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Reaction between two ^{6}Li at rest to give three α particles

E. Norbeck, C. R. Chen, M. D. Strathman,* and D. A. Fox†
Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242
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For ^{6}Li beam energies of 2 MeV, α-α coincidence measurements show a large, narrow peak due to the direct reaction $2^6\text{Li}\to 3\alpha$, where both ^{6}Li nuclei contribute one deuteron to make the third α particle. The large volume of three-body phase space allows the various parts of the reaction process to be separated from each other. Excellent fits to the data were obtained using a three-body, extended-range, distorted-wave Born-approximation model. Elastic scattering potentials for the calculation were generated by the double-folding technique in combination with $^6\text{Li} + {}^6\text{Li}$ elastic scattering measurements for ^{6}Li beam energies from 2 to 5.5 MeV.

[NUCLEAR REACTIONS $^6\text{Li}(^6\text{Li}, {}^4\text{He}){}^4\text{He}{}^4\text{He}$, $E = 2$ MeV; DWBA analysis. $^6\text{Li}(^6\text{Li}, {}^6\text{Li})$, $E = 2.0$–5.5 MeV; fitted $\sigma(\theta, E)$ with double-folding model potentials.]

I. INTRODUCTION

When three nuclei are the end products of a nuclear reaction, it is usually assumed that the reaction goes in two steps, where the second is the breakup of an unstable intermediate nucleus. However, in some cases, there is no intermediate nucleus. The model described here gives excellent fits to experimental data for the direct $2^6\text{Li}\Rightarrow 3\alpha$ reaction without invoking any kind of $\alpha+\alpha$ interaction. This work is a direct sequel to a 1972 paper[1] that presented a wide range of data and attempted to fit it with about the simplest possible plane-wave-Born-approximation model. The present work shows how quantitative fits to experimental data can be obtained with the same model but with distorted waves instead of plane waves.

The plane wave model gave rather good fits to the two peaks that appeared in the angular correlation data when the beam energy was well above the Coulomb barrier (6 to 13 MeV). At these energies, the peaks had an irregular shape because of the background from α particles produced by two-step reactions. At 2 MeV, which is well below the Coulomb barrier, the plane-wave model still predicted a pair of peaks; but the data showed a single, smooth peak that rose far above the surrounding background. It will be shown that this anomaly occurs when the Coulomb force stops the two ^{6}Li nuclei before the nuclear rearrangement takes place. To incorporate the Coulomb force, the plane waves must be replaced by distorted waves. The distorted wave model presented below gives an excellent fit to the single peak when sufficiently accurate $^6\text{Li}+{}^6\text{Li}$ elastic-scattering wave functions are used.

II. THEORETICAL MODEL

The main emphasis here will be on those features that must be added to the original plane-wave model[1] to incorporate distorted incoming waves and an extended interaction potential. For the reaction $^6\text{Li}_B + {}^6\text{Li}_T \Rightarrow \alpha_1 + \alpha_2 + \alpha_3$, the differential cross section for finding α_2 with E_2 at the solid angle Ω_2 and the simultaneous detection of α_3 at the solid angle Ω_3 is

$$\frac{d^3\sigma}{dE_2\,d\Omega_2\,d\Omega_3} = \frac{2\pi}{\hbar^2}\,\frac{\mu}{k}\,|M|^2\rho_f\,, \tag{1}$$

where $\hbar k$ is the relative momentum of the beam-target system which has a reduced mass μ. It is found that the phase space factor ρ_f for the three-body final state is essentially constant over the entire region where the matrix element $M = \langle\psi_f|V|\psi_i\rangle$ is large.

The initial-state wave function has the form

$$\psi_i = \chi_i(\vec{k},\vec{r})\varphi_{^6\text{Li}}(\vec{d}_B - \vec{\alpha}_B)\varphi_{^6\text{Li}}(\vec{d}_T - \vec{\alpha}_T)\,. \tag{2}$$

It is the first term, which governs the relative motion of the beam and the target, that is the most critical part of the calculation when the beam energy is near or below the Coulomb barrier. This term will be discussed in detail later. The d-α cluster wave functions for the two ^{6}Li nuclei were taken from Jain[2] and had the form

$$\varphi_{^6\text{Li}}(\vec{R}) = \begin{cases} R^2\exp(-\tfrac{2}{3}\beta R^2) & R<3.0\text{ fm}\,,\\ a\exp(-CR)/R & R>3.0\text{ fm}\,,\end{cases}$$

where $a=9.5493$, $\beta=0.32677$, and $c=0.30709$. The final-state wave function was

$$\psi_f=\exp(i\vec{k}_B\cdot\vec{\alpha}_B)\exp(i\vec{k}_T\cdot\vec{\alpha}_T)\exp(i\vec{k}_d\cdot\vec{\alpha}_d)\varphi_\alpha(\vec{d}_B-\vec{d}_T), \quad (3)$$

where the momentum and position of each α particle is identified as coming from the beam, the target, or from a combination of the two deuterons. Plane waves were adequate for the α particles because of the large Q value of 20.9 MeV and because the cross section is large only when the α particles are going in different directions, or in other words, the relative kinetic energy between any pair of α particles is well above the Coulomb barrier. With plane waves in the final state, all recoil effects are included exactly. The interaction potential was assumed to depend only on the distance between the two deuterons. This potential times the bound state of the third α particle was assigned the simple functional dependence

$$V(\vec{d}_B-\vec{d}_T)\varphi_\alpha(\vec{d}_B-\vec{d}_T)=A\exp[-\beta^2(\vec{d}_B-\vec{d}_T)^2], \quad (4)$$

where A is an arbitrary normalization factor and $1/\beta$ is an effective range for the interaction. For a beam energy of 2 MeV, any value of β greater than 0.5 fm^{-1} gave about the same result.

To evaluate the multidimensional integral in the matrix element, the ^6Li bound-state wave functions were expressed as a sum of Gaussian functions,

$$\varphi_{^6Li}(\vec{d}-\vec{\alpha})=\sum_j c_j\exp[-\gamma_j^2(\vec{d}-\vec{\alpha})^2]. \quad (5)$$

For ^6Li, 10 to 12 terms seemed to be about optimum. The details and validity of this type of expansion are given in Ref. 3. At this stage, the integrations over all of the variables except $\vec{r}$ can be done analytically. By using the partial wave expansion

$$\chi_i(\vec{k},\vec{r})=\sum_{lm}4\pi i^l e^{i\sigma_l}\frac{u_l(kr)}{kr}Y_l^m(\hat{r})Y_l^{m*}(\hat{k}) \quad (6)$$

the matrix was reduced to a sum of one-dimensional integrals over r which were evaluated numerically. In practice, this technique was found to be both efficient and accurate. The accuracy of the various expansions was tested by using the expansions to evaluate the matrix element with plane waves in the incident channel. For this case, the matrix element could be evaluated analytically. In addition, the accuracy in the distorted-wave program was tested by including more terms in the expansions until the change in the matrix element was less than the desired accuracy. After the integrations were finished, the matrix element was made symmetric with respect to the interchange of the three α particles. For the reaction to occur, the spins of the two ^6Li must be coupled to give zero so that the two deuterons can combine to form a spin zero α particle.

III. INTERFERENCE EFFECTS

The role of the Coulomb force in merging the two peaks into one is straightforward. As was shown in Ref. 1, the matrix element M is large only when the momentum transfer to one of the α particles is zero. There is a pole in the complex plane on the negative energy axis at the point corresponding to the binding energy of the deuteron in ^6Li. Of course, only one α particle at a time can experience zero momentum transfer and still have energy and momentum conserved for the overall reaction. Zero momentum transfer means that the α particle has the same velocity after the reaction as before. Figure 1(a) shows the case where this α particle, called α_1, originates in the target, which is at rest. In Fig. 1(b), α_1 originates in the beam and so leaves the reaction at zero degrees with the same velocity as the

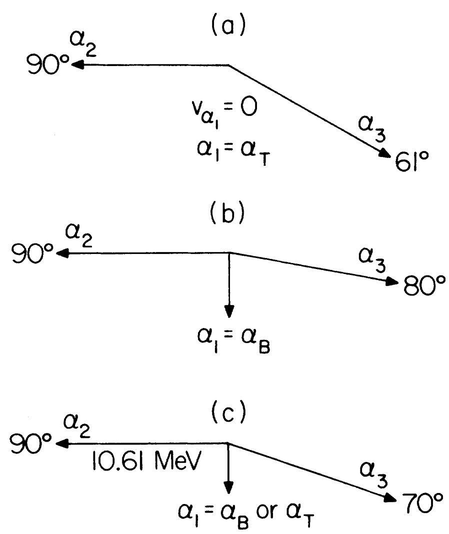

FIG. 1. Velocity diagrams in the laboratory system for $2^6Li\rightarrow 3\alpha$ assuming zero momentum transfer for the α particle cluster α_1 from one of the ^6Li nuclei when a second α particle α_2 is detected at an angle of 90°. For both b and c it can be shown from conservation of momentum and energy that the energy of α_2 at 90° must always be $Q/2-E_B/12$. For a it is $Q/2-E_B/4$. (a) α_1 is from the target and has zero energy for zero momentum transfer. (b) α_1 is from the ^6Li in the beam and continues with the original beam velocity. (c) α_1 has the center of mass velocity and could be from either beam or target.

incoming ^{6}Li. When the velocity of α_1 is determined by the requirement of zero momentum transfer, it is the other two α particles, α_2 and α_3, that must carry away the large amount of energy ($Q = 20.9$ MeV) released by the reaction. The quantity that is to be compared with the experiment is the angular distribution of α_3. The angle of α_3 is determined by conservation of energy and momentum. When α_2 is detected at 90°, all of the momentum of the beam must be divided between α_1 and α_3. In Fig. 1(a), α_1 carries off no energy or momentum, so α_3 must carry away all of the original beam momentum. In Fig. 1(b), $\frac{2}{3}$ of the beam momentum is carried off by α_1 so that the forward component of the α_3 velocity is only $\frac{1}{3}$ as great.

For beam energies below the Coulomb barrier, the relative motion of the beam and target is reduced to zero before the nuclear part of the interaction takes place. In this case, zero momentum transfer means that α_1 leaves the reaction with the same velocity as the center of mass as shown in Fig. 1(c), and there is no longer any distinction between α_B and α_T. The stopping of the relative motion before the reaction occurs is provided for automatically by the distorted-wave procedure.

Since all alpha particles are identical, there must be interference between the two peaks when they overlap. The two reaction amplitudes must be summed before they are squared to give the differential cross section, and the relative phase between the two amplitudes can be expected to be different for different energies and angles. Because our calculation is divided into many small parts that are summed at the end, it is not easy to determine the relative phase, but it does appear that the amplitudes are always in phase for the case where α_1 is at rest in the center of mass. Our calculations always showed some sort of peak at exactly the appropriate angle and energy for any beam energy up to 5.5 MeV, presumably because of the high degree of symmetry associated with such a configuration.

If the two peaks interfere constructively, the center of the combined peak should be four times as high as one of the separate peaks. We have not found a way to demonstrate this conclusively, although one possibility would be to compare the height of the peaks with the background of α particles produced via the various states in ^{8}Be. All that can be claimed at this stage is that the published data[1,4] is at least compatible with the conclusion that the ratio of the height of the direct peaks to the ^{8}Be background is about four times larger for $E_B = 2$ MeV than it is for $E_B = 6$ and 13 MeV.

IV. ELASTIC SCATTERING

The α_3 angular distribution is very sensitive to the form of the wave function responsible for bringing the two ^{6}Li nuclei together. In our model this is the elastic scattering wave function. There were a wide range of ^{6}Li+^{6}Li optical potentials that gave adequate fits to low energy ^{6}Li+^{6}Li elastic scattering data, but most of these gave fits to 2 ^{6}Li$\Rightarrow 3\alpha$ data that ranged from poor to terrible.[5] A potential calculated according to the double-folding model[6] and then adjusted to fit elastic scattering data gave excellent fits.

We folded a nucleon-nucleon potential of the form

$$V(r) = \left[6315\frac{e^{-4r}}{4r} - 1961\frac{e^{-2.5r}}{2.5r}\right]\text{MeV} \tag{7}$$

into a ^{6}Li nucleon distribution inferred from the scattering of electrons from ^{6}Li. This simple form for $V(r)$ has been used successfully in describing a wide range of heavy ion interactions. Reference 6 also gives several more complicated forms, but these all gave rise to ^{6}Li+^{6}Li potentials that were either too deep or too shallow to allow a good description of low energy ^{6}Li+^{6}Li elastic scattering. For most high-energy, heavy-ion scattering, only the outer edge of the potential affects the scattering, but for low energy lithium ions the entire potential is important.[7]

For the nucleon distribution in ^{6}Li, we used[8]

$$\rho(r) = \frac{A}{8\pi^{3/2}}\left[\frac{e^{-r^2/4a^2}}{a^3} + \frac{c^2}{b^5}\left(\frac{r^2}{4b^2} - \frac{3}{2}\right)e^{-r^2/4b^2}\right], \tag{8}$$

where $a^2 = 0.87$, $b^2 = 1.5$, and $c^2 = 0.14$ which gave an rms radius for ^{6}Li of 2.46 fm. With this simple form, we could evaluate the integrals in the double-folding model analytically. The resulting potential gave equally good fits to the data as a similar potential supplied by Satchler[9] that made use of shell model wave functions, including spin-orbit coupling, as described in Ref. 6.

The potential calculated from the folding model was multiplied by $V + iW$ to give a complex optical-model potential. The two constants V and W were adjusted to fit elastic-scattering differential cross sections for beam energies from 2.0 to 5.5 MeV. For $E_B = 2.0$ MeV, $V = 1.000$, and $W = 0.122$. For beam energies up to 3.0 MeV, the parameter V was unity to within a few percent. At higher energies, V decreased rapidly with energy so that by 5.5 MeV, the potential was only 36% as deep as predicted by the folding model. Many studies of higher energy ^{6}Li scattering from a variety of nuclei have found that the required potential is only about 60% as deep as predicted by the folding model.[6,10] When ^{6}Li

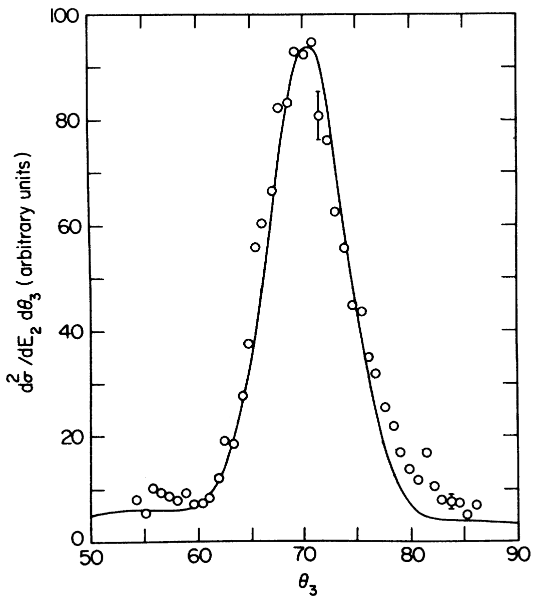

FIG. 2. $2\,^6Li \rightarrow 3\alpha$ for $E_B = 2.0$ MeV. Relative cross section for an α particle at an angle θ that is in time coincidence with an α particle of energy between 9.9 and 11.4 MeV found at 90° on the opposite side of the target chamber. Experimental uncertainty is indicated by sample error bars. The calculation of the solid curve is described in the text.

is used for both the beam and the target it could be expected that the discrepancy would be squared. It appears that our low energy scattering measurements have confirmed the surmise of Satchler and Love[6] that the reduction in well depth is caused by the break-up process $^6Li \rightarrow d + \alpha - 1.47$ MeV.

A detailed description of the $^6Li + {}^6Li$ elastic scattering experiments and calculations will be given in a forthcoming paper.

V. EXPERIMENT

The experimental apparatus and procedure have been described previously.[1] The ^6Li beam was produced by the University of Iowa HVEC Model CN 5.5 MV Van de Graaff accelerator. Two silicon surface-barrier detectors were used to detect the two high energy alpha particles at angles and energies in the region indicated in Fig. 1. Figure 2 shows those events for which α_2 at 90° had energies between 9.9 and 11.4 MeV in coincidence with α_3 at the indicated angle. The requirement that the α_3 energy be that required by conservation of energy and momentum eliminated all events that were not $2\,^6Li \Rightarrow 3\alpha$. The solid curve was calculated according to the model described above for an α_2 energy of 10.61 MeV. The irregular background at the edges of the large peak in Fig. 2 is due to α particles from various two step reactions involving energy levels in ^8Be. It can be seen that the agreement of the calculation with the data is excellent.

A plot of the energy of α_2 for the angles given in Fig. 1(c) would show a peak at 10.61 MeV. Such a peak has a shape that is not sensitive to the scattering wave functions. As was shown in Ref. 1, in this case, a good fit to data can be obtained even with plane waves.

VI. CONCLUSIONS

We have seen that the direct, two-to-three reaction $2\,^6Li \Rightarrow 3\alpha$ can be well described by a simple Born approximation is sufficiently accurate wave functions are used. At this point, the model has been tested only for energies well below the Coulomb barrier. A critical test of the model would be at energies close to the Coulomb barrier where the angular distribution is making the transition between one peak and two.

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*Present address: Lawrence Berkeley Laboratory, Berkeley, California 94720.

†Present address: Theological Seminary, University of Dubuque, Dubuque, Iowa 52001.

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