Isoscalar dipole resonance: Form factor and energy weighted sum rule

M. N. Harakeh and A. E. L. Dieperink
Kernfysisch Versneller Instituut, Groningen, The Netherlands
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A collective formalism first proposed by Deal to describe isoscalar dipole excitations in electron scattering is extended to include excitations of these modes by hadron scattering. Distorted-wave Born approximation calculations are performed using this collective isoscalar $L = 1$ form factor to describe the excitation of the 1^-, $T = 0$ state at $E_x = 7.12$ MeV ($1\hbar\omega$ character) in ^{16}O and a newly proposed candidate for the isoscalar giant dipole resonance ($3\hbar\omega$ character) at $E_x = 21.3$ MeV in ^{208}Pb excited in inelastic α scattering. A good description of the cross sections of the 1^-, 7.12 MeV state in ^{16}O and the $E_x = 21.3$ MeV bump in ^{208}Pb is obtained in this model exhausting 4.2% and 130% of the energy-weighted sum rule, respectively.

[NUCLEAR STRUCTURE Isoscalar dipole resonance; derived form factor, energy weighted sum rule. Excitation in hadron scattering.]

I. INTRODUCTION

Isoscalar dipole excitations have already been observed in many self-conjugate nuclei at low excitation energies by inelastic electron[1,2] and hadron[3-5] scattering. The relatively strong excitation of these isoscalar dipole states in inelastic electron[2] and hadron[3] scattering could be well understood in terms of plane wave Born approximation (PWBA) and distorted wave Born approximation (DWBA) calculations, respectively, using a microscopic form factor derived from $1\hbar\omega$ shell model wave functions and corrected for center of mass (c.m.) motion. This would correspond to the $1\hbar\omega$ (low-energy) isoscalar dipole resonance (LEDR), the strength of which should be fragmented over a small number of states for these light nuclei.

Recently, evidence was found[6] for strong excitation of isoscalar dipole states in ^{40}Ca at higher excitation energies ($E_x \sim 13$–17 MeV) at $E_\alpha = 104$ MeV. In addition, from an analysis of the forward-backward asymmetry observed in α_0 decay of the giant resonance (GR) in light nuclei it was conjectured[7] that an appreciable percentage of the isoscalar dipole sum rule could already be exhausted in the region of the giant quadrupole resonance in the sd-shell nuclei. For these nuclei, most of the $3\hbar\omega$ high energy isoscalar dipole resonance (HEDR) strength would be at higher excitation energies and would probably be spread over a wide energy region, similar to other giant resonances in this mass region. For heavier nuclei, it is expected that the HEDR strength is more concentrated in a narrow excitation region, and also that it could be rather strongly excited[7] in inelastic α scattering. Evidence for a possible excitation of the HEDR in ^{208}Pb at $E_x \simeq 21$ MeV has recently been found[8] from an inelastic α-scattering experiment on ^{208}Pb at $E_\alpha = 172$ MeV.

Although in principle one can perform DWBA calculations, using microscopic form factors, to describe these isoscalar dipole excitations, in practice these are difficult to perform, first, because for many nuclei such microscopic wave functions do not exist, and second, because if such wave functions do exist they most often are not properly corrected for the spurious c.m. motion. In view of this, and to facilitate future DWBA analysis of isoscalar dipole excitations, it is desirable to describe these excitations in terms of a collective model. In this way a collective form factor is obtained by deriving an energy weighted sum rule (EWSR) for these excitations. Such a formulation has been attempted by Deal,[9] where a collective form factor in q space for the excitation of these isoscalar dipole resonances in inelastic electron scattering has been obtained and successfully applied to the excitation of the lowest lying 1^-, $T = 0$ states in ^{12}C, ^{16}O, and ^{40}Ca. Unfortunately, in obtaining[9] the transition density in r space, an error occurred.

In this paper the transition density for the excitation of the isoscalar dipole resonance in inelastic electron and hadron scattering will be derived along lines similar to the formulation of Deal, but with a slightly different approach. Moreover, the EWSR for the isoscalar dipole excitations will be obtained. This collective formulation will be applied in DWBA calculations to describe the $J^\pi = 1^-$, $T = 0$ state at 7.12 MeV in ^{16}O (Refs. 3 and 7), which is supposedly a fragment of the LEDR, and the newly reported[8] possible candidate for the $3\hbar\omega$ HEDR in ^{208}Pb.

II. SUM RULES AND COUPLING POTENTIALS

In inelastic electron scattering and also in inelastic hadron scattering in PWBA, assuming[10] a

δ interaction, the isoscalar multipole spin-independent transition operator can be written

$$O^{(\lambda)}(q)\equiv\sum_{i=1}^{A} j_\lambda(qr_i')Y_\lambda(\hat{r}_i'),$$

where r_i' are the coordinates in the c.m. system ($\vec{r}_i'=\vec{r}_i-\vec{R}$). Whereas for $\lambda\geq 2$ in the limit of $q\to 0$ the first order terms in the expansion of $O^{(\lambda)}$ survive and are proportional to the electric spin-independent transition operators $E\lambda=\sum_i r_i'^{\lambda}Y_\lambda(\hat{r}_i')$, this is not the case for $\lambda=0$ and $\lambda=1$, which require a special treatment. For $\lambda=0$ the first term in the expansion is a constant which cannot induce any transitions from the ground state to excited states; the leading transition operator is due to the second term in the expansion in (qr).

For $\lambda=1$, the leading term for $q\to 0$ is proportional to the c.m. coordinate and should vanish if evaluated with translationally invariant wave functions. In this case the leading order transition operator is due to the second term in the expansion in (qr), but here the correct treatment (subtraction) of the c.m. motion is still of vital importance. This is treated in more detail in the Appendix; here we give the final results.

For $\lambda=1$ and $q_2\to 0$, the general c.m. adjusted sum rule for multipole λ [Eq. A11)] leads to the isoscalar dipole sum rule in q space:

$$\sum_n (E_n-E_0)\tilde{F}^{(1)}_{0n}(q)\tilde{P}^{(1)}_{n0}=-\frac{\hbar^2}{16m\pi}\left(3q^2\frac{d}{dq}\frac{1}{q}\frac{d}{dq}+5\frac{d}{dq}+\tfrac{5}{3}\langle r^2\rangle q+\epsilon q^2\frac{d}{dq}\right)F_{el}(q), \tag{1}$$

where $\epsilon=(4/E_2+5/E_0)\hbar^2/3mA$; $\tilde{F}^{(\lambda)}_{0n}(q)$ and $\tilde{P}^{(\lambda)}_{n0}$ are defined in the Appendix, and $F_{el}(q)$ is the isoscalar elastic form factor. A similar formula has been obtained by Deal [Eq. (15) of Ref. 9].

The corresponding sum rule in r space which can be obtained from Eq. (1) by a Fourier transform is

$$\sum_n (E_n-E_0)\tilde{P}^{(1)}_{n0}\rho^{(1)}_{0n}(r)=-\frac{\hbar^2}{4m}\left[3r^2\frac{d}{dr}+10r-\tfrac{5}{3}\langle r^2\rangle\frac{d}{dr}+\epsilon\left(r\frac{d^2}{dr^2}+4\frac{d}{dr}\right)\right]\rho_0(r)Y_1^0(\hat{r}), \tag{2}$$

where ρ_0 and $\rho^{(1)}_{0n}$ are the ground state and $\lambda=1$ the transition densities, respectively. The sign of the third term in Eq. (2) differs from that reported by Deal[9] and was also wrongly reported in Ref. 7.

In the limit of $q\to 0$, Eq. (1) leads to the static isoscalar dipole energy weighted sum rule:

$$\sum(E_n-E_0)[\tilde{P}^{(1)}_{0n}]^2=\frac{\hbar^2 A}{32m\pi}(11\langle r^4\rangle-\tfrac{25}{3}\langle r^2\rangle^2-10\epsilon\langle r^2\rangle). \tag{3}$$

If the isoscalar dipole energy weighted sum rule is exhausted by one state, then the transition density of this state can be expressed as

$$\rho^{(1)}(r)=-\frac{\beta_1}{R\sqrt{3}}\left[3r^2\frac{d}{dr}+10r-\tfrac{5}{3}\langle r^2\rangle\frac{d}{dr}+\epsilon\left(r\frac{d^2}{dr^2}+4\frac{d}{dr}\right)\right]\rho_0(r), \tag{4}$$

where

$$\beta_1^2=\frac{6\pi\hbar^2}{mAE_x}R^2/(11\langle r^4\rangle-\tfrac{25}{3}\langle r^2\rangle^2-10\epsilon\langle r^2\rangle), \tag{5}$$

and β_1 is the collective coupling parameter for the isoscalar dipole resonance and R is the half-density radius of the Fermi mass distribution.

In all the above equations ϵ, which depends inversely on A, is very small compared to $\langle r^2\rangle$, and for all practical purposes the terms depending on ϵ could be dropped in the above equations for $A\geq 20$.

In a semimicroscopic treatment of inelastic hadron scattering, one can, in principle, fold the projectile-nucleon interaction with the transition density of Eq. (4) to obtain the real form factor of isoscalar dipole excitations, which could then be used in DWBA calculations. However, in such a procedure there is no recipe to construct the imaginary form factor. In this paper we adopt another approach by assuming that both the real and imaginary parts of the optical model potential deform in the same way as the ground state density; in this way we obtain the transition potential form factor

$$\begin{aligned}\Delta U=&-\frac{\beta_R}{R_R\sqrt{3}}\left[3r^2\frac{d}{dr}+10r-\tfrac{5}{3}\{\langle r^2\rangle_R\}\frac{d}{dr}+\epsilon\left(r\frac{d^2}{dr^2}+4\frac{d}{dr}\right)\right]V(r)\\&-i\frac{\beta_I}{R_I\sqrt{3}}\left[3r^2\frac{d}{dr}+10r-\tfrac{5}{3}\{\langle r^2\rangle_I\}\frac{d}{dr}+\epsilon\left(r\frac{d^2}{dr^2}+4\frac{d}{dr}\right)\right]W(r).\end{aligned} \tag{6}$$

Here β_R and β_I are the coupling parameters for the real and imaginary parts of the transition potential. These can be related with the prescription that the deformation lengths for the real and imaginary potentials are equal: $\beta_R R_R=\beta_I R_I$; $\langle r^2\rangle_R$ and $\langle r^2\rangle_I$ are the mean square radii of the real and imaginary parts of the optical potential. These are taken in this manner to preserve the

correct c.m. adjustment for the real and imaginary form factors separately, which is rather essential in these calculations. For a Woods-Saxon shape potential

$$\langle r^2 \rangle = \tfrac{3}{5}R^2\left[1+\tfrac{7}{3}\left(\pi\frac{a}{R}\right)^2\right], \tag{7}$$

where a is the diffuseness parameter.

Moreover, in Eq. (5), $\langle r^2 \rangle$ and $\langle r^4 \rangle$ are calculated from the real part of the optical model potential to give the value of β_R, which corresponds to 100% of the exhaustion of the isoscalar dipole EWSR [Eq. (3)].

III. APPLICATIONS: DWBA ANALYSIS

The transition density in Eq. (6) has a node and satisfies the condition

$$\int_0^\infty \rho_{if}^{(1)} r^3 dr = 0 .$$

Equivalently this condition states that in the limit of momentum transfer $q \to 0$, the PWBA matrix element should converge rapidly to zero (proportional to q^3). In the DWBA, the distortion of the waves would alter the dependence of the isoscalar transition form factor from being $\propto j_\lambda(qr)$, and it is of interest to investigate the behavior of the DWBA transition matrix element for small q. In Fig. 1 we show two calculations: the PWBA calculations (dashed lines) and the DWBA calculations (solid lines) performed with the program DWUCK[11]

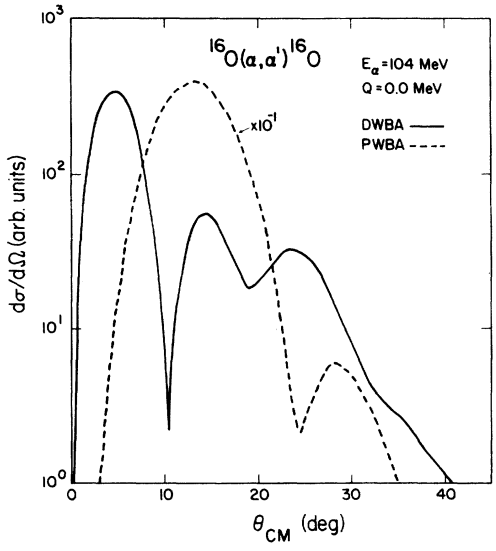

FIG. 1. DWBA (solid lines) and PWBA (dashed lines) calculations for a hypothetical 1^- state at $E_x = 0.0$ using the collective form factor described in the text. The PWBA calculations converge rapidly to zero as the momentum transfer $q \to 0$ ($\theta_{c.m.} \to 0°$).

for the reaction $^{16}O(\alpha,\alpha')^{16}O$ at $E_\alpha = 104$ MeV for a hypothetical $L=1$ transition at $E_x = 0$ MeV. For $\theta_{c.m.} = 0°$, this essentially ensures that $q=0$. It is shown in Fig. 1 that the differential cross section indeed converges quickly to zero as $\theta_{c.m.} \to 0°$. In the DWBA calculation ($E_x = 0$), the cross section for $\theta_{c.m.} \to 0°$ does not fall off to zero as rapidly as in the PWBA approximation, but at 0° it still vanishes. In a realistic situation $E_x \neq 0$, and hence the momentum transfer q is different from zero at 0°. In this case the cross section for an $L=1$ transition would be finite at $\theta_{c.m.} = 0°$.

We have performed DWBA calculations using the collective form factor [Eq. (7)] for the $L=1$ transition to the 7.12 MeV, $1^-(T=0)$ state in ^{16}O for the available experimental data at $E_\alpha = 75$ MeV (Ref. 7) and at $E_\alpha = 104$ MeV (Ref. 12). The optical model parameters[12] used were the same for both energies and are listed in Table I. Here we assumed $\beta_R R_R = \beta_I R_I$. The results of the calculations are shown in Fig. 2 and they are seen to fit the data at both energies rather well. Both fits were obtained with the same coupling parameter $\beta_R = 0.045$ corresponding to an energy weighted sum rule of $\simeq 4.2\%$. This is about half the value obtained by Deal[9] from the analysis of the electron scattering data to the same state. Such a discrepancy between $B(E\lambda)$ values obtained from inelastic electron and hadron scattering is well known[5,12] for light nuclei, and has been observed for various multipolarities λ.

As another test of this collective model for the excitation of isoscalar dipole states, we have performed DWBA calculations for the newly proposed candidate of the HEDR in ^{208}Pb at $E_x = 21.3$ MeV, observed[8] in inelastic α scattering at $E_\alpha = 172$ MeV. The optical potential used in this calculation was obtained[13] from optical model analysis of elastic α scattering at $E_\alpha = 139$ MeV and is listed in Table I. Here again $\beta_R R_R = \beta_I R_I$ was assumed. The results of the calculations with the collective $L=1$ form factor are shown in Fig. 3 (solid curve). They are seen to give a good fit to the data. The coupling parameter needed to fit the absolute cross section is $\beta_R = 0.036$, corresponding to 130% of the EWSR. This is in agreement with the result obtained in Ref. 8 with a different transition density. We have also performed a DWBA calculation for $L=3$ with the usual collective form factor for $L \geq 2$; namely $R dU_0/dr$. This is shown as dashed line in Fig. 3. Except for two points around 10°, the $L=3$ curve gives a reasonable fit to the data. The largest difference between $L=1$ and $L=3$ predictions is in the region of around 4°, where the $L=1$ DWBA curve goes through a local minimum while the $L=3$ curve goes through a local maximum. While an $L=1$ assignment for

TABLE I. Parameters of the optical model potentials of Woods-Saxon form used in the DWBA analysis; $R = rA^{1/3}$.

Nucleus	V (MeV)	r_R (fm)	a_R (fm)	W (MeV)	r_I (fm)	a_I (fm)	r_C (fm)	Ref.
^{16}O	− 88.64	1.457	0.693	−14.37	1.917	0.395	1.3	12
^{208}Pb	−155.	1.282	0.677	−23.26	1.478	0.733	1.4	13

the structure at $E_x = 21.3$ MeV is rather tempting, it would certainly be of great importance to measure inelastic cross sections in the region of 4°, which would make a definite assignment possible. Historically, this reminds one of the situation[14,15] of the monopole resonance in ^{208}Pb where a definite assignment of the $L = 0$ character could only be attained[15] from a forward angles measurement.

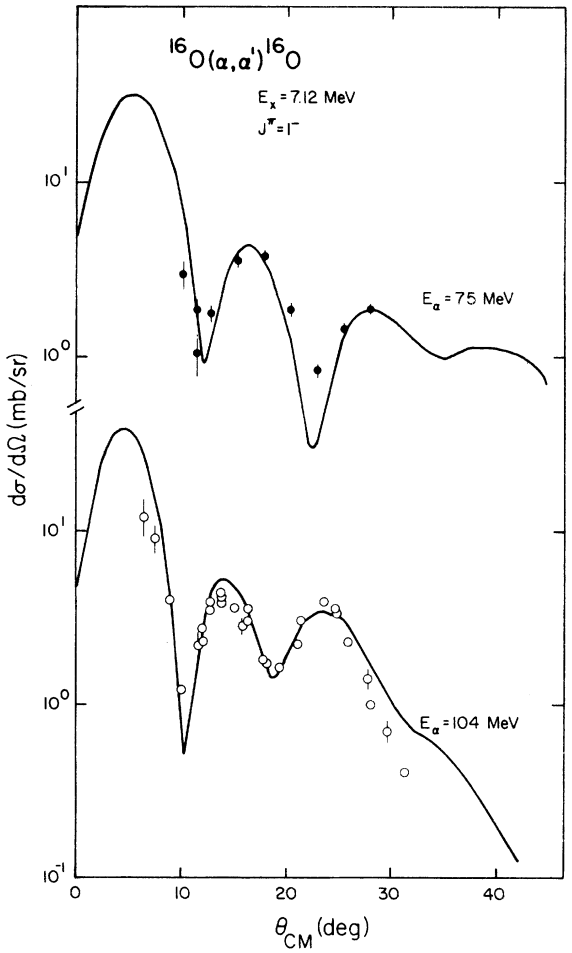

FIG. 2. Results of DWBA calculations using the collective form factor (see text) are drawn (solid curves) to fit the data from ^{16}O$(\alpha, \alpha')^{16}$O at $E_\alpha = 75$ MeV (Ref. 7) and $E_\alpha = 104$ MeV (Ref. 12).

IV. CONCLUSION

In this paper we have derived the energy weighted sum rule and the collective form factor for the excitation of isoscalar dipole resonances. This form factor was used in DWBA calculations for the 1^-, 7.12 MeV state in ^{16}O and the newly proposed HEDR in ^{208}Pb. In both cases the fits to the data were rather good, with the percentages of the isoscalar dipole EWSR exhausted by the 7.12 MeV level in ^{16}O to be 4.2% and the 21.3 MeV bump in ^{208}Pb to be 130%. It is further suggested that a definite assignment of $L = 1$ to the 21.3 MeV structure in ^{208}Pb depends strongly on the measurement of the forward angles.

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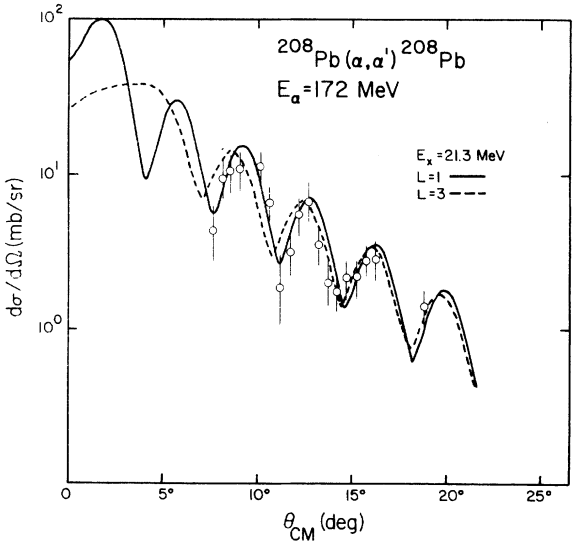

FIG. 3. Results of DWBA calculations using the $L = 1$ collective form factor (solid lines) and the $L = 3$ collective form factor (dashed lines) are drawn to fit the data to the proposed candidate of the HEDR observed (Ref. 8) in inelastic α scattering from ^{208}Pb at $E_\alpha = 172$ MeV.

APPENDIX

In this appendix an expression for the isoscalar dipole transition density will be derived. This derivation is based upon a generalization of the method used for other transition multipoles ($\lambda \geq 2$). For completeness we briefly summarize the main steps:

(i) Define the form factor operator

$$F(\vec{q}) = \frac{1}{A} \sum_i \exp(-i \vec{q} \cdot \vec{r}_i) . \tag{A1}$$

(ii) Assume that $F(\vec{q})$ commutes with the potential energy part of the Hamiltonian to derive the operator equation

$$[[F(\vec{q}_1), H], F^*(\vec{q}_2)] = \frac{\hbar^2}{mA} \vec{q}_1 \cdot \vec{q}_2 F(\vec{q}_2 - \vec{q}_1) , \tag{A2}$$

(iii) Take the expectation value of Eq. (A2) with respect to the nuclear ground state with $J=0$ and insert a complete set of intermediate states on the left hand side.

(iv) Integrate both sides of the resulting equation over $\int d(\cos\theta) Y^0_\lambda(\theta)$, where θ is the angle between $\vec{q}_1$ to $\vec{q}_2$, to project out a definite multipolarity λ.

(v) Take the long wavelength limit for $q_2 \to 0$.

The result can be expressed as

$$\sum_n (E_n - E_0) F^{(\lambda)}_{n0}(q) Q^{(\lambda)}_{0n} = (-)^{\lambda+1} \frac{\hbar^2}{2m} \frac{\lambda}{8\pi} q^\lambda \left(\frac{d}{qdq} \right)^{\lambda-1} F_{el}(q) , \tag{A3}$$

where $F_{el}(q)$ is the elastic form factor, and the multipole form factor

$$F^{(\lambda)}(q) = \frac{1}{A} \sum_i j_\lambda(qr_i) Y_\lambda(\hat{r}_i) \tag{A4}$$

has been expanded in terms of $(qr)^\lambda$:

$$AF^{(\lambda)}(q) = \frac{q^\lambda}{(2\lambda+1)!!} 2Q^{(\lambda)} - \frac{1}{2} \frac{q^{\lambda+2}}{(2\lambda+1)!!(2\lambda+3)} 2P^{(\lambda)} + \cdots , \tag{A5}$$

where

$$Q^{(\lambda)} = \tfrac{1}{2} \sum_i r_i^\lambda Y^0_\lambda(\hat{r}_i) \tag{A6a}$$

and

$$P^{(\lambda)} = \tfrac{1}{2} \sum_i r_i^{\lambda+2} Y^0_\lambda(\hat{r}_i) . \tag{A6b}$$

(vi) With the assumption that one single state n exhausts all the multipole strength, Eq. (A3) leads to the well known result

$$F^{(\lambda)}_{n0}(q) \sim q^\lambda \left(\frac{d}{qdq} \right)^{\lambda-1} F_{el}(q) , \text{ if } \lambda \geq 2 . \tag{A7}$$

The monopole ($\lambda=0$) and dipole ($\lambda=1$) cases require a more careful treatment. For the case of $\lambda=0$, the leading transition operator (for $q \to 0$) is $P^{(0)}$ rather than $Q^{(0)}$, and Eq. (A3) is to be replaced by

$$\sum (E_n - E_0) F^{(0)}_{n0}(q) P^{(0)}_{0n} = \frac{\hbar^2}{8m\pi} q \frac{d}{dq} F_{el}(q) . \tag{A8}$$

The case of $\lambda=1$ is more complicated, since the leading order contribution $\sum_i j_1(qr_i) Y_1(\hat{r}_i) \to q\vec{R}$, i.e., is proportional to the center of mass coordinate and therefore would give a completely spurious contribution if used with wave functions that are not translation invariant.

A correct treatment requires the use of the transition operator defined in terms of intrinsic coordinates $\vec{r}'_i = \vec{r} - \vec{R}$,

$$\tilde{F}(\vec{q}) = \frac{1}{A} \sum_i e^{-i\vec{q}\cdot\vec{r}'_i} = e^{i\vec{q}\cdot\vec{R}} F(\vec{q}) . \tag{A9}$$

The equivalent expression of Eq. (A2) for $\tilde{F}(q)$ can easily be obtained

$$[[\tilde{F}(\vec{q}), H], \tilde{F}^*(\vec{q}_2)] = \frac{\hbar^2}{mA} \vec{q}_1 \cdot \vec{q}_2 [\tilde{F}(\vec{q}_1 - \vec{q}_2) - \tilde{F}(\vec{q}_1)\tilde{F}(\vec{q}_2)] . \tag{A10}$$

Again taking the expectation value of (A10) with respect to a $J=0$ ground state, and making a multipole decomposition, yields

$$\sum (E_n - E_0) \tilde{F}^{(\lambda)}_{0n}(q_1) \tilde{F}^{(\lambda)}_{n0}(q_2) + \frac{\hbar^2}{2mA} \frac{q_1 q_2}{2\lambda+1} [(\lambda+1) \tilde{F}^{(\lambda+1)}_{0n}(q_1) \tilde{F}^{(\lambda+1)}_{n0}(q_2) + \lambda \tilde{F}^{\lambda-1}_{0n}(q_1) \tilde{F}^{\lambda-1}_{n0}(q_2)]$$
$$= \frac{\hbar^2}{2mA} \frac{q_1 q_2}{4\pi(2\lambda+1)} \left\langle 0 \left| \sum_i (\lambda+1) j_{\lambda+1}(q_1 r_i) j_{\lambda+1}(q_2 r_i) + \lambda j_{\lambda-1}(q_1 r_i) j_{\lambda-1}(q_2 r_i) \right| 0 \right\rangle . \tag{A11}$$

For $\lambda=1$ and $q_2 \to 0$ the leading order transition operator is $P^{(\lambda)}$

$$\sum (E_n - E_0) \tilde{F}^{(1)}_{0n}(q) \tilde{P}^{(1)}_{n0} = -\frac{\hbar^2}{16m\pi} \left(3q^2 \frac{d}{dq} \frac{1}{q} \frac{d}{dq} + 5 \frac{d}{dq} \right) \tilde{F}_{el}(q) + \frac{\hbar^2 q}{6mA} \sum_n \langle 0 | 4\tilde{Q}^{(2)}_{0n} \tilde{F}^{(2)}_{n0}(q) - 5\tilde{P}^{(0)}_{0n} \tilde{F}^{(0)}_{n0}(q) | 0 \rangle . \tag{A12}$$

Although the second term on the right-hand side of Eq. (A12) might appear to be a $1/A$ correction, the monopole contribution $\langle \tilde{P}^{(0)} \tilde{F}^{(0)} \rangle$ in fact goes like A since there is a diagonal contribution

$$\langle 0|\tilde{P}^{(0)}\tilde{F}^{(0)}|0\rangle=\tfrac{1}{2}\sum_i\langle 0|r_i^2|0\rangle\langle 0|\tilde{F}^{(0)}(q)|0\rangle+\sum_{n\neq 0}\langle 0|\tilde{P}^{(0)}_{0n}\tilde{F}_{n0}(q)|0\rangle\,, \tag{A13}$$

and one can write

$$\sum_n(E_n-E_0)\tilde{F}^{(1)}_{0n}(q)\tilde{P}^{(1)}_{n0}=-\frac{\hbar^2}{16m\pi}\left(3q^2\frac{d}{dq}\frac{1}{q}\frac{d}{dq}+5\frac{d}{dq}+\tfrac{5}{3}\langle r^2\rangle q+\epsilon q^2\frac{d}{dq}\right)\tilde{F}_{e1}(q)\,, \tag{A14}$$

where the coefficient of the correction term ϵ can be estimated by applying the sum rules Eq. (A3) (for $\lambda=2$) and Eq. (A8) (for $\lambda=0$) and assuming that all the isoscalar quadrupole and monopole strengths are exhausted by the respective quadrupole and monopole giant resonances (see Ref. 9). In this case we obtain

$$\epsilon=\left(\frac{4}{E_2}+\frac{5}{E_0}\right)\frac{\hbar^2}{3mA}\,, \tag{A15}$$

where E_2 and E_0 are the excitation energies of the giant quadrupole and monopole resonances, respectively.

The (once-integrated) isoscalar dipole sum rule in r space is obtained by Fourier transforming Eq. (A14):

$$\sum_n(E_n-E_0)\tilde{P}^{(1)}_{n0}\rho^{(1)}_{0n}=-\frac{\hbar^2}{4m}\left[3r^2\frac{d}{dr}+10r-\tfrac{5}{3}\langle r^2\rangle\frac{d}{dr}+\epsilon\left(r\frac{d^2}{dr^2}+4\frac{d}{dr}\right)\right]\rho_0(r)Y^0_1(\hat{r})\,, \tag{A16}$$

where $\rho_0(r)$ is the ground state distribution and $\rho^{(1)}_{0n}$ is the isoscalar dipole transition density. Note that Eq. (A16) agrees with the result of Ref. 9 except for a crucial (−) sign for the third term which ensures that $\int\rho^{(1)}_{0n}(r)r^3dr=0$, as is required by translational invariance.

The static energy weighted sum rule is obtained from Eq. (A14) in the limit of $q\to 0$:

$$\sum(E_n-E_0)(\tilde{P}^{(1)}_{0n})^2=-\frac{\hbar^2A}{32m\pi}(11\langle r^4\rangle-\tfrac{25}{3}\langle r^2\rangle^2-10\epsilon\langle r^2\rangle)\,. \tag{A17}$$

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