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Relativistic description of nuclear bound states

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The Tamm-Dancoff method is used to give a relativistic description of nuclear bound states; in particular we discuss exchange effects and deformation of the nucleon meson clouds stressing the importance of the self-energy effects. Application to hadronic atoms is also discussed.

[NUCLEAR STRUCTURE Relativistic treatment of nucleon interaction, Tamm-Dancoff, self-energy corrections to nucleon binding energy.]

I. INTRODUCTION

The fundamental problem of nuclear physics is to explain nuclei in terms of nucleons. The usual way to do this is to construct a static potential starting from an elementary particle Hamiltonian and to solve the Schrödinger equation for the state. This approach suffers from two fundamental limitations: the lack of relativistic covariance and the difficulties in describing processes in which the detailed nature of the exchanged objects is required (self-energies and exchange effects).

In order to overcome these limits a more fundamental formulation starting from a second quantized relativistic Hamiltonian would be required. The obvious drawback of such an approach is the difficulty in finding a reliable perturbation procedure valid for strong interactions. In fact one should compute, in addition to the properties of nuclei, the detailed properties of physical hadrons in terms of a few fundamental parameters. Such an objective, which was pursued in the forties and fifties,[1] turned out to be too ambitious to be attained.

The purpose of this paper is the resumption of these techniques in order to achieve much less general results, i.e., to calculate in a perturbative way the properties of nuclei starting from the *physical* properties of the free nucleons. Since nuclei are weakly bound objects these corrections should be small enough to be computed at least up to a certain degree of accuracy.

The first problem we have to face is of course the choice of the degrees of freedom relevant to the nucleon structure and the corresponding interactions; we choose to work with nucleon and light mesons (π, σ, ρ, ω, ...). The heaviest mesons and/or more fundamental constituents of the nucleons (quarks, gluons, ...) are expected to play a fundamental role in the explanation of the properties of the nucleon itself, but, due to the fact that they may travel a very short distance from the nucleons, they are expected to give a negligible contribution to the properties of the weakly bound state. In other words the average internucleon distance in nuclei fixes the mass scale of the relevant degrees of freedom in the range of the less than 1 GeV mesons, and the key assumption is that only such a peripheral meson cloud is modified by the nuclear binding.

In order to present the method, we discuss in this paper the case in which only π mesons are present besides the nucleons (in a forthcoming paper, in which numerical results will also be discussed, we will use a more complete Hamiltonian).

The structure of the paper is as follows: In Sec. II we give the explicit form of the Hamiltonian and discuss the structure of nuclear eigenstates stemming from a Tamm-Dancoff type eigenvalue equation. In Sec. III we show how to compute within this approach matrix elements of observable quantities in terms of the properties of physical nucleons. As a special example of a local current the trace of energy momentum tensor is discussed.[2] In Sec. IV this method is applied to the study of Lamb-shift-like[3] phenomena for strong interactions. In particular, the problem of a nucleon in an external long range field is discussed and the comparison with more standard treatment is given. This calculation helps us to check the range of validity of our approach. A few technical points are discussed in the appendices.

II. THE EIGENVALUE EQUATION

In order to be more specific let us consider the relativistic pion nucleon system defined by the Hamiltonian

$$H=\sum_{\alpha,a}\int d^3x\,\{\bar N^a(x)(-i\vec\gamma\cdot\vec\nabla+M)N^a(x)+\tfrac{1}{2}[\dot\phi^\alpha(x)\,\dot\phi^\alpha(x)+(\vec\nabla\phi^\alpha(x))^2+m_\pi^2\,\phi^\alpha(x)\,\phi^\alpha(x)] + ig\bar N^a(x)\gamma_5\tau^\alpha N^a(x)\,\phi^\alpha(x)\}\,, \tag{2.1}$$

where $N^a(x)$ is the isospin $\frac{1}{2}$ fermion field describing the nucleon with isospin index a ($a=1,2$) and $\phi^\alpha(x)$ is the isovector pseudoscalar pion field ($\alpha=1,2,3$). We choose to work in the Schrödinger picture in which the field operators do not evolve in time while state vectors do. Since we will also be mainly interested in the structure of energy eigenstates this time dependence will not be of concern to us.

An explicit expression of this Hamiltonian in terms of creation and annihilation operators is given in Appendix A. A graphical representation of the various interaction terms is given in Fig. 1.

The bare deuteron state $|\bar D\rangle$ can be described within this formalism as

$$|\bar D\rangle=\int\int d^3p_1d^3p_2\,\delta^{(3)}(\vec p_1+\vec p_2-\vec P)f_0(\vec p_1,\vec p_2)\times\tfrac{1}{2}(p^\dagger_{\vec p_1}n^\dagger_{\vec p_2}-n^\dagger_{\vec p_1}p^\dagger_{\vec p_2})|\bar 0\rangle\,, \tag{2.2}$$

where $|\bar 0\rangle$ is the bare vacuum state, $\vec P$ is the total center of mass momentum, $p^\dagger$ and $n^\dagger$ are the proton and neutron creation operators, $f_0(\vec p_1,\vec p_2)$ is the two body nuclear wave function, and spin indices are understood. To satisfy $\langle\vec p\,|\,\vec p\,'\rangle=\delta^3(\vec p-\vec p\,')$ we normalize f_0 as

$$\int\int d^3p_1d^3p_2|f_0(\vec p_1,\vec p_2)|^2\delta^3(\vec P-\vec p_1-\vec p_2)=1\,. \tag{2.3}$$

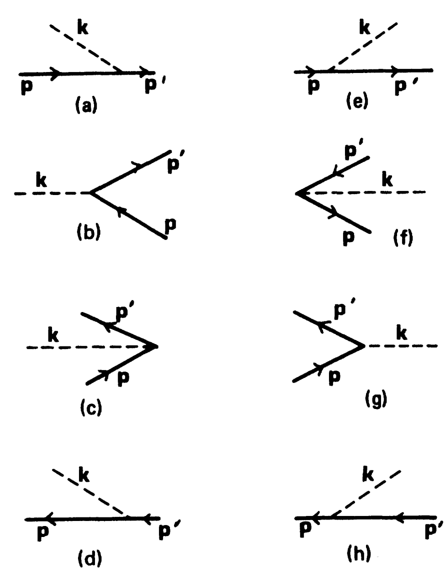


FIG. 1. Diagrammatic representation of the different terms of the interaction Hamiltonian H_I. The dotted line represents a pion, the right arrows nucleons, and the left arrows antinucleons.

Obviously the state defined in Eq. (2.2) cannot be an eigenstate of Hamiltonian (2.1) because a true eigenstate must contain an indefinite number of particles. So in this scheme the "true" deuteron state must be represented as

$$|D\rangle=\sqrt{Z_D}\,(f_0|N,N\rangle+f_1|N,N,\pi\rangle+f_2|N,N,\bar N,N,\pi\rangle+f_3|N,N,\bar N,N\rangle+f_4|N,N,\pi,\pi\rangle+\cdots)\,, \tag{2.4}$$

where f_i's are functions (to be determined) of the momenta of the particles indicated and Z_D is a normalization constant which assures the correct normalization of the states.

Although the series (2.4) does not come to an end, in order to be able to proceed we truncate it to a finite number of terms; this is usually known as the Tamm-Dancoff approximation.[4] We limit the series to the terms in Eq. (2.4) which, as will be seen later, is equivalent to expanding the wave function up to terms of the second order in the coupling constant g.

In order to calculate the f_i's and the binding energy of our system we use the eigenvalue equation

$$(H_0+H_I)|D\rangle=E_D|D\rangle \tag{2.5}$$

with $E_D=(M_D^2+\vec P^2)^{1/2}$ and project it on the states of our expansion. In this way we obtain a set of coupled homogeneous equations which allow us to express $f_{1,2,3,4}$ in terms of f_0 and E_D which are finally determined by a homogeneous integral equation. In Eq. (2.5) an additive constant in H_0+H_I is understood such that the energy of the physical vacuum is zero. This means that vacuum self-energy diagrams must be included in E_D. Such a choice guarantees, of course, the correct relativistic transformation of the energy and momentum.

Actually we get the system

$$(E_1+E_2)f_0-E_Df_0+f_1\langle N,N|H_I|N,N,\pi\rangle+f_2\langle N,N|H_I|N,N,\bar N,N,\pi\rangle=0\,, \tag{2.5a}$$

$$-\Delta_1 f_1+f_0\langle N,N,\pi|H_I|N,N\rangle+f_3\langle N,N,\pi|H_I|N,N,\overline{N},N\rangle +f_4\langle N,N,\pi|H_I|N,N,\pi,\pi\rangle\,, \quad (2.5b)$$

$$-\Delta_2 f_2+f_0\langle N,N,\overline{N},N,\pi|H_I|N,N\rangle +f_3\langle N,N,\overline{N},N,\pi|H_I|N,N,\overline{N},N\rangle +f_4\langle N,N,\overline{N},N,\pi|H_I|N,N,\pi,\pi\rangle\,, \quad (2.5c)$$

$$-\Delta_3 f_3+f_1\langle N,N,\overline{N},N|H_I|N,N,\pi\rangle +f_2\langle N,N,\overline{N},N|H_I|N,N,\overline{N},N,\pi\rangle\,, \quad (2.5d)$$

$$-\Delta_4 f_4+f_1\langle N,N,\pi,\pi|H_I|N,N,\pi\rangle +f_2\langle N,N,\pi,\pi|H_I|N,N,\overline{N},N,\pi\rangle\,, \quad (2.5e)$$

with

$$\Delta_1=E_D-E_1-E_2-\omega\,,\quad \Delta_2=E_D-E_1-E_2-E_3-E_4-\omega\,,$$
$$\Delta_3=E_D-E_1-E_2-E_3-E_4\,,\quad \Delta_4=E_D-E_1-E_2-\omega_1-\omega_2\,,$$

where the symbolic matrix elements stand for, e.g.,

$$f_1\langle N,N|H_I|N,N,\pi\rangle=\sum_{a,b,\gamma}\int\int\int\int\int d^3p_1\,d^3p_2\,d^3p_3\,d^3\kappa_1\,d^3\kappa_2\,f_1^{a,b,\gamma}(\vec{p}_1,\vec{p}_2,\vec{p}_3)\langle\overline{0}|b^c_{\vec{k}_1}b^d_{\vec{k}_2}H_I\,a^\dagger_{\vec{p}_3}b^{a\dagger}_{\vec{p}_2}b^{b\dagger}_{\vec{p}_1}|\overline{0}\rangle \quad (2.6)$$

to be evaluated by means of the expansion for H_I given in Appendix A. The solution of the system (2.5) is given up to second order in g by

$$f_1=f_0\langle N,N,\pi|H_I|N,N\rangle/\Delta_1+O(g^3)\,, \quad (2.7a)$$

$$f_2=f_0\langle N,N,\overline{N},N,\pi|H_I|N,N\rangle/\Delta_2+O(g^3)\,, \quad (2.7b)$$

$$f_3=f_0\langle N,N,\overline{N},N,\pi|H_I|N,N,\pi\rangle\langle N,N,\pi|H_I|N,N\rangle/(\Delta_1\Delta_3) +f_0\langle N,N,\overline{N},N|H_I|N,N,\overline{N},N,\pi\rangle\langle\pi,N,\overline{N},N,N|H_I|N,N\rangle/(\Delta_2\Delta_3)+O(g^3)\,, \quad (2.7c)$$

$$f_4=f_0\langle N,N,\pi,\pi|H_I|N,N,\pi\rangle\langle\pi,N,N|H_I|N,N\rangle/(\Delta_1\Delta_4) +f_0\langle N,N,\pi,\pi|H_I|N,N,\overline{N},N,\pi\rangle\langle\pi,N,\overline{N},N,N|H_I|N,N\rangle/(\Delta_2\Delta_4)+O(g^3)\,, \quad (2.7d)$$

where f_0 and E_D, as a consequence of Eq. (2.5a), satisfy the equation

$$(E_1+E_2-E_D)f_0+f_0\frac{\langle N,N,\pi|H_I|N,N\rangle}{\Delta_1}+f_0\frac{\langle N,N,\overline{N},N,\pi|H_I|N,N\rangle}{\Delta_2}=0\,. \quad (2.8)$$

Equation (2.7) gives f_i's in terms of f_0 algebraically. As an example we find

$$f_1^{1,2,\alpha}(\vec{k}_1,\vec{k}_2,\vec{k}_3)=\frac{g}{(2\pi)^{3/2}}\,\delta^3(\vec{P}-\vec{k}_1-\vec{k}_2-\vec{k}_3)\,\frac{1}{\sqrt{2\omega_{\vec{k}_3}}}\left(\frac{M}{E_{\vec{k}_1}}\right)^{1/2}\left(\frac{M}{E_{\vec{P}-\vec{k}_2}}\right)^{1/2}\frac{1}{\Delta_1} \times[\overline{n}_{\vec{k}_1}\gamma_5\tau^\alpha p_{-\vec{k}_2}f_0(\vec{P}-\vec{k}_2,\vec{k}_2)+\text{antisymmetrization}]\,. \quad (2.9)$$

Equation (2.8) symbolically denotes the integral equation (for deuteron at rest)

$$f_0(\vec{p},-\vec{p})(2E_p-M_D)+\frac{g^2}{(2\pi)^3}\int d^3k\,\frac{1}{2\omega_{\vec{k}+\vec{p}}}\,\frac{M}{E_{\vec{k}}}\,\frac{M}{E_{\vec{p}}} \times\left\{\frac{[\overline{u}_{\vec{k}}\gamma_5\tau^\alpha u_{-\vec{p}}\,\overline{u}_{-\vec{k}}\gamma_5 u_{\vec{p}}]\,f_0(\vec{k},-\vec{k})}{\Delta_1} +\left[\frac{\overline{u}_{\vec{p}}\gamma_5\tau^\alpha u_{\vec{k}}\,\overline{u}_{\vec{k}}\gamma_5\tau^\alpha u_{\vec{p}}}{\Delta_1}+\frac{u_{\vec{p}}\gamma_5\tau^\alpha v_{\vec{k}}\,\overline{v}_{\vec{k}}\gamma_5\tau^\alpha u_{\vec{p}}}{\Delta_2}\right]f_0(\vec{p},-\vec{p})\right\}=0\,. \quad (2.10)$$

The first integral in Eq. (2.10) represents the contribution of the pion exchange between the two nucleons, while the second and the third contain two self-energy terms (direct and Z diagram contribution). The approach we are following is not manifestly covariant, but since the theory is Lorentz invariant the same will be true for all the answers we will get.

In order to give an intuitive picture of these equations, we give a graphical representation in Fig. 2(a) for the state (2.4) in terms of the explicit solution (2.7a)–(2.7d) and in Fig. 2(b) the

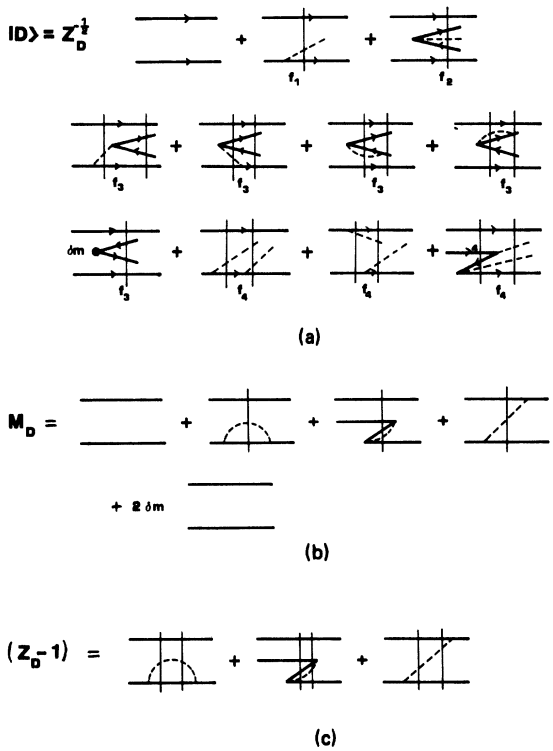


FIG. 2. (a) Structure of the deuteron state. The cut indicates where the energy pole denominators have to be computed. (b) Structure of the Hamiltonian expectation value. (c) Structure of the normalization constant.

representation of the integral equation (2.10). The analytic expression of a diagram turns out to be very similar to the corresponding expression in the old perturbation theory, except for the fact that the energy of the initial state can be put equal to E_D and then it must be averaged with the amplitude f_0. The cuts indicated show the points of the diagrams in which intermediate energy denominators must be computed.

Of course in Eq. (2.10) we have logarithmically divergent self-energy diagrams so that a subtraction procedure is required. When necessary we cut off the divergent integrals "à la Pauli Villars," namely introducing ghost pion fields coupled with an imaginary coupling constant to nucleons, whose mass λ plays the role of the cutoff. These "pion fields" give rise to subtraction terms which regularize the divergent integrals in Eq. (2.10). In order to eliminate the cutoff dependence in physical quantities we will replace the bare mass M in Eq. (2.1) by M-δm, where now M is the physical mass and δm is a quantity (cutoff dependent) which will be adjusted in such a way to eliminate the cutoff dependence. This means that a mass counterterm is added to Eq. (2.1):

$$H_{\delta m} = -\,\delta m \int d^3x\, \overline{N}(x) N(x)\,. \qquad (2.11)$$

This counterterm will also appear in the integral equation (2.10) in the form $-2\,\delta m\, M/E$ with

$$\delta m = \frac{g^2}{(2\pi)^3}\int d^3k\,\frac{M}{2\,\omega_{\vec k}E_{\vec k}}\left[\frac{\bar u\gamma_5\tau^\alpha u_{\vec k}\bar u_{\vec k}\gamma_5\tau^\alpha u}{(M-E_{\vec k}-\omega_{\vec k})}+\frac{\bar u\gamma_5\tau^\alpha v_{\vec k}\bar v_{\vec k}\gamma_5\tau^\alpha u}{(-M-E_{\vec k}-\omega_{\vec k})}\right]$$

$$-\,(\text{the same with } m_\pi\to\lambda)\,. \qquad (2.12)$$

With this subtraction (logarithmically divergent as $\lambda\to\infty$) all the terms in Eq. (2.10) are finite.

We remark that the presence of a mass counterterm in the interaction Hamiltonian modifies f_3 to order g^2 with a term

$$f_3 = \langle N, N, \overline{N}, N | H_{\delta m} | N, N\rangle / \Delta_4\,. \qquad (2.13)$$

The analytical expression for δm is given in Appendix A.

In Eq. (2.10) finite corrections remain after the subtraction of the self-mass. These contributions represent the deformation of the pion cloud of the nucleon owing to its being in a bound state.

The solution of eigenvalue equation (2.10) could give in principle the wave function of the system and its energy. Of course in order to obtain a realistic answer we should add to Hamiltonian (2.1) all the meson states which are known to be relevant to the nuclear binding. This is not our aim at present and this subject will be discussed in a forthcoming paper, where a variational formulation of the problem will be discussed.

III. MEAN VALUES OF OBSERVABLES

Once the f_i's are known, the deuteron state is completely determined (up to the relevant order) and we can proceed to compute the matrix elements of observables. We will explicitly discuss the mean value of the Hamiltonian operator which will give back expression (2.10) and, as an example of the evaluation of a local quantity, we will show how to compute the mean value of the trace of the energy momentum tensor $\theta = \theta^\mu_\mu$.

In order to normalize the state defined in Eq. (2.4) we have to multiply it by a constant Z_D with

$$Z_D = 1 - \frac{g^2}{(2\pi)^3} \int\int d^3k_1 d^3k_2 \frac{1}{2\omega_{\vec{k}_1+\vec{k}_2}} \frac{M}{E_1}\frac{M}{E_2} \times \left(\frac{S_1}{\Delta_1{}^2} + \frac{S_2}{\Delta_2{}^2} + \frac{S_3}{\Delta_1{}^2}\right), \tag{3.1}$$

where S_1, S_2, S_3—the nucleon matrix elements summed over spin and isospin—are given by

$$S_1 = -3/(2M^2)(M^2 - E_1E_2 - \vec{k}_1\vec{k}_2) f_0{}^2(-\vec{k}_1, \vec{k}_1), \tag{3.2a}$$

$$S_2 = -3/(2M^2)(M^2 + E_1E_2 - \vec{k}_1\vec{k}_2) f_0{}^2(-\vec{k}_1, \vec{k}_1), \tag{3.2b}$$

$$S_3 = \frac{(E_1+M)(E_2+M)}{4M^2} \times \left(\frac{\vec{k}_1{}^2}{(E_1+M)^2} + \frac{\vec{k}_2{}^2}{(E_2+M)^2} + \frac{2\vec{k}_1\vec{k}_2}{(E_1+M)(E_2+M)}\right) \times f_0(-\vec{k}_1, \vec{k}_1) f_0(-\vec{k}_2, \vec{k}_2). \tag{3.2c}$$

The graphical representation of (3.1) is given in Fig. 2(c). Z_D does not suffer from ultraviolet divergences and physically Z_D gives (up to g^2) the number of pions inside the nucleus, as can be seen computing the mean value of the operator $N_\pi = \int a_{\vec{k}}^{\dagger\alpha} a_{\vec{k}}^{\alpha} d^3k$. See Appendix B for a discussion of the relation between Z_D and the wave-function renormalization constant usually defined in quantum field theory.[5]

We are now ready to compute the mean value of H,

$$\langle D|H|D\rangle = Z_D(\langle \overline{D}|f_0 + \langle N,N,\pi|f_1 + \langle N,N,\overline{N},N,\pi|f_2) \times [H_0 + H_I + H_{\delta m}](f_0|\overline{D}\rangle + f_1|N,N,\pi\rangle + f_2|N,N,\overline{N},N,\pi\rangle). \tag{3.3}$$

Neglected terms in the wave function give rise to $O(g^3)$ terms. Using (3.1) for Z_D we get

$$M_D Z_D^{-1} = \langle \overline{D}|H_0|\overline{D}\rangle f_0{}^2 + \langle N,N,\pi|H_0|N,N,\pi\rangle f_1{}^2 + \langle N,N,\overline{N},N,\pi|H_0|N,N,\overline{N},N,\pi\rangle f_2{}^2 + 2\delta m f_0^2 + 2f_1f_0\langle N,N,\pi|H_I|\overline{D}\rangle + 2f_2f_0\langle N,N,\overline{N},N,\pi|H_I|\overline{D}\rangle. \tag{3.4}$$

The first three terms on the right-hand side represent the average kinetic energy of the nucleons and pions inside the deuteron and, together with the correction from the renormalization constant, exactly cancel the factor 2 in front of the off-diagonal matrix elements of the interaction Hamiltonian.

We finally get

$$M_D = \langle D|H_0|D\rangle + \sum_n |\langle D|H_I|n\rangle|^2/(E_D - E_n) + 2\delta m, \tag{3.5}$$

where $|n\rangle$ represent all the possible states connected to the bare deuteron by the interaction Hamiltonian up to order g^2 and E_n its kinetic energy. Equation (3.4) reads explicitly

$$M_D = \int d^3k_1\, 2E_1 f_0{}^2(-\vec{k}_1, \vec{k}_1) + \frac{g^2}{(2\pi)^3}\int\int d^3k_1 d^3k_2 \frac{1}{2\omega_{\vec{k}_1+\vec{k}_2}} \frac{M}{E_1}\frac{M}{E_2} \times \left[\left(\frac{S_1}{\Delta_1} - \frac{S_1}{\Delta_{1f}}\right) + \left(\frac{S_2}{\Delta_2} - \frac{S_2}{\Delta_{2f}}\right) + \frac{S_3}{\Delta_1}\right] \tag{3.6}$$

with $\Delta_{1f} = E_1 - E_2 - \omega_{\vec{k}_1+\vec{k}_2}$ and $\Delta_{2f} = -E_1 - E_2 - \omega_{\vec{k}_1+\vec{k}_2}$.

In Eq. (3.6) the first term is the kinetic energy contribution, the terms in S_1 and S_2 are self-energy subtracted pieces, and the last term in S_3 represents the exchanged pion contribution. This result coincides with the mean value of the integral equation (2.10) and is equivalent to the usual second order energy correction given in standard perturbation theory.

As another example let us illustrate the computation of the mean value of a local operator, namely θ.[2] It is well known that

$$(2\pi)^3\langle D|\theta|D\rangle = (E_D{}^2 - \vec{P}^2)/E_D = M_D^2/E_D. \tag{3.7}$$

On the other hand for the theory under consideration [Eq. (2.1)] one can define θ such that[6]

$$\theta = (M - \delta m)\overline{N}(x)N(x) + m_\pi{}^2\phi^\alpha(x)\phi^\alpha(x). \tag{3.8}$$

Sandwiching this equation between the deuteron state (2.4) we obtain the terms represented in Fig. 3.

Here too, we remark that finite corrections come from self-energy terms represented in Fig. 3. However, we would not trust such a finite value as a good representation of the mean value of the energy momentum tensor trace, because this number would represent the mean value of an observable computed in terms of the properties of the bare nucleons and pions. The big value of the pion-nucleon coupling constant indicates that such a computation would be meaningless.

However, we propose to compute only the differences between the value of physical quantities for a nucleon in the bound state and a free one. Owing to the weak binding hypothesis such a difference is small and could be a good approxima-

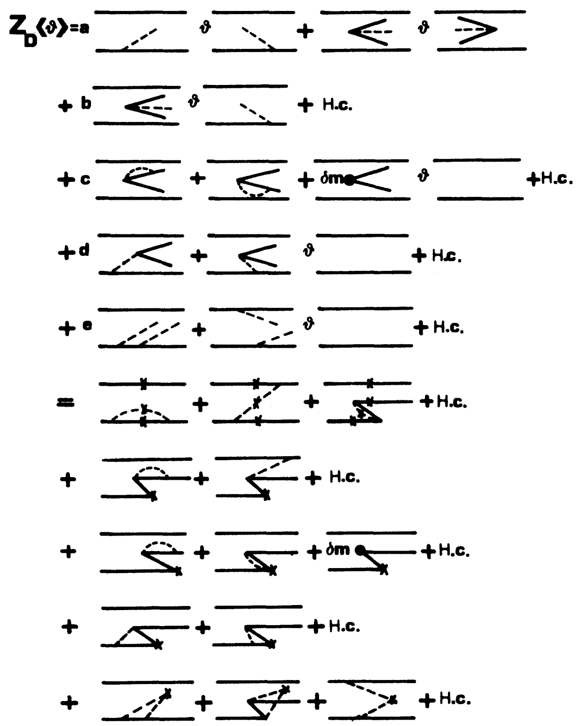


FIG. 3. Expectation value of the trace of the energy momentum tensor in the deuteron state. The crosses represent the points where the operator θ has to be inserted.

tion for the same difference of the actual physical quantities.

The philosophy is that the meson clouds, and in particular the pion one, are slightly modified by the presence of other nucleons and the contribution of heavier mesons are less and less modified by the bound state situation; so that in the difference with the free nucleon contribution they should cancel out almost exactly. By the same argument one can convince oneself that higher order in g, while contributing to the structure of nucleon itself, gives a small contribution to the difference between a physical free and a bound nucleon since they tend to modify its inner structure.

The actual procedure is as follows. On the one hand we must compute the matrix element in the given bound state. On the other hand we must perform the same calculation on free nucleons up to the same order in g, superpose with $f_0(\vec{k}_1, \vec{k}_2)$, and subtract from the above result. The computation of the nucleon matrix elements and the subtraction are carried out in Appendix C. In all computations we have neglected diagrams proportional to the volume of the space (vacuum type diagrams) according to general prescriptions of quantum field theory. As for the vacuum self-energy effects, these pose no problem and can be subtracted with the procedure indicated after Eq. (2.5). As for renormalization constants, the fact that a vacuum-type diagram can be neglected is more subtle and we show in Appendix B, in the particular case of a one nucleon state, how they can be handled. The subtraction of vacuum-type diagrams in renormalization constants gives rise to suprising effects on Z_D. The fact that $Z_D < 1$ is trivially guaranteed by the infinite vacuum-type diagrams. Once these are subtracted the remaining Z_D used in actual computation does not necessarily satisfy this condition.

Finally it is interesting to look at the numerical contribution of the various terms in Eq. (3.6) and Appendix C, to the mean value of the Hamiltonian and of the energy momentum tensor, taking the deuteron wave function from usual nonrelativistic approaches. In Table I we give these values for two typical wave functions (Hulthên modified and Hamada-Johnston; see Appendix D for their parametrization) neglecting d wave contribution and comparing it with the static limit.

Even if we are far from justifying the binding energy of the deuteron, which is given by the ρ, ω, σ, ... contribution, it is worth while to remark on the sizable contribution of the self-energy effects, and the repulsive character of the exchange diagrams, which is originated by the spread out of the delta singularity in the Yukawa potential due to relativistic effects.[7]

Relativistic calculations have been performed

TABLE I. Contribution to the expectation value of the trace of the energy momentum tensor and of the Hamiltonian coming from diagrams of Figs. 2(b) and 3. The two different wave functions used are given in Appendix D. Contributions coming from self-energy diagrams are given for each set of diagrams and total exchange contributions are evaluated both in the relativistic and static limit. Kinetic energy contributions are given for comparison in the last two lines. The energy units are expressed in MeV.

	Hamada-Johnston		Hulthên modified	
$\theta_{a\ \mathrm{self}}$	93.03	−1.66	83.30	−0.14
$\theta_{b\ \mathrm{self}}$	−94.69		−83.44	
$\theta_{c\ \mathrm{self}}$	−5.74		−2.70	
$\theta_{d\ \mathrm{self}}$	−2.76		−1.28	
$\theta_{e\ \mathrm{self}}$	0.55		−0.45	
$\theta_{\mathrm{tot\ self}}$	−9.59		−3.66	
θ_{exch}	−2.14		−3.01	
$\theta_{\mathrm{exch\ static}}$	2.20		1.99	
H_{self}	8.74		1.21	
H_{exch}	0.26		0.63	
$H_{\mathrm{exch\ static}}$	−2.20		−1.97	
$\theta_{\mathrm{kinetic}} - 2M$	−12.24		−9.90	
$H_{\mathrm{kinetic}} - 2M$	13.87		10.43	

by Levy,[7] and Partovi and Lomon.[8] Their main interest is in vertex and meson propagator radiative corrections (which are rather small) and in relativistic exchange where they point out the existence of a repulsive core. As far as the exchange is concerned, apart from the qualitative agreement, the result is rather sensitive to the structure of the energy denominators (we use Δ_1 against prescription of other authors; see for example, the relativistic exchange calculation by Ueda and Green[9]). Self-energy effects of the Lamb-shift type are not taken into account in the literature and give a substantial contribution to nuclear forces.

We further stress that in both the relativistic case and in the static limit $\langle D|\theta|D\rangle$ and $\langle D|H|D\rangle$ have different structure and are identical only on true eigenstates.[2] In our case we have chosen a good approximation for the physical state but a bad one for the Hamiltonian (only pions have been taken into account) so we cannot compare the results. The nonrelativistic connection between $\langle\theta\rangle$ and $\langle H\rangle$ has been discussed in Ref. 2.

IV. EXTERNAL POTENTIALS

In this section we want to study the strong corrections to the binding energy in the presence of an external potential $V(x)$. This is useful in order to compare the method given in Sec. II with the usual treatment of the Lamb-shift effect[10] and allows an investigation of some interesting physical systems such as antiproton in the Coulomb field of a nucleus.

In order to be more specific let us have an electrostatic potential $A_\mu = [\vec{0}, eV(x)]$. To describe the situation we have to add to the Hamiltonian Eq. (2.1) the following terms:

$$H_{\rm ext} = \int d^3x\, eV(x)\psi^\dagger(x)\left(\frac{1+\tau^3}{2}\right)\psi(x) + \epsilon_{3\alpha\beta}eV(x)\phi^\alpha(x)\dot{\phi}^\beta(x) + e^2[V(x)]^2[\phi^\alpha(x)\phi^\alpha(x) - \phi^3(x)\phi^3(x)]\,. \quad (4.1)$$

In this case we expand the field operators in terms of eigenfunctions of the Dirac and Klein-Gordon equation in the presence of the external field A_μ so that now the creation and annihilation operators create and annihilate particles in the presence of A_μ.

One can easily convince oneself that the Tamm-Dancoff procedure developed in the previous sections gives precisely the same answer as perturbation theory around the eigenstates and eigenvalue of $H_0 + H_{\rm ext}$.

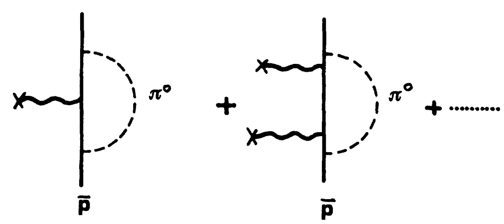
Following the techniques of Ref. 10 it is possible to cast this expression in the final form

$$\langle n|H_{\rm tot}|n\rangle = \langle n|H_0 + H_{\rm ext}|n\rangle + \langle n|H_{\rm int}|n\rangle = E_n + \Delta E_n \quad (4.2)$$

with

$$\Delta E_n = \frac{g^2}{(2\pi)^4}\int d^4k \frac{1}{(k^2 - m_\pi^2)}\left\langle n\left|\gamma_5 \frac{1}{(\not{\Pi} - \not{k} - M)}\gamma_5\right|n\right\rangle + \frac{g^2}{(2\pi)^4}\int d^4k \frac{1}{(\chi^2 - m_\pi^2)}\left\langle n\left|\gamma_5 \frac{1}{(\not{p} - \not{k} - M)}\gamma_5\right|n\right\rangle - \delta m, \quad (4.3)$$

where $\Pi_\mu = (p_\mu - eA_\mu)$ and $\chi_\mu = (k_\mu - eA_\mu)$ and where we have introduced the four-dimensional integration d^4k in order to deal with the more compact covariant propagators. The two terms in Eq. (4.3) correspond to the two self-energy diagrams of Fig. 4; in Fig. 4(a) the intermediate state is a charged nucleon in the external field plus a π^0, in Fig. 4(b) we have a neutron and a charged pion. The integral in (4.3) can be handled using the techniques of Ref. 10; we regularize the ultraviolet divergence and shift the k integration carefully treating the terms coming from the noncommutativity of the potential with the momentum. The procedure is cumbersome, but straightforward (more details are given in Ap-



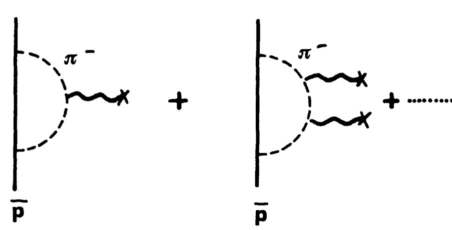


FIG. 4. (a) Self-energy contributions for an antiproton in a Coulomb field. A sum over an infinite number of interactions with the external field is understood. (b) Charged pion contribution to the antiproton self-energy.

pendix E) and gives the final result

$$\Delta E_n = \int_0^1 du \int_0^1 dz\, P(z,u) \left\langle n \left| p_i \frac{1}{\Delta} [p_i, V(x)] \right| n \right\rangle , \tag{4.4}$$

where $P(z,u)$ is rational in z and u and is given in Appendix E, and

$$\Delta = [z^2 M^2 + (1-z) m_\pi^2 + z(1-z) u H_{nr}] \tag{4.5}$$

with

$$H_{nr} = H_0 + V(x) - \langle n | H_0 + V(x) | n \rangle . \tag{4.6}$$

The expression is very similar to the one found in the electrodynamic case the main difference being, apart from the form of $P(z,u)$, the absence of infrared divergences and as a consequence of a term similar to the Bethe logarithm.

Expanding Eq. (4.4) in terms of H_{nr}, the zeroth order gives the contribution to the binding due to the physical proton radius and magnetic moment. These quantities are evaluated here in the second order perturbation theory and agree with the result given with a different method in Ref. 11. However, as already said, these quantities cannot be taken seriously and actually we know from Ref. 11 that the result is rather different from the experimental value and that higher order contributions give rise to big corrections and worsen the situation. The quantity to be evaluated is the difference in the binding of the antiproton due to the fact that its magnetic moment and radius are in turn modified by the potential itself. In order to do this, we have to expand the expression analyzed in Appendix E to higher order in the external field. The techniques are again those used by Ericson and Yennie and the leading corrections are of the order $Z\alpha$. This could give sizable contributions in the antiprotonic atoms and detailed calculations are in progress.

This particular application helps us to understand the validity of our general approach. In Sec. II corrections due to the propagation of intermediate states in the field generated by the other nucleon were not taken into account. Actually in the language of Secs. II and III this would correspond to a sum of an infinite number of diagrams. However, these two methods give comparable results when the virtual particle is massive in contrast with the electrodynamic case, as the following argument shows.

Following Yennie[12] we write the ratio of the first order to the zero order correction coming from expansion in power of the potential as

$$\delta = \int_0^1 dx \frac{x(1-x)\langle V \rangle M}{[x^2 M^2 + \langle V \rangle M x(1-x) + (1-x) m_\pi^2]} \tag{4.7}$$

when $m_\pi = 0$ (the electromagnetic case) the integral is dominated by the region $x \simeq 0$ and gives $\delta \simeq 1$. When $m_\pi \neq 0$ the full result is $\delta \simeq (\langle V \rangle / M) I$ with $I = 1.19$ for the actual value of the pion mass and strongly decreasing with the increase of the meson mass, so that $\delta \ll 1$ and it makes sense to stop the power expansion.

V. CONCLUSION

We have proposed in this paper a completely relativistic approach to the problem of weakly bound states which takes into account the modification of the structure of the component particles due to the binding. In particular, this method does not rely on the smallness of the underlying coupling constant which defines the theory, but only on the weakness of the binding energy. The method has been illustrated on the pseudoscalar pion theory which is known to provide a nonrealistic description of nuclear interaction. The inclusion of heavier mesons is straightforward and the problem of computing deuteron properties from a fundamental realistic Lagrangian is under study.

Within the approach we have shown how to compute physical quantities, in particular matrix elements of physical observables with a particular emphasis on the Hamiltonian and the trace of the energy momentum tensor. The study of matrix elements of vector and axial vector currents can be developed along the same lines and is currently pursued.

An interesting point which emerges from this approach is that self-energy effects, which are the result of the deformation of the meson cloud of a bound nucleon, turn out to be as important as the exchange effects usually taken into account. The approach proposed here is shown to be equivalent to the normal perturbative one when applied to external field problems. In particular, the standard treatment of the Lamb-shift is reproduced.

APPENDIX A

We introduce here the explicit expression of the Hamiltonian (2.1) in terms of creation and annihilation operators in momentum space, and give as an example the calculation of the self-mass term δm for a nucleon at rest. Equation (2.1) can be written

$$H = H_0 + H_I \tag{A1}$$

with

$$H_0 = \sum_{a,\alpha} \int d^3k \left[E_{\vec{k}}(b_{\vec{k}}^{a\dagger} b_{\vec{k}}^{a} + d_{\vec{k}}^{a\dagger} d_{\vec{k}}^{a}) + \omega_{\vec{k}} a_{\vec{k}}^{\alpha\dagger} a_{\vec{k}}^{\alpha}\right] \tag{A2}$$

and

$$H_I = \frac{ig}{(2\pi)^{3/2}} \sum_{a,b,\gamma} \int\int\int d^3p\, d^3p'\, d^3k\, \frac{1}{\sqrt{2\omega_k}} \left(\frac{M}{E_{\vec{p}}}\right)^{1/2} \left(\frac{M}{E_{\vec{p}'}}\right)^{1/2}$$

$$\times \left[a_{\vec{k}}^{\alpha} b_{\vec{p}}^{a\dagger} b_{\vec{p}'}^{b} \bar{u}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} u_{\vec{p}'}^{b} \delta^3(\vec{k} - \vec{p} + \vec{p}') \right. \tag{A3a}$$

$$+ a_{\vec{k}}^{\alpha} b_{\vec{p}}^{a\dagger} d_{\vec{p}'}^{b\dagger} \bar{u}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} v_{\vec{p}'}^{b} \delta^3(\vec{k} - \vec{p} - \vec{p}') \tag{A3b}$$

$$+ a_{\vec{k}}^{\alpha} d_{\vec{p}}^{a} b_{\vec{p}'}^{b} \bar{v}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} u_{\vec{p}'}^{b} \delta^3(\vec{k} + \vec{p} - \vec{p}') \tag{A3c}$$

$$+ a_{\vec{k}}^{\alpha} d_{\vec{p}}^{a} d_{\vec{p}'}^{b\dagger} \bar{v}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} v_{\vec{p}'}^{b} \delta^3(\vec{k} + \vec{p} + \vec{p}') \tag{A3d}$$

$$+ a_{\vec{k}}^{\alpha\dagger} b_{\vec{p}}^{a\dagger} b_{\vec{p}'}^{b} \bar{u}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} u_{\vec{p}'}^{b} \delta^3(-\vec{k} - \vec{p} + \vec{p}') \tag{A3e}$$

$$+ a_{\vec{k}}^{\alpha\dagger} b_{\vec{p}}^{a\dagger} d_{\vec{p}'}^{b\dagger} \bar{u}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} v_{\vec{p}'}^{b} \delta^3(-\vec{k} - \vec{p} - \vec{p}') \tag{A3f}$$

$$+ a_{\vec{k}}^{\alpha\dagger} d_{\vec{p}}^{a} b_{\vec{p}'}^{b} \bar{v}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} u_{\vec{p}'}^{b} \delta^3(-\vec{k} + \vec{p} + \vec{p}') \tag{A3g}$$

$$\left. + a_{\vec{k}}^{\alpha\dagger} d_{\vec{p}}^{a} d_{\vec{p}'}^{b\dagger} \bar{v}_{\vec{p}}^{a} \gamma_5 \tau^{\alpha} v_{\vec{p}'}^{b} \delta^3(-\vec{k} + \vec{p} - \vec{p}') \right] \tag{A3h}$$

where $a, a^\dagger$ are the pion annihilation and creation operators $(d, d^\dagger)$ and $(b, b^\dagger)$ are the antinucleon (nucleon) ones, u and v are the usual four component Dirac spinors, and summation on polarization indices is understood. $a = 1, 2$ and $\alpha = 1, 2, 3$ are the isospin indices of the nucleon and pion, respectively. The matrix elements can then be computed using the commutation relations of the creation and annihilation operators. As an example let us study the self-energy contribution

$$\delta m = \sum_n |\langle N|H_I|n\rangle|^2/(M - E_n), \tag{A4}$$

where n runs over all the allowed states, i.e., $|N, \pi\rangle$ and $|N, N, \bar{N}, N, \pi\rangle$. It is lengthy but straightforward to obtain Eq. (2.12), and summing over the intermediate spin we get

$$\delta m = \lim_{\lambda \to \infty} \frac{-g^2}{(2\pi)^3} 4\pi \int k^2 dk\, \frac{M}{2\omega_\pi E}\left[\frac{s_1}{(M - E - \omega)} + \frac{s_2}{(-M - E - \omega)}\right] - (\omega_\pi \to \omega_\lambda) \tag{A5}$$

with $\omega_\pi = (m_\pi^2 + k^2)^{1/2}$, $\omega_\lambda = (\lambda^2 + k^2)^{1/2}$, $E = (M^2 + k^2)^{1/2}$, and $s_1 = 3/2(E/M - 1)$, $s_2 = 3/2(-E/M - 1)$. This expression is easily reduced to

$$\delta m = \lim_{\lambda \to \infty} \frac{3g^2}{(2\pi^2)} M[I(m_\pi) - I(\lambda)] \tag{A6}$$

with

$$I(\mu) = \lim_{K_{\max} \to \infty} \int_0^{K_{\max}} \frac{k^2 dk}{E[(E + \omega_\mu)^2 - M^2]}, \tag{A7}$$

i.e.,

$$I(m_\pi) = \frac{1}{4}\left[\left(\frac{\eta}{2} - \frac{1}{2}\right) - \frac{\eta^2}{4}\ln(\eta) + \frac{\eta}{2}\ln(\eta) + \ln\frac{K_{\max}}{M} - \frac{\eta}{2}(4\eta - \eta^2)^{1/2}\cos^{-1}\frac{\eta^{1/2}}{2}\right] \tag{A8}$$

with $\eta = m_\pi^2/M^2$,

$$I(\lambda) = \frac{1}{4}\left[\ln\frac{K_{\max}}{M} - \ln\frac{\lambda}{M} - \frac{1}{4} + O\left(\frac{1}{\lambda}\right)\right] \tag{A9}$$

then

$$\delta m = \frac{3g^2}{2\pi^2}\frac{M}{4}\left[\ln\frac{\lambda}{M} - \frac{1}{4} + \frac{\eta}{2} + \frac{\eta}{2}\ln\eta - \frac{\eta^2}{4}\ln\eta - \eta(\eta - \eta^2/4)^{1/2}\cos^{-1}\frac{\eta^{1/2}}{2}\right] \quad \text{(A10)}$$

in agreement with the more standard explicitly covariant evaluation of the self-energy contributions.[13]

APPENDIX B

In this appendix we want to discuss a few points in connection with vacuum effects in normalization factors and the relation of these factors with the renormalization constants in field theory. For simplicity we will discuss the case of a single nucleon. In the more complex case of a bound state the same arguments can be carried through.

The physical one nucleon state [eigenstate of Eq. (2.1)] can be written as

$$|N\rangle_{\rm ph} = Z_N^{1/2}|N\rangle + \phi_1|\pi, N\rangle + \phi_2|\pi, N, \bar{N}, N\rangle + \cdots, \quad \text{(B1)}$$

where $|N\rangle_{\rm ph}$ is the physical nucleon state and the kets in the rhs of Eq. (B1) are bare states; $\phi_1, \phi_2, \ldots$ can be computed by the eigenvalue equation

$$(H_0 + H_I)|N\rangle_{\rm ph} = E_N|N\rangle_{\rm ph}, \quad \text{(B2)}$$

for example, in a power series in the coupling constant g. The value of Z_N is graphically represented in Fig. 5(a). The diagram (α) in Fig. 5(a) is obviously proportional to the volume of the space. However, in the computation of the expectation value of any observable O (for simplicity bilinear in the nucleon fields) we have to compute

$$_{\rm ph}\langle N|O|N\rangle_{\rm ph} = Z_N\langle N|O|N\rangle + \phi_1^2\langle N, \pi|O|\pi, N\rangle + \phi_2^2\langle N, \bar{N}, N, \pi|O|N, \bar{N}, N, \pi\rangle + \cdots. \quad \text{(B3)}$$

The last term indicated in Eq. (B3) contains a contribution proportional to the volume of the space which is depicted in Fig. 5(b). It is evident that its contribution is exactly canceled by the vacuum term in Z_N.

As for the connection of our normalization factor Z_N with the wave function renormalization constant in quantum field theory, let us recall the definition of the latter

$$_{\rm ph}\langle 0|\psi|N\rangle_{\rm ph} = \tilde{Z}_N^{1/2}u_N, \quad \text{(B4)}$$

where $|0\rangle_{\rm ph}$ is the physical vacuum. In analogy

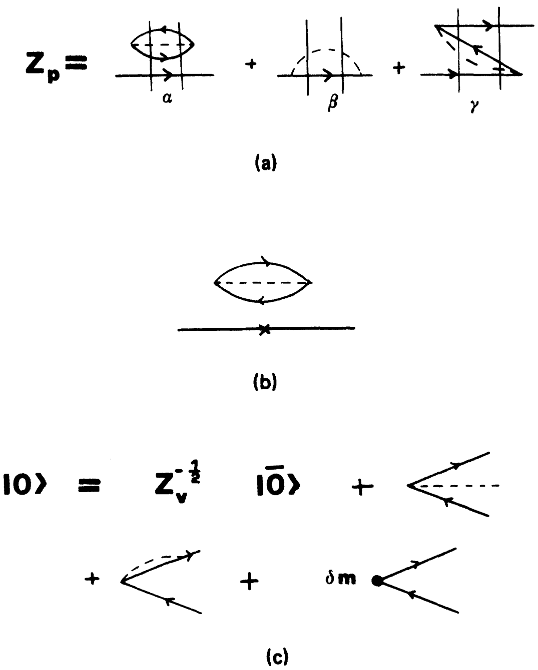


FIG. 5. (a) Structure of the normalization constant. (b) Vacuum-type contribution to the mean value of an operator. (c) Structure of the vacuum.

with the one nucleon physical state the vacuum has the following structure:

$$|0\rangle_{\rm ph} = Z_v^{1/2}|0\rangle + \varphi_1|N, \bar{N}, \pi\rangle + \varphi_2|N, \bar{N}\rangle + \cdots, \quad \text{(B5)}$$

where φ_1 and φ_2 can be computed as the f_i's in Sec. II and Z_v is the factor which guarantees the normalization of the physical vacuum. We represent Eq. (B4) diagrammatically in Fig. 5(c).

Using this equation the contribution to Z_N comes from the same kind of diagrams which contribute to our Z_N. The only difference is the relative sign of the two which turns out to be positive, giving rise to the usual ultraviolet logarithmically divergent expression, to be handled with the standard regularization techniques. In Eq. (B4) we have two vacuum-type contributions. The first one comes from Z_v, the other one is contained in the term

$$\varphi_1^*\phi_2\langle\pi, N, \bar{N}|\psi|\pi, N, N, \bar{N}\rangle. \quad \text{(B6)}$$

It is easy to check that they compensate each other.

In our expression, Eq. (3.1) the sign of the Z diagram terms [Fig. 5(a), γ] is negative and Z_N turns out to be finite:

$$Z_N - 1 = -\frac{3g^2}{2\pi^2}\frac{m_\pi^2}{M^2}\int_0^\infty \frac{M^2\kappa^2}{[(E+\omega)^2 - M^2]}\frac{M}{E}\frac{dk}{\omega}$$

$$= -\frac{3g^2}{4\pi^2}\frac{m_\pi^2 0.374}{M^2} \quad . \tag{B7}$$

Its value is numerically small, $(Z_N - 1) = 0.124$, due to the fact that it is proportional to the effective coupling constant $g^2/4\pi\ m_\pi^2/4M^2 = f^2 = 0.08$.

APPENDIX C

In evaluating the mean value of the energy momentum tensor we give separately the contributions from the different terms in Fig. 3. The first one coming from the term a with the correction due to the normalization constant is

$$\theta_a = \int\int d^3k_1 d^3k_2 \frac{M^2}{E_1E_2\omega}\Bigg\{\Bigg[-M_D\left(\frac{S_1}{\Delta_1^2} + \frac{S_2}{\Delta_2^2} + \frac{S_3}{\Delta_1^2}\right) + \left(\frac{M^2}{E_1} + \frac{M^2}{E_2} + \frac{m_\pi^2}{\omega}\right)\frac{S_1}{\Delta_1^2} + \left(\frac{3M^2}{E_1} + \frac{M^2}{E_2} + \frac{m_\pi^2}{\omega}\right)\frac{S_2}{\Delta_2^2}$$
$$\times\left(\frac{M^2}{E_1} + \frac{M^2}{E_2} + \frac{m_\pi^2}{\omega}\right)\frac{S_3}{\Delta_1^2}\Bigg]$$
$$-\Bigg[\frac{M^2}{E_2}\left(\frac{S_1}{\Delta_{1f}^2} + \frac{S_2}{\Delta_{2f}^2}\right) + \left(\frac{M^2}{E_2} + \frac{m_\pi^2}{\omega}\right)\frac{S_1}{\Delta_{1f}^2} + \left(\frac{2M^2}{E_1} + \frac{M^2}{E_2} + \frac{m_\pi^2}{\omega}\right)\frac{S_2}{\Delta_{2f}^2}\Bigg]\Bigg\}. \tag{C1}$$

The contribution coming from Fig. 3(b) is

$$\theta_b = 2\int\int d^3k_1 d^3k_2 \frac{M^2}{E_1E_2\omega}\frac{M^2}{E_1}\left(\frac{S_4}{\Delta_1\Delta_2} + \frac{S_5}{\Delta_1\Delta_2} - \frac{S_4}{\Delta_{1f}\Delta_{2f}}\right) \tag{C2}$$

where

$$S_4 = \frac{1}{2M^2}(\vec{k}_1^2 + \vec{k}_1\vec{k}_2) f_0^2(\vec{k}_1, -\vec{k}_1)\,, \tag{C3a}$$

$$S_5 = \frac{1}{2M^2}(\vec{k}_1^2 + \vec{k}_1\vec{k}_2) f_0(\vec{k}_1, -\vec{k}_1) f_0(\vec{k}_2, -\vec{k}_2)\,, \tag{C3b}$$

and from Fig. 3(c) is

$$\theta_c = 2\int\int d^3k_1 d^3k_2 \frac{M^2}{E_1E_2\omega}\frac{M^2}{E_1}\left[\frac{S_4}{(M_D - 4E_1)\Delta_2} - \frac{S_4}{(M_D - 4E_1)\Delta_{2f}} - \frac{S_4}{(-2E_1)\Delta_{2f}}\right]. \tag{C4}$$

We stress that in the absence of the second term in the last equation which corresponds to the self-mass subtraction, the full expression would diverge logarithmically with coefficients proportional to $(M_D - 2E_1)$.

The fourth diagram contribution with Fig. 3(d) is

$$\theta_d = 2\int\int d^3k_1 d^3k_2 \frac{M^2}{E_1E_2\omega}\left\{\frac{M^2}{E_1}\left(\frac{S_5}{\Delta_1\Delta_3} + \frac{S_5}{\Delta_2\Delta_3}\right) + \frac{M^2}{E_2}\left[\frac{S_6}{\Delta_1\Delta_3} - \frac{S_6}{(-2E_2)\Delta_{1f}} + \frac{S_6}{\Delta_2\Delta_3} - \frac{S_6}{(-2E_2)\Delta_{2f}}\right]\right\}, \tag{C5}$$

where

$$\Delta_3 = M_D - 2E_2 - 2E_1 \text{ and } S_6 = 3/2M^2(\vec{k}_2^2 + \vec{k}_1\vec{k}_2) f_0^2(\vec{k}_1, -\vec{k}_1)\,. \tag{C6}$$

The last contribution [Fig. 3(e)] is

$$\theta_e = 2\int\int d^3k_1 d^3k_2 \frac{M^2}{E_1E_2\omega}\frac{m_\pi^2}{\omega}\left[\frac{S_1}{\Delta_1\Delta_4} + \frac{S_2}{\Delta_2\Delta_4} - \frac{S_1}{(-2\omega)\Delta_{1f}} - \frac{S_2}{(-2\omega)\Delta_{2f}} + \frac{S_3}{\Delta_1\Delta_4}\right] \tag{C7}$$

with $\Delta_4 = M_D - 2\omega - 2E_1$.

APPENDIX D

In this appendix we give two of the more standard and simple analytical parametrizations for the s wave part of the deuteron wave function, which we use in Secs. II and III, to compute the mean value of the Hamiltonian and of the trace of the energy-momentum tensor.

1. Hulthén modified[14]

$$\psi(r) = N_1(e^{-ar} - e^{-dr})(1 - e^{-\gamma r})(1 - e^{-cr})/r\,, \tag{D1}$$

where N_1 is a normalization constant $N_1 = 0.2583\ \text{fm}^{-1/2}$.

$$a=0.232\ \mathrm{fm}^{-1},\ d=1.9\ \mathrm{fm}^{-1},\ g=2.5\ \mathrm{fm}^{-1},\ c=1.54\ \mathrm{fm}^{-1}. \tag{D2}$$

The Fourier transform is given by

$$\psi(k)=N_1\left(\frac{2}{\pi}\right)^{1/2}\left\{\frac{1}{(k^2+a^2)}-\frac{1}{(k^2+d^2)}-\frac{1}{[(c+a)^2+k^2]}+\frac{1}{[(c+d)^2+k^2]}+\frac{1}{[(d+g)^2+k^2]}-\frac{1}{[(a+g)^2+k^2]}+\frac{1}{[(c+a+g)^2+k^2]}-\frac{1}{[(c+g+d)^2+k^2]}\right\}. \tag{D3}$$

2. Hamada Johnston truncated[15]

$$\psi(r)=N_2\frac{e^{-r}}{r}[1-e^{\delta(r-r_c)}]\sum_{i=0}^{3}\beta_i\, e^{-ri\,m_\pi}\quad \text{for } r>r_c,$$
$$=0\quad \text{for } r<r_c, \tag{D4}$$

where N_2 is a normalization constant $N_2=0.2903\ \mathrm{fm}^{-1/2}$ and

$$r_c=0.3\ \mathrm{fm},\ \delta=3.8\ \mathrm{fm}^{-1},\ \alpha=0.232\ \mathrm{fm}^{-1},\ \beta_0=0.884,\ \beta_1=-0.011,\ \beta_2=-1.245,\ \beta_3=0.350. \tag{D5}$$

The Fourier transform is given by

$$\psi(k)=N_2\left(\frac{2}{\pi}\right)^{1/2}\sum_{i=0}^{3}[I(k,\alpha+im_\pi)\beta_i-I(k,\alpha+im_\pi+\delta)\beta_i\, e^{\delta r_c}] \tag{D6}$$

with

$$I(k,\gamma)=\frac{e^{-\gamma r_c}}{(k^2+\gamma^2)k}[\gamma\sin(kr_c)+k\cos(kr_c)]. \tag{D7}$$

APPENDIX E

In this appendix we evaluate the contribution ΔE_n^N and ΔE_n^π coming from diagrams in Fig. 4(a) and 4(b), respectively. Following Ref. 10 we write

$$\Delta E_n^N=\frac{g^2}{8\pi^2}\int_0^1 dz\int_0^{\Lambda^2(1-z)}dK\int d^4k\frac{\langle n|\gamma_5(\not\Pi-\not k+m)\gamma_5|n\rangle}{[K-(k-z\Pi)^2+z(1-z)H_{nr}-z^2M^2+(1-z)m_\pi^{\,2}]^3}-\delta m, \tag{E1}$$

where for simplicity we have neglected in the denominator terms which give rise to the hadronic magnetic moment interaction with the external field. Bypassing the complex techniques which allow us to handle the shift $(k-z\Pi)$ in the denominator, the final result is expressed in terms of the commutator $\Pi_\nu[\Pi_\nu,\not\Pi]\simeq p_i[p_i,\ eV(x)]$ and can be cast in the form given in Eq. (4.4). The algebraic function $P(z,u)$ is given by the expression

$$P(z,u)=-2(1-z)z^2(1-u)u\left[1-\frac{(1-z)m_\pi^{\,2}}{z^2M^2+(1-z)m_\pi^{\,2}}\right]+z^2(1-z)(1-u)+z^3(1-u). \tag{E2}$$

The zero order result is obtained by setting $H_{nr}=0$ in the denominator of Eq. (4.4) and is given by

$$\Delta E_n^N=\langle n|\{p_i,[p_i,V(x)]\}|n\rangle\tfrac{1}{6}\langle r_p^{N^2}\rangle=\frac{2\pi}{3}Z\alpha\,|\psi_n(0)|^2\langle r_p^{N^2}\rangle \tag{E3}$$

with

$$-\langle r_p^{N^2}\rangle=\frac{1}{M^2}\frac{1}{3}\frac{g^2}{8\pi^2}\left[\frac{\eta}{4\eta-\eta^2}(6+\tfrac{33}{2}\eta-5\eta^2)+(2\eta-\tfrac{5}{2}\eta^2)\ln(\eta)+(-54+34\eta-5\eta^2)\frac{\eta^3}{(4\eta-\eta^2)^{3/2}}\cos^{-1}\left(\frac{\eta^{1/2}}{2}\right)\right]\simeq\frac{1.16}{M^2} \tag{E4}$$

and $\eta=m_\pi^{\,2}/M^2$. Analogously the diagram 5(b) gives

$$\Delta E_n^\pi=\frac{g^2}{8\pi^2}\int_0^1 dz\int_0^{\Lambda^2(1-z)}dK\int d^4k\frac{\langle n|\not k-\not A-z\not\Pi+zM|n\rangle(1-z)}{[K-(k-A-z\Pi)^2-z(1-z)H_{nr}+z^2M^2+(1-z)m_\pi^{\,2}]^3}-\delta m. \tag{E5}$$

Shifting as before and performing the multiple integration we get

$$\Delta E_n^\pi=\frac{2\pi}{3}Z\alpha\,|\psi_n(0)|^2\langle r_p^{\pi 2}\rangle \tag{E6}$$

with

$$-\langle r_p^{\pi 2}\rangle = \frac{1}{M^2}\frac{g^2}{16\pi^2}\bigg[(40-49\eta+10\eta^2)\frac{1}{(4-\eta)}+(3-12\eta+5\eta^2)\ln(\eta) - (140-210\eta+84\eta^2-10\eta^3)\frac{\eta^2}{(4\eta-\eta^2)^{3/2}}\cos^{-1}\left(\frac{\eta^{1/2}}{2}\right)\bigg] \simeq \frac{-5.51}{M^2}\,. \tag{E7}$$

These results for $\langle r_p^{\pi 2}\rangle$ and $\langle r_p^{N 2}\rangle$ are in agreement with those given in Ref. 12 for the nucleon form factors; however, as already said they are not reliable since they are the first order expansion of a meaningless perturbative series. Actually we are interested only in corrections to these expressions due to the external field distortion. If we expand the denominator in Eq. (4.4) the first correction is proportional to the mean value of $H_{\pi\pi}\alpha(Z\alpha)^2$; however, since the main contribution comes from the relativistic ultraviolet region it is possible to prove only that the real behavior is of order $(Z\alpha)$. This correction is rather important, especially in heavy antiprotonic atoms, where the strength of the field induces strong distortions of the mesonic cloud. The full result is, however, rather complicated since it depends both on the denominator expansion in Eq. (4.4) and on the other higher order contributions coming from the noncommutativity of the field with the momentum operator. An accurate analysis is in progress.

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