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Dirac phenomenology in nuclear structure and reactions

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A recently developed relativistic theory of nuclear matter and of finite nuclei is extended to allow for consideration of the nucleon-nucleus interaction at positive energy. A new method of reducing the Dirac equation to a nonrelativistic Schrödinger equation is presented. This scheme is used, along with the results of calculations performed earlier for nuclear matter, to determine the parameters of the nuclear optical model. The results are in good agreement with the phenomenological local potentials conventionally used to analyze nucleon-nucleus scattering.

[NUCLEAR REACTIONS Relativistic effects and the nuclear optical potential, reduction of Dirac equation to Schrödinger form.]

I. INTRODUCTION

Recently we have developed a theory of nuclear structure which provides a unified framework for incorporating both the effects of nucleon-nucleon correlations and a relativistic description of nucleon motion.[1] The derivation of the equations of the theory is presented in Ref. 2, where use is made of some of the relativistic Green's function relations given by Wilets.[3] The calculations reported in Ref. 1 are for nuclear matter. We find that negative-energy states induced in the nucleon wave function play an essential role in changing the character of the saturation curve for nuclear matter. The inclusion of such states leads to a strongly density dependent interaction which takes one off the Coester line[4] and leads to results in reasonable agreement with the generally accepted values for the binding energy and density of nuclear matter. (Our work may be contrasted to that of Walecka[5] in which the coupling constants of the meson fields are adjusted to yield the correct saturation properties of nuclear matter.) In our analysis we find that momentum-space potentials, such as the potential HM2 of Holinde and Machleidt,[6] which have a weak tensor force and would, in a standard calculation, lead to significant overbinding of nuclear matter at much too high a density, can be brought into agreement with the data if a relativistic calculation is made.[1] There is some evidence in favor of such forces, which have weak tensor interactions, from the analysis of forward proton production in the photodisintegration of the deuteron.[7] In addition, this type of force is favored on the basis of other theoretical studies.[8-10]

We are presently applying our model to the study of finite nuclei and will report on our results elsewhere. In this paper we extend our analysis to include a description of nucleon-nucleus scattering. In particular, we present a novel reduction of the Dirac equation to a Schrödinger form. Once this reduction is made we can compare our results for the parameters of the optical potential with the values obtained in phenomenological studies. In Sec. II we review the standard reduction scheme used to obtain a Schrödinger equation from the Dirac equation and then present our approach to this problem. In Sec. III we develop various connections between our wave function and the T matrix associated with the optical potential for nucleon-nucleus scattering. In Sec. IV we extrapolate our results for nuclear matter to yield the parameters of the optical potential. We provide some discussion of our results and some concluding remarks in Secs. V and VI.

II. REDUCTION OF THE DIRAC EQUATION TO SCHRÖDINGER FORMS

Using the techniques of Refs. 1 and 11 we may introduce a Dirac equation which describes the scattering of a nucleon of momentum $\vec{k}$ and spin projection s from a spin-zero target. If we neglect recoil we have

$$E(\vec{k})\gamma^0\psi_{\vec{k},s}(\vec{p}) = (\vec{\gamma}\cdot\vec{p}+m)\psi_{\vec{k},s}(\vec{p}) + \int \Sigma(\vec{p}|k|\vec{p}\,')\psi_{\vec{k},s}(\vec{p}\,')d\vec{p}\,'. \quad (2.1)$$

Here

$$E(\vec{k}) = (\vec{k}^2+m^2)^{1/2}, \quad (2.2)$$

and $\Sigma(k)$ is the nucleon self-energy operator evaluated at the energy $E(\vec{k})$. In Fig. 1 we depict various (relativistic) approximations which may be used to construct the self-energy. For example,

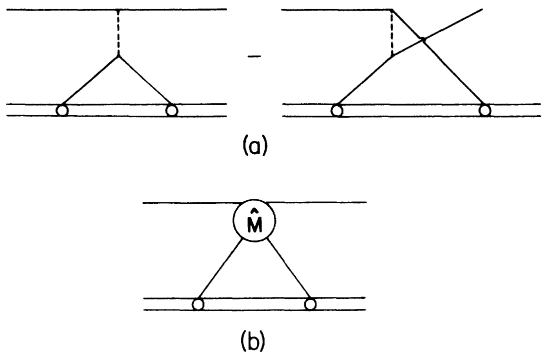

FIG. 1. Diagrammatic representations of approximations that may be used in the calculation of the nucleon self-energy. The dashed line represents a meson propagator and the single lines represent nucleon propagators. The double line represents a nucleus of A or $A-1$ particles and the small circles are vertex functions. (a) The Hartree-Fock approximation is obtained if both the direct and exchange terms are calculated. The Hartree approximation corresponds to the calculation of only the direct term. (b) A calculation of the self-energy which includes correlation effects. Here $\hat{M}$ represent a nucleon-nucleon scattering amplitude in the medium. This amplitude is modified from the free-space amplitude by the inclusion of Pauli principle effects, etc.

one may use the Hartree, Hartree-Fock, or Brueckner-Hartree-Fock approximations. (The use of the first two of these approximations requires the introduction of *effective* coupling constants.)

The self-energy is, in general, a nonlocal, energy-dependent operator.[1,11] However, if use is made of the Hartree approximation,[12,16] Σ is both local and energy independent. The use of the Hartree approximation therefore allows for a relatively simple reduction of the Dirac equation to a Schrödinger form. We will first review the techniques used in this case and then go on to introduce some new techniques appropriate to the theories where Σ is a nonlocal operator.

In the next section we will show that the wave function introduced in Eq. (2.1) is related to a coordinate-space wave function as [see Eqs. (3.6) and (3.7)]

$$\psi_{\vec{k},s}(\vec{p})=\int \frac{e^{-i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}\,\psi_{\vec{k},s}(\vec{x})\left[\frac{E(\vec{k})}{m}\right]^{1/2} d\vec{x}, \tag{2.3}$$

where

$$\psi_{\vec{k},s}(\vec{x})=\langle g|\Psi(\vec{x},0)|\vec{k},s;g\rangle_{\text{in}}. \tag{2.4}$$

Here $\Psi(\vec{x},t)$ is the nucleon field operator, $|g\rangle$ is the ground state of the target nucleus, and $|\vec{k},s;g\rangle_{\text{in}}$ is the *exact* scattering state of the nucleon-nucleus system which, when evolved back in time to $t=-\infty$, reduces to a plane wave and the target in its ground state.

In Refs. 12–16, where use is made of a (static) *Hartree* approximation for the exchange of isoscalar scalar and vector mesons, Σ is represented as

$$\Sigma(\vec{x})=A(|\vec{x}|)+\gamma^0 B(|\vec{x}|). \tag{2.5}$$

This results in a Dirac equation of the form

$$E(\vec{k})\gamma^0\psi_{\vec{k},s}(\vec{x})=[\vec{\gamma}\cdot(-i\vec{\nabla})+m+\Sigma(\vec{x})]\psi_{k,s}(\vec{x}). \tag{2.6}$$

This is a simplified coordinate-space version of Eq. (2.1) which is obtained in the Hartree approximation. One may then write the wave function as

$$\psi_{\vec{k},s}(\vec{x})=\begin{pmatrix} F_{\vec{k}}(\vec{x})\chi_s \\ G_{\vec{k}}(\vec{x})\chi_s \end{pmatrix}. \tag{2.7}$$

Now if, as is usual, one eliminates the lower components and sets

$$F_{\vec{k}}(\vec{x})=\frac{1}{x}\left[\frac{E(\vec{k})+m+A(x)-B(x)}{E(\vec{k})+m}\right]^{1/2}\hat{F}_{\vec{k}}(\vec{x}), \tag{2.8}$$

one finds that $\hat{F}_{\vec{k}}(\vec{x})$ obeys a Schrödinger equation of the form

$$\frac{k^2}{2m}\hat{F}_{\vec{k}}(\vec{x})=\left(-\frac{\partial^2}{\partial x^2}+\frac{\vec{1}_{\rm op}{}^2}{x^2}\right)\frac{1}{2m}\hat{F}_{\vec{k}}(\vec{x}) + U[E(\vec{k}),x]\hat{F}_{\vec{k}}(\vec{x}), \tag{2.9}$$

where

$$U(E,x)=V_c(E,x)+V_{\rm so}(E,x)\vec{\sigma}\cdot\vec{1}_{\rm op}. \tag{2.10}$$

Here the central and spin-orbit potentials are given by

$$V_c(E,x)=A(x)+\frac{E}{m}B(x)+[A^2(x)-B^2(x)]/2m - \frac{1}{2m}\left[\overline{\frac{\mathfrak{D}^{1/2}(E,x)}{x^2}\frac{d}{dx}\frac{x^2}{\mathfrak{D}(E,x)}\frac{d\mathfrak{D}^{1/2}}{dx}(E,x)}\right], \tag{2.11}$$

and

$$V_{\rm so}(E,x)=-\frac{1}{2mx}\frac{\mathfrak{D}'(E,x)}{\mathfrak{D}(E,x)}, \tag{2.12}$$

where

$$\mathfrak{D}(E,x)=E+m+A(x)-B(x). \tag{2.13}$$

The bar in Eq. (2.11) indicates that the differential operators do not act on the wave function. Equation (2.9) is in a form that may be compared to the forms used in conventional optical-model analyses. In Refs. 12–16, the numerical values of A

and B are related to potentials describing the exchange of scalar and vector mesons, respectively. Values of these parameters obtained from nuclear matter studies are then able to reproduce, in an approximate fashion, the phenomenological optical potentials employed in nucleon-nucleus analyses below 100 MeV. We point out that for any realistic evaluation of Σ which does not use the Hartree approximation, the technique used in arriving at Eq. (2.9) is inapplicable. We also note that $\hat{F}(x)$ and $xF(x)$ have the same asymptotic form. However $F(x)$ is an auxiliary quantity and Eq. (2.8) must be used to construct the wave function of interest, $\psi_{\vec{k},s}(\vec{x})$.

In what follows we suggest an alternative reduction of Eq. (2.1) to a nonrelativistic form. This new reduction scheme is more generally applicable than the one considered above.

Let us expand $\psi_{\vec{k},s}(\vec{p})$ in the complete set of free-particle spinors, i.e.,

$$\psi_{\vec{k},s}(\vec{p})=\sum_{s'}\psi^{+(s')}_{\vec{k},s}(\vec{p})\,u^{(s')}(\vec{p})+\psi^{-(s')}_{\vec{k},s}(\vec{p})w^{(s')}(\vec{p}), \quad (2.14)$$

where (using the Bjorken and Drell convention)

$$w^{(s)}(\vec{p})=v^{(-s)}(-\vec{p}). \quad (2.15)$$

The u's and w's are such that

$$u^{(s)\dagger}(\vec{p})w^{(s')}(\vec{p})=0;$$
$$u^{(s)\dagger}(\vec{p})\,u^{(s')}(\vec{p})=w^{(s)\dagger}(\vec{p})w^{(s')}(\vec{p})=\delta_{ss'}\frac{E(\vec{p})}{m} \quad (2.16)$$

and

$$\left[\frac{m}{E(\vec{p})}\right]\left[\sum_s u^{(s)}(\vec{p})u^{(s)\dagger}(\vec{p})+w^{(s)}(\vec{p})w^{(s)\dagger}(\vec{p})\right]=\gamma^0. \quad (2.17)$$

If Eq. (2.14) is substituted into Eq. (2.1), one finds that the expansion coefficients obey the equations

$$\frac{E(\vec{p})}{m}[E(\vec{k})-E(\vec{p})]\psi^{+(s')}_{\vec{k},s}(\vec{p})=\int d\vec{p}'\,\Sigma^{++}_{s's''}(\vec{p}|k|\vec{p}')\psi^{+(s'')}_{\vec{k},s}(\vec{p}')+\int d\vec{p}'\,\Sigma^{+-}_{s's''}(\vec{p}|k|\vec{p}')\psi^{-(s'')}_{\vec{k},s}(\vec{p}') \quad (2.18)$$

and

$$\frac{E(\vec{p})}{m}[E(\vec{k})+E(\vec{p})]\psi^{-(s')}_{\vec{k},s}(\vec{p})=\int d\vec{p}'\,\Sigma^{--}_{s's''}(\vec{p}|k|\vec{p}')\psi^{-(s'')}_{\vec{k},s}(\vec{p}')+\int d\vec{p}'\,\Sigma^{-+}_{s's''}(\vec{p}|k|\vec{p}')\psi^{+(s'')}_{\vec{k},s}(\vec{p}'), \quad (2.19)$$

where, for example,

$$\Sigma^{++}_{ss'}(\vec{p}|k|\vec{p}')=\bar{u}^{(s)}(\vec{p})\Sigma(\vec{p}|k|\vec{p}')u^{(s')}(\vec{p}'),$$
$$\Sigma^{-+}_{ss'}(\vec{p}|k|\vec{p}')=\bar{w}^{(s)}(\vec{p})\Sigma(\vec{p}|k|\vec{p}')u^{(s')}(\vec{p}'), \quad (2.20)$$

etc. In a manner analogous to the elimination of the lower components in the conventional treatment, we eliminate the coefficients of $w^{(s)}$ in Eq. (2.18) using Eq. (2.19). Further we note that

$$\frac{E(\vec{p})}{m}[E(\vec{k})-E(\vec{p})]=\frac{(k^2-p^2)}{[E(\vec{p})+E(\vec{k})]}\cdot\frac{E(\vec{p})}{m}\equiv\frac{(k^2-p^2)}{2m}R^{-2}(p,k). \quad (2.21)$$

We then define a wave function $\hat{\psi}$ and a potential $V(k)$:

$$\psi^{+(s')}_{\vec{k},s}(\vec{p})=R(p,k)\hat{\psi}^{(s')}_{\vec{k},s}(\vec{p}) \quad (2.22)$$

and

$$\langle\vec{p},s|V(k)|\vec{p}',s'\rangle=R(p,k)\Sigma^{++\text{eff}}_{ss'}(\vec{p}|k|\vec{p}')R(p',k). \quad (2.23)$$

Here

$$\Sigma^{++\text{eff}}_{ss'}(\vec{p}|k|\vec{p}')=\Sigma^{++}_{ss'}(\vec{p}|k|\vec{p}')+\int d\vec{q}\,d\vec{q}'\,\Sigma^{+-}_{sr}(\vec{p}|k|\vec{q})G^{--}_{rr'}(\vec{q}|k|\vec{q}')\times\Sigma^{-+}_{r's'}(\vec{q}'|k|\vec{p}'), \quad (2.24)$$

where

$$G^{---1}_{ss'}(\vec{q}|k|\vec{q}')=\delta(\vec{q}-\vec{q}')\delta_{ss'}\frac{E(\vec{q})}{m}\cdot[E(\vec{k})+E(\vec{q})]-\Sigma^{--}_{ss'}(\vec{q}|k|\vec{q}'). \quad (2.25)$$

If we make use of Eqs. (2.18)–(2.25) we find

$$\frac{k^2-p^2}{2m}\hat{\psi}^{(s'')}_{\vec{k},s}(\vec{p})=\int d\vec{p}'\langle\vec{p},s''|V(k)|\vec{p}',s'\rangle\hat{\psi}^{(s')}_{\vec{k},s}(\vec{p}'), \quad (2.26)$$

which is a conventional Schrödinger equation (in momentum space) with a nonlocal, energy-dependent potential. Equation (2.26) is a central result of this section. In Sec. III we will discuss the use of $\hat{\psi}$ in the description of nucleon-nucleus scattering. We may note at this point that like $\hat{F}$, $\hat{\psi}$ is an auxiliary quantity and is related to $\psi^{+(s')}_{\vec{k},s}(\vec{p})$ by the function $R(p,k)$—see Eq. (2.22). Since $R^{-2}(p,k)=2E(\vec{p})/[E(\vec{p})+E(\vec{k})]$, if $|\vec{p}|$ and $|\vec{k}|$ are small compared to M, we have $\hat{\psi}_{\vec{k},s}(s')(\vec{p})\simeq\psi^{+}_{\vec{k},s}(s')(\vec{p})$. This correspondence is not achieved in a comparison of $F(x)$ and $xF(x)$ since $A(x)-B(x)$ is a large quantity (of the order of -700 MeV) and cannot be neglected in Eq. (2.8).

Our basic equation, Eq. (2.26), is nonlocal. It is useful to construct a local equation that can be compared to the local phenomenological forms conventionally used. To this end we construct a *simple* nonlocal form which approximates $\langle \vec{p}, s|V(k)|\vec{p}', s'\rangle$ of Eq. (2.26). We chose to consider

$$\langle \vec{p}, s|V(k)|\vec{p}', s'\rangle = \delta_{ss'}\left[\langle \vec{p}|\hat{V}_C|\vec{p}'\rangle + \left(\frac{\vec{p}+\vec{p}'}{2k_f}\right)^2 \langle \vec{p}|\hat{V}_{NL}|\vec{p}'\rangle + i(\vec{p}\times\vec{p}')\cdot\vec{\sigma}_{ss'}\langle \vec{p}|\hat{V}_{SP}|\vec{p}'\rangle/k_f^2\right], \quad (2.27)$$

where $\hat{V}_C$, $\hat{V}_{NL}$, and $\hat{V}_{SP}$ are *local* potentials:

$$\langle \vec{p}|\hat{V}_C|\vec{p}'\rangle = \int \frac{d\vec{x}}{(2\pi)^3} e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}\hat{V}_C(\vec{x}), \quad (2.28)$$

$$\langle \vec{p}|\hat{V}_{NL}|\vec{p}'\rangle = \int \frac{d\vec{x}}{(2\pi)^3} e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}\hat{V}_{NL}(\vec{x}), \quad (2.29)$$

and

$$\langle \vec{p}|\hat{V}_{SP}|\vec{p}'\rangle = \int \frac{d\vec{x}}{(2\pi)^3} e^{-i(\vec{p}-\vec{p}')\cdot\vec{x}}\hat{V}_{SP}(\vec{x}). \quad (2.30)$$

We extrapolate our nuclear results to the finite system by introducing form factors

$$\hat{V}_C(\vec{x}) = V_C^{NM} f(\vec{x}), \quad (2.31)$$

$$\hat{V}_{NL}(\vec{x}) = V_{NL}^{NM} f(\vec{x}), \quad (2.32)$$

with $f(\vec{x}) = [1+\exp(x-R)/a]^{-1}$. Approximate values for V_C^{NM} and V_{NL}^{NM} may be taken from a parametrization of the nuclear matter results for $\Sigma^{++\text{eff}}(\vec{p})$. As we will discuss in more detail later [see Eq. (4.4)], $\Sigma^{++\text{eff}}(\vec{p}) \simeq [-90+29(p/k_F)^2]$ MeV. Thus we can put $V_C^{NM} \simeq -90$ MeV and $V_{NL}^{NM} \simeq 29$ MeV. Thus if $f(\vec{x})=1$, as is appropriate for nuclear matter, Eq. (2.27) becomes

$$\langle \vec{p}, s|V(k)|\vec{p}', s'\rangle = \delta_{ss'}\delta(\vec{p}-\vec{p}')\Sigma^{++\text{eff}}(\vec{p}). \quad (2.33)$$

From this discussion we can understand the choice made for the first two terms on the right hand side of Eq. (2.27). The specification of the spin-orbit term is more complicated and we defer discussion of that term to the end of Sec. IV. However, we note that the spin-orbit term has a rather complicated dependence on the nuclear form factors and writing $V_{SP}(\vec{x}) \sim 1/xf'(x)$ would be a somewhat crude approximation—see Eqs. (4.25) and (4.26). Similar objections can be made to the use of Eq. (2.31) and (2.32) since these approximations are linear in the form factors while there are some terms in $\Sigma^{++\text{eff}}(\vec{p})$ that are of a more complex structure. These terms are grouped together as $\Sigma_{RC}(\vec{p})$ in Eqs. (4.2) and (4.3); however, they are a relatively small part of $\Sigma^{++\text{eff}}(\vec{p})$, and thus the approximation of Eqs. (2.31) and (2.32) is adequate for the qualitative discussion of this section.

With these complications in mind, the potential of Eq. (2.27) becomes

$$\langle \vec{x}, s|V(k)|\vec{x}', s'\rangle = \delta(\vec{x}-\vec{x}')V_{ss'}(\vec{x}), \quad (2.34)$$

where

$$2mV_{ss'}(\vec{x}) = \delta_{ss'}\left\{2m\hat{V}_C(x) + \frac{2m}{k_f^2}\left[\hat{V}_{NL}(x)\vec{p}_{op}^{\,2} - \overline{\frac{\partial}{\partial x}\hat{V}_{NL}(x)}\frac{\partial}{\partial x} - \overline{\frac{1}{4x^2}\frac{\partial}{\partial x}x^2\frac{\partial}{\partial x}\hat{V}_{NL}(x)}\right]\right\} + \frac{2m}{k_f^2}\frac{1}{x}\frac{\partial}{\partial x}\hat{V}_{SP}(x)\vec{\sigma}_{ss'}\cdot\vec{l}_{op}. \quad (2.35)$$

Equations (2.34) and (2.35) may be used to construct a coordinate-space version of Eq. (2.26). Further, if we define functions $\phi_{\vec{k},s}^{(s')}(\vec{x})$ and $\phi_{\vec{k},s}(\vec{x})$:

$$\phi_{\vec{k},s}^{(s')}(\vec{x}) = x\hat{\psi}_{\vec{k},s}^{(s')}(\vec{x}) = x\int d\vec{p}\langle \vec{x}|\vec{p}\rangle\hat{\psi}_{\vec{k},s}^{(s')}(\vec{p}), \quad (2.36)$$

$$\phi_{\vec{k},s}(\vec{x}) \equiv \sum_{s'} \chi_{s'}\phi_{\vec{k},s}^{(s')}(\vec{x}), \quad (2.37)$$

we find that $\phi_{\vec{k},s}(\vec{x})$ obeys the Schrödinger equation

$$\frac{k^2}{D(x)}\phi_{\vec{k},s}(\vec{x}) = \left\{-\frac{1}{D(x)}\frac{\partial}{\partial x}D(x)\frac{\partial}{\partial x} + \frac{\vec{l}_{op}^{\,2}}{x^2} + 2m\tilde{V}_C(x) + \frac{2m}{k_f^2}\tilde{V}_{SP}(x)\vec{\sigma}\cdot\vec{l}_{op} - \frac{2m}{k_f^2}\frac{1}{4D(x)} \times\left[\overline{x^2\frac{\partial}{\partial x}\frac{1}{x^2}\frac{\partial}{\partial x}\hat{V}_{NL}(x)}\right]\right\}\phi_{\vec{k},s}(\vec{x}), \quad (2.38)$$

where

$$D(x) = 1 + \frac{2m}{k_f^2}\,\hat{V}_{\rm NL}(x)\;, \tag{2.39}$$

and

$$\tilde{V}_C(x) \equiv \hat{V}_C(x)/D(x)\;, \tag{2.40}$$

$$\tilde{V}_{\rm SP}(x) \equiv \left[\frac{1}{x}\,\frac{\partial}{\partial x}\,\hat{V}_{\rm SP}(x)\right]\Big/D(x)\;. \tag{2.41}$$

In Sec. IV we will provide some numerical estimates for $D(x)$, $V_C(x)$, and $V_{\rm SP}(x)$ based upon our recent nuclear matter calculations.

We have not as yet achieved our goal of writing an equation of the Schrödinger form. To this end, we extract an "effective-mass" factor $D(x)$ from $\phi_{\vec{k},s}^{\dagger}(\vec{x})$ and introduce the function $\hat{\phi}_{\vec{k},s}^{\dagger}(\vec{x})$:

$$\phi_{\vec{k},s}^{\dagger}(\vec{x}) = \hat{\phi}_{\vec{k},s}^{\dagger}(\vec{x})/[D(x)]^{1/2}\;. \tag{2.42}$$

The auxiliary function $\phi_{\vec{k},s}^{\dagger}(\vec{x})$ obeys the equation

$$\begin{aligned} k^2\hat{\phi}_{\vec{k},s}^{\dagger}(\vec{x}) = \Bigg\{ & -\frac{\partial^2}{\partial x^2} + \frac{\vec{l}_{\rm op}^{\,2}}{x^2} + 2m\,\overline{V}_C(x) \\ & + \frac{2m}{k_f^2}\,\tilde{V}_{\rm SP}(x)\vec{\sigma}\cdot\vec{l}_{\rm op} \\ & + k^2\,\frac{[D(x)-1]}{D(x)}\Bigg\}\hat{\phi}_{\vec{k},s}^{\dagger}(\vec{x})\;, \end{aligned} \tag{2.43}$$

where

$$2m\,\overline{V}_C(x) \equiv 2m\hat{V}_C(x)/D(x) + \overline{\frac{1}{4x^2}\,\frac{\partial}{\partial x}\,x^2\,\frac{\partial}{\partial x}\,[\ln D(x)]}\,. \tag{2.44}$$

We note that Eq. (2.43), if used to analyze data, will yield a wave function $\hat{\phi}$ which is phase-shift equivalent to the wave function ϕ. [See Eq. (2.42).] It is the latter wave function, however, which is more closely related to the fundamental quantity $\psi_{\vec{k},s}^{\ddagger}(\vec{p})$ appearing in Eq. (2.14). [See Eqs. (2.22), (2.36), (2.37), and (2.42).]

We note that the potential $\overline{V}_C(x)$ of Eq. (2.43) is related to $\hat{V}_C(x)$ through the effective-mass factor $D(x)$—see Eq. (2.44). Further, an energy-dependent potential, the last term on the right-hand side of Eq. (2.43), appears. This letter term has its origin in the requirement that Eq. (2.26) be transformed to appear as a local nonrelativistic Schrödinger equation.[17] We finally note that we may write $D(x)=m/m^*(x)$, where $m^*(x)$ is a density-dependent effective mass. An expression for $m^*(x)$ may be obtained from Eq. (2.39):

$$\frac{m^*(x)}{m} = \frac{1}{1+\dfrac{2m}{k_f^2}\,\hat{V}_{\rm NL}(x)}\;. \tag{2.45}$$

III. THE S AND T MATRICES

In this section we shall develop the connection between $\hat{\psi}_{\vec{k},s}$, introduced in the last section, and the S and T matrices describing nucleon-nucleus scattering. (We continue to neglect target recoil.) The S matrix for nucleon nucleus scattering is given by

$$\langle \vec{k}'s'|S|\vec{k},s\rangle = \langle g|a_{\vec{k}',s'}^{(+)}a_{\vec{k},s}^{(-)\dagger}|g\rangle\;, \tag{3.1}$$

where $|g\rangle$ is the ground state of the target and $a_{\vec{k},s}^{(-)\dagger}$ is a creation operator for the incoming nucleon in the Heisenberg representation and is related to the nucleon field operator $\Psi(\vec{x},t)$ by

$$a_{\vec{k},s}^{(\pm)} = \int_{t\to\pm\infty} d\vec{x}\,\bar{u}^{(s)}(\vec{k})\gamma^0\Psi(\vec{x},t)\,\frac{e^{ik\cdot x}}{(2\pi)^{3/2}}\left[\frac{m}{E(\vec{k})}\right]^{1/2}\,. \tag{3.2}$$

One may develop $a_{\vec{k},s}^{(+)}$ as follows:

$$\begin{aligned} a_{\vec{k},s}^{(+)} = a_{\vec{k},s}^{(-)} - i\int d\vec{x}\,dt\,\bar{u}^{(s)}(\vec{k})\gamma^0\left[\left(\frac{1}{-i}\,\frac{\partial}{\partial t} - k^0\right)\Psi(\vec{x},t)\right] \\ \times\frac{e^{ik\cdot x}}{(2\pi)^{3/2}}\left[\frac{m}{E(\vec{k})}\right]^{1/2}\,. \end{aligned} \tag{3.3}$$

Since

$$\Psi(\vec{x},t) = e^{iHt}\Psi(\vec{x},0)e^{-iHt} \tag{3.4}$$

and therefore

$$\langle g|\Psi(\vec{x},t)|\vec{k},g\rangle = e^{-iE(\vec{k})t}\langle g|\Psi(\vec{x},0)|\vec{k},g\rangle_{\rm in}\;, \tag{3.5}$$

we find

$$\begin{aligned} \langle \vec{k}'s'|S|\vec{k},s\rangle & = \delta(\vec{k}-\vec{k}')\delta_{ss'} - 2\pi i\delta[E(\vec{k})-E(\vec{k}')] \\ & \times\lim_{k\to k'}[E^+(\vec{k})-E(\vec{k}')]\,\frac{1}{(2\pi)^{3/2}}\left[\frac{m}{E(\vec{k}')}\right]^{1/2} \\ & \times\bar{u}^{(s')}(\vec{k}')\gamma^0\int d\vec{x}\,e^{-i\vec{k}'\cdot\vec{x}}\langle g|\Psi(\vec{x},0)|\vec{k},g\rangle_{\rm in}\;. \end{aligned} \tag{3.6}$$

It is now convenient to introduce Eqs. (2.3) and (2.4) so that

$$\langle \vec{k}',s'|S|\vec{k},s\rangle = \delta(\vec{k}-\vec{k}')\delta_{ss'} - 2\pi i\delta[E(\vec{k})-E(\vec{k}')]\lim_{k\to k'}[E^+(\vec{k})-E(\vec{k}')]\left[\frac{m^2}{E(\vec{k})E(\vec{k}')}\right]^{1/2}\bar{u}^{(s')}(\vec{k}')\gamma^0\psi_{\vec{k},s}(\vec{k}')\;. \tag{3.7}$$

Therefore, the T matrix is given by

$$\langle \vec{k}', s' | T_E | \vec{k}, s\rangle = \left[\frac{m^2}{E(\vec{k})E(\vec{k}')}\right]^{1/2} \lim_{k\to k'} [E^+(\vec{k}) - E(\vec{k}')] \frac{E(\vec{k}')}{m} \psi^{+(s')}_{\vec{k},s}(\vec{k}') = \lim_{k\to k'} [E^+(\vec{k}) - E(\vec{k}')] \hat{\psi}^{(s')}_{\vec{k},s}(\vec{k}') = \lim_{k\to k'} \frac{(k^2 - k'^2 + i\epsilon)}{2E(\vec{k})} \hat{\psi}^{(s')}_{\vec{k},s}(\vec{k}')$$

$$= \int \frac{1}{2E(\vec{k}')} \langle \vec{k}', s' | 2mV(k) | \vec{k}'', s''\rangle \hat{\psi}^{(s'')}_{\vec{k},s}(\vec{k}'') d\vec{k}'' \ . \tag{3.8}$$

If we rewrite S as

$$\langle \vec{k}', s' | S | \vec{k}, s\rangle = \frac{\delta(k-k')}{kk'} [\delta(\hat{k} - \hat{k}') - \pi i k \langle \vec{k}', s' | T(k) | \vec{k}, s\rangle] \ , \tag{3.9}$$

we find that

$$\langle \vec{k}', s' | T(k) | \vec{k}, s\rangle = \int d\vec{k}'' \langle \vec{k}', s' | 2mV(k) | \vec{k}'', s''\rangle \hat{\psi}^{(s'')}_{\vec{k},s}(\vec{k}'') \ . \tag{3.10}$$

We should now like to invert Eq. (2.1) or alternatively Eqs. (2.18) and (2.19) or (2.26). Clearly, a properly normalized solution to Eq. (2.1) with Σ equal to zero is

$$\psi^0_{\vec{k},s}(\vec{k}') = u^{(s)}(\vec{k})\delta(\vec{k}' - \vec{k}) \ . \tag{3.11}$$

Correspondingly, we have

$$\psi^{+(s')0}_{\vec{k},s}(\vec{k}') = \delta_{s',s}\delta(\vec{k}' - \vec{k}) \ , \tag{3.12}$$

$$\psi^{-(s')0}_{\vec{k},s}(\vec{k}') = 0 \ , \tag{3.13}$$

and

$$\hat{\psi}^{(s')0}_{\vec{k},s}(\vec{k}') = \delta_{s',s}\delta(\vec{k}' - \vec{k}) \ . \tag{3.14}$$

Therefore,

$$\hat{\psi}^{(s')}_{\vec{k},s}(\vec{k}') = \delta_{s',s}\delta(\vec{k}' - \vec{k}) + \frac{\langle \vec{k}', s' | T(k) | \vec{k}, s\rangle}{k^2 - k'^2 + i\epsilon} \ , \tag{3.15}$$

where T for $k' \neq k$ is given by the same formula as in Eq. (3.10). Finally with $\overline{V}(k) = 2mV(k)$ we find, after introducing Eq. (3.15) into Eq. (3.10), that

$$T(k) = \overline{V}(k) + \overline{V}(k) \left(\frac{1}{k^2 - p_{op}^{\ 2} + i\epsilon}\right) T(k) \ . \tag{3.16}$$

IV. NUMERICAL RESULTS AND INTERPRETATION

We have solved Eq. (2.1) for values of energy parameters appropriate to occupied states in nuclear matter.[1] These calculations may be extended to obtain both bound and continuum solutions of this equation for finite nuclei. At this state, however, it seems appropriate to attempt to extrapolate our nuclear matter results to the optical-model regime of 0–100 MeV projectile energy in a manner similar to that used by other authors. We devote this section to this application of our nuclear matter results to the determination of some optical-model parameters.

Our best fit to nuclear matter properties was accomplished by using the potential of Holinde and Machleidt designated HM2.[6] This potential has a large tensor coupling for the ρ meson and hence a small net tensor force. This small tensor force is currently favored on the basis of other considerations.[7-10] (Results for other potentials we have considered are similar to those for HM2 but do not fit nuclear matter quite as well.[1]) In Figs. 2, 3, and 4 we exhibit $\Sigma^{++}(p)$, $\Sigma^{-+}(p)$, and $\Sigma^{--}(p)$. The quantities were calculated in nuclear matter for the potential HM2 (Ref. 1) and are analogous to the Σ^{++}, Σ^{-+}, and Σ^{--} considered here. The calculations of Ref. 1 include full two-body correlation effects and the effects of the exchange of all the various mesons of the one-boson-exchange model of nuclear forces ($\sigma, \omega, \pi, \rho, \ldots$).

For the analysis of this section it is useful to parametrize the nucleon self-energy using the nuclear matter results. To this end we depict various contributions to the nucleon self-energy in Fig. 5. In this figure single lines refer to states described by positive-energy spinors and double lines refer to negative-energy states. The wavy lines are reaction matrices. (We have only shown the direct terms for simplicity.) Numerical results for the quantity $\Sigma^{++}(p)$ shown in Fig. 2 represent the results of a calculation of the process depicted in Fig. 5(a) (plus the associated exchange term). Figure 5(b) is a diagrammatic representation of the second term on the right-hand side of Eq. (2.24) and can be estimated from the knowledge of $\Sigma^{-+}(p)$ and $\Sigma^{--}(p)$. (See Figs. 3 and 4.) The values for $\Sigma^{++}(p)$ shown in Fig. 2 do not contain the relativistic correction

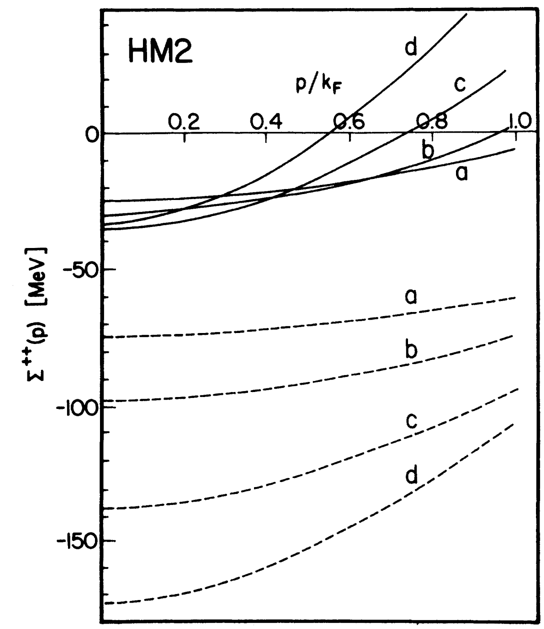

FIG. 2. The self-energy $\Sigma^{++}(p)$ calculated for the potential HM2. The calculation is made for symmetric nuclear matter for various values of k_F. (a) $k_F = 1.2\ \text{fm}^{-1}$, (b) $k_F = 1.36\ \text{fm}^{-1}$, (c) $k_F = 1.6\ \text{fm}^{-1}$, and (d) $k_F = 1.8\ \text{fm}^{-1}$. The solid lines are the result of calculations made in the Hartree-Fock approximation [Fig. 1(a)] while the dashed lines are the results for calculations which include correlation effects [Fig. 1(b)].

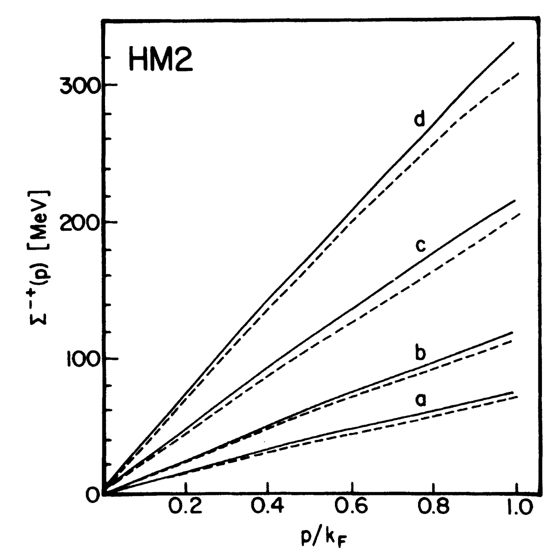

FIG. 3. The results of calculations made for symmetric nuclear matter for the transition potential $\Sigma^{-+}_{1/2\,1/2}(p)$ are shown in this figure. (See caption to Fig. 2).

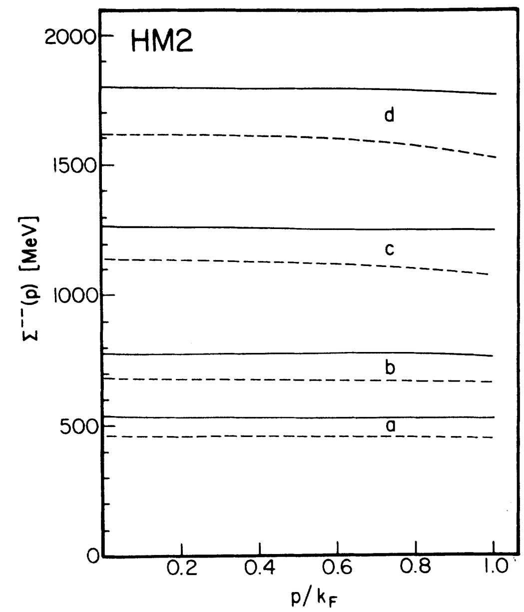

FIG. 4. The results of calculations of the quantity $\Sigma_{ss}^{--}(p) = \bar{w}^{(s)}(\vec{p})\Sigma(p)w^{(s)}(p)$ made for symmetric nuclear matter are shown in this figure. (See caption to Fig. 2).

terms exhibited in Fig. 5(c). These can be estimated from other considerations. The results for the leading term of Σ^{++} [Fig. 5(a)], which are given in Fig. 2, may be approximated as follows

$$\Sigma^{++}(p) = [-97.5 + 22.5(p/k_f)^2]\ \text{MeV}\,. \tag{4.1}$$

The remaining terms in the self-energy, depicted in Figs. 5(b) and 5(c), can be grouped together as "relativistic corrections" and designated as $\Sigma_{\text{RC}}(p)$. The correction shown as Fig. 5(b) is essentially $|\Sigma^{-+}(p)|^2/(2m)$ and is approximately equal to $(5/3)(3.86)(p/k_F)^2$ MeV. The other two terms in Fig. 5(c) are less strongly momentum dependent and may be approximated as constants $\simeq 3.86$ MeV. Therefore we have for the relativistic correction terms of Figs. 5(b) and 5(c)

$$\Sigma_{\text{RC}}(p) = [2(3.86) + 3.86(5/3)(p/k_f)^2]\ \text{MeV} \tag{4.2}$$

$$= [7.72 + 6.43(p/k_f)^2]\,\text{MeV}\,, \tag{4.3}$$

and therefore, combining Eq. (4.1) and Eq. (4.3), we have

$$\Sigma^{++\text{eff}}(p) = \Sigma^{++}(p) + \Sigma_{\text{RC}}(p)$$

$$= [-89.8 + 28.9(p/k_f)^2]\ \text{MeV}\,. \tag{4.4}$$

We now wish to use these results to determine the parameters of our simplified nonlocal potential of Eq. (2.27). The first term in Eq. (4.4) will be associated with V_C and the second with V_{NL}. We write $V_{\text{NL}} = 28.9 f(x)$ MeV, where $f(x)$ is a form factor normalized such that $f(0) = 1$. Thus

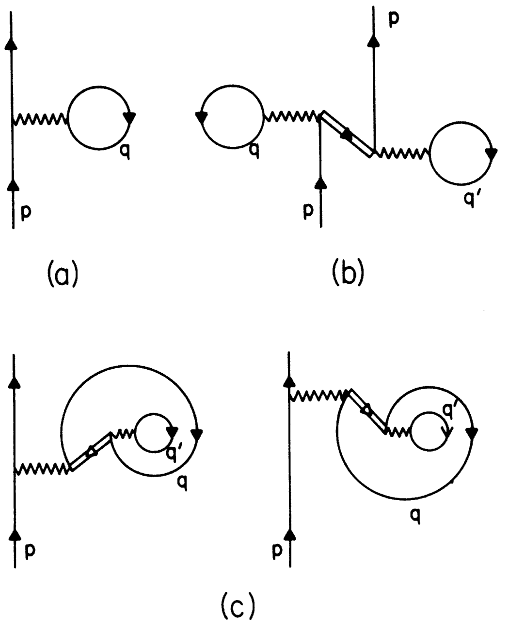

FIG. 5. Various contributions to the self-energy $\Sigma^{++}_{eff}(p)$ in nuclear matter. (Only direct diagrams are shown for simplicity.) The processes shown in (b) and (c), are grouped together as relativistic corrections which involve negative-energy states; the sum of these terms is called $\Sigma^{++}_{RC}(p)$ in the text. In these figures the wavy line is a reaction matrix which includes the effects of nucleon-nucleon correlations and the single lines denote states represented by positive-energy spinors. (a) The evaluation of this diagram (plus the exchange counterpart) yields the values of $\Sigma^{++}(p)$ shown in Fig. 2. (An integral over $\vec{q}$, where $|\vec{q}| \leq k_f$, is performed in the evaluation of this diagram.) (b) This part of $\Sigma^{++}_{RC}(p)$, plus the corresponding exchange terms, is equal to $|\Sigma^{+-}_{1/2\,1/2}(p)|^2/2m$ and may be estimated using the results for $\Sigma^{+-}_{1/2\,1/2}(p)$ given in Fig. 3. (c) These contributions to $\Sigma^{++}_{RC}(p)$ arise when one takes into account the negative-energy states in the density matrix of the target. [In a calculation of Fig. 1 (a), using the self-consistent density matrix for the target which contains negative-energy state components, one would include both the processes shown in Fig. 5(a) and Fig. 5(c) in a single calculation.]

$$\frac{m^*(x)}{m} = \frac{1}{1+28.9\left(\frac{2m}{k_f^2}\right)f(x)}\,, \qquad (4.5)$$

and

$$\frac{m^*(0)}{m} = \frac{1}{1+28.9\left(\frac{2m}{k_f^2}\right)} = \frac{1}{1+0.76} = 0.57\,. \quad (4.6)$$

In a similar fashion we put $V_C = -89.8f(x)$ MeV. Therefore from Eq. (2.40) we have

$$\tilde{V}_C(x) = -\frac{89.8f(x)}{D(x)} = -\frac{89.8f(x)}{1+28.9\left(\frac{2m}{k_f^2}\right)f(x)}\,, \quad (4.7)$$

$$\tilde{V}_C(0) = -89.8(0.57)\ \text{MeV} = -51.1\ \text{MeV}\,. \quad (4.8)$$

This number represents the strength of the central potential to be used in the Schrödinger equation [Eq. (2.43)] if we neglect the second term in Eq. (2.44).

We next consider the parametrization of the energy-dependent term in the potential appearing in Eq. (2.43). We have

$$V_E(x) = \frac{k^2}{2m}\left[\frac{D(x)-1}{D(x)}\right] = \frac{k^2}{2m}\left[1-\frac{m^*(x)}{m}\right], \quad (4.9)$$

with

$$V_E(0) = 0.43(k^2/2m)\,. \quad (4.10)$$

Before discussing the spin-orbit term of Eq. (2.43), we remark on the self-energy in nuclear matter. We note that we may write using general invariance principles

$$\Sigma(p) = A(p) + \gamma^0 B(p) + \frac{\vec{\gamma}\cdot\vec{p}}{m}C(p)\,. \quad (4.11)$$

Here A appears as a kind of *effective* scalar field and B as an *effective* vector field. We have calculated these quantities for nuclear matter including effects of correlations. One finds that $C(p)$ is small and may be neglected in these considerations. We can parametrize our results as follows:

$$A(p) \simeq [-393 + 21(p/k_f)^2]\ \text{MeV}\,, \quad (4.12)$$

$$B(p) \simeq [294 - 12(p/k_F)^2]\ \text{MeV}\,, \quad (4.13)$$

and

$$\frac{E(\vec{p})}{m}B(p) \simeq [294 - 0.02(p/k_f^2)]\ \text{MeV}\,. \quad (4.14)$$

It is the latter quantity which appears directly in $\Sigma^{++\text{eff}}(p)$. Comparing Eqs. (4.12) and (4.14) we see that the energy-dependent term in the nonrelativistic Lippmann-Schwinger equation has its origin in the effective scalar field's nonlocality. The energy dependence associated with the vector field, i.e., the $E(\vec{p})/m$ factor in Eq. (4.11), serves to reduce the role of the nonlocality of the effective vector field when calculating the parameters of the optical potential.

The spin-orbit potential is more difficult to extract in our scheme. The terms in $V(k)$ or $\Sigma^{++\text{eff}}$ corresponding to Fig. 5(c) require detailed calculations. In addition, the correlations in finite nuclei may introduce additional spin-orbit terms in Σ^{++}. However, without these nontrivial complications we may still make a crude estimate of the spin-orbit potential. We recall that [See Eqs. (2.38) and (2.41)]

$$\frac{1}{k_f^2}\tilde{V}_{SP}(x)\vec{\sigma}\cdot\vec{l}_{op}=\frac{1}{k_f^2}\left[\frac{1}{D(x)}\frac{1}{x}\frac{\partial}{\partial x}\hat{V}_{SP}(x)\right]\vec{\sigma}\cdot\vec{l}_{op}\ , \tag{4.15}$$

where one can identify

$$\left\langle\vec{p}\left|\frac{1}{x}\frac{\partial}{\partial x}\overline{\frac{\hat{V}_{SP}}{k_f^2}}(x)\vec{\sigma}\cdot\vec{l}_{op}\right|\vec{p}\,'\right\rangle=\vec{\sigma}\cdot[\tfrac{1}{2}\mathrm{Tr}\vec{\sigma}\langle\vec{p}\,|V(k)|\,\vec{p}\,'\rangle]\ . \tag{4.16}$$

Using Eqs. (2.23), (2.24), and (2.25) we may obtain an explicit formula for $\langle\vec{p}\,|V(k)|\,\vec{p}\,'\rangle$. To estimate the magnitude of the spin-orbit potential in a finite system we take the self-energy operator to be

$$\Sigma(\vec{p}\,|\,\vec{p}\,')=\langle\vec{p}\,|A\,|\,\vec{p}\,'\rangle+\gamma^0\langle\vec{p}\,|B\,|\,\vec{p}\,'\rangle\ , \tag{4.17}$$

where

$$\langle\vec{p}\,|A\,|\,\vec{p}\,'\rangle=A_0\langle\vec{p}\,|f|\,\vec{p}\,'\rangle\ , \tag{4.18}$$

$$\langle\vec{p}\,|B\,|\,\vec{p}\,'\rangle=B_0\langle\vec{p}\,|f|\,\vec{p}\,'\rangle\ , \tag{4.19}$$

with

$$f(x)=\{1+\exp[-(R-x)/a]\}^{-1}\ . \tag{4.20}$$

We identify A_0 and B_0 as follows:

$$A_0=A(\vec{p}=0)[\rho_0/\rho_{NM}]\simeq-413\text{ MeV}\ , \tag{4.21}$$

$$B_0=B(\vec{p}=0)[\rho_0/\rho_{NM}]\simeq 310\text{ MeV}\ . \tag{4.22}$$

Here $\rho_0=(4\pi r_0^3/3)^{-1}=0.179\text{ fm}^{-3}$ at $r_0=1.1$ fm, and $\rho_{NM}=2k_f^3/(3\pi^2)=0.17\text{ fm}^{-3}$ for $k_f=1.36\text{ fm}^{-1}$. [We remind the reader that the values quoted in Eqs. (4.21) and (4.22) are based upon our relativistic Brueckner-Hartree-Fock calculations and include the full effects of two-body correlations, etc.]

For simplicity we shall neglect a term in Σ of the form $\vec{\gamma}\cdot(\vec{p}+\vec{p}\,')\times\langle\vec{p}\,|C\,|\,\vec{p}\,'\rangle/m$. From our previous calculations we can expect the contributions coming from this term will increase the result to be calculated here by at most 10%. If in the evaluation of Eqs. (2.23), (2.24), and (2.25) we ignore corrections of order $(p/m)^2$ or $(k/m)^2$ we find upon using Eq. (4.16)

$$\begin{aligned}\left\langle\vec{p}\left|\frac{1}{x}\frac{\partial}{\partial x}\overline{\frac{V_{SP}(x)}{k_f^2}}\vec{\sigma}\cdot\vec{l}_{op}\right|\vec{p}\,'\right\rangle&=-i\frac{\vec{\sigma}}{(2m)^2}\cdot\left\langle\vec{p}\left|\left(\{\vec{p}_{op}[A(x)-B(x)]\}\times\vec{p}_{op}-\{\vec{p}_{op}[A(x)-B(x)]+[A(x)+B(x)]\vec{p}_{op}\}\frac{1}{d(x)}\right.\right.\right.\\&\qquad\left.\left.\left.\times\{[A(x)-B(x)]\vec{p}_{op}+\vec{p}_{op}[A(x)+B(x)]\}\right)\right|\vec{p}\,'\right\rangle\\&=-\frac{1}{(2m)^2}\left\langle\vec{p}\left|\frac{1}{x}[A'(x)-B'(x)]\left\{1-\frac{4A(x)}{d(x)}\left[1-\frac{A(x)}{d(x)}\right]\right\}\vec{\sigma}\cdot\vec{l}_{op}\right|\vec{p}\,'\right\rangle\ ,\end{aligned} \tag{4.23}$$

where

$$d(x)=2m+A(x)-B(x)\ . \tag{4.24}$$

Therefore

$$\frac{1}{k_f^2}\tilde{V}_{SP}(x)\vec{\sigma}\cdot\vec{l}_{op}=\frac{1}{m_\pi^2}\frac{1}{D(x)}\left(-\left(\frac{m_\pi}{m}\right)^2\left\{1-\frac{4A(x)}{d(x)}\left[1-\frac{A(x)}{d(x)}\right]\right\}\frac{1}{x}[A'(x)-B'(x)]\right)\vec{\sigma}\cdot\vec{l}_{op}\ . \tag{4.25}$$

For $D(x)$ we take $D(x)=1+0.76f(x)$. [See Eqs. (4.5) and (4.6).] If we evaluate $D(x)$ and A/d at $x=R$, i.e., $D^{-1}=0.73$ and $(A/d)=-0.136$, we find

$$\frac{1}{k_f^2}\tilde{V}_{SP}(x)\vec{\sigma}\cdot\vec{l}_{op}\simeq\frac{1}{m_\pi^2}v_{so}\frac{1}{x}f'(x)\vec{\sigma}\cdot\vec{l}_{op} \tag{4.26}$$

with $v_{so}\simeq 4.7$ MeV. This value is reasonably close to the experimental value,[17] 7.5 to 6 MeV. It is to be noted that we have left out the contributions to the spin-orbit potential corresponding to Fig. 5(c) and have neglected the nonlocalities associated with A and B. The terms shown in Fig. 5(c) may be estimated to contribute a few tenths of an MeV to v_{so}. Indeed our calculated values for A_0 and B_0 given in Eqs. (4.21) and (4.22) are very close to the *phenomenological* parameters chosen by Noble[13]: $A_0=-420$ MeV and $B_0=328$ MeV, in our notation. These parameters are determined in Noble's work by fitting to the empirical optical-potential parameters. His value for the parameter v_{so} is 5.1 MeV. If we use Noble's parameters in our Eq. (4.25) we obtain $v_{so}=4.9$ MeV. The small remaining descrepancy between Noble's analysis and ours can be ascribed to the different formalisms used in reducing the Dirac equation to a Schrödinger form.

V. DISCUSSION

We have discussed two methods for reducing the Dirac equation to a Schrödinger form. The first method is summarized in Eqs. (2.5)–(2.13) and is suited to the case in which the self-energy is a *local* function such as that given in Eq. (2.5). The second method is more general in that it may be used when the self-energy is nonlocal as it would be in a relativistic Hartree-Fock or a relativistic Brueckner-Hartree-Fock theory.

A few comments are in order concerning the second method. Even if one were to start with a local form for Σ such as that given in Eq. (2.5), Σ_{eff} of Eq. (2.24) would be nonlocal. If we work in the approximation $R(p,k)\simeq 1$ this nonlocality would arise from the second term in Eq. (2.24). Therefore to obtain a *local* Schrödinger form one would still use the procedures discussed in Eqs. (2.27)–(2.44). We recall that one obtains a local, energy-dependent interaction in Eq. (2.43) after introducing the transformation given in Eq. (2.42).

We note that in the numerical studies presented in Sec. IV we found that the momentum dependence of $\Sigma^{++}(p)$ played a major role in determining the value of the effective mass. The role of $\Sigma_{RC}(p)$ was relatively less important as may be seen by comparing Eqs. (4.1) and (4.3). The contribution of $\Sigma_{RC}(p)$ to the nonlocality and the effective mass is specifically a relativistic effect due to the inclusion of negative-energy states in the theory. The nonlocality implied by the momentum dependence of $\Sigma^{++}(p)$ is a feature that already appears in the *nonrelativistic* Brueckner theory of nuclear matter.

VI. CONCLUSIONS

We have presented a procedure for reducing the Dirac equation to a Schrödinger form which is suited to the nonlocal potentials which arise naturally in relativistic Hartree-Fock or relativistic Brueckner-Hartree-Fock calculations. As we have seen, the nonlocality of the potential plays an important role in explaining the energy dependence of the effective *local* potential. It is also interesting to note that about 25% of the nonlocality can be ascribed to relativistic effects involving negative-energy states. [These effects were grouped together in $\Sigma_{RC}(p)$. In addition, approximately 40% of the spin-orbit strength is due to these effects.] [See the terms proportional to A/d in Eq. (4.25).]

The fact (a) that the central potential's strength and energy dependence and the spin-orbit potential's strength have reasonable values when our results in nuclear matter are extrapolated to finite nuclei and (b) that our relativistic Brueckner-Hartree-Fock model is able to deal successfully with correlations and nonlocality warrants the detailed finite-nucleus calculations we are presently undertaking.

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