# Static quadrupole moments for the $2^+$ states of $^{54,56,58}$Fe

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The static quadrupole moments of the first $2^+$ excited states of $^{54}$Fe, $^{56}$Fe, and $^{58}$Fe were measured utilizing the reorientation effect in Coulomb excitation. Results for $B(E2;0^+\rightarrow 2^+)$ and $Q(2^+)$ values were obtained by combining, for each Fe isotope, light heavy ion ($^{12}$C or $^{16}$O) and heavier heavy ion ($^{52}$Cr or $^{40}$Ca) Coulomb excitation results. Scattered $^{12}$C or $^{16}$O ions or recoiling Fe ions were detected in a quadrupole-dipole-dipole-dipole spectrometer. The results for the $B(E2;0^+\rightarrow 2^+)$, $Q(2^+)$ pairs (in $e^2$fm$^4$ and $e$fm$^2$) are as follows: $^{54}$Fe, $676\pm 38$, $-5\pm 14$; $^{56}$Fe, $1022\pm 55$, $-19\pm 8$; and $^{58}$Fe, $1234\pm 36$, $-27.3\pm 5.0$.

[NUCLEAR REACTIONS $^{56,58}$Fe($^{12}$C,$^{12}$C), $E=22$ MeV; $^{56,58}$Fe($^{52}$Cr,$^{56,58}$Fe)$^{52}$Cr, $E=110$–$120$ MeV; $^{54}$Fe($^{16}$O,$^{16}$O)$^{54}$Fe, $E=28$ MeV; $^{54}$Fe($^{40}$Ca,$^{40}$Ca)$^{54}$Fe, $E=86$ MeV; measured relative cross section, deduced $Q(2^+)$ and $B(E2;0^+\rightarrow 2^+)$. Enriched targets.]

## I. INTRODUCTION

Electric quadrupole matrix elements, diagonal and nondiagonal, provide a valuable measure of the gross shape of nuclei. The static quadrupole moment is of particular importance because its sign differentiates between prolate and oblate quadrupole deformations.

The Fe isotopes provide an excellent example of the importance of this parameter. Experimental[1,2] and theoretical[2,3] studies show clearly the coexistence of low-lying prolate and oblate states in $^{56}$Fe. The ground state band is rather strongly deformed with measured values[4,5] for the static quadrupole moment of the first $2^+$ state of $Q(2^+) = -(23\pm 3)$ and $-(25\pm 6)$ $e$ fm$^2$. A band crossing occurs at about $J=6^+$ between the ground state band and a band of oblate deformation with its $0^+$ state at 2941 keV. On the other hand, a measurement[6] of $Q(2^+)=+(29\pm 8)$ $e$ fm$^2$ has been reported[7] for the first-excited state of $^{58}$Fe leading to the speculation[8] that a reversal of the prolate and oblate bands takes place between $^{56}$Fe and $^{58}$Fe.

Such a reversal is in basic disagreement with shell-model calculations of McGrory[9,10] which predict a $Q(2^+)$ value for $^{58}$Fe of $-27$ $e$ fm$^2$ and a band crossing similar to that seen and calculated in $^{56}$Fe. As is the case for $^{56}$Fe, the $B(E2)$ values for the $^{58}$Fe $4_1^+\rightarrow 2_1^+$ and $2_1^+\rightarrow 0_1^+$ transitions (see Fig. 1) are large and indicative of a rather stiff rotor. These nondiagonal $E2$ properties do not distinguish between oblate and prolate deformation; thus the importance of the $Q(2^+)$ measurements.

Because of the crucial role of the $^{58}$Fe $Q(2^+)$ value in understanding this problem, we have undertaken a further measurement using the reorientation effect[13] in Coulomb excitation. As a check on the experimental techniques and analysis, $Q(2^+)$ values were also measured for $^{54}$Fe and $^{56}$Fe. In the method used the $Q(2^+)$ and $B(E2)$ values are dependent and $Q(2^+)$ is very sensitive to $B(E2)$.

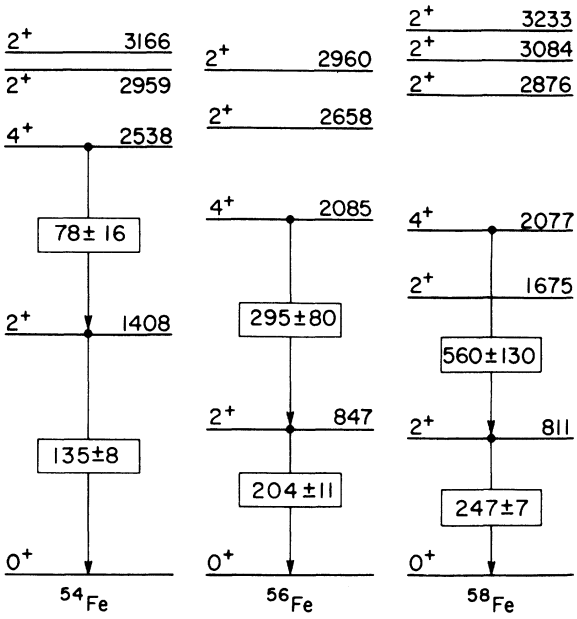


FIG. 1. Energy levels of $^{54,56,58}$Fe pertinent to the present studies. The excitation energies and spin-parity assignments are from Refs. 7, 11, and 12. $B(E2)$ values (in $e^2$fm$^4$) are indicated in the boxes for the $4^+\rightarrow 2^+$ and $2^+\rightarrow 0^+$ transitions. The $4^+\rightarrow 2^+$ values were obtained from an assessment of previous measurements as discussed in Sec. IV. The $2^+\rightarrow 0^+$ results are from the present work. For comparison, single-particle (Weisskopf units) estimates for the $B(E2)$ values are 12.12, 12.73, and 13.34 $e^2$fm$^4$ for $^{54}$Fe, $^{56}$Fe, and $^{58}$Fe, respectively.

Thus, $B(E2)$ values were also measured for $^{54,56,58}$Fe since previous measurements were rather inconsistent for all three isotopes (see Sec. III A for a discussion).

## II. EXPERIMENT

The excitation probability in Coulomb excitation of a spin-zero nucleus displays a sensitivity to the static quadrupole moment of the excited $2^+$ state which is given approximately by

$$Y_{2^+}/Y_{0^+} \equiv P_2 = P_2^{(0)}[1 + b \cdot Q(2^+)], \quad (1)$$

where the sensitivity parameter $b$ is largest for heavy projectiles and backward angles. The first-order excitation probability $P_2^{(0)}$ is proportional to the $B(E2; 0^+ \rightarrow 2^+)$ for the transition; we use $P_2^{(0)} = B(E2)/R_2$, where the constant $R_2$ is given by the dynamical factors. A complete determination of the moments involved thus requires a minimum of two measurements with different sensitivities to the static quadrupole moment.

We have chosen to measure the moments of the stable even-$A$ iron isotopes using paired measurements: A light heavy-ion projectile ($^{12}$C or $^{16}$C) was used to measure the excitation probability with low sensitivity to the static moment, $b \sim 2 \times 10^{-3}$ $(e\,\mathrm{fm}^2)^{-1}$, and a heavier projectile ($^{52}$Cr or $^{40}$Ca) was used to determine the excitation probability with $b \sim 8 \times 10^{-3}$ $(e\,\mathrm{fm}^2)^{-1}$.

Beams were provided by the BNL Tandem Van de Graaff facility. Excitation probabilities were measured using thin targets (5–10 $\mu$g/cm$^2$) and by measuring the scattered particle momentum spectrum in a 65-cm long detector in the focal plane of the BNL quadrupole-dipole-dipole-dipole (QDDD) spectrometer.

### A. The small sensitivity measurements

A 22.0-MeV $^{12}$C beam was used to bombard a 5-$\mu$g/cm$^2$ target, enriched to 73.3% in $^{58}$Fe, supported on a 10-$\mu$g/cm$^2$ carbon foil. The QDDD spectrometer, positioned at 95° to the beam direction, viewed the target in reflection geometry, with a 5-msr solid angle. The focal plane detector, composed of separate position-sensitive $\Delta E$ and $E$ volumes, allowed unique identification of

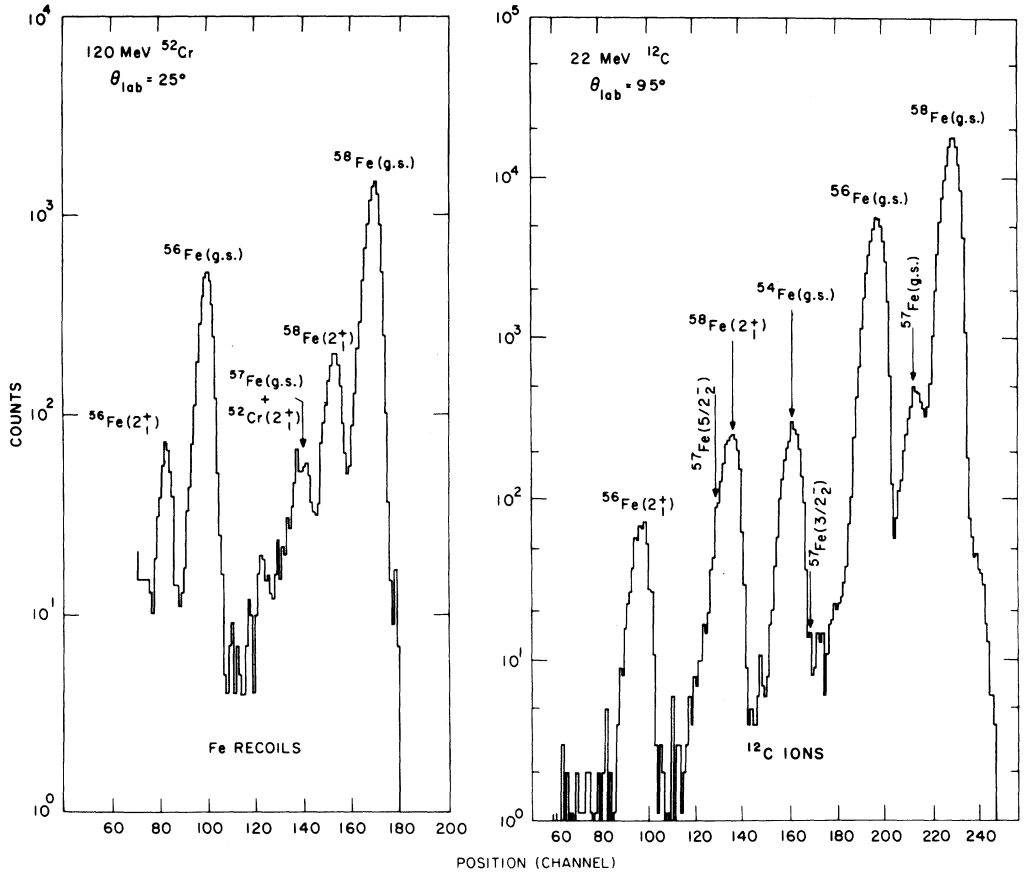


FIG. 2. $^{58}$Fe and $^{56}$Fe results from $^{56,58}$Fe($^{52}$Cr, $^{56,58}$Fe)$^{52}$Cr and $^{56,58}$Fe($^{12}$C,$^{12}$C)$^{56,58}$Fe at the bombarding energies and detection angles indicated.

TABLE I. Experimental excitation probabilities for $^{54,56,58}$Fe (corrected for charge state distributions).

| Target | Beam | Energy (MeV) | $\theta_{c.m.}$ (elastic) (deg) | $Y_{2^+}/Y_{0^+}$ |
|---|---|---|---|---|
| $^{54}$Fe | $^{16}$O | 28 | 73 | $(8.10 \pm 0.31)\times 10^{-2}$ |
| $^{54}$Fe | $^{40}$Ca | 86 | 131 | $1.013 \pm 0.090$ |
| $^{56}$Fe | $^{12}$C | 22 | 107 | $1.242 \pm 0.050$ |
| $^{56}$Fe | $^{52}$Cr | 120 | 130 | $10.49 \pm 0.51$ |
| $^{56}$Fe | $^{52}$Cr | 110 | 130 | $5.92 \pm 0.17$ |
| $^{58}$Fe | $^{12}$C | 22 | 107 | $1.607 \pm 0.035$ |
| $^{58}$Fe | $^{52}$Cr | 120 | 130 | $13.65 \pm 0.39$ |

the scattered $^{12}$C ions. The resulting spectrum is shown in the right hand panel of Fig. 2.

The contribution to the $^{58}$Fe($2_1^+$) due to the $^{57}$Fe($\frac{5}{2}^-$, 707 keV) state[14] was calculated to represent a 0.5% effect; no correction for it has been made. Yields for the elastic and inelastic scattering were corrected for the different efficiencies arising from the difference in scattered ion energies. For this purpose, charge state distributions were taken from the compilation of Marion and Young.[15] Corrections to the excitation probabilities amounted to less than 1%. The excitation probabilities for $^{56,58}$Fe($2_1^+$), thus corrected, are shown in Table I.

A 10-$\mu$g/cm$^2$ thick target of isotopically enriched $^{54}$Fe (97.2%) was bombarded with a 28.0-MeV $^{16}$O beam. The particle spectrum, measured at $\theta_{lab} = 58.3°$, is shown in the right hand panel of Fig. 3. In this case, the charge state distribution was measured for the energies corresponding to elastic and inelastic scattering; the distribution predicted by Marion and Young agrees with our measurements to 0.3%. The excitation probability is given in Table I.

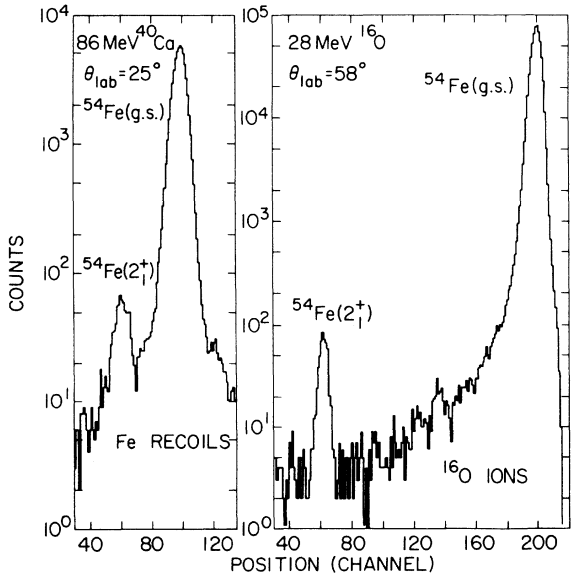


FIG. 3. $^{54}$Fe results from $^{54}$Fe($^{40}$Ca, $^{54}$Fe)$^{40}$Ca and $^{54}$Fe($^{16}$O, $^{16}$O)$^{54}$Fe at the bombarding energies and detection angles indicated.

The errors shown in Table I for these light heavy-ion measurements reflect statistics primarily. The uncertainty in determining backgrounds under the inelastic peaks is not a major factor here.

The laboratory angle was checked for the measurements described by comparing spectra measured at small angles on either side of the beam axis; zero angle could be determined in this way to less than 0.05°. Calculations show that the fractional changes in excitation probabilities are 0.08% ($^{12}$C) and 0.25% ($^{16}$O) for an angular error of 0.05°.

### B. The large sensitivity measurements

The large sensitivity measurements were made using the technique of kinematic reversal. In this technique, the recoiling target nucleus is observed at a forward angle $\theta_{lab}$. Such an event corresponds to scattering through the c.m. angle $\theta = \pi - 2\theta_{lab}$ for the case of elastic scattering.

$^{58}$Fe targets 10-$\mu$g/cm$^2$ thick and enriched to 73.3% were bombarded with a 120-MeV $^{52}$Cr beam. Scattered particles at $\theta_{lab} = 25°$ were detected in the QDDD focal plane detector. Identification of the ions was sufficient to cleanly separate Cr from Fe recoils. The resulting position spectrum is shown in the left hand panel of Fig. 2. $^{56}$Fe and $^{58}$Fe spectra are completely separated because the position measured reflects momentum ($\sim\sqrt{mE}$). The sharply increasing background in the $^{56}$Fe spectrum towards the left from the $2_1^+$ peak arises because of the presence of a higher charge state $^{58}$Fe recoil which could not be separated entirely from the $^{56}$Fe recoils. This background resulted in a large uncertainty in determining the $^{56}$Fe($2_1^+$) yield. Therefore, further measurements were made using a target enriched to 99.87% in $^{56}$Fe. Spectra were measured for the $^{56}$Fe target, as described above, using both 110- and 120-MeV $^{52}$Cr beams.

Excitation probabilities were corrected for the energy dependence of charge state distributions. These distributions were calculated following Betz.[16] Corrections were 2.3% or less. The corrected excitation probabilities are shown in Table I.

A beam of $^{52}$Cr was not useful for the $^{54}$Fe measurement because of the proximity of the $^{52}$Cr 1.43-MeV state to the $^{54}$Fe $2^+$ 1.408-MeV state.

Instead, a beam of 86-MeV $^{40}$Ca was used to bombard a 10-$\mu$g/cm$^2$ thick $^{54}$Fe target enriched to 97.23%. Spectra of $^{54}$Fe recoils scattered at $\theta_{lab} = 25°$ were measured as described above. Identification in the focal plane detector was sufficient to separate Fe recoils from Ca ions with no difficulty. The resulting position (momentum) spectrum is shown in the left hand panel of Fig. 3. This spectrum, like the others shown in Figs. 2 and 3, is a projection on the position (focal plane) axis of a two-dimensional energy versus position spectrum. Because of a strong skewness in this plot, the position resolution in each energy slice of this plot is considerably better than in the total projection shown in Fig. 3. Thus, analysis of this data was carried out for individual energy slices or groups of slices and the results summed to obtain the excitation probability. Because of the small excitation probability for $^{54}$Fe($2_1^+$), the uncertainty due to the background dominates the statistical uncertainty and, in fact, is large enough so that this measurement is of lower accuracy than those for the other isotopes. Charge state corrections[16] were applied, resulting in the excitation probability in Table I.

A zero angle determination was made for this measurement. While no measurements of zero angle were made for the $^{52}$Cr bombardments, this angle is estimated to be no more than 0.2° in error, corresponding to a fractional change in the excitation probability of 0.36%.

## III. SAFE ENERGIES

Yields in Coulomb excitation grow rapidly with increased bombarding energy. On the other hand, energies which are too large will result in excitation probabilities which are influenced by the nuclear potential.

Our bombarding energies were chosen such that the distance of closest approach $D_s$ for the classical orbit corresponding to the scattering angle of interest would provide at least 5.5 fm between the nuclear surfaces

$$D_s \geq 1.25(A_1^{1/3} + A_2^{1/3}) + 5.5 \text{ fm}.$$

To provide confidence in this recipe, calculations were carried out using the coupled-channels code CCC.[17] Using a radius $R = 1.25(A_1^{1/3} + A_2^{1/3})$, a real Woods-Saxon potential well depth $V = 40$ MeV, diffuseness $a = 0.6$ fm, and a Woods-Saxon volume imaginary potential depth $W = 10$ MeV, the case where the nuclear surfaces came closest, 120-MeV $^{52}$Cr + $^{58}$Fe, was investigated. The nuclear potential caused a change in the excitation probability of less than 0.1%.

The measurement on $^{56}$Fe at 110 MeV was carried out as a further test of the above relationship. The static quadrupole moment derived from the lower energy measurement is not significantly different from that determined at 120 MeV (see the discussion in Sec. V).

## IV. ANALYSIS

### A. The Fe spectra

Pertinent information on the energy spectra of $^{54,56,58}$Fe is shown in Fig. 1. The data are from the NDS compilations for $^{54}$Fe,[11] $^{56}$Fe,[12] and $^{58}$Fe (Ref. 7) as well as from the present results. Aside from $2_1^+$ and $4_1^+$ states, the next 2 or 3 $2^+$ states which might conceivably have some influence on the reorientation measurement are included. Shown in Fig. 1 are our adopted averages of previous measurements for $B(E2; 4_1^+ \rightarrow 2_1^+)$ values and the $B(E2; 2_1^+ \rightarrow 0_1^+)$ results of the present study. Previous results for $^{54}$Fe, $^{56}$Fe, and $^{58}$Fe are summarized in Tables II, III, and IV, respectively. For $^{56}$Fe the various previous determinations of the $B(E2)$ values are in quite poor agreement, i.e., many of the assigned uncertainties are too small. This is not surprising since the measurements summarized go back to 1964 and were made during a period when the methods used were being perfected. In our treatment of the data we have attempted to assign relative uncertainties which reflect the reliability of the method at the time it was used with the aim of arriving at a final normalized chi-squared $\chi_D^2$ not too far from unity.

TABLE II. Resume of transition strengths previously measured for the $^{54}$Fe $2_1^+ \rightarrow 0_1^+$ transition.

| $B(E2)$ ($e^2$ fm$^4$) | Method[a] | Reference |
|---|---|---|
| 102 ± 10 | CE | b |
| 122 ± 24 | CE | c |
| 158 ± 24 | DSA | d |
| 126 ± 10 | CE | d |
| 130 ± 8 | weighted mean | e |

[a] CE (Coulomb excitation); DSA (Doppler shift attenuation).

[b] J. J. Simpson, J. A. Cockson, D. Eccleshall, and M. J. L. Yates, Nucl. Phys. 62, 385 (1965). This value was excluded from the weighted average because later results (Ref. 4) indicate that the ($^{16}$O, $^{16}$O′) measurements of Simpson *et al.* were performed above the safe energy.

[c] O. F. Afonin, A. P. Grinberg, I. K. Lemberg, I. N. Chugunov, Yad. Fiz. 6, 219 (1967) [Soviet J. Nucl. Phys. 6, 160 (1968)].

[d] D. Ward, I. M. Szoghy, J. S. Forster, and W. G. Davies, Report No. AECL-4314, 1972, p. 9.

[e] $\chi_D^2 = 0.82$.

TABLE III. Resume of $E2$ transition strengths previously measured for the $^{56}$Fe $2_1^+ \rightarrow 0_1^+$ transition, from results summarized in Ref. 12.

| $B(E2)$ ($e^2$ fm$^4$) | Method[a] | Number of measurements ($N_e$) | Assigned uncertainty[b] (Units of $\chi_D^2$) |
|---|---|---|---|
| 190 ± 22 | $(e, e')$ | 2[c] | 2.0 |
| 195 ± 42 | $(^3\text{He}, ^3\text{He}')$, $(\alpha, \alpha')$ | 2 | 3.0 |
| 194 ± 35 | $(\gamma, \gamma')$ | 1 | 1.5 |
| 176 ± 41 | DSA | 2 | 1.5 |
| 236 ± 39 | RDM | 1 | 1.0 |
| 210 ± 16 | CE | 3 | 2.0 |
| 202 ± 11 | weighted mean | | 1.0[d] |

[a] DSA (Doppler shift attenuation); RDM (Recoil distance method); CE (Coulomb excitation).

[b] The larger of the internal and external errors from the mean of the $N_e$ determinations is multiplied by this factor to arrive at the assigned uncertainty for $B(E2)$. This factor reflects our subjective assessment of the rigour and reliability of the method at the time it was performed.

[c] The value of Ref. 18, 136 ± 10 $e^2$fm$^4$, has been omitted.

[d] $\chi_D^2 = 0.36$.

The previous value listed for the $B(E2)$ of the $^{58}$Fe $2_1^+ \rightarrow 0_1^+$ transition is the weighted mean of the three Coulomb excitation measurements and one $(e, e')$ measurement[18] quoted by Kocher and Auble[7] and listed in Table IV. A recent determination[19] via the Doppler shift attenuation (DSA) method gives 685 ± 160 $e^2$fm$^4$; this value was not included in the average.

The $4_1^+ \rightarrow 2_1^+$ $B(E2)$ value of $^{54}$Fe is that of Brown, Fossan, McDonald, and Snover.[20] The adopted value of Auble[12] is given for $^{56}$Fe, and for $^{58}$Fe we give the mean of two recent DSA measurements.[8,19]

The data used in estimating effects of the higher-lying $2^+$ states are given in Table V. Also included in this table are shell-model calculations of McGrory[9,10] for the configuration space

$$(1f_{7/2})^{-2}(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^m , \qquad (2)$$

where $m = N - 28$. A modified surface $\delta$ interaction was used for the $n$-$p$ matrix elements and an effective charge parameter $x$ [i.e., $e_p = (1 + x)e$, $e_n = xe$] of $x = 1.0$ was chosen. For $^{54}$Fe, the space of Eq. (2) is just $(f_{7/2})^{-2}$ and there is only one $0^+$, $2^+$, and $4^+$ state.

TABLE IV. Resume of transition strengths previously measured for the $^{58}$Fe $2_1^+ \rightarrow 0_1^+$ transition.

| $B(E2)$ ($e^2$ fm$^4$) | Method[a] | Reference |
|---|---|---|
| 400 ± 100 | CE | b |
| 220 ± 40 | CE | c |
| 188 ± 16 | $(e, e')$ | d |
| 172 ± 10 | CE | e |
| 180 ± 12 | weighted mean | f |

[a] CE (Coulomb excitation).

[b] D. G. Alkhazov, A. P. Grinberg, K. I. Erokhina, and I. Kh. Lemberg, Izvest. Akad. Nauk SSSR, Ser. Fiz. 23, 223 (1959); Columbia Tech. Transl. 23, 215 (1960).

[c] D. S. Andreyev, A. P. Grinberg, K. I. Erokhina, and I. Kh. Lemberg, Nucl. Phys. 19, 400 (1960).

[d] Reference 18.

[e] Reference 6.

[f] $\chi_D^2 = 2.24$.

### B. Quadrupole moments

The bulk of the analysis was performed using the semiclassical code of de Boer and Winther.[22] In addition to the ground state and $2_1^+$ state, the $2_2^+$ state and $4_1^+$ state were included in the calculations for each nucleus. The projectile energies used in the calculations correspond to bombarding energies corrected for energy loss in the target, vacuum polarization,[23] and atomic screening.[23] These latter two corrections amount to about 2 keV and 75 to 100 keV increases in bombarding energy for the light and heavy projectiles, respectively.

Sensitivity parameters $b$ were extracted from 2-level semiclassical calculations performed for each experimental measurement. These were compared to the results of quantal calculations.[17] The sensitivities calculated by the latter means were used to replace the sensitivity parameters calculated semiclassically. This was done since the relationship frequently used to correct for quantal effects, suggested by Alder and Winther,[24] was not reproduced by our calculations. It should also be noted that the value of the correction term $e(\theta, \xi)$ given in Ref. 24 is not in agreement with

TABLE V. $E2$ transition strengths in $^{54,56,58}$Fe.

| Transition $(J_i \to J_f)$ | $E_1$ (MeV) | $B(E2)$ [a] ($e^2$ fm$^4$) | McGrory [b] (shell model) |
|---|---|---|---|
| | | $^{54}$Fe | |
| $2_1^+ \to 0_1^+$ | 1408 | 130 ± 8 [c] (135 ± 8) | (+)90 |
| $2_2^+ \to 0_1^+$ | 2959 | 26 [d] | |
| $2_2^+ \to 2_1^+$ | 2959 | 6 [d] | |
| $2_3^+ \to 0_1^+$ | 3166 | 9 [d] | |
| $2_3^+ \to 2_1^+$ | 3166 | 11 [d] | |
| $4_1^+ \to 2_1^+$ | 2538 | 78 ± 16 [c] | (+)89 |
| | | $^{56}$Fe | |
| $2_1^+ \to 0_1^+$ | 847 | 202 ± 11 [c] (204 ± 11) | (+)176 |
| $2_2^+ \to 0_1^+$ | 2658 | 3.4 ± 1.1 [e] | (−)0.87 |
| $2_2^+ \to 2_1^+$ | 2658 | 34 ± 11 [e] | (−)0.02 |
| $2_3^+ \to 0_1^+$ | 2960 | 2.0 ± 0.7 [e] | |
| $2_3^+ \to 2_1^+$ | 2960 | 36 ± 12 [e] | |
| $4_1^+ \to 2_1^+$ | 2085 | 295 ± 80 [c] | (+)224 |
| | | $^{58}$Fe | |
| $2_1^+ \to 0_1^+$ | 811 | 180 ± 12 [c] (247 ± 7) | (+)227 |
| $2_2^+ \to 0_1^+$ | 1675 | 10 [f] | (+)2 |
| $2_2^+ \to 2_1^+$ | 1675 | 87 [f] | (−)55 |
| $2_3^+ \to 0_1^+$ | 2876 | 31BR [f,g] | |
| $2_3^+ \to 2_1^+$ | 2876 | 13 [e] | |
| $2_4^+ \to 0_1^+$ | 3084 | 81BR [f,g] | |
| $2_4^+ \to 2_1^+$ | 3084 | 1 [e] | |
| $2_5^+ \to 0_1^+$ | 3233 | 7.4BR [f,g] | |
| $2_5^+ \to 2_1^+$ | 3233 | <11 [e] | |
| $4_1^+ \to 2_1^+$ | 2074 | 560 ±130 [b] | (+)233 |

[a] $B(E2)$ values are from previous measurements and, for the $2_1^+ \to 0_1^+$ transitions, from the present measurements (in parentheses).

[b] See the text and Refs. 9 and 10. The relative phase of the $E2$ matrix element is given in parentheses.

[c] From Fig. 1 and Sec. IV A.

[d] From the lifetimes, $E2/M1$ mixing ratios and branching ratios of Ref. 11.

[e] From Ref. 21.

[f] From the lifetimes of Ref. 19 and the $E2/M1$ mixing ratios and branching ratios of Ref. 7.

[g] A ground state branch has not been observed; the $B(E2)$ is proportional to this branching ratio which is probably <10%(BR < 0.1) for $2_3^+$, $2_4^+$, and $2_5^+$.

the values for $e(\theta, \xi)$ implied in Ref. 25. Finally, we note that our quantal calculations performed for $Q(2^+) = 0$ agree with semiclassical calculations to 1%; when the static moment vanishes only second-order quantal corrections come into play. The quantal correction to $b$ is largest for the light ion cases, where it typically adds 10% to the value of $b$. The suggested corrections of Ref. 23 vary by as much as a factor of 2 from our corrections.

TABLE VI. Parameters of $Y_{2^+}/Y_{0^+} = [B(E2; 0^+ \to 2^+)/R_2][1 + b \cdot Q(2^+)]$ for the 2-level calculations corresponding to the present measurements.

| Nucleus | Exp | $R_2$ ($e^2$ fm$^4$) | $b$ ($e$ fm$^2$)$^{-1}$ |
|---|---|---|---|
| $^{54}$Fe | 28-MeV $^{16}$O | $8.210 \times 10^5$ | $1.957 \times 10^{-3}$ |
| | 86-MeV $^{40}$Ca | $6.394 \times 10^4$ | $8.440 \times 10^{-3}$ |
| $^{56}$Fe | 22-MeV $^{12}$C | $7.896 \times 10^4$ | $2.068 \times 10^{-3}$ |
| | 120-MeV $^{52}$Cr | $8.361 \times 10^3$ | $8.056 \times 10^{-3}$ |
| | 110-MeV $^{52}$Cr | $1.401 \times 10^4$ | $7.503 \times 10^{-3}$ |
| $^{58}$Fe | 22-MeV $^{12}$C | $7.182 \times 10^4$ | $2.129 \times 10^{-3}$ |
| | 120-MeV $^{52}$Cr | $7.046 \times 10^3$ | $7.940 \times 10^{-3}$ |

The parameters $R_2[\equiv B(E2)/P_2^{(0)}]$ and $b$ of Eq. (1) are collected in Table VI for the measurements made on all three Fe isotopes. The values given are for the 2-level case including the corrections outlined above. We now consider the effects of the $4_1^+$ and other $2_1^+$ states.

The matrix elements used for other states are derivable from the $B(E2)$ values in Table V. The effect of including the $4_1^+$ states is independent of the sign of the matrix elements and is to change the $Q(2^+)$ by −0.3, −0.9, and −1.9 $e$ fm$^2$ for $^{54,56,58}$Fe, respectively. This correction for $4^+$ was made with no assumed error. The increasing effect with increasing $A$ reflects the increasing $B(E2; 4^+ \to 2^+)$ value and the decreasing $4_1^+ - 2_1^+$ separation energy (see Fig. 1).

The effect of including the $2_2^+$ states in $^{54,56,58}$Fe is to change $Q(2_1^+)$ by ±1.7, ±0.6, and ±2.4 $e$ fm$^2$, respectively. In principle, nuclear model calculations can provide a guide to the choice of sign of these $2_2^+$ corrections. For instance, using the signs given in Table V for the predictions of McGrory, we find +0.6 and −2.4 $e$ fm$^2$ corrections for $^{56,58}$Fe, respectively. However, the very small values of the $B(E2)$ values predicted for the $2_2^+ \to 0_1^+$ and $2_2^+ \to 2_1^+$ transitions in $^{56}$Fe and the $2_2^+ \to 0_1^+$ transition in $^{58}$Fe reflect a high degree of cancellation between the contributions to the matrix elements; this cancellation causes the predictions to be too sensitive to the details of the calculation to be reliable. This sensitivity was also found for $^{56}$Fe by Lesser *et al.*[5]

The effects of $2_3^+$ states in $^{54,56}$Fe are expected to be comparable to those of $2_2^+$ states, while the $2_3^+$ state should have a negligible effect in $^{58}$Fe. These qualitative effects are predictable from the relative $B(E2)$ values and separation energies

TABLE VII. $B(E2)$ and $Q(2^+)$ values from Coulomb excitation measurements. The analysis includes effects of interference with the $4_1^+$ states as well as estimates of the contribution from the giant dipole resonance. The uncertainty due to the sign of the interference with the higher $2^+$ states is incorporated in the estimated error.

| Nucleus | $B(E2; 0^+ \rightarrow 2^+)$ ($e^2$ fm$^4$) this work | $B(E2; 0^+ \rightarrow 2^+)$ ($e^2$ fm$^4$) previous work[a] | $Q(2^+)$ ($e$ fm$^2$) this work | $Q(2^+)$ ($e$ fm$^2$) previous work | Remarks (for this work) |
|---|---|---|---|---|---|
| $^{54}$Fe | $676 \pm 38$ | $648 \pm 39$ | $-5 \pm 14$ | | Adopted value |
| $^{56}$Fe | $1019 \pm 55$ | $1010 \pm 55$ | $-16 \pm 9$ | $-23 \pm 3$[b,c] | 120-MeV $^{52}$Cr |
| | $1044 \pm 61$ | | $-28 \pm 13$ | $-25 \pm 6$[c,d] | 110-MeV $^{52}$Cr |
| | $1022 \pm 55$ | | $-19 \pm 8$ | | Combined value |
| $^{58}$Fe | $1234 \pm 36$ | $900 \pm 60$ | $-27.3 \pm 5.0$ | $+29 \pm 8$[e] | Adopted value |

[a] From Table V. $B(E2; 0^+ \rightarrow 2^+) = 5 \cdot B(E2; 2^+ \rightarrow 0^+)$.
[b] Reference 4.
[c] No correction has been made for the giant dipole resonance. We calculate an effect of +2.5 $e$ fm$^2$. The effect of higher $2^+$ states is neglected.
[d] Reference 5. No quantal corrections have been applied.
[e] Reference 6.

shown in Fig. 1 and Table V. The estimated uncertainties in $Q(2^+)$ due to $2_n^+$ states with $n > 1$ were added in quadrature for all three isotopes; the results shown in Table VII have not been corrected for the effects of higher $2^+$ states.

Finally, the effects of the giant dipole resonance have been estimated using results of de Boer and Eichler.[26] These corrections, which increase $B(E2; 0^+ \rightarrow 2^+)$ by 1–2 $e^2$ fm$^4$ and the $Q(2^+)$ value by +2.6 $e$ fm$^2$ for all cases, have been included in the results quoted in Table VII.

## V. DISCUSSION

### A. The present results

The moments resulting from our analysis are given in Table VII. For the case of $^{56}$Fe, three sets of numbers are shown. The first number results from analyzing the 120-MeV $^{52}$Cr and 22-MeV $^{12}$C bombardments, the second from analyzing the 110-MeV $^{52}$Cr and the 22-MeV $^{12}$C bombardments, and the third results from analyzing all the data together. While the reduced excitation probability at 110 MeV results in an uncertainty for the $Q(2^+)$ value which is larger than that at 120 MeV, the value obtained is slightly more negative than the 120-MeV value. Coulomb-nuclear interference just above the Coulomb barrier always results in a $Q(2^+)$ which is too negative. Thus, comparison of the 120- and 110-MeV results leads to the conclusion that the derived $Q(2^+)$ values are consistent, and that there is no evidence for 120 MeV being an unsafe energy.

### B. Comparison with previous results

The emphasis in the present work was heavily on $^{58}$Fe and the statistical accuracy obtained for this nucleus was greatly superior to that for $^{54}$Fe and $^{56}$Fe. There is no doubt that we obtain a negative value for the $^{58}$Fe $Q(2^+)$ value, in essential agreement with McGrory's shell-model predictions but in disagreement with the previous value of Ref. 6. We note that this previous value was never published because it was suspected[27] that the measurement was flawed.

Our $B(E2; 0_1^+ \rightarrow 2_1^+)$ value for $^{58}$Fe, $1234 \pm 36$ $e^2$ fm$^4$, is also in strong disagreement with the weighted mean of the four previous measurements, $900 \pm 60$ $e^2$ fm$^4$. This value is dominated by the Coulomb excitation result[6] of $860 \pm 50$ $e^2$ fm$^4$ obtained simultaneously with $Q(2^+) = +29 \pm 8$ $e$ fm$^2$ and thus equally suspect,[27] and by an $(e, e')$ measurement of $940 \pm 80$ $e^2$ fm$^4$ obtained[18] simultaneously with an anomalously low $B(E2)$ value for $^{56}$Fe of $680 \pm 50$ $e^2$ fm$^4$. Since this $(e, e')$ result for $^{56}$Fe is in strong disagreement with present and previous results (see Table VII), the $^{58}$Fe result is also suspect. In summary, then, there are good reasons to discard the previous $^{58}$Fe results for both $Q(2^+)$ and $B(E2)$ and to adopt the present results as they stand. Note that, for our $^{58}$Fe measurements, if the $B(E2)$ value had been fixed at 900 $e^2$ fm$^4$ in the analysis, (i.e., the best previous value) we would have obtained $Q(2^+) = +17$ $e$ fm$^2$. This point is made to illustrate the importance of $B(E2)$ values in the reorientation method and why we felt it necessary to measure both $Q(2^+)$ and $B(E2)$.

For $^{56}$Fe the agreement with previous work (Table VII) is quite satisfactory especially if the correction for the giant dipole resonance of +2.5 $e$ fm$^2$ is made to the previous values. However, the accuracy of our $Q(2^+)$ determination was limited by statistics and is not very competitive with the more accurate previous results.

No previous measurement of $Q(2^+)$ exists for $^{54}$Fe. Our result for the $^{54}$Fe $B(E2;0^+\rightarrow 2^+)$ value is in good agreement with the weighted average of Table III.

### C. Comparison with shell-model predictions

Recent shell-model calculations of Vennink and Glaudemans[28] for $^{54}$Fe give $Q(2^+)=-15\ e\,\mathrm{fm}^2$ for a basis consisting of $f_{7/2}{}^{-2}$ and an additional effective charge $x$ of 1.0 [i.e., the same calculation as that of McGrory (Sec. III)] and $Q(2^+)=-20\ e\,\mathrm{fm}^2$ for a basis which allows a neutron or proton excitation out of the $f_{7/2}$ shell. Since this latter calculation is generally quite successful, it would appear that a prolate deformation is strongly indicated for $^{54}$Fe $2_1^+$ as well as for $^{56,58}$Fe. Our result for $^{54}$Fe of $Q(2^+)=-5\pm 14\ e\,\mathrm{fm}^2$ is consistent with this shell-model prediction but due to its limited accuracy, does not represent a serious test of this value.

The experimental result of $Q(2^+;{}^{58}\mathrm{Fe})=-28\pm 5\ e\,\mathrm{fm}^2$ lays to rest a possible discrepancy with shell-model predictions. The discrepancy would have been very severe indeed because the many calculations[29] and interpretations[2,9,28,30,31] of the energy spectra of the even-$A$ Fe isotopes imply a very stable prolate deformation for the first $0^+-2^+-4^+$ states with quite highly collective $E2$ transitions between them.

In one quite enlightening interpretation, the collectivity of these states is shown[30,31] to follow largely from the dominance of the quadrupole-quadrupole force in the $n$-$p$ interactions. $^{56}$Fe, for instance, is viewed as $^{54}\mathrm{Fe}\otimes{}^{58}\mathrm{Ni}$ and the $Q_p\cdot Q_n$ force pushes the resulting nearly degenerate $2^+_{1,2}$ states apart and endows the lower one with most of the $E2$ strength connecting to the ground state. The similarity of the low-lying spectra of the even Ni isotopes—particularly $^{58,60,62}$Ni—guarantees the stability of the $E2$ collectivity from $^{56}$Fe to $^{60}$Fe.

This stability is also borne out by shell-model predictions. Most calculations have been performed in the configurational space of Eq. (2). These various calculations scan a range of single-particle energies and nucleon-nucleon interactions. In all of them the $B(E2;2^+\rightarrow 0^+)$ and $Q(2^+)$ values are collective and the various contributions add coherently thus ensuring insensitivity to the wave functions. The various calculations are in quite good agreement.

The recent extension for $^{54,55,56}$Fe of the configurational basis to include a neutron or additional proton hole in the $f_{7/2}$ shell and an additional nucleon in the $(2p_{3/2}, 1f_{7/2}, 2p_{1/2})$ shell[28] also is enlightening. It is found that in this basis the $\pi f_{7/2}{}^{-3}$ components in the wave functions of $^{54,55,56}$Fe are all ~30%. The near constancy of this percentage is indicative of the stability of the basic character of the low-lying even-$A$ Fe structure. Also of pertinence is the fact that the experimental $B(E2)$ and $Q(2^+)$ values of the low-lying levels are now reproduced with an effective charge parameter $x=0.5$ rather than the $x=1.0$ needed for the space of Eq. (2). This would indicate the validity of the shell model in the sense that the effective parameters are converging rapidly towards the free nucleon values as the configurational space is expanded.

The observables most important to this discussion are $Q(2^+)$, $B(E2;2^+\rightarrow 0^+)$, and $B(E2;4^+\rightarrow 2^+)$. How do the current experimental values for these observables compare to shell-model predictions? Because the effective charge is allowed to vary in order to compensate for the limited configurational space, close *quantitative* comparisons are not very meaningful. The currently measured $Q(2^+)$ and $B(E2;2^+\rightarrow 0^+)$ values for $^{54,56,58}$Fe are consistent with predictions as are the average values for $B(E2;4_1^+\rightarrow 2^+)$ listed in Table V for $^{54,56}$Fe. Only the measured[2,18] $B(E2;4^+\rightarrow 2^+)$ value for $^{58}$Fe is out of line with systematics and specific predictions.[28]

The use of an additional effective charge $x$ in the calculation of $E2$ moments is a highly successful method of compensating for the use of too restricted a configurational basis, but it does not allow the prediction of absolute $E2$ moments—only of relative ones. One might ask then: What can we learn from a relative comparison of $Q(2^+)$ for the even-Fe isotopes? The predictions of Raman and McGrory[10] for $^{56,58,60}$Fe are $Q(2^+)=-26$, $-27$, and $-29\ e\,\mathrm{fm}^2$, respectively, for the space of Eq. (2); while Vennink and Glaudemans obtained $Q(2^+)=-15$ and $-24\ e\,\mathrm{fm}^2$ for $^{54,56}$Fe in the same space. At its present level of accuracy it does not seem likely that the reorientation technique is capable of testing the differences between these predictions—at least for $A>56$. For instance, the second-order corrections discussed in Sec. III B are of the order of the differences and have large uncertainties. It would seem that the most we can do is to regard $Q(2^+)$ as an indicator of the sense of the $E2$ deformation and to test its magnitude with measurements of the $B(E2)$ values.

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