# Relativistic model of interacting nucleons and mesons

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We construct a relativistic model of interacting nucleons and mesons. The character of the model is phenomenological in that we make extensive use of the relativistic quasipotentials obtained in the study of the one-boson-exchange model of nuclear forces. We show that in the static limit for the meson field we can derive a relativistic version of the Bethe-Brueckner theory of nuclear matter. (The corrections due to the nonstatic aspects of the field are expected to be small.) The method may be extended to provide a basis for a relativistic Brueckner-Hartree-Fock theory of finite nuclei.

[NUCLEAR STRUCTURE Relativistic many-body theory; derivation of relativistic Bethe-Brueckner theory of nuclear matter.]

## I. INTRODUCTION

There has been a good deal of interest in recent years in the formulation of a relativistic theory of nuclear matter and of finite nuclei.[1] The mean-field approximation of Walecka has been extensively investigated in this connection. (The mean-field approximation corresponds closely to the Dirac-Hartree approximation.) Various calculations which we have made have convinced us that correlation corrections are quite important and the Hartree or Hartree-Fock approximations are only of limited applicability.[2] We are therefore interested in formulating a relativistic theory of interacting nucleons and mesons that includes short-range correlation effects. We should note that one may approach this problem from two points of view. For example, if one uses the relativistic mean-field theory of Walecka one must introduce *effective* coupling constants for the $\sigma$ and $\omega$ fields which contain implicitly the effects of correlations and the effects of mesons (such as the $\pi$ meson) which do not appear explicitly in the theory.[2] One can correlate a good deal of experimental data if one uses the Walecka theory with effective coupling constants determined phenomenologically. Alternatively, one may attempt to calculate the properties of nuclear matter and finite nuclei using a theory that has no adjustable parameters. In this case one uses nucleon-nucleon interactions that have been determined from the study of nucleon-nucleon scattering and from the properties of the deuteron.[3] In this work we develop a program based upon the latter scheme.

All previous attempts to calculate the properties of nuclear matter and/or finite nuclei using interactions obtained from the study of nucleon-nucleon scattering have used nonrelativistic many-body theories or theories in which the nucleon wave function is written in terms of positive-energy spinors only.[3] Our studies have shown that this program has failed (on the whole) because of the use of the nonrelativistic approximation or the use of an incomplete relativistic description of the nucleon wave function. In this work we hope to provide a theoretical basis for a relativistic theory of nuclear matter (and of finite nuclei). In the following work we show that a systematic application of these ideas leads to a reasonably successful description of the properties of nuclear matter. Since negative energy states may be shown to play an extremely important role in the saturation properties of nuclear matter, the reason for the lack of success of the nonrelativistic theories is clear.[4]

Of the various attempts to formulate a relativistic field-theoretic model of interacting nucleons and mesons, the most ambitious is that of Wilets and collaborators.[5] This model had only limited success in that an attempt was made, as a first step, to provide a theory of the nucleon based upon a model of a nucleon field interacting with a pion field. Extensive effort was devoted to determine the single-nucleon Green's function and only limited application was made in the study of the many-body system. Our goal is much less ambitious; since the nucleons and mesons are now believed to be composite objects we do not expect to be able to develop a theory of interacting nucleons and mesons from first principles. Rather, we are interested in constructing a *model* which is patterned after the models used to understand nucleon-nucleon scattering and the properties of the deuteron. The simplest relativistic model we consider is the one-boson-exchange model of nuclear forces.[3] In the application of this model one chooses a particular set of mesons and associated interaction Lagrangians. The mesons $(\sigma, \pi, \rho, \omega, \ldots)$ do not represent fundamental fields and therefore we do not concern ourselves with the question of renormalization, etc. After a series of approximations (use of positive-energy intermediate states, summation of ladder graphs, etc.) one is able to determine, in the context of the one-boson exchange (OBE) model, a set of masses, coupling constants, and form factors for the various mesons of the theory ($\sigma, \pi, \omega, \rho, \ldots$). Since we hope to construct a model that incorporates this phenomenological information, we believe it is necessary to make approximations in the many-body theory which are similar to those used in the OBE model. For example, in our analysis, the use of a complicated form for the nucleon's Green's function (such as that discussed in Ref. 5) is inappropriate since only simple propagators are used in the OBE model analysis of the scattering data.[3] The detailed correspondence between our approximations for the description of the interacting nucleon-nucleon system in the medium (nuclear matter) and the procedures used in the OBE model will become apparent as we proceed.

The plan of our work is as follows: In Sec. II we review some aspects of the OBE model of nuclear forces that are pertinent to our work. In Sec. III we write the Hamiltonian for a system of nucleons interacting with a scalar meson field. [The form of the equations of the theory for the interaction of nucleons with pseudoscalar mesons ($\pi$) and vector mesons ($\rho, \omega$) will be given in an Appendix. The structure of the theory and the nature of our approximations can be adequately described if we limit the discussion to the consideration of nucleons interacting with scalar mesons.] In Sec. III we show that the Hamiltonian can be put into an especially simple form if one makes the *static* approximation for the meson field. (Nonstatic corrections to the energy can be calculated, but these are expected to be small.)

In Sec. IV we define various many-particle Green's functions following the notation of Wilets.[5] Various useful relations between these Green's functions are noted.

In Sec. V we discuss the energy of the system and show how the energy may be expressed in terms of the Green's functions defined in Sec. IV. Section VI is concerned with the structure of the simplest many-body Green's function $G(x,x')$. We write a simple form for this Green's function appropriate to the formulation of a relativistic Bethe-Brueckner theory of nuclear matter.

In the static approximation, the Hamiltonian may be written as a sum of a "Dirac Hamiltonian" and an interaction term. In Sec. VII we obtain the expectation value of the Dirac Hamiltonian. (See Sec. III.) In the nonrelativistic limit this expectation value is equal to the mass of the system plus the kinetic energy of a noninteracting Fermi gas. In the relativistic theory there are important corrections due to the presence of negative-energy components in the nucleon wave function.

Section VIII is devoted to a discussion of a relativistic Hartree-Fock approximation. In Sec. IX we show how we may go beyond the Hartree-Fock approximation and introduce correlation effects. Based upon the discussion of Sec. IX, we introduce the equations of a relativistic Brueckner-Hartree-Fock theory of nuclear matter in Sec. X. [We note that while there is no question as to wave-function self-consistency in the nonrelativistic Bethe-Brueckner theory, in the relativistic theories of nuclear matter the (relativistic) wave function of the nucleon should be determined in a self-consistent fashion.] In a future work, we make use of a different formulation and will show that our choice of the self-consistent representation can be related to the requirement that the residual interaction not admix one-particle one-hole states into the wave function in perturbation theory.[7] (Here "hole" refers to both positive *and* negative energy occupied states.) The basic result of our analysis is given by Eq. (10.5) which is the expression for the energy in terms of the self-consistent spinor wave functions. This expression goes over to that of the Bethe-Brueckner theory at low densities, where one can neglect the negative-energy spinor components in the self-consistent spinor wave functions. The essential point is that the transition potentials which excite pair currents are large and coherent and that therefore there are very large corrections to the standard nonrelativistic theories arising from admixtures of the particle-hole type, where hole here refers to an occupied negative-energy state. (We remark that in nuclear matter, particle-hole admixtures, where the holes have positive energy, are excluded by momentum conservation.)

Finally in Sec. XI we make some concluding comments and describe some applications of the theory. We reserve detailed investigation of the nonstatic aspects of the theory to a future work.

## II. THE ONE-BOSON-EXCHANGE MODEL OF NUCLEAR FORCES

In this section we provide a few comments on the one-boson-exchange model of nuclear forces[3] since we will make some correspondence between the approximations used in this model and the model for the many-body system we discuss in this work.

In discussing nucleon-nucleon scattering one may start with the Bethe-Salpeter equation

$$M = K + K G_F M , \tag{2.1}$$

where $G_F$ is the Feynman propagator for the intermediate state nucleons and $M$ is the invariant scattering amplitude. The kernal $K$ may be eliminated in favor of a quasipotential $U$ by writing two equations which are equivalent to Eq. (2.1):

$$M = U + UgM , \tag{2.2}$$

$$U = K + K(G_F - g)U . \tag{2.3}$$

The propagator $g$ is taken to have the same right-hand cut as $G_F$ and is usually chosen to contain spinor projection operators which restrict the intermediate nucleons to positive-energy states. In making the latter choice, we write $g = g^{++}$. Indeed, for the study of nucleon-nucleon scattering with $g = g^{++}$ we may write Eq. (2.2) as

$$M^{++++} = U^{++++} + U^{++++}g^{++}M^{++++} , \tag{2.4}$$

where the notation is such that

$$\langle \vec{p}s_1\vec{q}s_2 | M^{++++} | \vec{p}'s_1'\vec{q}'s_2' \rangle \equiv \langle \bar{u}^{(s_1)}(\vec{p})\bar{u}^{(s_2)}(\vec{q}) | M | u^{(s_1')}(\vec{p}')u^{(s_2')}(\vec{q}') \rangle , \tag{2.5}$$

etc. Therefore we see that with $g = g^{++}$ we need only matrix elements of $M$ and $U$ taken between *positive-energy* spinor states in order to study the nucleon-nucleon scattering problem.

Now the procedure used in the OBE model of nuclear forces is to parametrize the *quasipotential* $U$ in terms of coupling constants, masses, and form factors of the exchanged mesons ($\pi, \sigma, \rho, \omega, \ldots$). As remarked above, for the study of nucleon-nucleon scattering one requires only a limited set of matrix elements of $U$, i.e., those sufficient to construct $U^{++++}$.

Now in our analysis of the relativistic many-body problem, which is based upon phenomenological information obtained from the OBE model, we should be careful to use similar approximations to introduce correlation effects. Since ours is a phenomenological theory it would be inappropriate to use propagators and form factors of greater complexity than those used in the OBE model. As an example, we remark that when performing ladder summations of interactions in the many-body theory we restrict ourselves to positive-energy intermediate states as in the OBE model. (The attempt to use more complicated nucleon propagators in the intermediate states in the many-body theory would require us to go back and reanalyze the nucleon-nucleon scattering problem.)

For future reference we define a reaction matrix $M$ for use in many-body studies *via* the equation

$$\hat{M} = U + U\hat{g}^{++}\hat{M}. \tag{2.6}$$

The discussion of Secs. IX and X and the Appendices will provide support for the use of Eq. (2.6) in the calculation of the properties of nuclear matter. Here $\hat{g}^{++}$ differs from $g^{++}$ in that $\hat{g}^{++}$ contains a Pauli operator appropriate to the calculation of the nucleon-nucleon scattering amplitude in the medium. This Pauli operator restricts the intermediate state nucleons to have momenta greater than $k_F$. We further note that we are assuming that we need not modity the quasipotential $U$ in the medium. We will again refer to $\hat{M}$ in Secs. IX and X when we discuss the role of two-body correlations in a relativistic Brueckner-Hartree-Fock theory of nuclear matter. In Appendix A we present an alternate approach to the derivation of Eq. (2.6). (We exhibit explicit forms for $g^{++}$ and $\hat{g}^{++}$.)

## III. A HAMILTONIAN FOR A SYSTEM OF INTERACTING NUCLEONS AND MESONS

We consider a model in which the nucleon interacts with various mesons such as the $\sigma$, $\pi$, $\rho$, and $\omega$ mesons. Of these the $\rho$ and $\pi$ mesons have $T=1$ and the $\sigma$ and $\omega$ have $T=0$. Now in order to avoid writing an excessive number of formulas in the text we will limit our consideration to nucleons interacting with the scalar $\sigma$ field. (For completeness we present some additional formulas in Appendix B appropriate to the more complicated couplings of the $\pi$, $\rho$, and $\omega$ mesons.) The essential points can be made by considering the simplest case.

We begin by considering the following Lagrangian densities,

$$\mathcal{L} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_\sigma + \mathcal{L}_\sigma^{\text{int}} , \tag{3.1}$$

where

$$\mathcal{L}_{\text{Dirac}} = -\bar{\psi}(x)\left(\gamma^0 i\frac{\partial}{\partial t} - \vec{\gamma}\cdot\frac{1}{i}\vec{\nabla} - m\right)\psi(x) , \tag{3.2}$$

$$\mathcal{L}_\sigma = -\tfrac{1}{2}\{-[\dot{\phi}(x)]^2 + [\vec{\nabla}\phi(x)]^2 + \mu^2\phi^2(x)\} , \tag{3.3}$$

$$\mathcal{L}_\sigma^{\text{int}} = -g_\sigma\bar{\psi}(x)\psi(x)\phi(x) . \tag{3.4}$$

The field equation for the scalar field is ($\Box = \partial_t^2 - \vec{\nabla}^2$)

$$(\Box + \mu^2)\phi(x) = -g_\sigma\bar{\psi}(x)\psi(x) . \tag{3.5}$$

The Hamiltonian density is constructed as usual,

$$\mathcal{H} = \pi(x)\dot{\phi}(x) + \pi_\psi(x)\dot{\psi}(x) - \mathcal{L}(x) \tag{3.6}$$

$$= \mathcal{H}_{\text{Dirac}}(x) + \mathcal{H}_\sigma(x) + \mathcal{H}_{\text{int}}(x) , \tag{3.7}$$

where

$$\mathcal{H}_{\text{Dirac}} = \bar{\psi}(x)\left(\vec{\gamma}\cdot\frac{1}{i}\vec{\nabla} + m\right)\psi(x) , \tag{3.8}$$

$$\mathcal{H}_\sigma = \tfrac{1}{2}[\pi^2(x) + \vec{\nabla}\phi(x)\cdot\vec{\nabla}\phi(x) + \mu^2\phi^2(x)] , \tag{3.9}$$

and

$$\mathcal{H}_{\text{int}} = g_\sigma\bar{\psi}(x)\psi(x) . \tag{3.10}$$

The Hamiltonian is obtained from the Hamiltonian density by integrating over the spatial coordinate:

$$H = H_{\rm Dirac} + H_\sigma + H_{\rm int}\,, \tag{3.11}$$

where

$$H_{\rm Dirac} = \int d\vec{x}\,\bar{\psi}(x)\left(\vec{\gamma}\cdot\frac{1}{i}\vec{\nabla}+m\right)\psi(x)\,, \tag{3.12}$$

$$H_\sigma = \tfrac{1}{2}\int d\vec{x}\,[\pi^2(x)+\vec{\nabla}\phi(x)\cdot\vec{\nabla}\phi(x)+\mu^2\phi^2(x)]\,, \tag{3.13}$$

$$H_{\rm int} = g_\sigma\int d\vec{x}\,\bar{\psi}(x)\psi(x)\phi(x)\,. \tag{3.14}$$

It is useful at this point to note the relations which follow if we assume that the meson field is *static*. In this case we can set $\pi=\dot{\phi}=0$ and use the equation of motion [Eq. (3.5)] in the form

$$(\vec{\nabla}^2-\mu^2)\phi(x) = g_\sigma\bar{\psi}(x)\psi(x)$$

to rewrite $H_\sigma$ as follows

$$\begin{aligned} H_\sigma^{\rm stat} &= \tfrac{1}{2}\int d\vec{x}\,[\vec{\nabla}\phi(x)\cdot\vec{\nabla}\phi(x)+\mu^2\phi^2(x)] \\ &= \tfrac{1}{2}\int d\vec{x}\,[-\phi(x)\vec{\nabla}^2\phi(x)+\mu^2\phi^2(x)] \\ &= -\tfrac{1}{2}g_\sigma\int \bar{\psi}(x)\psi(x)\phi(x) \\ &= -\tfrac{1}{2}H_{\rm int}\,. \end{aligned} \tag{3.15}$$

Equations such as (3.15) are rather general for fields coupled linearly to the source. (See Appendix B.) Thus we find, in the static limit for the meson field,

$$H^{\rm stat} = H_{\rm Dirac} + \tfrac{1}{2}H_{\rm int}\,. \tag{3.16}$$

We will make important use of the static limit in the following. We will also discuss the nonstatic terms of the theory; however, we defer that discussion to a point after the introduction of many-body Green's functions.

We note that if one takes the expectation value of $\frac{1}{2}H_{\rm int}$ in the ground state,

$$\tfrac{1}{2}\langle H_{\rm int}\rangle = \tfrac{1}{2}\int d\vec{x}\,\langle\bar{\psi}(x)\psi(x)\phi(x)\rangle\,, \tag{3.17}$$

and use the simplest factorization, $\langle\bar{\psi}(x)\psi(x)\phi(x)\rangle \simeq \langle\bar{\psi}(x)\psi(x)\rangle\langle\phi(x)\rangle$, one readily derives the Hartree-Dirac theory. Note that $\langle\phi(x)\rangle$ can be obtained from Eq. (3.5) after taking the ground state matrix element of that equation. The introduction of many-body Green's functions allows us to go beyond this approximation. (We will return to a discussion of the Hartree approximation in Sec. VIII.)

## IV. GREEN'S FUNCTIONS FOR THE MANY-BODY SYSTEM

We again follow the notation and conventions of Ref. 5. It is useful to introduce Green's functions which describe the properties of the many-body system. We will work in the zero temperature limit. (The nucleon coordinates are denoted by integers, e.g., 1 stands for $\vec{x}_1$ and $t_1$.)

The nucleon Green's function is given by

$$G_n(1,\ldots,n;1'\ldots n') \equiv i^n\langle T\{\psi(1)\ldots\psi(n)\bar{\psi}(n')\ldots\psi(1')\}\rangle\,, \tag{4.1}$$

where $T$ is the Wick time-ordering symbol which includes a factor $(-1)^p$. Here $p$ is the number of permutations required to go from the time-ordered sequence to that shown above. The expectation value is taken with respect to the ground state of the system. For simplicity we can consider symmetric nuclear matter ($N=Z$). Similarly, one introduces meson Green's functions which are defined as

$$\mathcal{G}_n(\xi_1\ldots\xi_n;\xi_1'\ldots\xi_n') = i^n\langle T\{\phi(\xi_1)\ldots\phi(\xi_n)\phi(\xi_n')\ldots\phi(\xi_1')\}\rangle\,, \tag{4.2}$$

where again $T$ denotes the Wick time-ordering operator and contains no factor of $(-1)^p$. Further, one introduces mixed Green's functions which are defined as

$$G_{n,m/2}(1\ldots n;1'\ldots n';\xi_1\ldots\xi_m) = i^{n+[m/2]}\langle T\{\psi(1)\ldots\psi(n)\bar{\psi}(n')\ldots\bar{\psi}(1')\phi(\xi_1)\ldots\phi(\xi_m)\}\rangle\,. \tag{4.3}$$

Here the Wick time-ordering operator contains a factor $(-1)^p$, where $p$ counts the permutations for the fermion fields only. Further, $[m/2]$ is the largest integer less than or equal to $m/2$. The following Green's functions will be of particular importance for our work:

$$G(11') = i\langle T\{\psi(1)\bar{\psi}(1')\}\rangle\,, \tag{4.4}$$

$$\mathcal{G}(\xi\xi') = i\langle T\{\phi(\xi)\phi(\xi')\}\rangle\,, \tag{4.5}$$

$$G_{1,1/2}(11'\xi_1) = i\langle T\{\psi(1)\bar{\psi}(1')\phi(\xi_1)\}\rangle\,, \tag{4.6}$$

$$G_2(12;1'2') = i^2\langle T\{\psi(1)\psi(2)\bar{\psi}(2')\bar{\psi}(1')\}\rangle\,. \tag{4.7}$$

Various relations among these functions may be found in Ref. 5. For example, we note that, with

$$\begin{aligned} \Omega(34\xi') &= g_\sigma\delta(\vec{x}_3-\vec{x}_4)\delta(\vec{x}_3-\vec{\xi}') \\ &\quad\times\delta(t_3-t_4-0^+)\delta(t_3-t_{\xi'})\,, \\ G_{1,1/2}(21';\xi) &= i\mathcal{G}^0(\xi\xi')\Omega(34\xi')G_2(24;31')\,, \end{aligned} \tag{4.8}$$

where $\mathcal{G}^0(\xi\xi')$ is the free meson's Green's function which satisfies the wave equation

$$(\Box+\mu^2)\mathcal{G}^0(\xi\xi')=\delta(\xi\xi')\,. \tag{4.9}$$

The solution of this equation which has the appropriate boundary conditions may be written

$$\mathcal{G}^0(\xi\xi')=\int\frac{d^4p}{(2\pi)^4}\mathcal{G}^0(\vec{p},p^0)e^{i\vec{p}\cdot(\vec{\xi}-\vec{\xi}')-ip^0(t_\xi-t_{\xi'})} \tag{4.10}$$

with

$$\mathcal{G}^0(\vec{p},p^0)=\left(\frac{1}{\vec{p}^2-(p^0)^2+\mu^2-i\eta}\right). \tag{4.11}$$

Another important relation which we will need is[5]

$$(\Box+\mu^2)\mathcal{G}(\xi\xi')=\delta(\xi\xi')-i\Omega(12\xi)\Omega(34\xi'')\times G_2(24;13)\mathcal{G}^0(\xi''\xi')\,. \tag{4.12}$$

This follows from the use of the field equation for the meson field and Eq. (4.8).

Let us now consider the equation for the one nucleon Green's function,[5]

$$(-\gamma\cdot p_1+m)G(11')=\delta(11')-\Omega(12\xi)G_{1,1/2}(21'\xi)\,. \tag{4.13}$$

(Here $m$ denotes the renormalized nucleon mass.) Now if we use Eq. (4.8) this can be written as

$$[G^0(12)]^{-1}G(21')=\delta(11')+i\langle 12|v|34\rangle G_2(34;21') \tag{4.14}$$

with

$$\langle 12|v|34\rangle=-\Omega(13\xi)\mathcal{G}^0(\xi\xi')\Omega(24\xi')\,, \tag{4.15}$$

and

$$[G^0(12)]^{-1}=(-\gamma\cdot p_1+m)\delta(x_1-x_2)\,. \tag{4.16}$$

Of particular importance is the *mass operator* which one defines through the relation

$$\Sigma(12)G(21')=-i\langle 12|v|34\rangle G_2(34;21')\,. \tag{4.17}$$

Thus, one may write Eq. (4.14) as

$$[G^0(12)]^{-1}G(21')=\delta(11')-\Sigma(12')G(2'1')\,, \tag{4.18}$$

or

$$(-\gamma\cdot p_1+m)G(11')+\Sigma(12)G(21')=\delta(11')\,. \tag{4.19}$$

The mass operator is particularly useful in the construction of $G(11')$. We may write Eq. (4.19) in momentum space as

$$[\vec{\gamma}\cdot\vec{p}-\gamma^0p^0+m+\Sigma(\vec{p},p^0)]G(\vec{p},p^0)=1\,. \tag{4.20}$$

One may consider various approximations for the construction of $\Sigma(\vec{p},p^0)$ which correspond to various approximations to $G_2(34,21')$ of Eq. (4.17). For example, $\Sigma(\vec{p},p^0)$ may be calculated in the Hartree or Hartree-Fock approximation or in an approximation which includes correlation effects. (See Fig. 1.)

In general one may write

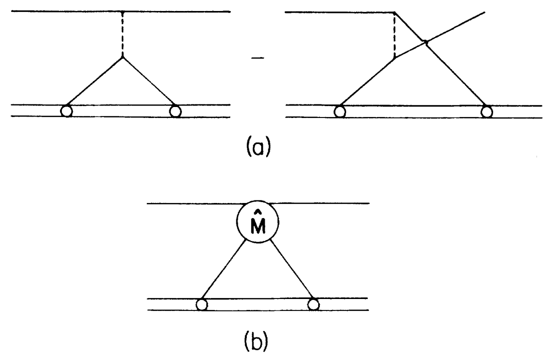


FIG. 1. Various approximations used in the construction of the mass operator $\Sigma(\vec{p},p^0)$. (a) The Hartree-Fock approximation is obtained from the calculation of these diagrams. (The Hartree approximation is obtained if one neglects the exchange diagram.) Here the dashed lines represent the meson propagator and the small open circles are vertex functions. The calculation is made in a self-consistent fashion since the vertex functions depend upon the mass operator. In general the nucleon of momentum $(\vec{p},p^0)$ is off its mass shell. (b) A relativistic Brueckner approximation is obtained by calculating the mass operator for an off-mass-shell nucleon of momentum $(\vec{p},p^0)$ using a reaction matrix $\hat{M}$. Procedures for constructing the reaction matrix are discussed in the text.

$$\Sigma(\vec{p},p^0)=A(\vec{p},p^0)+\gamma^0B(\vec{p},p^0)+\frac{\vec{\gamma}\cdot\vec{p}}{m}C(\vec{p},p^0)\,, \tag{4.21}$$

with the result that $\gamma^0\Sigma(\vec{p},p^0)$ is a Hermitian operator. [We remark that the effective single-body Hamiltonian is $H_{sp}=\vec{\alpha}\cdot\vec{p}+\gamma^0m+\gamma^0\Sigma(\vec{p},p^0)$ and therefore we see that $\gamma^0\Sigma(\vec{p},p^0)$ must be Hermitian.] The procedures used in the construction of $\Sigma(\vec{p},p^0)$ will be discussed in the following paper. At this point it is useful to consider the various eigenfunctions of $H_{sp}$. It will be quite useful to consider $H_{sp}$ as depending on $p^0$ as a parameter and therefore we will write $H_{sp}(p^0)$. We may introduce the solution of the equation

$$H_{sp}(p^0)\phi^{(s)}(\vec{p},p^0)=\epsilon(\vec{p},p^0)\phi^{(s)}(\vec{p},p^0)\,, \tag{4.22}$$

where $\epsilon(\vec{p},p^0)>0$. We remark that a negative-energy solution of Eq. (4.22) may be obtained from $\phi^{(s)}(\vec{p},p^0)$:

$$\theta^{(s)}(\vec{p},p^0)=\gamma^5\gamma^0\phi^{(s)}(\vec{p},p^0)\,, \tag{4.23}$$

and since, with $\bar{H}_{sp}(p^0)\equiv H_{sp}(p^0)-B(\vec{p},p^0)$,

$$\bar{H}_{sp}(p^0)\gamma^5\gamma^0=-\gamma^5\gamma^0\bar{H}_{sp}(p^0)\,, \tag{4.24}$$

we see that

$$H_{sp}(p^0)\theta^{(s)}(\vec{p},p^0)=-\bar{\epsilon}(\vec{p},p^0)\theta^{(s)}(\vec{p},p^0)\,, \tag{4.25}$$

where $\bar{\epsilon}(\vec{p},p^0)\equiv\epsilon(\vec{p},p^0)-2B(\vec{p},p^0)$. It is also useful to introduce a quantity $\epsilon(\vec{p})$ which is obtained from the solution of the equation $p^0=\epsilon(\vec{p},p^0)$. We define $\epsilon(\vec{p})\equiv p^0(\vec{p})$. Similarly we can also introduce $\bar{\epsilon}(\vec{p})$

which is obtained from the solution of $\bar{p}^0+\bar{\epsilon}(\vec{p},\bar{p}^0)=0$, that is, $\bar{\epsilon}(\vec{p})\equiv-\bar{p}^0(\vec{p})$. It is also useful to introduce the wave functions $\phi^{(s)}(\vec{p})$ and $\theta^{(s)}(\vec{p})$ defined by the relations

$$\phi^{(s)}(\vec{p})\equiv\phi^{(s)}[\vec{p},\epsilon(\vec{p})], \tag{4.26}$$

$$\theta^{(s)}(\vec{p})\equiv\theta^{(s)}[\vec{p},-\bar{\epsilon}(\vec{p})]. \tag{4.27}$$

These wave functions arise when one evaluates residues of the single-particle Green's function [see Eqs. (6.12) and (6.13)] at the quasiparticle poles located at $\epsilon(\vec{p})$ and at $-\bar{\epsilon}(\vec{p})$.

One can show that with $\vec{p}$ along the $z$ axis we may write

$$\phi^{(s)}(\vec{p})=\left[\frac{m}{E(\vec{p})}\right]^{1/2}[a(\vec{p})u^{(s)}(\vec{p})+b(\vec{p})(-1)^{1/2-s}v^{(-s)}(-\vec{p})], \tag{4.28}$$

where $u^{(s)}(\vec{p})$ and $v^{(-s)}(-\vec{p})$ are those solutions of the Dirac equation without interaction as defined by Bjorken and Drell.[6] For convenience we also define

$$w^{(s)}(\vec{p})\equiv v^{(-s)}(-\vec{p})=\gamma^5\gamma^0u^{(s)}(\vec{p}). \tag{4.29}$$

For $\vec{p}$ in an arbitrary direction we may write Eq. (4.28) as

$$\phi^{(s)}(\vec{p})=\left[\frac{m}{E(\vec{p})}\right]^{1/2}\left[a(\vec{p})u^{(s)}(\vec{p})+b(\vec{p})\sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w^{(s')}(\vec{p})\right]. \tag{4.30}$$

Of course, Eq. (4.30) reduces to Eq. (4.23) if $\vec{p}$ is taken along the $z$ axis.

Now we note that there is little that we know about $\Sigma(\vec{p},p^0)$ for $p^0<0$, and therefore we will further define our model by assuming that $\Sigma(\vec{p},p^0)$ is negligible for $p^0<0$. Therefore for $p^0<0$ we have

$$\phi^{(s)}(\vec{p},p^0)=\left[\frac{m}{E(\vec{p})}\right]^{1/2}u^{(s)}(\vec{p}),\quad p^0<0 \tag{4.31}$$

and

$$\theta^{(s)}(\vec{p},p^0)=\left[\frac{m}{E(\vec{p})}\right]^{1/2}w^{(s)}(\vec{p}),\quad p^0<0. \tag{4.32}$$

The neglect of $\Sigma(\vec{p},p^0)$ for $p^0<0$ is somewhat arbitrary. In an alternate formulation[7] we drop this assumption and formulate a theory where states with $p^0>0$ and $p^0<0$ are treated more symmetrically. (A particularly simple theory results if one completely neglects the dependence of the self-energy on $p^0$.) The essential features of the theory are unmodified in the new formulation. The remaining ambiguities have their origin in our inability to provide a satisfactory calculation of the mass of the nucleon at this time.

For future reference we may define spinors $f^{(s)}(\vec{p},p^0)$ and $h^{(s)}(\vec{p},p^0)$:

$$\phi^{(s)}(\vec{p},p^0)=\left[\frac{m}{E(\vec{p})}\right]^{1/2}f^{(s)}(\vec{p},p^0), \tag{4.33}$$

$$\theta^{(s)}(\vec{p},p^0)=\left[\frac{m}{E(\vec{p})}\right]^{1/2}h^{(s)}(\vec{p},p^0), \tag{4.34}$$

and make the further definitions:

$$f^{(s)}(\vec{p})=f^{(s)}[\vec{p},\epsilon(\vec{p})], \tag{4.35}$$

$$h^{(s)}(\vec{p})=h^{(s)}[\vec{p},-\bar{\epsilon}(\vec{p})]. \tag{4.36}$$

Using Eqs. (4.26) and (4.27) we see that

$$\phi^{(s)}(\vec{p})=\left[\frac{m}{E(\vec{p})}\right]^{1/2}f^{(s)}(\vec{p}), \tag{4.37}$$

$$\theta^{(s)}(\vec{p})=\left[\frac{m}{E(\vec{p})}\right]^{1/2}h^{(s)}(\vec{p}). \tag{4.38}$$

If we do not make the Hartree, Hartree-Fock, or (relativistic) Brueckner-Hartree-Fock approximation we must label our solutions in a more complex fashion since, in general, the interacting system can have many states of momentum $\vec{p}$ and spin $s$. We discuss this general situation in Sec. VI and then go on to write $G(11')$ in the form appropriate to the various relativistic self-consistent field approximations mentioned above (Hartree, Hartree-Fock, and Brueckner-Hartree-Fock).

## V. THE ENERGY OF THE INTERACTING SYSTEM

In this section we wish to express the expectation value of the energy in terms of the Green's functions introduced previously. We note that

$$iG^{<}(11')=\langle\bar{\psi}(1')\psi(1)\rangle, \tag{5.1}$$

so that

$$\langle H_{\rm Dirac}\rangle=\lim_{\vec{x}'\to\vec{x}}\int d\vec{x}\left(\vec{\gamma}\cdot\frac{1}{i}\vec{\nabla}_x+m\right)[iG^{<}(\vec{x}t;\vec{x}'t)]. \tag{5.2}$$

Now we consider[5]

$$\langle H_\sigma\rangle=\langle\tfrac{1}{2}\int d\vec{x}\,[\pi^2(x)+\vec{\nabla}\phi(x)\cdot\vec{\nabla}\phi(x)+\mu^2\phi^2(x)]\rangle$$

$$=\frac{-i}{2}\int d\vec{x}\left(\Box+\mu^2-\frac{2\partial^2}{\partial t^2}\right)\mathcal{G}(\xi\xi')\Big|_{t'=t}. \tag{5.3}$$

Using Eq. (4.12) we may write

$$\mathcal{G}(\xi\xi')=\mathcal{G}^0(\xi\xi')-i\mathcal{G}^0(\xi\xi'')\Omega(12\xi'')\Omega(34\xi''')\times G_2(24;13)\mathcal{G}^0(\xi'''\xi'), \tag{5.4}$$

so that Eq. (5.3) may be rewritten as

$$\langle H_\sigma\rangle = i\int d\vec{x}\frac{\partial^2}{\partial t^2}\mathcal{G}^0(\xi\xi')\Big|_{\xi'=\xi} + \int d\vec{x}\left[\frac{\partial^2\mathcal{G}^0(\xi\xi'')}{\partial t^2}-\tfrac{1}{2}\delta(\xi\xi'')\right]\Omega(12\xi'')\Omega(34\xi''') \times G_2(24;13)\mathcal{G}^0(\xi'''\xi')\Big|_{\xi'=\xi}. \quad (5.5)$$

As Wilets remarks,[5] the first term on the right-hand side is the vacuum energy for the noninteracting meson field and can be dropped.

We now turn to the interaction term $H_{\rm int}$ of Eq. (3.14). It is not difficult to see that the ground state matrix element can be expressed immediately in terms of $G_{1,1/2}$ of Eq. (4.6). If use is made of Eq. (4.8) we can then write

$$\langle H_{\rm int}\rangle = \int d\vec{x}\,\Omega(12\xi)\Omega(34\xi')G_2(24;13)\mathcal{G}^0(\xi'\xi). \quad (5.6)$$

Combining this with Eq. (5.5) we have

$$\langle H_\sigma\rangle + \langle H_{\rm int}\rangle = \int d\vec{x}\;\frac{\partial^2\mathcal{G}^0(\xi\xi'')}{\partial t^2}\;\Omega(12\xi'')\Omega(34\xi''') \times G_2(24;13)\mathcal{G}^0(\xi'''\xi')\Big|_{\xi'=\xi} + \tfrac{1}{2}\int d\vec{x}\,\Omega(12\xi)\Omega(34\xi')G_2(24;13)\mathcal{G}^0(\xi'\xi). \quad (5.7)$$

In Eq. (5.7), the first term on the right hand side is a specific *nonstatic* contribution to the energy and the second term is a "potential energy" term that survives in the static limit. We will return to the evaluation of this term in Secs. VIII–X. However, we will first provide a more detailed discussion of the Green's function $G(11')$.

## VI. CONSTRUCTION OF THE GREEN'S FUNCTION $G(11')$ - SELF-CONSISTENT FIELD APPROXIMATIONS

In this section we discuss procedures for constructing the Green's function $G(x,x')$. We define

$$G^{<}_{\alpha\alpha'}(x,x') = -i\langle\bar\psi_{\alpha'}(x')\psi_\alpha(x)\rangle, \quad (6.1)$$

where $\alpha,\alpha'$ are spinor and isospin indices. One approach to the construction of this quantity is the introduction of a complete set of states between $\psi_\alpha(x)$ and $\bar\psi_{\alpha'}(x')$ and the use of the relation

$$\psi(\vec{x},t) = e^{iHt-i\vec{P}\cdot\vec{x}}\psi(0)e^{-iHt+i\vec{P}\cdot\vec{x}}. \quad (6.2)$$

Thus one can write

$$\begin{aligned}G^{<}_{\alpha\alpha'}(x,x') &= -i\sum_{n,s}\int d\vec{p}\langle 0|\bar\psi_{\alpha'}(x')|n,-\vec{p},-s\rangle\langle n,-\vec{p},-s|\psi_\alpha(x)|0\rangle\\ &= -i\sum_{n,s}\int d\vec{p}\langle 0|\bar\psi_{\alpha'}(0)|n,-\vec{p},-s\rangle\langle n,-\vec{p},-s|\psi_\alpha(0)|0\rangle\exp[i(E_0^A-E_n^{A-1})(t'-t)-i\vec{p}\cdot(\vec{x}'-\vec{x})]\\ &= -i\sum_{n,s}\int\frac{d\vec{p}}{(2\pi)^3}\exp[-i(E_0^A-E_n^{A-1})(t-t')+i\vec{p}\cdot(\vec{x}-\vec{x}')]\phi_{\alpha'}^{(n,s)}(\vec{p})\bar\phi_{\alpha}^{(n,s)}(\vec{p}),\end{aligned} \quad (6.3)$$

where

$$\phi_\alpha^{(n,s)}(\vec{p}) \equiv \langle n,-\vec{p},-s|\psi_\alpha(0)|0\rangle(2\pi)^{3/2}. \quad (6.4)$$

In these equations the index $n$ serves to differentiate the various states of momentum $-\vec{p}$ and spin projection $-s$.

If we also define the Fourier transform of $G^{<}_{\alpha\alpha'}(x,x')$ through the relation

$$G^{<}_{\alpha\alpha'}(x,x') = \frac{1}{(2\pi)^4}\int d^4p\,G^{<}_{\alpha\alpha'}(\vec{p},p^0)e^{i\vec{p}\cdot(\vec{x}-\vec{x}')-ip^0(t-t')}, \quad (6.5)$$

we find that

$$G^{<}_{\alpha\alpha'}(\vec{p},p^0) = (-1)\sum_{n,s}\frac{\phi_\alpha^{(n,s)}(\vec{p})\bar\phi_{\alpha'}^{(n,s)}(\vec{p})}{p^0+[E_n^{A-1}(\vec{p})-E_0^A]-i\eta}. \quad (6.6)$$

This function is singular in the upper half of the $p^0$ plane. It is also useful to write

$$E_n^{A-1}(\vec{p})-E_0^A = (E_0^{A-1}-E_0^A)+[E_n^{A-1}(\vec{p})-E_0^{A-1}] = -\hat\mu+\hat\epsilon_n(\vec{p}), \quad (6.7)$$

where $\hat\mu = E_0^A - E_0^{A-1}$ is defined as the *chemical potential*. The $\hat\epsilon_n(\vec{p})$ are the excitation energies in the system with baryon number $A-1$. It is useful to divide $G^<(\vec{p},p^0)$ into two parts. With the definition

$$\phi^{(n,s)}(\vec{p}) = \left[\frac{m}{E(\vec{p})}\right]^{1/2}f^{(n,s)}(\vec{p}), \quad (6.8)$$

for $[E_0^A - E_n^{A-1}(\vec{p})]>0$ and with

$$\phi^{(n,s)}(\vec{p}) = \left[\frac{m}{E(\vec{p})}\right]^{1/2}h^{(n,s)}(\vec{p}) \quad (6.9)$$

for $[E_0^A - E_n^{A-1}(\vec{p})]<0$,

we have

$$G^{<}_{\alpha\alpha'}(\vec{p},p^0)=(-1)\left[\frac{m}{E(\vec{p})}\right]\left\{\sum_{n,s}\frac{f^{(n,s)}_{\alpha}(\vec{p})\bar{f}^{(n,s)}_{\alpha'}(\vec{p})}{p^0-\hat{\mu}+\hat{\epsilon}_n(\vec{p})-i\eta}\theta[\hat{\epsilon}(\vec{p})-\hat{\mu}]+\sum_{n,s}\frac{h^{(n,s)}_{\alpha}(\vec{p})\bar{h}^{(n,s)}_{\alpha'}(\vec{p})}{p^0-\hat{\mu}+\hat{\epsilon}_n(\vec{p})-i\eta}\theta[\hat{\mu}-\hat{\epsilon}(\vec{p})]\right\}. \quad (6.10)$$

It can be seen that the two parts of Eq. (6.10) correspond to the division of $G^{<}_{\alpha\alpha'}(x,x')$ of Eq. (6.1) into terms determined from the negative and positive frequency parts of the field $\psi(x)$:

$$G^{<}_{\alpha\alpha'}(x,x')=-i\langle\bar{\psi}^{(+)}_{\alpha'}(x')\psi^{(+)}_{\alpha}(x)\rangle-i\langle\bar{\psi}^{(-)}_{\alpha'}(x')\psi^{(-)}_{\alpha}(x)\rangle\,. \quad (6.11)$$

We now turn from these rather general considerations to the construction of $G^{<}(\vec{p},p^0)$ in a *self-consistent field approximation*. In this case there is only a *single state* for each momentum $\vec{p}$ and spin $s$. We assume $\Sigma(\vec{p},p^0)$ is constructed in a relativistic self-consistent field approximation (Hartree, Hartree-Fock, or Brueckner-Hartree-Fock). One simple way to obtain $G^{<}(\vec{p},p^0)$ is to write

$$G^{<}(p^0)=(-1)\left[\frac{1}{p^0-H_{sp}(p^0)-i\eta}\right]\gamma^0 . \quad (6.12)$$

Note the completeness relation

$$\sum_s[f^{(s)}(\vec{p},p^0)f^{(s)\dagger}(\vec{p},p^0)+h^{(s)}(\vec{p},p^0)h^{(s)\dagger}(\vec{p},p^0)]=\frac{E(\vec{p})}{m}\,. \quad (6.13)$$

In this manner we find for $p^0>0$,

$$G^{<}(\vec{p},p^0)=(-1)\left[\frac{m}{E(\vec{p})}\right]\left[\sum_s\frac{f^{(s)}_{\alpha}(\vec{p},p^0)\bar{f}^{(s)}_{\alpha'}(\vec{p},p^0)}{p^0-\epsilon(\vec{p},p^0)-i\eta}\theta(k_F-|\vec{p}|)+\sum_s\frac{h^{(s)}_{\alpha}(\vec{p},p^0)\bar{h}^{(s)}_{\alpha'}(\vec{p},p^0)}{p^0+\bar{\epsilon}(\vec{p},p^0)-i\eta}\right], \quad (6.14)$$

where we have kept in mind the fact that only *occupied* states (of positive and negative energy) contribute to $G^{<}(\vec{p},p^0)$. Noting that only unoccupied states contribute to $G^{>}$ we have

$$G^{>}_{\alpha\alpha'}(\vec{p},p^0)=(-1)\left[\frac{m}{E(\vec{p})}\right]\sum_s\frac{f^{(s)}_{\alpha}(\vec{p},p^0)\bar{f}^{(s)}_{\alpha'}(\vec{p},p^0)}{p^0-\epsilon(\vec{p},p^0)+i\eta}\times\theta(|\vec{p}|-k_F)\,. \quad (6.15)$$

We now may write

$$G_{\alpha\alpha'}(\vec{p},p^0)=G^{>}_{\alpha\alpha'}(\vec{p},p^0)+G^{<}_{\alpha\alpha'}(\vec{p},p^0)\,. \quad (6.16)$$

We remark that for $p^0<0$ we have neglected $\Sigma(\vec{p},p^0)$ so that *for* $p^0<0$,

$$G_{\alpha\alpha'}(\vec{p},p^0)=(-1)\left[\frac{m}{E(\vec{p})}\right]\left[\sum_s\frac{u^{(s)}_{\alpha}(\vec{p})\bar{u}^{(s)}_{\alpha'}(\vec{p})}{p^0-E(\vec{p})+i\eta}\theta(|\vec{p}|-k_F)+\sum_s\frac{u^{(s)}_{\alpha}(\vec{p})\bar{u}^{(s)}_{\alpha'}(\vec{p})}{p^0-E(\vec{p})-i\eta}\theta(k_F-|\vec{p}|)+\sum_s\frac{w^{(s)}_{\alpha}(\vec{p})\bar{w}^{(s)}_{\alpha'}(\vec{p})}{p^0+E(\vec{p})-i\eta}\right],\quad p^0<0\,. \quad (6.17)$$

We further note that if we only have a single free nucleon, i.e., $k_F=\hat{\mu}=0$, we have $G_{\alpha\alpha'}(\vec{p},p^0)\to G^0_{\alpha\alpha'}(\vec{p},p^0)$, where

$$G^0_{\alpha\alpha'}(\vec{p},p^0)=(-1)\left[\frac{m}{E(\vec{p})}\right]\left[\sum_s\frac{u^{(s)}_{\alpha}(\vec{p})\bar{u}^{(s)}_{\alpha'}(\vec{p})}{p^0-E(\vec{p})+i\eta}+\sum_s\frac{w^{(s)}_{\alpha}(\vec{p})\bar{w}^{(s)}_{\alpha'}(\vec{p})}{p^0+E(\vec{p})-i\eta}\right] \quad (6.18)$$

$$=(-1)(\gamma^0p^0-\vec{\gamma}\cdot\vec{p}-m+i\eta)^{-1}_{\alpha\alpha'}\,. \quad (6.19)$$

We are now able to use Eqs. (6.14)–(6.17) to evaluate various expressions for the energy derived previously.

## VII. EVALUATION OF THE DIRAC HAMILTONIAN

We consider in this section the evaluation of $\langle H_{\rm Dirac}\rangle$ given in Eq. (5.2). Making use of Eq. (6.14) we find

$$\langle H_{\rm Dirac}\rangle=(-i)\sum_s\int\frac{d^4p}{(2\pi)^4}\left[\frac{m}{E(\vec{p})}\right]\frac{\bar{f}^{(s)}(\vec{p},p^0)(\vec{\gamma}\cdot\vec{p}+m)f^{(s)}(\vec{p},p^0)}{p^0-\epsilon(\vec{p},p^0)-i\eta}\theta(k_F-|\vec{p}|)+(-i)\sum_s\int\frac{d^4p}{(2\pi)^4}\left[\frac{m}{E(\vec{p})}\right]\frac{\bar{h}^{s}(\vec{p},p^0)(\vec{\gamma}\cdot\vec{p}+m)h^{(s)}(\vec{p},p^0)}{p^0+\bar{\epsilon}(\vec{p},p^0)-i\eta}\,. \quad (7.1)$$

If we evaluate the integrals over $p^0$ and close the contours in the upper-half $p^0$ plane we have, neglecting the renormalization of the quasiparticle pole,

$$\langle H_{\rm Dirac}\rangle=\sum_s\int\frac{d\mathrm{p}}{(2\pi)^3}\left[\frac{m}{E(\vec{p})}\right]\bar{f}^{(s)}(\vec{p})(\vec{\gamma}\cdot\vec{p}+m)f^{(s)}(\vec{p})\theta(k_F-|\vec{p}|)+\sum_s\int\frac{d\mathrm{p}}{(2\pi)^3}\left[\frac{m}{E(\vec{p})}\right]\bar{w}^{(s)}(\vec{p})(\vec{\gamma}\cdot\vec{p}+m)w^{(s)}(\vec{p})\,, \quad (7.2)$$

where we have made use of the relations $f^{(s)}(\vec{p}) = f^{(s)}[\vec{p},\epsilon(p)]$ and $w^{(s)}(\vec{p}) = h^{(s)}[\vec{p},-E(\vec{p})]$. [We recall that for $p^0 < 0$ we considered $\Sigma(\vec{p},p^0)$ to be negligible and therefore $h^{(s)}(\vec{p},p^0)$ is a negative-energy solution of the free Dirac equation.] The second term in Eq. (7.2) is an infinite quantity which is associated with the energy of the vacuum and may be discarded. Thus

$$\langle H_{\text{Dirac}}\rangle = \sum_s \int \frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}\bar{f}^{(s)}(\vec{p})(\vec{\gamma}\cdot\vec{p}+m)f^{(s)}(\vec{p})\theta(k_F-|\vec{p}|). \tag{7.4}$$

We may now use [see Eq. (4.30)],

$$f^{(s)}(\vec{p}) = a(\vec{p})u^{(s)}(\vec{p}) + b(\vec{p})\sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w^{(s')}(\vec{p}) \tag{7.5}$$

to obtain

$$\langle H_{\text{Dirac}}\rangle = \sum_s \int\frac{d\vec{p}}{(2\pi)^3}E(\vec{p})a^2(\vec{p}) - \sum_s\int\frac{d\vec{p}}{(2\pi)^2}E(\vec{p})b^2(\vec{p})$$
$$= \sum_s\left(\frac{1}{2\pi^2}\right)\int_0^{k_F}p^2dp\,E(\vec{p})[a^2(\vec{p})-b^2(\vec{p})] \tag{7.6}$$
$$= \sum_s\left(\frac{1}{2\pi^2}\right)\int_0^{k_F}p^2dp\,E(\vec{p})[1-2b^2(\vec{p})], \tag{7.7}$$

where we have used the fact that

$$a^2(\vec{p}) + b^2(\vec{p}) = 1. \tag{7.8}$$

If we divide $\langle H_{\text{Dirac}}\rangle$ by the number of nucleons, $A$, we have

$$\frac{\langle H_{\text{Dirac}}\rangle}{A} = \frac{3}{k_F^{\,3}}\int_0^{k_F}p^2dp\,E(\vec{p})[1-2b^2(\vec{p})]. \tag{7.9}$$

The last term in Eq. (7.9) reflects the reduction of the energy of the Dirac field when negative-energy states are introduced into the wave function.

## VIII. THE RELATIVISTIC HARTREE-FOCK APPROXIMATION

We consider the second term in Eq. (5.7) which we denote as $V$:

$$V = \tfrac{1}{2}\int d\vec{x}\,\Omega(12\xi'')\mathcal{G}^0(\xi''\xi''')\Omega(34\xi''')G_2(24;13). \tag{8.1}$$

For the moment we do not consider the nonstatic term in Eq. (5.7),

$$V^{\text{ns}} \equiv \int d\vec{x}\left[\frac{\partial^2\mathcal{G}^0(\xi\xi'')}{\partial t_\xi^{\,2}}\right]\Omega(12\xi'')\Omega(34\xi''')$$
$$\times\, G_2(24;13)\mathcal{G}^0(\xi'''\xi')\Big|_{\xi'=\xi}. \tag{8.2}$$

The relativistic Hartree-Fock approximation is obtained *via* the factorization of $G_2$:

$$G_2^{\text{HF}}(24;13) = G^<_{\text{HF}}(21)G^<_{\text{HF}}(43) - G^<_{\text{HF}}(23)G^<_{\text{HF}}(41). \tag{8.3}$$

Indeed, inserting Eq. (8.3) into Eq. (8.1) we have

$$V_{\text{HF}} = \tfrac{1}{2}\int d\vec{x}\,G^<_{\text{HF}}(21)\Omega(12\xi')\mathcal{G}^0(\xi'\xi''')\Omega(34\xi''')G^<_{\text{HF}}(43)$$
$$-\tfrac{1}{2}\int d\vec{x}\,G^<_{\text{HF}}(23)\Omega(12\xi')\mathcal{G}^0(\xi'\xi''')\Omega(34\xi''')G^<_{\text{HF}}(41). \tag{8.4}$$

If this relation is rewritten in momentum space one obtains various terms which may be represented diagrammatically. In Fig. 2(a) we show the terms referring to the exchange of mesons between particles in positive-energy states and in Fig. 2(b) we show the exchange of mesons between particles in negative-energy states. These latter effects can be discarded since they represent part of the vacuum energy.

Note that the factorization given in Eq. (8.3) implies that the self-energy operator $\Sigma(\vec{p},p^0)$ is calculated in the Hartree-Fock approximation. [See Fig. 1(a).]

Evaluation of Eq. (8.4), with neglect of retardation in the exchange term, leads to,

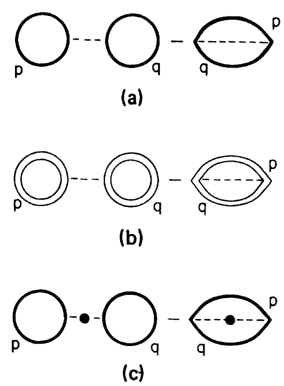

FIG. 2. Diagrammatic representation of terms appearing when use is made of the relativistic Hartree-Fock approximation in the evaluation of the expectation value of the interaction Hamiltonian. The heavy solid lines represent self-consistent spinor wave functions $\phi^{(s)}(\vec{p})$ and the dashed lines are meson propagators. (a) Direct and exchange contribution to the Hartree-Fock interaction energy. [See Eq. (8.5).] (b) Here the double lines represent negative-energy states. These terms are dropped in passing from Eq. (8.4) to Eq. (8.5) since they represent part of the vacuum energy. (c) Diagrammatic representation of the two terms obtained from the evaluation of Eq. (8.7) in momentum space. These represent *nonstatic* contributions to the energy in the Hartree-Fock approximation. [See discussion in the text after Eq. (8 7).]

$$V_{\rm HF}=\frac{1}{2}\sum_{ss'}\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{m}{E(\vec{p})}\frac{m}{E(\vec{q})}[\langle\bar{f}^{(s)}(\vec{p})\bar{f}^{(s')}(\vec{q})\,|\,U|f^{(s)}(\vec{p})f^{(s')}(\vec{q})\rangle-\langle\bar{f}^{(s)}(\vec{p})\bar{f}^{(s')}(\vec{q})\,|\,U|f^{(s')}(\vec{q})f^{(s)}(\vec{p})\rangle]$$
$$\times\,\theta(k_F-|\vec{p}|)\theta(k_F-|\vec{q}|)\,. \tag{8.5}$$

Further, the neglect of retardation in the calculation of the exchange term in Eq. (8.5) allows us to also write

$$V_{\rm HF}=\frac{1}{2}\sum_{ss'}\int\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{m}{E(\vec{q})}\frac{m}{E(\vec{p})}\langle\bar{f}^{(s)}(\vec{p})\bar{f}^{(s')}(\vec{q})\,|\,U(1-P_{12})|f^{(s)}(\vec{p})f^{(s')}(\vec{q})\rangle\theta(k_F-|\vec{p}|)\theta(k_F-|\vec{q}|)\,. \tag{8.6}$$

Here $P_{12}$ is the operator which exchanges the coordinates of particles 1 and 2.

We now turn to a consideration of the nonstatic term $V^{\rm ns}$ given by Eq. (8.2). Using Eq. (8.3) we have

$$V^{\rm ns}=\int d\bar{x}\,G^{<}_{\rm HF}(21)\Omega(12\xi'')\frac{\partial^2}{\partial t_\xi^{\,2}}\mathcal{G}^0(\xi\xi'')\mathcal{G}^0(\xi'''\xi')\Omega(34\xi''')G^{<}_{\rm HF}(43)\Big|_{\xi'=\xi}$$
$$-\int d\bar{x}\,G^{<}_{\rm HF}(23)\Omega(12\xi'')\frac{\partial^2}{\partial t_\xi^{\,2}}\mathcal{G}^0(\xi\xi'')\mathcal{G}^0(\xi'''\xi')\Omega(34\xi''')G^{<}_{\rm HF}(41)\Big|_{\xi'=\xi}\,. \tag{8.7}$$

Making use of Eqs. (4.10) and (4.11) we may write

$$\frac{\partial^2}{\partial t_\xi^{\,2}}\mathcal{G}^0(\xi\xi')=\int\frac{d^4p}{(2\pi)^4}\tilde{\mathcal{G}}^0(\vec{p},p^0)e^{i\vec{p}\cdot(\vec{\xi}-\vec{\xi}')-ip^0(t_\xi-t_{\xi'})}\,, \tag{8.8}$$

where we have defined

$$\tilde{\mathcal{G}}^0(\vec{p},p^0)=\frac{-(p^0)^2}{\vec{p}^2-(p^0)^2+\mu^2-i\epsilon}\,. \tag{8.9}$$

Again we can rewrite this expression in momentum space. The new feature is the appearance of the propagator $\tilde{\mathcal{G}}^0(\vec{p},p^0)$. We denote this propagator by a dashed line with a black dot at the end—see Fig. 2(c). (The ordinary propagator $\mathcal{G}^0$ is denoted by a dashed line as before.) It is not difficult to see that the first diagram of Fig. 2(c), which represents the nonstatic effect in the Hartree term, is equal to zero. This is clear since the meson being exchanged in the Hartree term corresponds to "forward scattering" of the nucleons and therefore carries zero four-momentum. In the exchange term shown in Fig. 2(c) the meson carries four-momentum $p-q=(\vec{p}-\vec{q},\ p^0-q^0)$. Thus, relative to the Fock term shown in Fig. 2(a), the nonstatic correction is of the order $[(p^0-q^0)/\mu]^2$. This is a small quantity and therefore we conclude that we can drop the correction to the theory described by $V^{\rm ns}$. This is consistent with our neglect of retardation effects in the evaluation of the exchange term of Eq. (8.4).

## IX. INCLUSION OF CORRELATION EFFECTS

In order to evaluate the expression given in Eq. (8.1) and include the effects of correlations we require a more general factorization of $G_2(24;13)$. One writes[5]

$$G_2(24;13)=G^{<}(21)G^{<}(43)-G^{<}(23)G^{<}(14)$$
$$+G(25)G(46)(-i)\langle56|\mathfrak{M}|78\rangle_A G(71)G(83)\,. \tag{9.1}$$

In Eq. (9.1) we see that the first two terms will, if taken alone, reproduce the Hartree-Fock results. The last term introduces a generalized scattering amplitude in the medium $\mathfrak{M}$. [Our $\mathfrak{M}$ is equal to the quantity $iT$ defined in Eq. (5.22) of Ref. 5.]

We note that there is no restriction on whether, for example, $t_5$ is greater or less than $t_2$. Similarly $t_6$, $t_7$, $t_8$ can be greater or less than $t_4$, $t_1$, and $t_3$. Therefore the second term in Eq. (9.1) describes a very large number of processes and a multitude of correlation effects. Our basic philosophy in this work, however, is to include the *same* type of correlations and ladder summations in our study of the many-body system as in the OBE model of $N$-$N$ scattering. To implement this idea it is necessary to have $t_2<t_5$, $t_4<t_6$, $t_7>t_1$, and $t_8>t_3$. We therefore write

$$G_2(24;13)=G^{<}(21)G^{<}(43)-G^{<}(23)G^{<}(14)$$
$$+G^{<}(25)G^{<}(46)(-i)\langle56|\mathfrak{M}|78\rangle_A G^{>}(71)G^{>}(83) \tag{9.2}$$

Now we insert Eq. (9.2) in Eq. (8.1) to obtain

$$V=\frac{1}{2}\int d\bar{x}\,G^{<}(21)\Omega(12\xi'')\mathcal{G}^0(\xi'''\xi'')\Omega(34\xi''')G^{<}(43)$$
$$-\frac{1}{2}\int d\bar{x}\,G^{<}(23)\Omega(12\xi'')\mathcal{G}^0(\xi'''\xi'')\Omega(34\xi''')G^{<}(41)$$
$$+\frac{1}{2}\int d\bar{x}\,\Omega(12\xi'')\mathcal{G}^0(\xi'''\xi'')\Omega(34\xi'')G^{<}(25)G^{<}(46)$$
$$\times(-i)\langle56|\mathfrak{M}|78\rangle_A G^{>}(71)G^{>}(83)\,. \tag{9.3}$$

The first two terms of Eq. (9.3) have the appearance of the previous Hartree-Fock result for $V$; however, that is deceptive since the one-body Green's functions of Eq. (9.3) are *not* equal to

$G_{HF}(12)$. This is obvious since the mass operator which is used in the construction of $G(12)$ is not calculated in the Hartree-Fock approximation.

Equation (9.3) can be rewritten in a more transparent form by making a change of variables in the first two terms of Eq. (9.3); that is, one can replace 1 by 5 and 3 by 6. For simplicity we can consider only the direct terms and write

$$V_{\text{direct}}=\tfrac{1}{2}\int d\vec{x}\,G^{<}(25)[\Omega(25\xi')\mathcal{G}^0(\xi'''\xi')\Omega(64\xi''')+(-i)\langle 56|\mathfrak{M}|78\rangle G^{>}(71)G^{>}(83)\Omega(12\xi')\mathcal{G}^0(\xi'''\xi')\Omega(34\xi''')]G^{<}(46)\,. \tag{9.4}$$

Now if we put

$$\langle 56|U|24\rangle=\Omega(52\xi')\mathcal{G}^0(\xi'''\xi')\Omega(64\xi''')\,, \tag{9.5}$$

$$\begin{aligned}V_{\text{direct}}&=\tfrac{1}{2}\int d\vec{x}\,G^{<}(25)[\langle 56|U|24\rangle+\langle 56|\mathfrak{M}|78\rangle(-i)G^{>}(71)G^{>}(83)\langle 13|U|24\rangle]G^{<}(46)\\&=\tfrac{1}{2}\int d\vec{x}\,G^{<}(25)\langle 56|\mathfrak{M}|24\rangle G^{<}(46)\,.\end{aligned} \tag{9.6}$$

In passing from Eq. (9.6) to Eq. (9,7) we have used the equation satisfied by $\mathfrak{M}$ in the medium which we define to be

$$\langle 56|\mathfrak{M}|24\rangle=\langle 56|U|24\rangle+\langle 56|\mathfrak{M}|78\rangle(-i)G^{>}(71)G^{>}(83)\langle 13|U|24\rangle\,. \tag{9.8}$$

Alternatively we can write

$$\langle 56|\mathfrak{M}|24\rangle=\langle 56|U|24\rangle+\langle 56|U|13\rangle(-i)G^{>}(17)G^{>}(38)\langle 78|\mathfrak{M}|24\rangle\,. \tag{9.9}$$

Restoring the exchange terms ($\mathfrak{M}\to\mathfrak{M}_A$) we can rewrite Eq. (9.7) in momentum space as

$$V=\tfrac{1}{2}\sum_{ss}\int\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{m}{E(\vec{p})}\frac{m}{E(\vec{q})}\times\langle\bar{f}^{(s)}(\vec{p})\bar{f}^{(s')}(\vec{q})|\mathfrak{M}(1-P_{12})|f^{(s)}(\vec{p})f^{(s')}(\vec{q})\rangle\,, \tag{9.10}$$

where $P_{12}$ indicates the appropriate exchange operation.

As stated previously, we wish to introduce correlation effects into the theory in the same fashion that such effects are introduced in the OBE model of nuclear forces. Therefore we wish to relate $\mathfrak{M}$ to the relativistic ampltude $M$ constructed in the OBE model. (See Sec. II.) More precisely we wish to show that $\mathfrak{M}$ can be replaced by $M$ introduced in Eq. (2.6). We discuss this matter in the next section and in Appendix A.

## X. RELATIVISTIC BRUECKNER-HARTREE-FOCK THEORY

In the last section we saw how the potential energy term $V$ could be written in terms of a scattering amplitude $\mathfrak{M}$. In this section we wish to discuss a further approximation which should be quite accurate in the limit of small retardation effects. We can rewrite Eq. (9.9) as

$$\mathfrak{M}=U+U(-i)G^{>}G^{>}\mathfrak{M} \tag{10.1}$$

in a schematic notation.

This equation may be compared to the equation determining the quantity $\hat{M}$,

$$\hat{M}=U+U\hat{g}^{++}\hat{M}\,, \tag{10.2}$$

which was given previously [see Eq. (2.6)]. Using standard operator algebra we can write

$$\mathfrak{M}=\hat{M}+\hat{M}[(-i)G^{>}G^{>}-\hat{g}^{++}]\mathfrak{M}. \tag{10.3}$$

*If retardation effects are small,* and we approximate $G^{>}(\vec{p},p^0)\simeq G_0^{>}(\vec{p},p^0)$, where

$$G_0^{>}(\vec{p},p^0)\equiv(-1)\sum_s\left[\frac{m}{E(\vec{p})}\right]\frac{u^{(s)}(\vec{p})\bar{u}^{(s)}(\vec{p})}{p^0-E(\vec{p})+i\eta}\,\theta(|\vec{p}|-k_F)\,, \tag{10.4}$$

we can show that $\mathfrak{M}=\hat{M}$. (See Appendix A.) Indeed there is little point in providing a more elaborate treatment of retardation effects in the study of the two-body problem in the OBE approximation. Disregarding the second term in Eq. (10.3) is consistent with our basic philosophy that the calculation of the reaction matrix in the many-body system should parallel the calculation made for the two-body scattering problem to the greatest degree possible. (See Appendix A.)

Thus with the negelct of retardation effects, i.e., $\mathfrak{M}=\hat{M}$, we have

$$V=\tfrac{1}{2}\sum_{ss}\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{m}{E(\vec{p})}\frac{m}{E(\vec{q})}\theta(k_F-|\vec{p}|)\theta(k_F-|\vec{q}|)\times\langle\bar{f}^{(s)}(\vec{p})f^{(s')}(\vec{q})|\hat{M}(1-P_{12})|f^{(s)}(\vec{p})f^{(s')}(\vec{q})\rangle\,. \tag{10.4}$$

Therefore we have for the energy of the system

$$E=\sum_{s}\int\frac{d\vec{p}}{(2\pi)^3}\frac{m}{E(\vec{p})}\bar{f}^{(s)}(\vec{p})(\vec{\gamma}\cdot\vec{p}+m)f^{(s)}(\vec{p})\theta(k_F-|\vec{p}|)$$
$$+\tfrac{1}{2}\sum_{ss'}\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\frac{m}{E(\vec{p})}\frac{m}{E(\vec{q})}\times\langle\bar{f}^{(s)}(\vec{p})\bar{f}^{(s')}(\vec{q})|\hat{M}(1-P_{12})|f^{(s)}(\vec{p})f^{(s')}(\vec{q})\rangle\times\theta(k_F-|\vec{p}|)\theta(k_F-|\vec{q}|)\,. \quad (10.5)$$

Equation (10.5) is the expression for the energy in what we may term the relativistic Brueckner-Hartree-Fock theory. We remark that this expression goes over to the usual Bethe-Brueckner result at low densities where we can neglect the negative energy spinors in $f^{(s)}(\vec{p})$. If $f^{(s)}(\vec{p})=u^{(s)}(\vec{p})$ and $E(\vec{p})\simeq p^2/2m+m$ we have

$$E\simeq\sum_{s}\int\frac{d\vec{p}}{(2\pi)^3}\left(\frac{p^2}{2m}+m\right)\theta(k_F-|\vec{p}|)$$
$$+\tfrac{1}{2}\sum_{ss'}\int\frac{d\vec{p}}{(2\pi)^3}\frac{d\vec{q}}{(2\pi)^3}\langle\vec{p}s,\vec{q}s'|\hat{G}[\epsilon(\vec{p})+\epsilon(\vec{q})]|\vec{p}s,\vec{q}s'\rangle_A\times\theta(k_F-|\vec{p}|)\theta(k_F-|\vec{q}|)\,, \quad (10.6)$$

where we have defined

$$\langle\vec{p}s,\vec{q}s'|\hat{G}[\epsilon(\vec{p})+\epsilon(\vec{q})]|\vec{p}s,\vec{q}s'\rangle_A=\left[\frac{m}{E(\vec{q})}\frac{m}{E(\vec{p})}\right]\langle\bar{u}^{(s)}(\vec{p})\bar{u}^{(s')}(\vec{q})|\hat{M}(1-P_{12})|\times u^{(s)}(\vec{p})u^{(s')}(\vec{q})\rangle\,. \quad (10.7)$$

The quantity $\hat{G}$ is to be identified with the Bethe-Brueckner reaction-matrix, and therefore Eq. (10.6) represents the energy in the Bethe-Brueckner theory of nuclear matter. The approximation given in Eq. (10.6) has been extensively used by the Bonn group[3]; however, our calculations indicate that one *must* use the more accurate expression, Eq. (10.5), for values of the density appropriate to the study of nuclear matter. This will be seen most clearly in the following paper, where we report calculations based upon the use of both Eq. (10.5) and the low-density approximation to Eq. (10.5) given by Eq. (10.6).

## XI. SUMMARY AND DISCUSSION

We have shown how the use of the static approximation for the meson field enables one to derive a relativistic version of the Bethe-Brueckner theory of nuclear matter. (The extension of this theory to the study of finite nuclei is reasonably straightforward and will be the subject of a future publication.) Our numerical results which are reported in the following paper show that relativistic effects are quite important in explaining the saturation properties in nuclear matter. Further we can conclude that the nonrelativistic Bethe-Brueckner theory is good approximation only at *low densities* and is inadequate for $k_F \gtrsim 1.2\ \mathrm{fm}^{-1}$. In forming these conclusions we have made use of the simplest version of the OBE model of nuclear forces. It will be of interest to see if similar conclusions follow from the use of other models of the nucleon-nucleon force, for example those where the $\Delta$ plays an important role in intermediate states.

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## APPENDIX A

In this appendix we discuss the derivation of the quasipotential equation (2.4) from a somewhat different point of view than that presented in Sec. II. We return to Eq. (2.1) and write $G_F=(-i)G(1)G(2)$, where $G(1)$ and $G(2)$ are Feynman propagators for nucleons 1 and 2. Equation (2.1) may be rewritten as two equivalent equations:

$$M=U'+U'(-i)G^{>}(1)G^{>}(2)M\,, \quad (A1)$$

$$U'=K+K\left[G_F-\frac{1}{i}G^{>}(1)G^{>}(2)\right]U'\,. \quad (A2)$$

In these equations $G^{>}(1)$ refers to that part of $G(1)$ in which the nucleons propagate forward in time. Thus in Eq. (A1) we only have *positive-energy* intermediate states. The effects of propagation in negative-energy states is contained indirectly in the quasipotential $U'$.

We now wish to show that if we neglect retardation we can identify the quasipotential $U'$ with the quasipotential $U$. (Recall we had written $M=U+Ug^{++}M$.) In order to make this identification we wish to show that $(-i)G^{>}(1)G^{>}(2)$ may be replaced by $g^{++}$ if one neglects retardation.

We refer to Fig. 3 for a pictorial representation of Eq. (A1). We consider the scattering amplitude

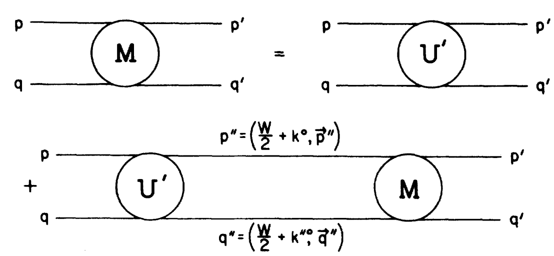


FIG. 3. Diagrammatic representation of Eq. (A1). The intermediate states of momentum $p''=(W/2+k''^0,\vec{p}'')$ and $q''=(W/2-k''^0,\vec{q}'')$ are states of positive energy. The corresponding propagators may be found by inspection of Eq. (A4). The relative momenta are given by $k=(k^0,\vec{k})=(p-q)/2=[(p^0-q^0)/2,(\vec{p}-\vec{q})/2]$, etc. The total four-momentum is $P=(W,\vec{P})=(p^0+q^0,\vec{p}+\vec{q})$.

for two nucleons of four-momenta, $p=\{p^0,\vec{p}\}$ and $q=\{q^0,\vec{q}\}$. Note that $s=(p^0+q^0)^2-(\vec{p}+\vec{q})^2$. We define $W=p^0+q^0$ and $\vec{P}=\vec{p}+\vec{q}$. The intermediate state nucleons have four-momenta $p''=(p''^0,\vec{p}'')=(W/2+k''^0,\vec{p}'')$ and $q''=(q''^0,\vec{q}'')=(W/2-k''^0,\vec{q}'')$. Here we have introduced the relative frequency $k''^0=(p''^0-q''^0)/2$, etc.

Now if we neglect the dependence of $U'$ and $M$ on the relative frequencies, that is, we assume that $U'$ and $M$ depend on the three momenta such as $\vec{p}$ and $\vec{q}$ and the variables $W$ and $\vec{P}$, we can make a change of variables to the relative momenta in Eq. (A1) and then complete the integral over the quantity $k''^0$. Thus

$$(-i)\int\frac{dk^0}{2\pi}G^>(p'')G^>(q'')=(-i)\int\frac{dk^0}{(2\pi)}\left[\frac{m}{E(\vec{p}'')}\frac{m}{E(\vec{q}'')}\right]\sum_{ss'}\frac{u^{(s)}(\vec{p}'')\bar{u}^{(s)}(\vec{p}'')}{(W/2+k^0)-E(\vec{p}'')+i\eta}\cdot\frac{u^{(s')}(\vec{q}'')\bar{u}^{(s')}(\vec{q}'')}{(W/2-k^0)-E(\vec{q}'')+i\eta} \tag{A3}$$

$$=\frac{m}{E(\vec{p}'')}\frac{m}{E(\vec{q}'')}\sum_{ss'}\frac{[u^{(s)}(\vec{p}'')\bar{u}^{(s)}(\vec{p}'')][u^{(s')}(\vec{q}'')\bar{u}^{(s')}(\vec{q}'')]}{W-E(\vec{p}'')-E(\vec{q}'')+i\eta} \tag{A4}$$

$$=g^{++}(W;\vec{p}'',\vec{q}'')\,. \tag{A5}$$

Therefore we can write

$$\begin{aligned}\langle\bar{u}^{(s_1)}(\vec{p})\bar{u}^{(s_2)}(\vec{q})|M|u^{(s_3)}(\vec{p}')u^{(s_4)}(\vec{q}')\rangle &=\langle\bar{u}^{(s_1)}(\vec{p})\bar{u}^{(s_2)}(\vec{q})|U'|u^{(s_3)}(\vec{p}')u^{(s_4)}(\vec{q}')\rangle\\ &+\sum_{ss'}\int\int\langle\bar{u}^{(s_1)}(\vec{p})\bar{u}^{(s_2)}(\vec{q})|U'|u^{(s_3)}(\vec{p}'')u^{(s_4)}(\vec{q}'')\rangle\frac{d\vec{p}''}{(2\pi)^3}\frac{m}{E(\vec{p}'')}\\ &\times\frac{d\vec{q}''}{(2\pi)^3}\frac{m}{E(\vec{q}'')}\frac{1}{W-E(\vec{p}'')-E(\vec{q}'')+i\eta}\langle\bar{u}^{(s)}(\vec{p}'')\bar{u}^{(s')}(\vec{q}'')|M|u^{(s_3)}(\vec{p}')u^{(s_4)}(\vec{q}')\rangle\end{aligned} \tag{A6}$$

or in a more abstract notation,

$$M=U+U\,g^{++}M\,. \tag{A7}$$

In Eq. (A7) we have now identified $U'$ with $U$, the quasipotential determined when use is made of the OBE model and the propagator $g^{++}$ of Eq. (A5).

The same derivation may be made using the many-body Green's functions. Equations (A3)–(A5) would read [see Eq. (10.4)]

$$(-i)\int\frac{dk^0}{(2\pi)}G_0^>(p'')G_0^>(q'')=\frac{1}{i}\int\frac{dk^0}{2\pi}\left[\frac{m}{E(\vec{p}'')}\frac{m}{E(\vec{q}'')}\right]\sum_{ss'}\left[\frac{u^{(s)}(\vec{p}'')\bar{u}^{(s)}(\vec{p}'')\theta(|\vec{p}''|-k_F)}{(W/2+k^0)-E(\vec{p}'')+i\eta}\times\frac{u^{(s')}(\vec{q}'')\bar{u}^{(s')}(\vec{q}'')\theta(|\vec{q}''|-k_F)}{(W/2-k^0)-E(\vec{q}'')+i\eta}\right] \tag{A8}$$

$$=\frac{m}{E(\vec{p}'')}\frac{m}{E(\vec{q}'')}\sum_{ss'}\frac{u^{(s)}(\vec{p}'')\bar{u}^{(s)}(\vec{p}'')u^{(s')}(\vec{q}'')\bar{u}^{(s')}(\vec{q}'')\theta(|\vec{p}''|-k_F)\theta(|\vec{q}''|-k_F)}{W-E(\vec{p}'')-E(\vec{q}'')+i\eta} \tag{A9}$$

$$=\hat{g}^{++}(W;\vec{p}'',\vec{q}'')\,. \tag{A10}$$

Further, Eqs. (A6) and (A7) now become

$$\hat{M}=U+\sum_{ss'}\int\int U|u^{(s)}(\vec{p}'')u^{(s')}(\vec{q}'')\rangle\theta(|\vec{p}''|-k_F)\theta(|\vec{q}''|-k_F)\times\frac{\frac{d\vec{p}''}{(2\pi)^3}\frac{d\vec{q}''}{(2\pi)^3}\left[\frac{m}{E(\vec{p}'')}\frac{m}{E(\vec{q}'')}\right]\langle\bar{u}^{(s)}(\vec{p}'')\bar{u}^{(s')}(\vec{q}'')|\hat{M}}{W-E(\vec{p}'')-E(\vec{q}'')+i\eta} \tag{A11}$$

and

$$\hat{M}=U+U\,\hat{g}^{++}\hat{M}\,. \tag{A12}$$

We recall Eq. (10.1). If we replace $G^>$ in this equation by $G_0^>$ of Eq. (10.4) we have

$$\mathfrak{M}=U+U(-i)G_0^>G_0^>\mathfrak{M}\,. \tag{A13}$$

Now we have seen that if we neglect retardation we can effectively replace $(-i)G_0^>G_0^>$ by $g^{++}$. Therefore comparing Eq. (A12) with Eq. (A13) we can identify $\mathfrak{M}$ with $\hat{M}$. In this manner we can justify the passage from Eq. (9.10) to Eq. (10.4).

## APPENDIX B

In this appendix, we exhibit the Hamiltonian and field equations for a system of nucleons with a pseudoscalar ($\pi$ meson) or a vector ($\rho$ and $\omega$ mesons) field. In the case of the pion with pseudovector coupling (the case of pseudoscalar coupling is given in Ref. 5), the Lagrangian densities are

$$\mathcal{L}_{\pi(\mathrm{pv})}=\tfrac{1}{2}[\partial_\mu\vec{\phi}(x)\cdot\partial^\mu\vec{\phi}(x)-\mu^2\vec{\phi}(x)\cdot\vec{\phi}(x)] \quad \text{(B1)}$$

and

$$\mathcal{L}^{\mathrm{int}}_{\pi(\mathrm{pv})}=-\frac{f_\pi}{\mu}\bar{\psi}(x)\gamma^5\gamma^\mu\vec{\tau}\cdot\psi(x)\partial_\mu\vec{\phi}(x), \quad \text{(B2)}$$

where the isovector nature of the pion is exhibited explicitly. The Dirac part of the Lagrangian and Hamiltonian is unchanged from Eqs. (3.2) and (3.12) and will not be repeated here. The Hamiltonian is then

$$H_{\pi(\mathrm{pv})}=\tfrac{1}{2}\int d\vec{x}\,[\vec{\pi}(x)\cdot\vec{\pi}(x)+\partial_j\vec{\phi}(x)\cdot\partial_j\vec{\phi}(x)+\mu^2\vec{\phi}(x)\cdot\vec{\phi}(x)] \quad \text{(B3)}$$

and

$$H_{\mathrm{int}}=\int d\vec{x}\left\{\frac{f_\pi}{\mu}\bar{\psi}(x)\gamma^5\gamma^\mu\vec{\tau}\cdot\psi(x)\partial_\mu\vec{\phi}(x)-\tfrac{1}{2}\left[\frac{f_\pi}{\mu}\bar{\psi}(x)\gamma^5\gamma^0\vec{\tau}\psi(x)\right]^2\right\} \quad \text{(B4)}$$

with

$$\vec{\pi}(x)=\dot{\vec{\phi}}(x)-\frac{f_\pi}{\mu}\bar{\psi}(x)\gamma^5\gamma^0\vec{\tau}\psi(x) \quad .$$

In the static limit for the pion field

$$\vec{\pi}\to-\frac{f_\pi}{\mu}\bar{\psi}\gamma^5\gamma^0\vec{\tau}\psi \ ,$$

and the first term in Eq. (B3) cancels the last term in Eq. (B4). Thus we have

$$H^{\mathrm{stat}}_{\pi(\mathrm{pv})}=\int d\vec{x}\,[\partial_j\vec{\phi}(x)\cdot\partial_j\vec{\phi}(x)+\mu^2\vec{\phi}(x)\cdot\vec{\phi}(x)], \quad \text{(B5)}$$

and

$$H^{\mathrm{stat}}_{\mathrm{int}}=-\int d\vec{x}\frac{f_\pi}{\mu}\bar{\psi}(x)\gamma^5\gamma^i\vec{\tau}\cdot\psi(x)\partial_i\vec{\phi}(x). \quad \text{(B6)}$$

In the same limit, the field equation for the pion field is

$$(\vec{\nabla}^2-\mu^2)\vec{\phi}(x)=\frac{f_\pi}{\mu}\partial_i[\bar{\psi}(x)\gamma^5\gamma^i\vec{\tau}\psi(x)] \quad . \quad \text{(B7)}$$

As with the scalar field [see Eq. (3.15)], it is a simple exercise, using Eq. (B7), to rewrite Eq. (B5) as

$$H^{\mathrm{stat}}_{\pi(\mathrm{pv})}=-\tfrac{1}{2}H^{\mathrm{stat}}_{\mathrm{int}} \ . \quad \text{(B8)}$$

For a vector meson, the appropriate Lagrangian densities are

$$\mathcal{L}_v=\tfrac{1}{4}G_{\mu\nu}(x)G^{\mu\nu}(x)-\tfrac{1}{2}\mu^2\phi_\mu(x)\phi^\mu(x) \quad \text{(B9)}$$

and

$$\mathcal{L}^{\mathrm{int}}_v=-g_V\bar{\psi}(x)\gamma^\mu\psi(x)\phi_\mu-\frac{f_V}{4m}\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)G_{\mu\nu}(x), \quad \text{(B10)}$$

with

$$G^{\mu\nu}(x)=\partial^\mu\phi^\nu(x)-\partial^\nu\phi^\mu(x) \ , \quad \text{(B11)}$$

$$\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu] \ , \quad \text{(B12)}$$

and $m$ is the renormalized nucleon mass. For the $\omega$ meson $f_V$ is small and usually taken as $f_V=0$, whereas for the $\rho$ meson $f_V\neq 0$. The isovector nature of the $\rho$ field must be incorporated into Eqs. (B9)–(B11). The Hamiltonian for nucleons interacting with vector mesons is then (recall that the Dirac part is the same as in Sec. III)

$$H_V=\int d\vec{x}\,\{\pi_\mu(x)\pi^\mu(x)-\tfrac{1}{2}[\partial_\mu\phi_\nu(x)G^{\mu\nu}(x)]-\mu^2\phi_\mu(x)\phi^\mu(x)\} \quad \text{(B13)}$$

and

$$H_{\mathrm{int}}=\int d\vec{x}\left\{g_V\bar{\psi}(x)\gamma^\mu\psi(x)\phi_\mu(x)+\frac{f_V}{2m}\bar{\psi}(x)\sigma^{\mu j}\psi(x)\partial_\mu\phi_j(x)+\pi_\nu(x)\left[\partial^\nu\phi^0(x)+\frac{f_V}{2m}\bar{\psi}(x)\sigma^{0\nu}\psi(x)\right]\right\} \ , \quad \text{(B14)}$$

with

$$\pi^\mu(x)=G^{0\mu}(x)-\frac{f_V}{2m}\bar{\psi}(x)\sigma^{0\mu}\psi(x) \ .$$

In the static limit

$$\pi^\mu\to-\partial^\mu\phi^0-\frac{f_V}{2m}\bar{\psi}\sigma^{0\mu}\psi$$

and the first term in Eq. (B13) cancels the last term in Eq. (B14). Thus we have

$$H^{\mathrm{stat}}_V=\tfrac{1}{2}\int d\vec{x}\,[\vec{\nabla}\phi_\mu(x)\cdot\vec{\nabla}\phi^\mu(x)+\mu^2\phi_\mu(x)\phi^\mu(x)] \quad \text{(B15)}$$

and

$$H^{\mathrm{stat}}_{\mathrm{int}}=\int d\vec{x}\left[g_V\bar{\psi}(x)\gamma^\mu\psi(x)\phi_\mu(x)+\frac{f_V}{2m}\epsilon^{ijk}\bar{\psi}(x)\Sigma^k\psi(x)\partial_i\phi^j(x)+i(f_V/2m)\bar{\psi}(x)\gamma^5\Sigma^i\psi(x)\partial_i\phi^0(x)\right], \quad \text{(B16)}$$

where $\epsilon^{ijk}\Sigma^k=\sigma^{ij}$. In this limit the meson field equation is

$$(\vec{\nabla}^2-\mu^2)\phi^i(x)=g_V\bar{\psi}(x)\gamma^i\psi(x)-\frac{f_V}{2m}\epsilon^{ijk}\partial_i[\bar{\psi}(x)\Sigma^k\psi(x)], \quad \text{(B17)}$$

$$(\vec{\nabla}^2-\mu^2)\phi^0(x)=g_V\bar{\psi}(x)\gamma^0\psi(x)-i\frac{f_V}{2m}\partial_i[\bar{\psi}(x)\Sigma^i\gamma^5\psi(x)],$$

so that Eq. (B15) can be rewritten as

$$H^{\mathrm{stat}}_v=-\tfrac{1}{2}H^{\mathrm{stat}}_{\mathrm{int}} \ . \quad \text{(B18)}$$

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