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Recent nucleon-nucleon scattering data and the Paris potential predictions

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New data (cross section, polarization, Wolfenstein parameters, and spin-correlations) on proton-proton and neutron-proton scattering have been recently published by different groups. These include high precision and/or original measurements covering the energy range 6 MeV $< T_{\mathrm{lab}} <$ 800 MeV. A direct comparison of these data with the values produced by the Paris *NN* potential for energies $T_{\mathrm{lab}} <$ 350 MeV is reported here. The agreement between theory and experiment is very satisfactory both for low and medium energies. The total χ^2 for the world *NN* data set for $T_{\mathrm{lab}} <$ 350 MeV is also reported and compared with those given by the phase shift analysis of Arndt *et al.* and by the phenomenological Reid soft-core potential.

[NUCLEAR REACTIONS Nucleon-nucleon interaction, comparison of the Paris *NN* potential predictions with new experimental data.]

INTRODUCTION

During the last two years, a great deal of new measurements on cross sections, polarization, Wolfenstein parameters, and spin correlations for *NN* scattering have been performed both at low[1-5] and intermediate[6-10] energies. The reported data are, in the *pp* case, both new and very accurate and in the *np* case, although of lesser precision nevertheless crucial for the resolution of the long standing ambiguities in the phase-shift analyses at 50 and 325 MeV. In all cases, they provide the possibility for testing with accuracy the quality of the different theoretical models of the *NN* interaction, especially for the long and medium range parts.

The purpose of this paper is to present a direct comparison of these data with the predictions of the Paris *NN* potential. We would like to emphasize that, for an accurate quantitative test of theoretical models, it is more decisive to compare theoretical predictions and experimental data directly rather than through phase shifts. Of course a phase-shift representation is useful and gives a good idea of the overall properties of the *NN* interaction. It is, however, well known that the solution is not necessarily unique, especially in the case of fixed-energy phase-shift analysis. In the present work, some of the new data were used to get a better determination of the phenomenological core parameters while others are simply compared with our predictions. In the latter case, the agreement between theory and experiments is very good.

THE PARIS POTENTIAL

Details on the construction of the Paris potential and its final parametrization can be found in Refs. 11 and 12. Let us only recall that this potential contains a theoretical long and medium range part due to one-pion, two-pion, and parts of the three-pion exchange contributions. The two-pion exchange contribution was derived, via dispersion relations, from pion-nucleon phase shifts and pion-pion *S*- and *P*-wave amplitudes. The ω and A_1 mesons were included as parts of the three-pion exchange contribution. The short range part was constructed phenomenologically with the requirement that the long and medium range theoretical part is preserved for internucleon distances beyond 0.8 fm. The parameters were adjusted to fit a set of 913 data points between 3 and 330 MeV for *pp* scattering and a set of 2239 data points between 13 and 350 MeV for *np* scattering. We restrict ourselves here to energies not too far above the π production threshold as we are using a nonrelativistic potential picture. The analysis of higher energy data is currently under investigation with the same dynamical input but in a relativistic scheme. In this regard, let us recall that our uncorrelated two pion-exchange provides inelasticity parameters that are expected to give a good description of the high partial waves.

PROTON-PROTON SCATTERING

The new *pp* experimental data discussed in this paper were not available when our fit was performed and hence not used for the determination of our

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core parameters. Our *pp* results are therefore predictions.

At low energies, the comparison of the new high precision polarization data at 6.141 MeV (Ref. 1) and of earlier data at 10 MeV (Ref. 13) and 16 MeV (Ref. 14) with the Paris potential predictions has been reported elsewhere.[15] Let us only recall that the χ^2/data is 0.08, 3.31, and 1.61 at 6.141, 10, and 16 MeV, respectively. A very recent high precision measurement at 9.88 MeV was reported lately,[16] yielding results compatible with those at 10 MeV. These two sets of measurements[13,16] are more accurate than those we have used in our core parameters search. In view of that, the value 3.31 for the χ^2/data at 10 MeV should be considered as being satisfactory. A slight adjustment of the short range part in our isovector potential should be sufficient to improve the agreement at 10 MeV without spoiling the results at 6.141, 16 MeV, and above. The spin correlation parameter A_{yy} at 9.57 MeV and 90° c.m. angle has been measured by Obermeyer *et al.*[2] The value they found is -0.978 ± 0.012, to be compared with our result of -0.969. They also performed a single energy phase-shift analysis which yields results differing from ours and from those of Arndt *et al.*[17] This is an illustration of an ambiguous situation, as mentioned in the introduction, where the data agree without an agreement of the phase shifts.

At medium energies, the polarization P and Wolfenstein parameters D, R, R', and A have now been measured: P, D, R, R' at TRIUMF by the Basque group[6] for 209 and 324 MeV and c.m. angles between 13° and 58° and P, D, R, A at SIN by the Geneva group[7] for 312 MeV and c.m. angle between 3° and 33°. These data are compared with our predictions in Figs. 1–3. We have also plotted the results obtained with the phase-shift analysis of Arndt *et al.*[17] No renormalization has been done. The TRIUMF data on P and R' are in agreement with the theoretical results at both energies. At 209 MeV, R is well reproduced, but our values for D are slightly lower than the TRIUMF measurements. This could be traced back to the fact that the D values at 213 MeV of the Rochester group[18] which we used, are in general lower than the TRIUMF values while the R values of the two groups are nearly the same. At 324 MeV, the results of Arndt *et al.* are in better agreement with the TRIUMF values than ours as they included in their phase-shift analysis the TRIUMF data. These are much more accurate than the earlier R and D data[19] we used at 310 MeV. The accord of our results with the Geneva measurements is excellent for P and very good for R, A, and D, considering the spread of the

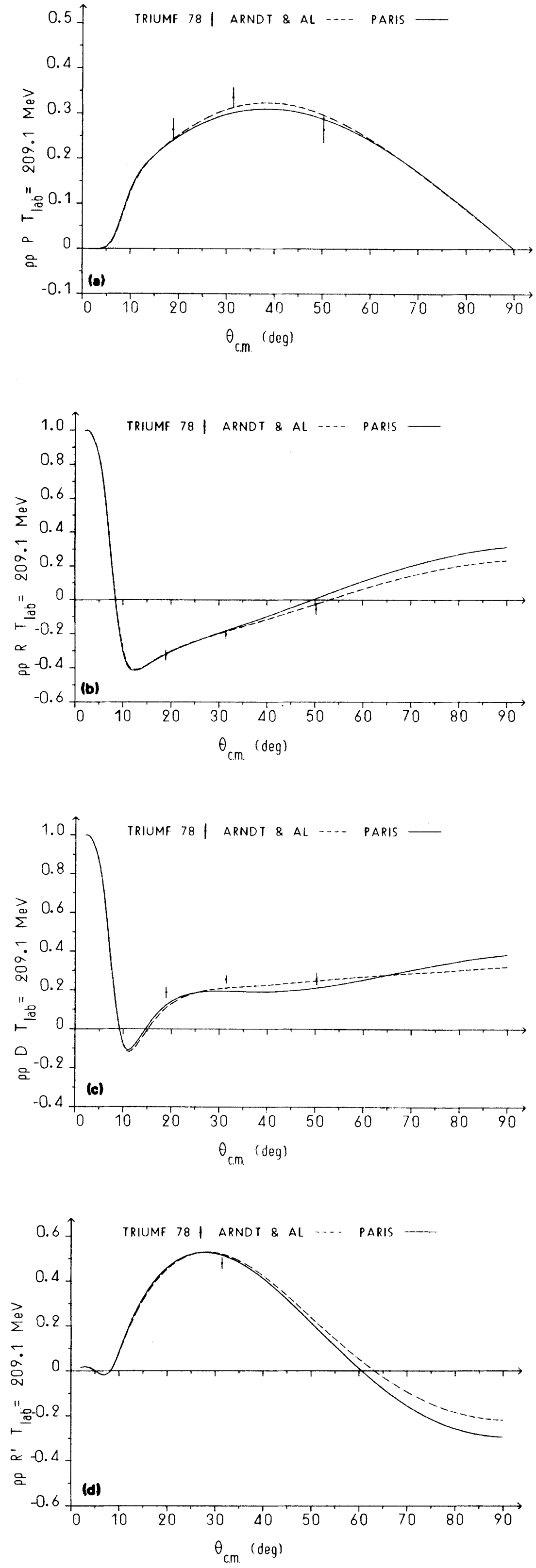

FIG. 1. *pp* polarization and Wolfenstein parameters.

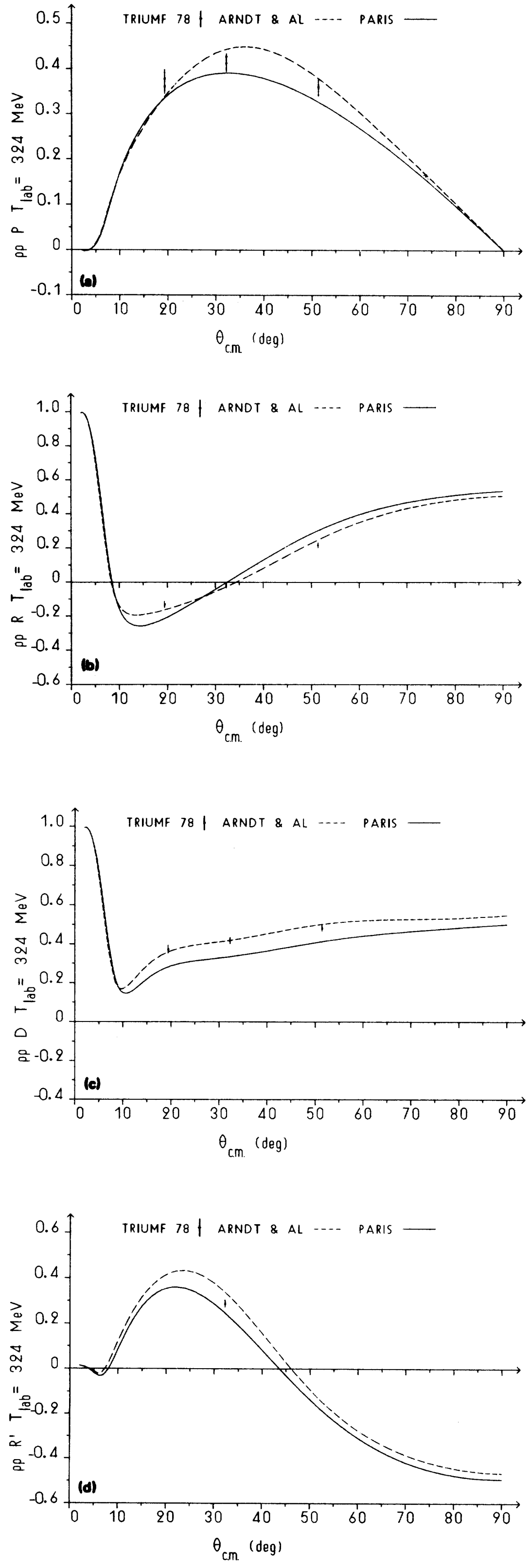

FIG. 2. *pp* polarization and Wolfenstein parameters.

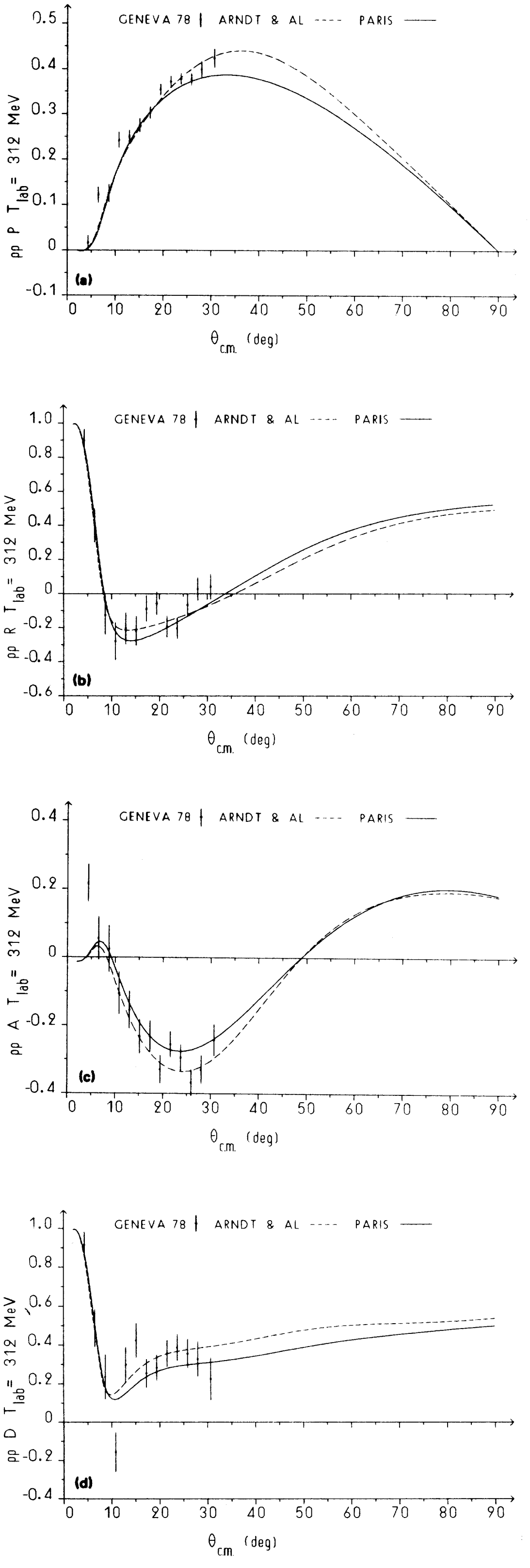

FIG. 3. *pp* polarization and Wolfenstein parameters.

data.

Finally, in view of future experiments with polarized beams and targets, the longitudinal and transverse cross sections $\Delta\sigma_L$ and $\Delta\sigma_T$($\Delta\sigma_L = \sigma_{\rightleftarrows} - \sigma_{\rightrightarrows}$, $\Delta\sigma_T = \sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow}$) as well as σ_{tot} are displayed in Fig. 4 along with the preliminary results of Aprile *et al.*[20] for $\Delta\sigma_L$ at 201 and 280 MeV. We have calculated these total cross sections via the optical theorem without Coulomb interaction.

NEUTRON-PROTON SCATTERING

At low energies, new measurements of the polarization between 13.5 and 16.9 MeV (Ref. 3) and at 14.2 MeV (Ref. 4) have been recently reported. At 16.9 MeV, Tornow *et al.*[3] found that their data disagree with the results of Yale-IV,[21] MAW-X,[22] and Arndt *et al.*[17] phase-shift analyses. Again, these data were not included in our core parameters search. The details of our predictions and analysis have been reported in Ref. 23, and the results are summarized in Fig. 5. The agreement with these data is excellent for $P(90^\circ)$ with a χ^2/data of 0.7 and for $P(\theta)$ at 14.2 MeV with a χ^2/data of 0.1, and good at 16.9 MeV with a χ^2/data of 2.6. At these energies, the polarization depends mostly on the *P*, *D*, and *F* waves, an interesting feature, since these waves, in particular the *F* wave, are mainly given by the intermediate and long range part of the interaction, that part which is well founded theoretically in our potential. Let us note furthermore that our χ^2/data is 1.1 for 400 data points in *np* scattering between 11 and 40 MeV.

The Davis group[5] has measured $d\sigma/d\Omega$, *P*, and A_{yy} at 50 MeV and $d\sigma/d\Omega$ at 63.1 MeV. The comparison of their data with our fit is shown in Fig. 6. At 50 MeV the agreement is quite good for $d\sigma/d\Omega$, *P*, and A_{yy} but not so good at 63.1 MeV for the backward points of $d\sigma/d\Omega$. Their results stabilize the values for ϵ_1 around 3° and for ${}^1P_1$ around -6.5° at 50 MeV, more compatible than before with those we found (1.9° and -11°, respectively). The difference, however, might be at the origin of the discrepancy at 63.1 MeV in $d\sigma/d\Omega$ at backward angles. It should be recalled that the values for ϵ_1 and ${}^1P_1$ obtained successively in different phase-shift analyses have varied widely.[24]

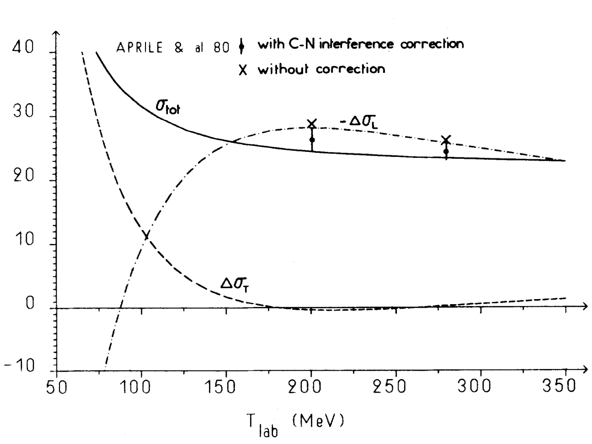

FIG. 4. *pp* total cross section σ_{tot} and longitudinal and transverse *pp* total cross section differences $\Delta\sigma_L$ and $\Delta\sigma_T$ (in mb) as function of laboratory kinetic energy. The experimental points of Ref. 20 correspond to 201 and 280 MeV.

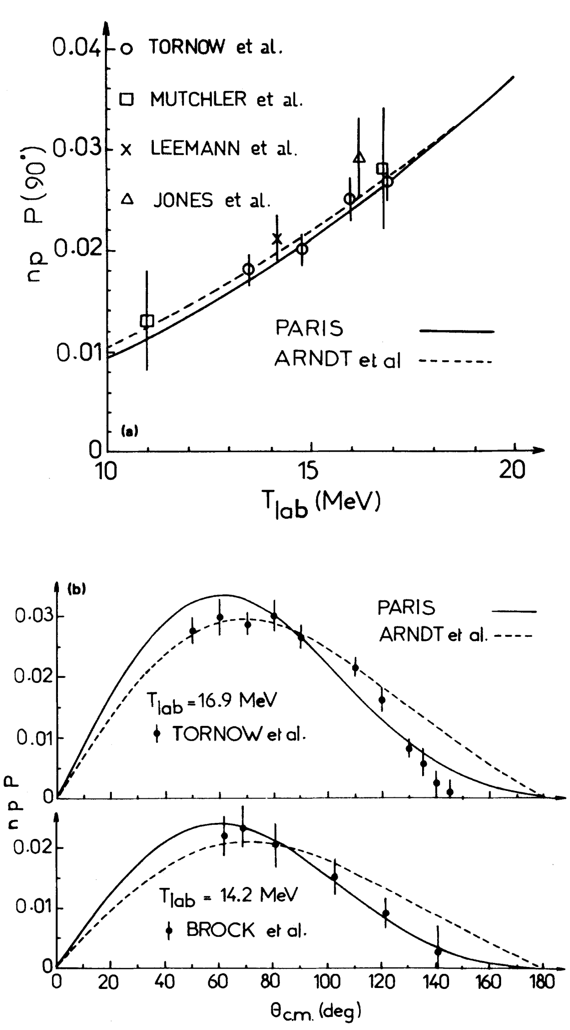

FIG. 5. (a) *np* polarization at 90° c.m. angle as function of laboratory kinetic energy. The references for experimental points can be found in Tornow *et al.* (Ref. 3). (b) *np* polarization at 14.2 and 16.9 MeV as function of c.m. angle.

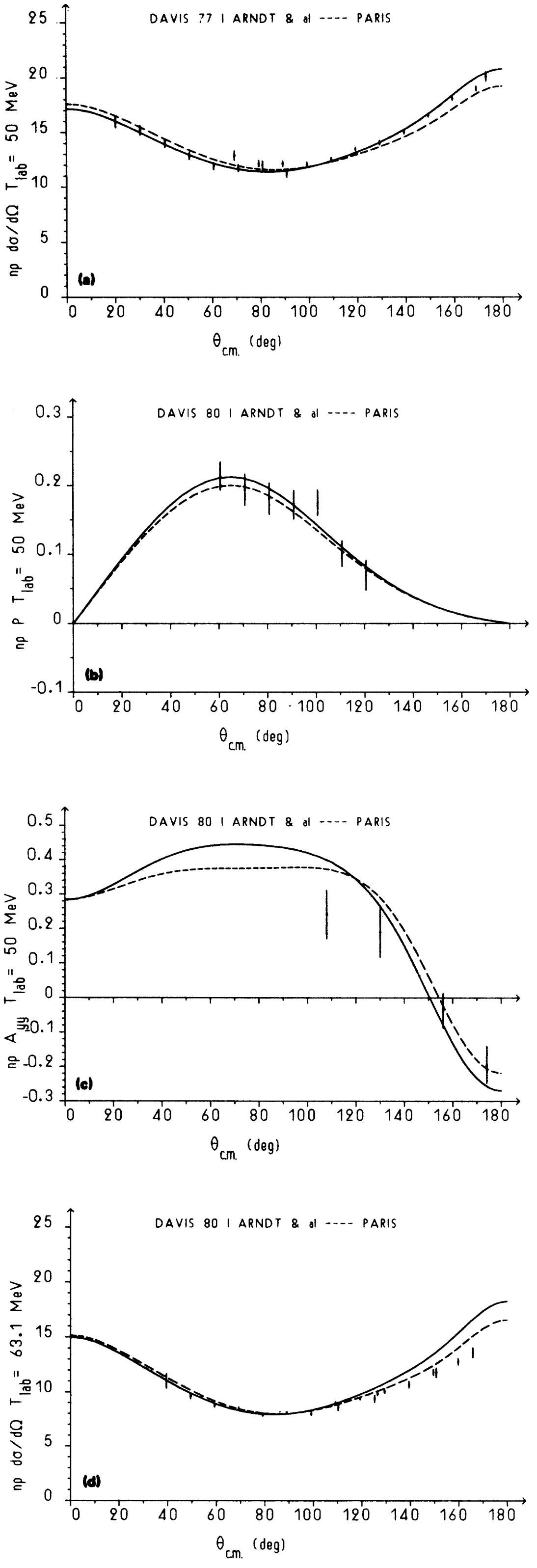

FIG. 6. *np* $d\sigma/d\Omega$ (in mb), P, and A_{yy} at 50 MeV; *np* $d\sigma/d\Omega$ at 63.1 MeV (in mb).

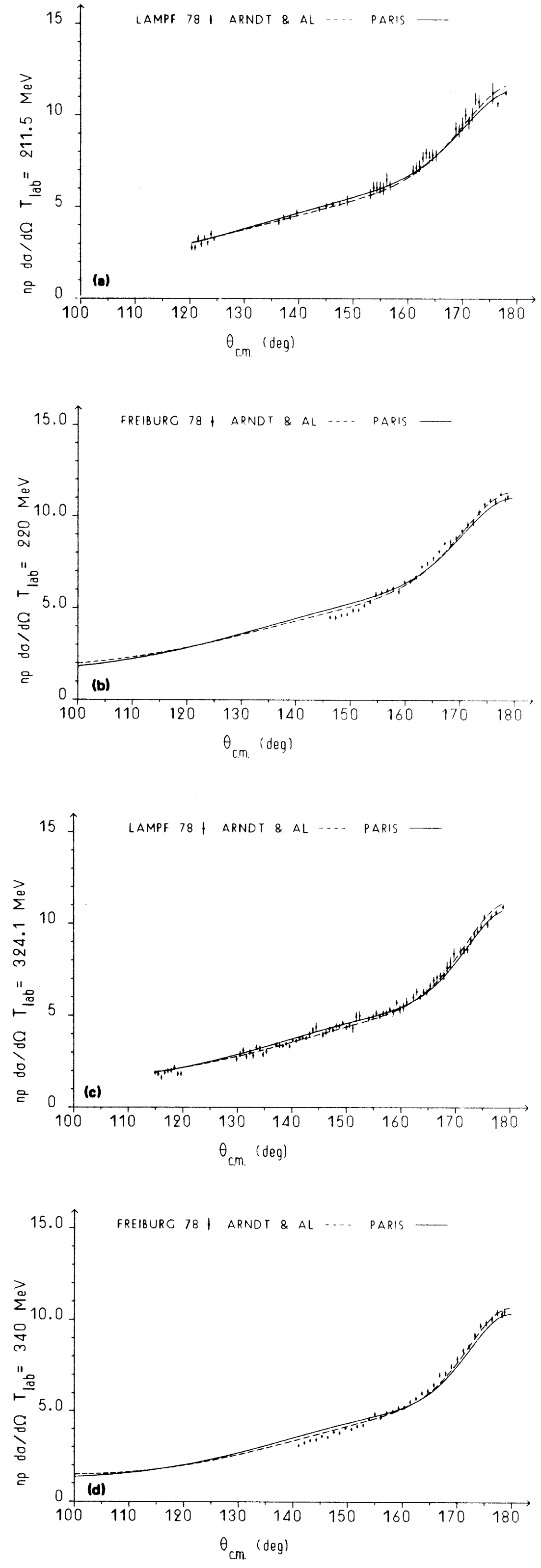

FIG. 7. *np* backward cross sections at medium energy (in mb).

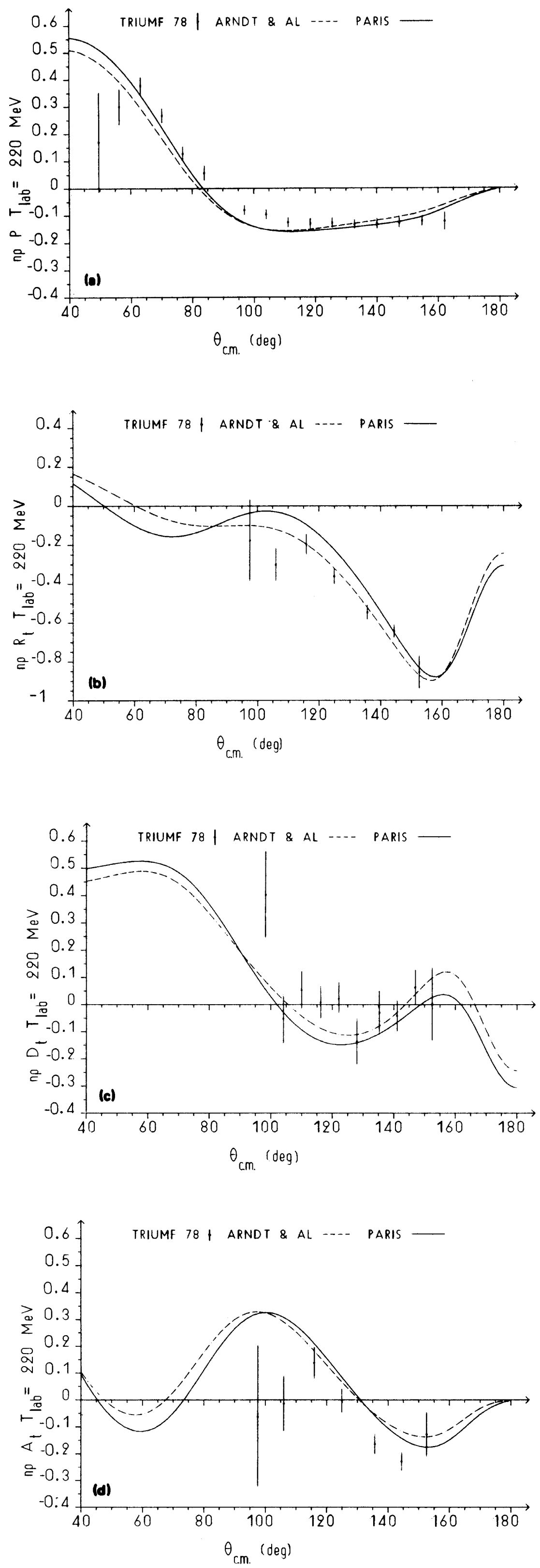

FIG. 8. *np* polarization and Wolfenstein parameters.

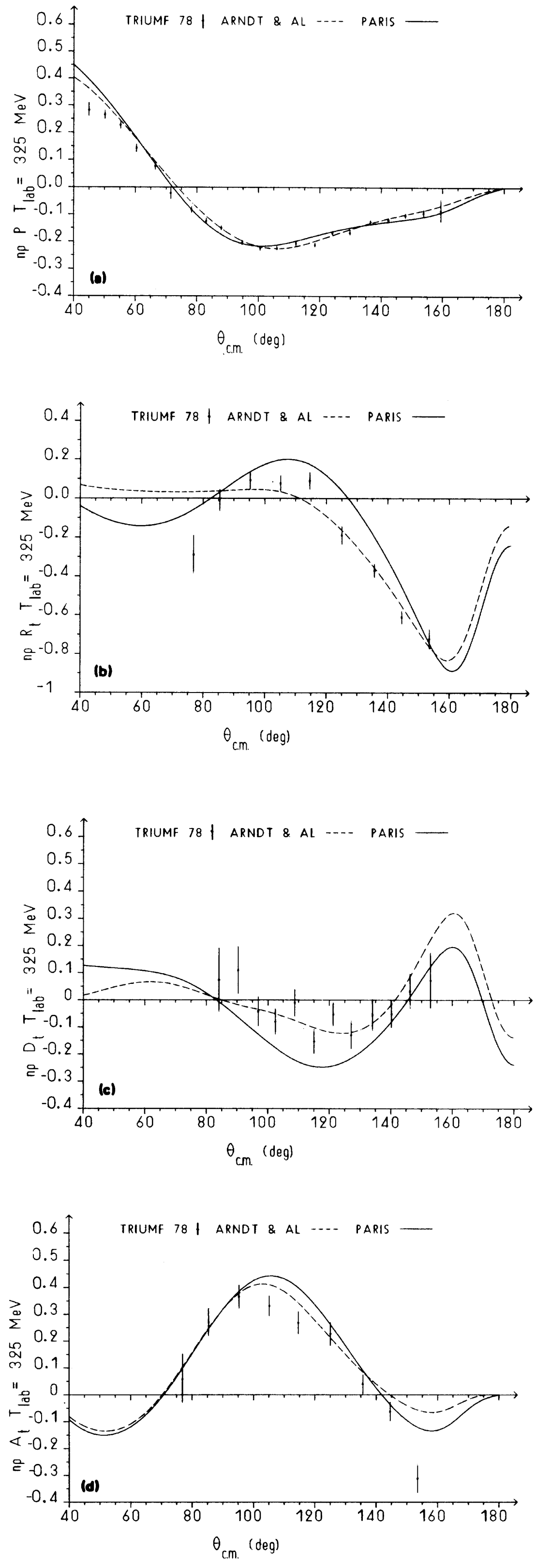

FIG. 9. *np* polarization and Wolfenstein parameters.

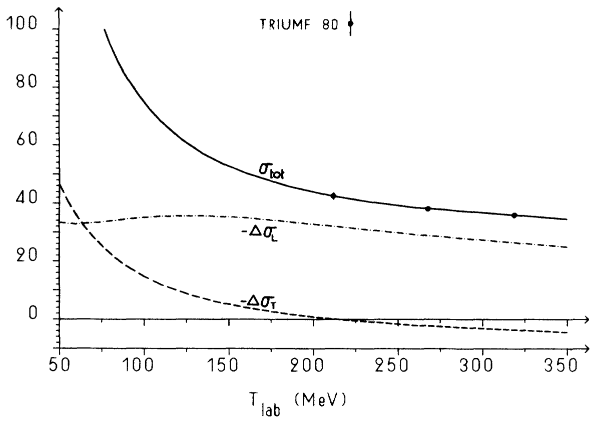

FIG. 10. *np* total cross section σ_{tot} and longitudinal and transverse *np* total cross section differences $\Delta\sigma_L$ and $\Delta\sigma_T$ (in mb) as function of laboratory kinetic energy. The experimental points of Ref. 26 correspond to 212, 268, and 319 MeV.

At medium energies, backward *np* cross sections have been measured both at LAMPF (Ref. 8) (150–800 MeV) and by the Freiburg group[9] at SIN (200–600 MeV). Absolute normalization is only known for energies above 300 MeV and data at lower energies are only relative. We floated all these data. Our results, shown in Fig. 7, fit easily the LAMPF data (χ^2/data$\simeq 1.7$ for energies between 150 and 350 MeV) with the correct normalization but not so easily the Freiburg results (χ^2/data$\simeq 16$ for energies between 200 and 350 MeV). However, in the latter case, if data at angles below 154° are left out, the χ^2/data drops from 16 to 2. Also at medium energies, measurement of polarization P and Wolfenstein parameters D_t, R_t, and A_t has been performed at TRIUMF by the Basque group[10] at 220 and 325 MeV. Their results are plotted in Fig. 8 and 9. Here again they are well reproduced by ours, without any renormalization of the data. At 325 MeV, the new data permit the elimination of the long standing ambiguities in the phase-shift analyses. The ϵ_1 and ϵ_3 mixing parameters are stabilized at 8.4° and 7.3°,[25] to be compared with our values $\epsilon_1 = 5.3°$ and $\epsilon_3 = 7.7°$. Provided that the triplet G wave phase shifts are constrained by theory the solution is now unique.

Also, in view of future experiments with polarized beams and targets, $\Delta\sigma_T$, $\Delta\sigma_L$, and σ_{tot} are displayed in Fig. 10 along with results of the Basque group[26] for σ_{tot} at 212, 268, and 319 MeV.

THE QUALITY OF THE FIT

In order to provide an overview of the goodness of fit of the Paris potential on the *NN* data below 350 MeV we introduce an accumulated chi-squared per degree of freedom $\chi^2(T)$ defined as

$$\chi^2(T) = \chi_{tot}{}^2(T)/[N(T) - \Lambda(T)]\,,$$

where $\chi_{tot}{}^2(T)$, $N(T)$, and $\Lambda(T)$ are the total χ^2, the number of data points and the number of renormalizations up to the laboratory energy T, respectively. The expression for $\chi_{tot}{}^2(T)$ is

$$\chi_{tot}{}^2(T) = \sum_{j=1}^{N(T)} \left(\frac{O_j^{theo} - \lambda_{i(j)} O_j^{exp}}{\Delta O_j^{exp}}\right)^2 + \sum_{i=1}^{\Lambda(T)} \left(\frac{1-\lambda_i}{\Delta\lambda_i^{exp}}\right)^2,$$

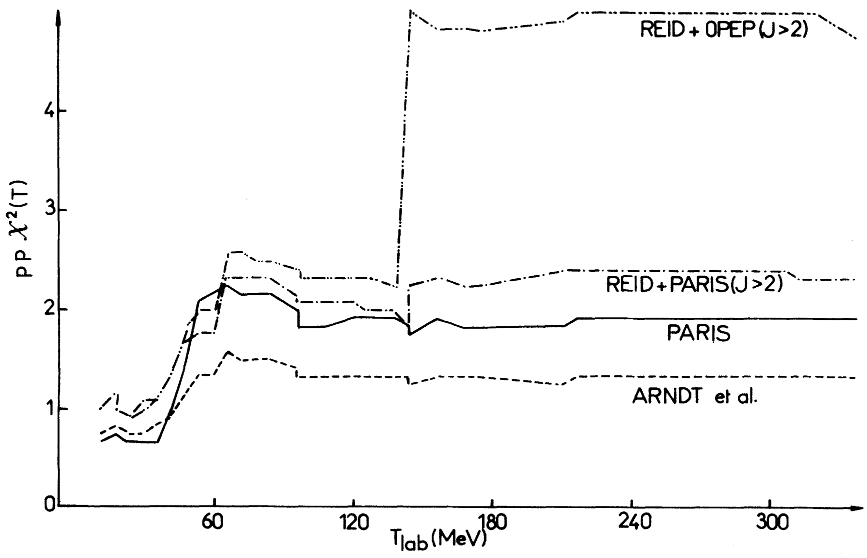

FIG. 11. The accumulated chi-squared per degree of freedom for *pp* scattering.

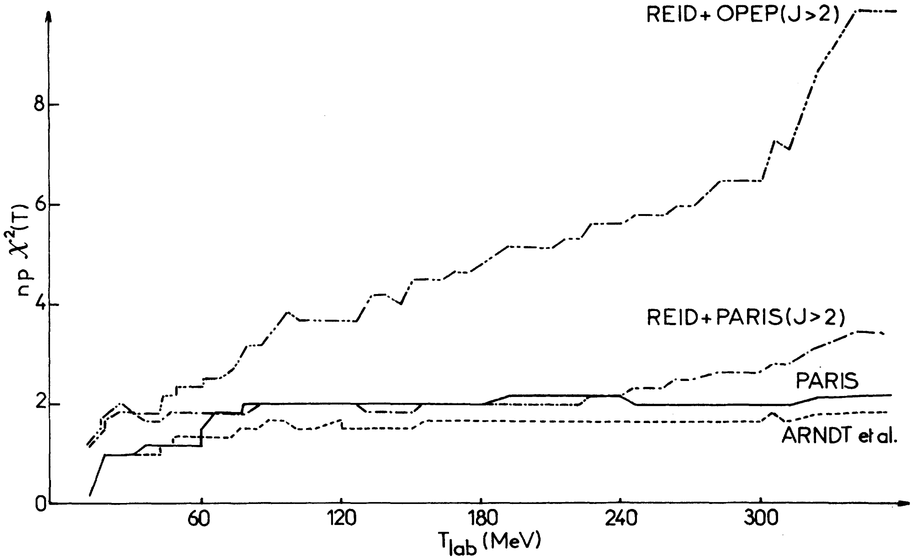

FIG. 12. The accumulated chi-squared per degree of freedom for *np* scattering.

where λ_i and $\Delta\lambda_i^{exp}$ are the predicted normalizations and the experimental normalization uncertainties. In this work only the differential cross-sections and polarizations were renormalized using in most cases the $\Delta\lambda_i^{exp}$ of the Livermore compilation[27] and of the papers of Arndt *et al.*[17,28] The floating of the data corresponds to an infinite $\Delta\lambda_i^{exp}$, i.e., a zero penalty term. The λ_i are obtained by minimization of $\chi_{tot}^2(T)$. The $\chi^2(T)$ is shown in Fig. 11 for *pp* scattering and in Fig. 12 for *np* scattering. For comparison, we also plot the $\chi^2(T)$ obtained with the Arndt *et al.* phase-shift analysis, with the Reid soft-core potential[29] supplemented, as usually done in the literature, by the high partial waves ($J>2$) of the OPEP and with the Reid soft core potential whose high partial waves ($J>2$) are replaced by those of the Paris potential. As can be seen, this replacement yields a dramatic improvement of the fit and indicates rather strongly that our medium range forces, well founded theoretically, are needed to get a good fit of the data for energies $T_{lab} > 60$ MeV. The result, however, is still not as good as that given by the full Paris potential.

CONCLUSIONS

In view of the reasons discussed in the introduction, we chose to test the quality of the Paris *NN* potential by a direct comparison with the *NN* scattering data rather than with the phase shifts. The endeavor is very time consuming but we believe it is worth the effort since it leads to more clear-cut conclusions. Considering the wealth of the data and the high accuracy attached to many of them, our accumulated chi-squared per degree of freedom reaches very satisfactory values: 1.99 for *pp* scattering and 2.17 for *np* scattering to be compared with 4.76 and 9.99 for the Reid soft-core potential and with 1.33 and 1.80 for the phase-shift analysis of Arndt *et al.* The comparison of the Paris potential results with those of the Reid soft core potential indicates that the two-pion exchange contribution (TPEC) is necessary for the medium range forces to get a good fit of the data. This suggests also that the TPEC should be a useful constraint for phase-shift analyses.

The analysis of higher energy data (e.g., 350 to 800 MeV) can be performed in the same spirit either by using a relativistic wave equation with an optical potential to account for inelasticity or via a phase-shift analysis with inelasticity parameters. We are currently carrying out this program by using either the long range imaginary part of the optical potential or the peripheral inelasticity parameters, both extracted from our TPEC.

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