formula to the description of n-n scattering. The singularity properties are exact and independent of such assumptions.

As for the practical possibilities, the $t(d, {}^3\mathrm{He})2n$ reaction seems to be suitable for our purpose. At $E_d = 10.9$ MeV it was studied by Larson *et al.*[1] in detail. At forward angles they found a considerable contribution from the Watson-Migdal mechanism. This means that the pole contribution, which is contained by the Watson-Migdal formula, is strong there. It is sufficient to measure the energy spectrum of the ^{3}He particle because the internal kinetic energy of the n-n subsystem E_{nn} can be calculated with the aid of kinematics. As for the statistical errors of the measured data, the high intensity bombarding beam, the detection of only one charged particle, and the large—about 150 mb/sr MeV—cross section all provide a considerable advantage. We think that they compensate for the difficulties connected with a tritium target. The data measured by Larson *et al.*,[1] however, cannot be used because of the insufficient

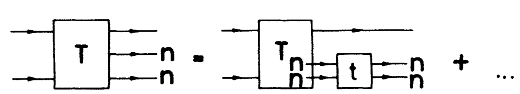

FIG. 1. That contribution to the total amplitude which contains the pole.

energy resolution (≈ 60 keV) and statistics. Because of this they had to fit the parameters of the Watson-Migdal formula not in that region where the virtual bound state pole is dominant, but in a much wider region where one has a diminishing pole contribution. Therefore it is necessary to remeasure the $t(d, {}^3\text{He})2n$ reaction cross section.

To study the potentialities of empirical continuation methods, we applied methods similar to those used by Cutkosky and Deo[7] and by Borbély[8] for determining the residue in a pole of fixed position. The more sophisticated schemes reviewed by Ciulli *et al.*,[9] for instance, need additional information on the continued function outside the physical region, but such information is unavailable. We analyzed a model spectrum provided by the Watson-Migdal formula[1] with $a = -16.0$ fm for the scattering length and $r_0 = 2.7$ fm for the effective range. In this case the cross section is a function of E_{nn} rather than one of $k_{nn} = (E_{nn})^{1/2}$ as is a real cross section.[10] If data for $(E_{nn} - \epsilon_{nn})$ $\sigma(E_{nn})$ with a guessed position of the pole ϵ_{nn} are fitted by polynomials, then at the correct position the value of the higher index coefficients (which are dominated by the pole contribution since the pole is the nearest singularity[11]) is drastically decreased. Those coefficients which became smaller than their errors are practically defined by the pole; therefore, their zeros are suitable for determining its position.[8] The minimum in the number of significant coefficients provides another possibility: With a fixed number of fitted coefficients the value of χ^2 also shows a minimum.[7] These methods are not equivalent in the sense that their results can differ in value, but the possible difference should be in agreement with the statistical errors.[5] In the first case the error of the result can be calculated directly from the statistical error of the coefficients, whereas in the second case it is given by the ϵ_{nn} values where $\chi^2 = \chi^2_{min} + 1$.[7] The crucial point of these methods is the correct determination of whether a given coefficient is dominated by the pole. This problem directly concerns the magnitude of the possible systematic error. For a detailed discussion of it and for methods of checking the results, see Ref. 5.

To imitate the conditions of a real experiment we not only formally attributed an error provided by the corresponding number of counts to a given calculated data value, but we virtually modified it in a random manner. For the peak in the spectrum we assumed a height of 1.1×10^4 counts/100 keV. From $E_{nn} = 10$ keV we used 14 points with steps of 20 keV, whereas from 270 to 1990 keV we used 43 points with steps of 40 keV. For the fit we chose a special polynomial system which gives uncorrelated coefficients A_n with an rms

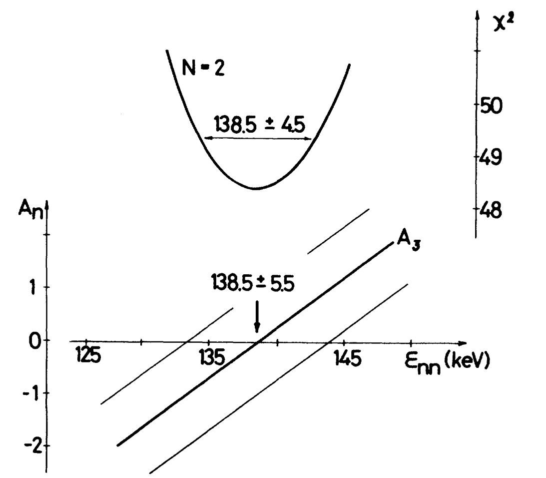

FIG. 2. Dependence of the fitted coefficient A_3 and the value of χ^2 with $N = 2$ fitted parameters on the guessed position of the pole ϵ_{nn}.

error $\Delta A_n = \pm 1$. (For details see Ref. 5.) It was found that at the correct position of the pole ($\epsilon_{nn} = -139.25$ keV) the third and higher index coefficients are insignificant; therefore, they are dominated by the pole contribution. This was checked by analyzing data which were not modified by their errors. From the zeros of the A_n coefficients the results are $\epsilon_{nn} = 138.5 \pm 5.5$, 133 ± 9, and 153 ± 16 keV with $n = 3$, 4, and 5; whereas from the minima of the χ^2 values they are $\epsilon_{nn} = 138.5 \pm 4.5$, 138 ± 7, and 149 ± 13 keV with $N = 2$, 3, and 4 fitted coefficients, respectively. (See also, Fig. 2.) The results provided by the zeros of the coefficients are statistically independent, whereas a result provided by the minimum of χ^2 with a given N is statistically equivalent to the weighted average of the results from the zeros with $n \geq N + 1$.[5] For illustration, in Fig. 2 we give the dependence of A_3 and χ^2 with $N = 2$ fitted parameters. The expectation value of $\chi_{min}{}^2$ is 55 ± 10.5, which is in agreement with the observed value of 48.4; the small difference is explained by the random character of data modification.

In view of the above, we feel justified in concluding that the methods proposed here are suitable for determining the position of the pole. As $\Delta a = \pm 1$ fm corresponds to $\Delta\epsilon_{nn} = \pm 15$ keV (and there is no practical sensitivity to the value of r_0), in this way one can accurately determine the value of the scattering length too. But one should be aware that the real cross section contains a different background contribution, which could alter the conditions of the analysis; in particular, the number of terms necessary to describe the background. Therefore our analysis, based as it is on model data, illustrates the feasibility of the method.

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[10] Strictly speaking, the cross section contains a phase space factor (see also Ref. 1) proportional to k_{nn}, but in this paper it is considered to be automatically removed.
[11] The Watson-Migdal formula gives another singularity, an effective pole at -26 MeV with the given parameters. Roughly speaking it corresponds to the two pion exchange cut. For the singularity structure of the nucleon-nucleon elastic scattering amplitudes see J. Hamilton, Phys. Rev. 114, 1170 (1959).